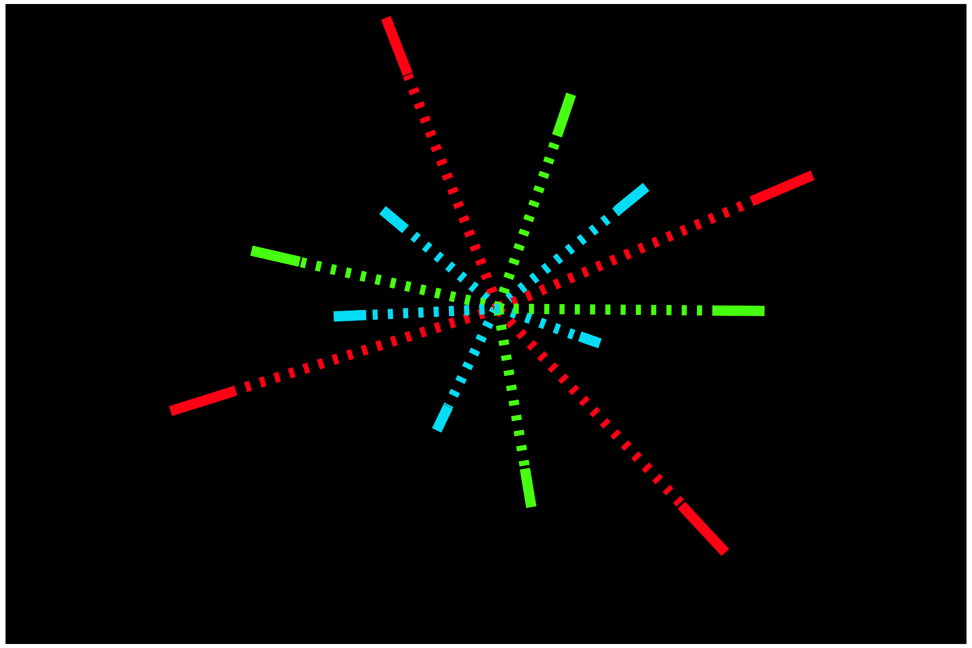
Cosmology, Gravitational waves, Inflation & Dark Matter

Jean Orloff
U. Blaise Pascal, Clermont-Ferrand

Preamble: the sleepy July 14 spectator

- Fell asleep on his terrace waiting for the fireworks
- Suddenly awaken by the first shot
- Q: Can he make up for his absence during the explosion?
- A: Thanks to
  - mechanical laws
  - observations
- He can:
  - reconstruct the fragments' trajectories
  - notice that the fastest are the furthest away
  - establish that they seem to come from one point
  - evaluate the moment of the explosion



Transposed to the Universe, this is cosmology's program

Cosmological Hypotheses

Cosmology = madly ambitious endeavor (Einstein):

