CP-odd invariants for multi-Higgs models and applications with discrete symmetry

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Motivation

CP violation is important

BSM with extra scalars is well motivated

Invariant approach

Physics does not depend on choice of basis

CP violation is subtle
(e.g. theories with complex parameters that conserve CP)

Better to rely on an Invariant Approach to CP
E.g. in the SM the Jarlskog invariant (Yukawa sector)

Yukawa sector - not this talk

Recent work on IA for Yukawa sector

Branco, IdMV, King

http://arxiv.org/abs/1502.03105

http://arxiv.org/abs/1505.06165

Scalar sector - this talk

IdMV, King, Luhn, Neder

http://arxiv.org/abs/1603.06942

Standard form

$$V = \phi^{*a} \, Y_a^b \, \phi_b + \phi^{*a} \phi^{*c} \, Z_{ac}^{bd} \, \phi_b \phi_d$$

For n Higgs doublets $H_{i\alpha} = (h_{i,1}, h_{i,2})$, where $\alpha = 1, 2$ denotes the $SU(2)_L$ index and i goes from 1 to n

$$\phi = (\varphi_1, \varphi_2, \ldots, \varphi_{2n-1}, \varphi_{2n}) = (h_{1,1}, h_{1,2}, \ldots, h_{n,1}, h_{n,2})$$

Complex conjugation

Complex conjugation:

$$\phi_a \mapsto (\phi_a)^* \equiv \phi^{*a}$$
$$\phi^{*a} \mapsto (\phi^{*a})^* \equiv \phi_a$$

$V = V^*$ therefore one can check that

$$(Y^a_b)^* = Y^b_a$$

and

$$(Z^{ac}_{bd})^* = Z^{bd}_{ac}$$

General CP transformation

$$\phi_a \mapsto {\phi^*}^{a'} U^a_{a'}$$
$$\phi^{*a} \mapsto {U^\dagger}^{a'}_{a} \phi_{a'}$$

Basis change

$$\phi_a \mapsto V_a^{a'} \phi_{a'}$$
$$\phi^{*a} \mapsto \phi^{*a'} {V^\dagger}^{a}_{a'}$$

$$Y_a^b \mapsto V_a^{a'} \; Y_{a'}^{b'} \; {V^\dagger}^{b}_{b'}$$
$$Z_{ac}^{bd} \mapsto V_a^{a'} \; V_c^{c'} \; Z_{a'c'}^{b'd'} \; {V^\dagger}^{b}_{b'} \; {V^\dagger}^{d}_{d'}$$

Invariants just with Y

$$Y^a_a$$

$$Y^a_a Y^b_b \text{ and } Y^a_b Y^b_a$$

Note that

$$Y^a_a Y^b_b \Leftrightarrow a \mapsto a \text{ and } b \mapsto b$$

and

$$Y^a_b Y^b_a \Leftrightarrow a \mapsto b \text{ and } b \mapsto a$$

Formally:

$$Y^a_{\sigma(a)} Y^b_{\sigma(b)} \text{ with } \sigma \in S_2$$

Invariants just with Z

$$Z^{ab}_{ab} \text{ and } Z^{ab}_{ba}$$

Formally:

$$Z^{ab}_{\sigma(a)\sigma(b)} \text{ with } \sigma \in S_2$$

$$Z^{ab}_{\sigma(a)\sigma(b)} Z^{cd}_{\sigma(c)\sigma(d)} \text{ with } \sigma \in S_4$$

Just two new ones, e.g.:

$$Z^{ab}_{bd} Z^{cd}_{ac} \text{ and } Z^{ab}_{cd} Z^{cd}_{ab}$$

Many invariants are equivalent, or products of smaller invariants

General CPI

$$I_\sigma^{(n_Z,m_Y)} \equiv Y^{a_1}_{\sigma(a_1)} \dots Y^{a_{m_Y}}_{\sigma(a_{m_Y})} Z^{b_1 b_2}_{\sigma(b_1)\sigma(b_2)} \dots Z^{b_{2n_Z-1} b_{2n_Z}}_{\sigma(b_{2n_Z-1})\sigma(b_{2n_Z})}$$

with $\sigma \in S_{m_Y+2n_Z}$

Many basis invariants are CP-even (all examples so far)

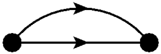
$$\mathcal{I} = I - I^*$$

Complex conjugation swaps upper and lower indices.