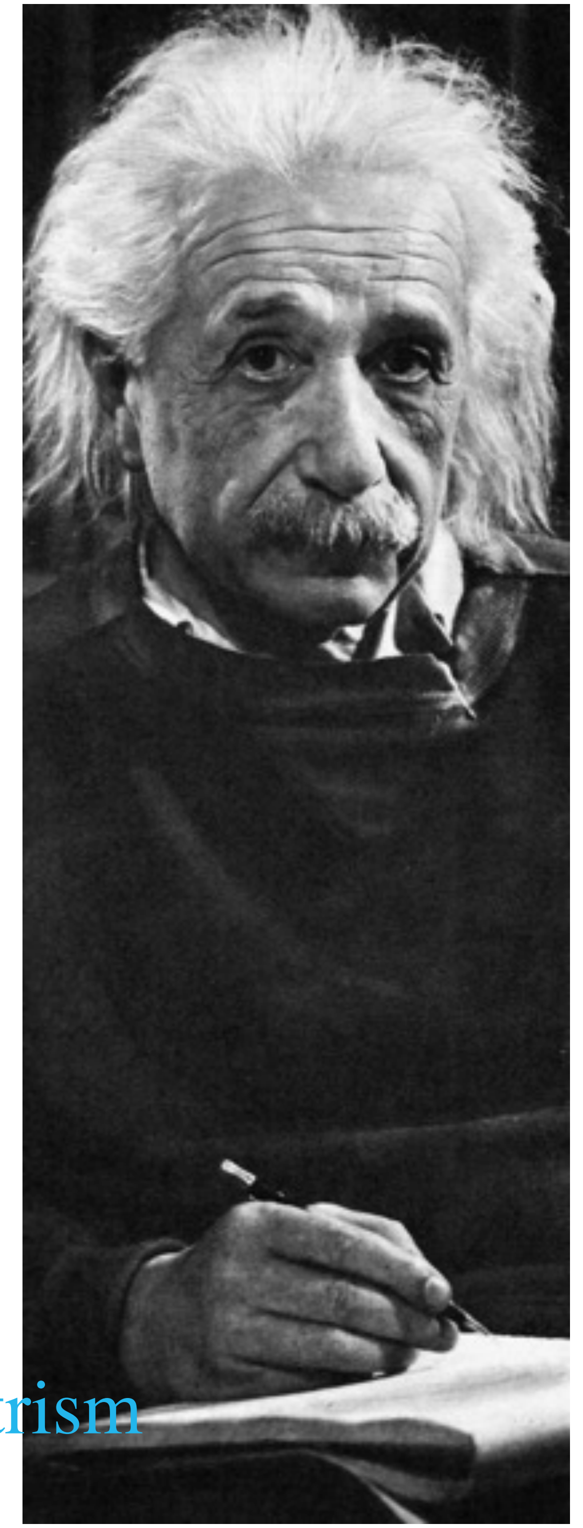
Huge universe, not fully accessible

⇒ starting hypotheses necessary;

check for coherence afterwards

The Universe is :

- simpler than its parts (earth, sun,... = details)
- governed everywhere by same physical laws
  fixed by measurements on earth
  (not directly observable)
- isotropic ⇔ no privileged direction (observable)
- homogeneous ⇔ no privileged places = anti-geocentrism
  (not directly observable: further = earlier)

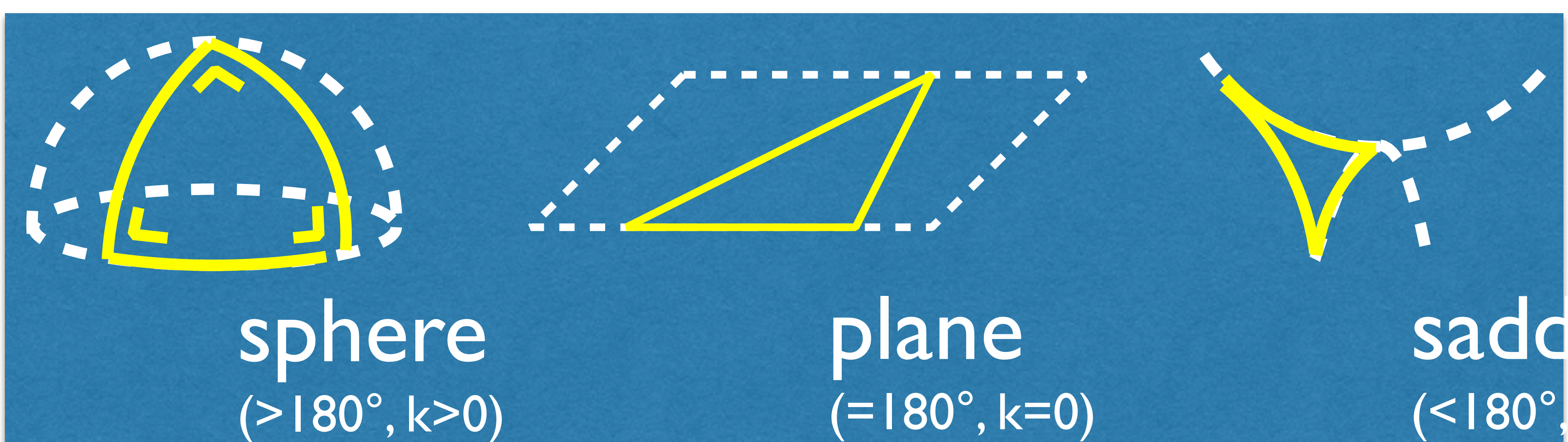


⇒ **very constrained system, predictive and testable**

Example of such hypotheses: Is the Earth a sphere?

If you suppose the earth surface to be :

- isotropic around a town
  ⇔ exactly concentric mountains
- homogeneous ⇔ same landscape around every town
- both ⇒ surface with curvature k=1/R = cte= single parameter



Earth: validity of the hypotheses

- single local measurement of R(earth): validates nothing

  Eratosthenes deduction from Alexandria & Asswan's wells
- many local measurements: better (if they agree!!!)

  ⇒ **importance of widening the horizon:**

Ideal: global measurement (shadow of Earth on Moon (Aristotle), plane, satellite…), but requires a zoom-out impossible in cosmology

Remark: forget foregrounds (= "annoying details"!!!)

Homogeneity of the Universe

Not globally testable: you can only assume homogeneity and later test the coherence of its implications:

- Isotropy+homogeneity at given time ⇒ matter distribution (stars, galaxies...) is constant (ϱ=ct), and infinite (no boundaries)
- The only compatible movements preserve ratios of distances, the "comovements":

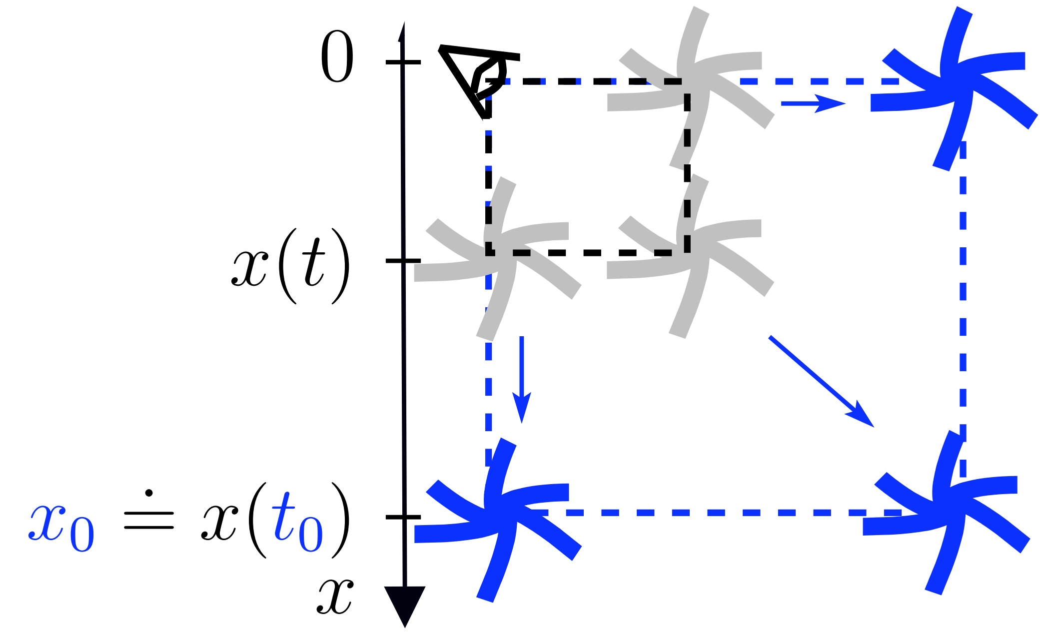
$$x_0 \doteq cte$$

$$a(t) < a(t_0) \doteq a_0 \doteq 1$$

$$\Rightarrow x(t) = a(t) x_0$$

$$\Rightarrow \dot{x}(t) = \dot{a}(t) x_0 = \frac{\dot{a}(t)}{a(t)} x(t)$$