Diagram rules

$$X^{a.}_{..} X^{..}_{a.} =$$ 

Lines don't need to be distinguished:

$$Z^{ab}_{..} Z^{..}_{ab} =$$ 

Contracting indices on same tensor:

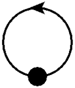
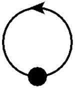
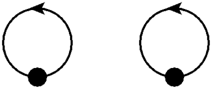
$$X^{a.}_{a.} =$$ 

Diagram and matrix examples

$Y^a_a =$  $= \begin{pmatrix} 1 \end{pmatrix}$

$Y^a_a Y^b_b =$  $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

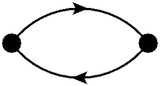
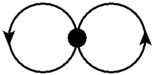
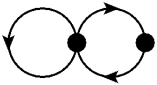
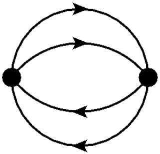
$Y^a_b Y^b_a =$  $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Diagram and matrix examples (cont.)

$Z^{ab}_{ab} =$  $= \begin{pmatrix} 2 \end{pmatrix}$

$Z^{ac}_{bc} Y^{b}_{a} =$  $= \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$Z^{ab}_{cd} Z^{cd}_{ab} =$  $= \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

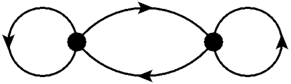
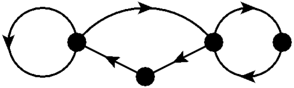
$Z^{ab}_{ac} Z^{cd}_{bd} =$  $= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Diagram CPI example

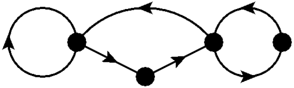
$I_1 \equiv Z^{ab}_{ae} Z^{cd}_{bf} Y^{e}_{c} Y^{f}_{d} =$



$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

and its CP conjugate

$I_1^* \equiv Z^{ae}_{ab} Z^{bf}_{cd} Y^{c}_{e} Y^{d}_{f} =$



Potential $\Delta(27)$

$$V_0(\varphi) = -\, m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2 \,,$$

$$V_{\Delta(27)}(\varphi) = \quad V_0(\varphi) + \left[d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) + \text{h.c.} \right] \,.$$

Calculated CPI example Δ(27)

Applied to $V_{\Delta(27)}(\varphi)$ and $V_{\Delta(27)}(H)$

$$\mathcal{I}_{4,5}^{(6)} = -\frac{3}{32}\left(d^3 - d^{*3}\right)\left(d^3 + 6dd^*s + d^{*3} - 8s^3\right)$$

6 solutions for this $\mathcal{I}_{4,5}^{(6)} = 0$ matched 12 distinct CP symmetries compatible with a Δ(27) triplet

Nishi
`http://arxiv.org/abs/1306.0877`

Calculated CPI example $\Delta(3n^2)$

Applied to $V_{\Delta(3n^2)}(\varphi, \varphi')$

$$\mathcal{I}_2^{(6)} = \frac{3}{512} i\tilde{s}_2\tilde{s}_3(-3\tilde{r}_2^2 + \tilde{s}_3^2)[-\tilde{s}_1^2 + \tilde{s}_1\tilde{s}_2 + \tilde{r}_2(-2\tilde{s}_1 + \tilde{s}_2) + \tilde{s}_3^2]$$

Matched solutions for this $\mathcal{I}_2^{(6)} = 0$ to some CP symmetries:
E.g. trivial CP_0 forces $\tilde{s}_3 = 0$

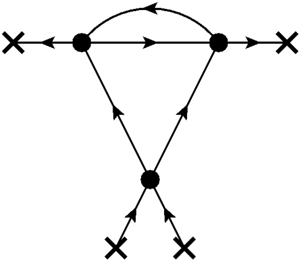
We also obtained expressions $V_{\Delta(3n^2)}(H, H')$ etc.