$$\Leftrightarrow \dot{x}(t) = H(t) x(t)$$



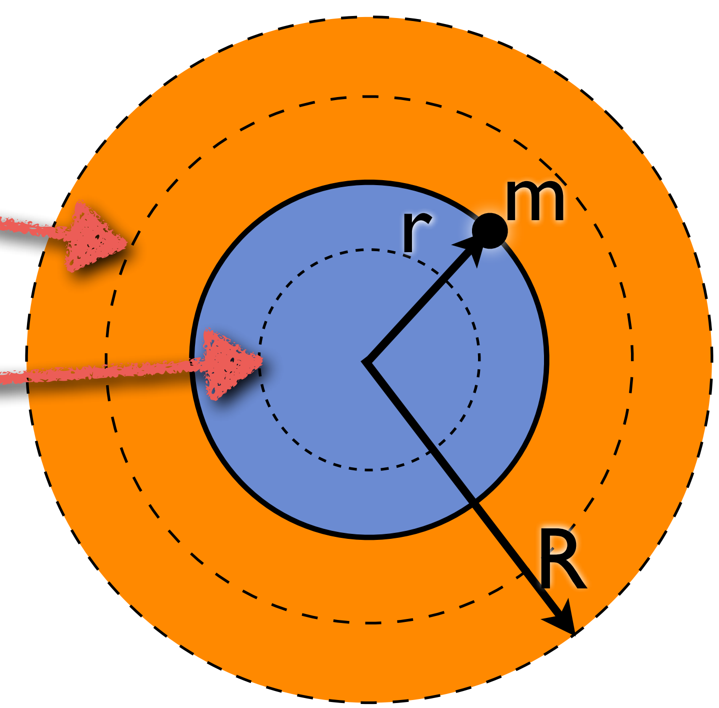
Hubble law: speed increases linear with distance

Newtonian Dynamics (0):

2 properties of gravitation

For any force ~ $1 / r^2$ like gravity (or electricity), the attraction of a spherical shell of mass *M* and radius *R* is: (Newton)

- vanishing on a mass m located
  inside the sphere ($R > r$)
- identical to a point mass *M*
  located at the center of the sphere,
  for any mass m outside the sphere ($R < r$)



Thus, for a spherical mass distribution, only the blue shells attract the mass *m*, with a total force

$$F_m(r) = G_N m M(r) \frac{1}{r^2} = m G_N \frac{4\pi \rho r^3}{3} \frac{1}{r^2}$$

Newtonian Dynamics (1)

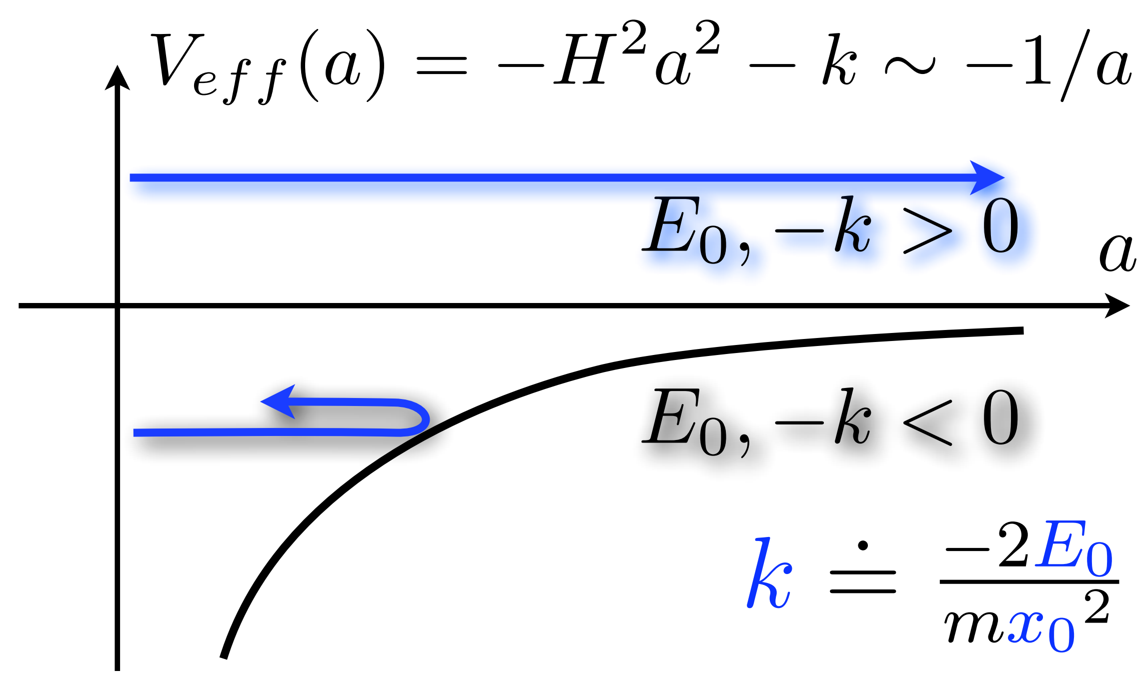
- Let's choose a point (the earth) as a center
- Consider a star *m* at distance $x(t)$ of the earth:
- it is only attracted by the constant mass $M(x) = 4\pi/3\ x_0^3\rho_0$ inside a sphere of radius $x(t)$, that attracts it towards the earth and slows its escape (energy conservation)
- $a(t)$ obeys the equation of motion of a 1-d point particle in the potential $V_{eff}(a)$
- Sign of k decides whether expansion stops or goes forever

$$E_0 = \frac{m}{2}\dot{x}^2 - mG\frac{M(x)}{x}$$

$$= \frac{m}{2}x_0^2\dot{a}^2 - mG\ \frac{4\pi}{3}x_0^2\ \frac{\rho_0^M}{a}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \boxed{H^2 = \frac{8\pi G}{3}\frac{\rho_0^M}{a^3} - \frac{k}{a^2}}\ ;$$

1[st] Friedman-Lemaître eqn



Newtonian Dynamics (2)

- Today: Hubble constant H_0=70 km/s/Mpc
- =1/(15 Gyears)
  ⇔ in a year, the distance between 2 galaxies increases by 1/15 billionth
- Critical density:

$$\rho_0^c \doteq \; 3H_0^2/8\pi G = h^2[10 m_p/\mathrm{m}^3]$$

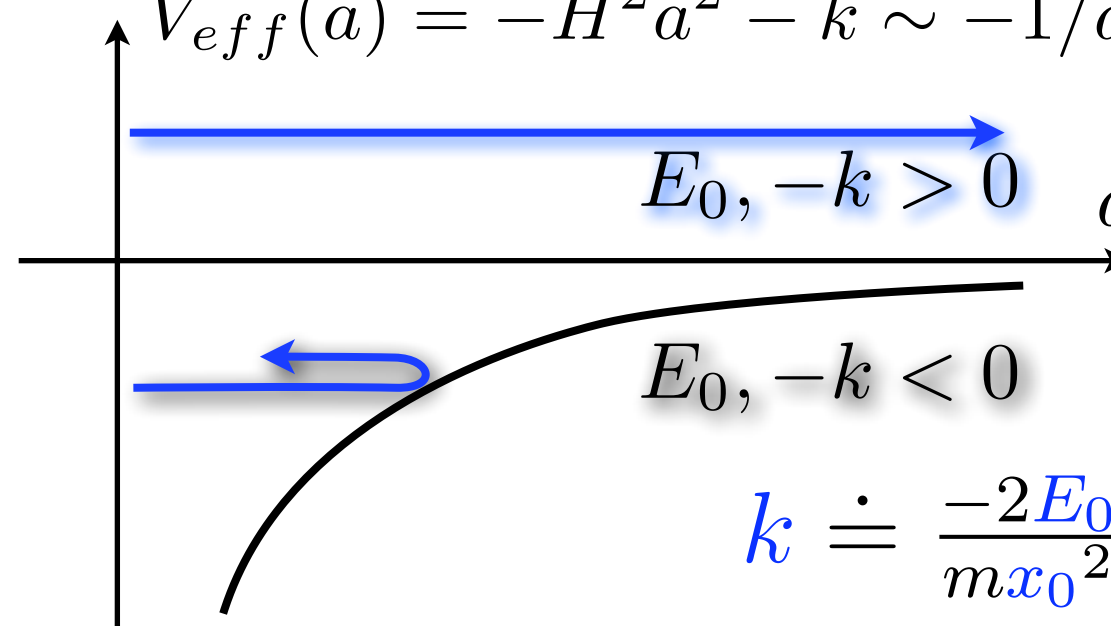
- Matter density, w.r.t. critical density:

$$\Omega^M \doteq \rho_0^M/\rho_0^c \approx 0.3$$

$$\begin{aligned} E_0 &= \frac{m}{2}\dot{x}^2 - mG\frac{M(x)}{x} \\ &= \frac{m}{2}x_0^2\dot{a}^2 - mG\,\frac{4\pi}{3}x_0^2\,\frac{\rho_0^M}{a} \\ \left(\frac{\dot{a}}{a}\right)^2 &= \boxed{H^2 = \frac{8\pi G}{3}\frac{\rho_0^M}{a^3} - \frac{k}{a^2}}\;; \end{aligned}$$

1st Friedman-Lemaître eqn

$$V_{eff}(a) = -H^2a^2 - k \sim -1/a$$



$$k \doteq \frac{-2E_0}{m{x_0}^2}$$

Discussion

Is this construction really homogeneous???

$F_{C|A}$ =Force on object C computed from spheres around A ?=?$F_{C|B}$?

Is F_C mathematically well-defined ???

$F_{C-B|A}=(F_{C|A}-F_{B|A})$?=? $F_{C-B|B}$

Are differences of forces well-defined? (hint: absolute convergence)

Are relative accelerations well-defined?

$F_{A|A}=F_{B|B}=0$; can both A and B be at rest in an inertial frame?

Which one is « right »???

Need more general frames… ⇒ **General relativity!!!**

General Relativity (in 1 slide…)

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \doteq dx_\nu dx^\nu \quad \text{Metric (0,2)-tensor}$$

$$D_\mu V_\nu(x) \doteq \partial_\mu V_\nu - \Gamma^\alpha_{\mu\nu} V_\alpha \quad \text{Covariant derivative}$$

$$\Gamma_{\beta\mu\nu} \doteq g_{\alpha\beta}\Gamma^\alpha_{\mu\nu} \doteq (-\partial_\beta g_{\mu\nu} + \partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu})/2$$

$$R^\beta_{\nu\rho\sigma} = \partial_\sigma \Gamma^\beta_{\nu\rho} + \Gamma^\alpha_{\nu\sigma}\Gamma^\beta_{\alpha\rho} - (\rho \leftrightarrow \sigma) \quad \text{Curvature (1,3)-tensor}$$

$$G_{\nu\rho} = R^\mu_{\nu\rho\mu} - g_{\nu\rho}(R^\mu_{\alpha\beta\mu} g^{\alpha\beta})/2 \quad \text{Einstein (0,2)-tensor}$$

$$G^{\mu\nu} = -8\pi G_N T^{\mu\nu} \quad \text{Einstein's equations}$$

$$T^{\mu\nu} = \rho v^\mu v^\nu = \rho \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} \quad \text{Energy-momentum tensor}$$

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0 \quad \text{Geodesic matter motion}$$

Gravitational waves

Harmonic coordinates

Under a coordinate transformation, the metric transforms as a (0,2)-tensor:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

or for $x'^\mu = x^\mu + \epsilon \xi^\mu(x)$

$$g'_{\mu\nu} = g_{\mu\nu} - \epsilon(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) + O(\epsilon^2)$$

Harmonic coordinates are defined to satisfy the 4 equations:

$$g^{\mu\nu}(x)\Gamma^\lambda_{\mu\nu}(x) = 0$$

→ for scalars, covariant == ordinary D'Alembertian:

$$\Box\phi \doteq g^{\mu\nu} D_\mu D_\nu \phi = g^{\mu\nu}(\partial_\mu \partial_\nu \phi - \Gamma^\lambda_{\mu\nu} \partial_\lambda \phi) = g^{\mu\nu} \partial_\mu \partial_\nu \phi$$

Each coordinate satisfies the harmonic equation $\Box\phi = 0$, and is defined up to a harmonic function:

$$x^\mu \Leftrightarrow x'^\mu = x^\mu + \phi^\mu$$

Weak field wave solutions

For $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$ with $h_{\mu\nu}; h \doteq \eta^{\mu\nu} h_{\mu\nu} \ll 1$:

$$2G_{\mu\nu} = \partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu} + \eta_{\mu\nu}(\Box h - \partial_{\alpha\beta} h^{\alpha\beta})$$