More results

	$\mathcal{I}_2^{(6)}$	$\mathcal{I}_3^{(6)}$	$\mathcal{I}_4^{(6)}$	$\mathcal{I}_5^{(6)}$	CP
$(\mathbf{3}_{A_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	Eq. (4.5)
$(\mathbf{3}_{A_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	Eq. (4.10)
$2 \times (\mathbf{3}_{A_4}, \mathbf{1}_{SU(2)_L})$	*	*	*	*	NA
$2 \times (\mathbf{3}_{A_4}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{\Delta(27)}, \mathbf{1}_{SU(2)_L})$	0	0	Eq. (5.3)	Eq. (5.3)	NA
$(\mathbf{3}_{\Delta(27)}, \mathbf{2}_{SU(2)_L})$	0	0	Eq. (5.13)	Eq. (5.13)	NA
$2 \times (\mathbf{3}_{\Delta(27)}, \mathbf{1}_{SU(2)_L})$	*	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(27)}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{\Delta(3n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	Eq. (4.5)
$(\mathbf{3}_{\Delta(3n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	Eq. (4.10)
$2 \times (\mathbf{3}_{\Delta(3n^2)}, \mathbf{1}_{SU(2)_L})$	Eq. (6.5)	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(3n^2)}, \mathbf{2}_{SU(2)_L})$	*	*	*	*	NA
$(\mathbf{3}_{S_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.5)
$(\mathbf{3}_{S_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.10)
$2 \times (\mathbf{3}_{S_4}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.15)
$2 \times (\mathbf{3}_{S_4}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.20)
$(\mathbf{3}_{\Delta(54)}, \mathbf{1}_{SU(2)_L})$	0	0	*	*	NA
$(\mathbf{3}_{\Delta(54)}, \mathbf{2}_{SU(2)_L})$	0	0	*	*	NA
$2 \times (\mathbf{3}_{\Delta(54)}, \mathbf{1}_{SU(2)_L})$	0	*	*	*	NA
$2 \times (\mathbf{3}_{\Delta(54)}, \mathbf{2}_{SU(2)_L})$	0	*	*	*	NA
$(\mathbf{3}_{\Delta(6n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.5)
$(\mathbf{3}_{\Delta(6n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.10)
$2 \times (\mathbf{3}_{\Delta(6n^2)}, \mathbf{1}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.15)
$2 \times (\mathbf{3}_{\Delta(6n^2)}, \mathbf{2}_{SU(2)_L})$	0	0	0	0	CP_0 & Eq. (4.20)

SCPI

$$J_1^{(3,2)} \equiv Z_{a_4 a_5}^{a_1 a_2} Z_{a_2 a_6}^{a_3 a_4} Z_{a_7 a_8}^{a_5 a_6} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$



Calculated example, $V_{\Delta(27)}(\varphi)$

$$\mathcal{J}_1^{(3,2)} = \frac{1}{4}(d^{*3} - d^3)Q(|v_i|)$$
$$+\frac{1}{2}(dd^{*2} - 2d^* s^2 + d^2 s)(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2})$$
$$-\frac{1}{2}(d^2 d^* - 2ds^2 + d^{*2} s)(v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2)\ .$$

Impose CP_0 (forces $\mathrm{Arg}(d) = 0$)

$$\mathcal{J}_1^{(3,2)} = \frac{1}{2}(d^3 - 2ds^2 + d^2 s)$$
$$\left[(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2}) - (v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2)\right]$$

Matches known results
(e.g. $\langle\varphi\rangle = (1, \omega, \omega^2)$ conserves CP_0 but $\langle\varphi\rangle = (\omega, 1, 1)$ does not)

IdMV, Emmanuel-Costa
`http://arxiv.org/abs/1106.5477`

Summary

- Developed formalism to find CPIs and performed systematic search up to 6 Z tensors
- Methods used are valid for any potential when brought to standard form
- Verified the CP properties of 3HDM and 6HDM symmetric under $\Delta(3n^2)$ and $\Delta(6n^2)$ groups

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