In harmonic coordinates, $\partial^\nu h_{\mu\nu} - \partial_\mu h/2 = 0$ leaving 10 - 4 = 6 components, obeying in the vacuum:

$$\Box h_{\mu\nu} = 0 \rightarrow h_{\mu\nu}(x) = C_{\mu\nu} e^{ik_\mu x^\mu}$$

Exercise: for $k^\mu = \omega(1,0,0,1)$ use the harmonic condition $k^\nu C_{\mu\nu} - k_\mu C/2 = 0$ to express $C_{0\mu}$ in terms of spatial components, and make them vanish using the harmonic transformations

$$x'^\mu = x^\mu + Y^\mu e^{ik_\mu x^\mu} \rightarrow C'_{\mu\nu} = C_{\mu\nu} - iY_\mu k_\nu - iY_\nu k_\mu$$

Show that the remaining independent components are

$$\begin{cases} C'_{11} = -C'_{22} \doteq C_+ \\ C'_{12} = C'_{21} \doteq C_\times \end{cases} \Leftrightarrow \begin{cases} C_R = \frac{1}{\sqrt{2}}(C_+ + iC_\times) \\ C_L = \frac{1}{\sqrt{2}}(C_+ - iC_\times) \end{cases}$$

which come back to after a180° rotation around z-axis (spin 2).

GWs in a nutshell

Gravitational waves are dynamic fluctuations in the fabric of space-time, propagating at the speed of light

Predicted by Einstein 100 years ago; first indirect confirmation by Hulse & Taylor (Nobel Prize in 1993)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

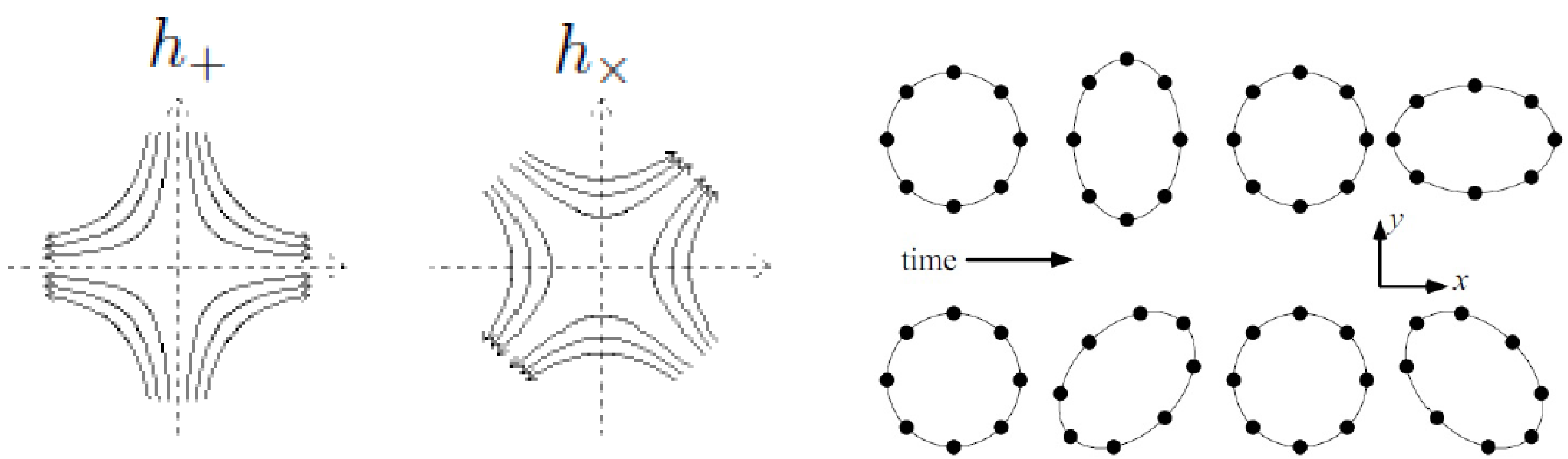
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$|h_{\mu\nu}| << 1 \qquad \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

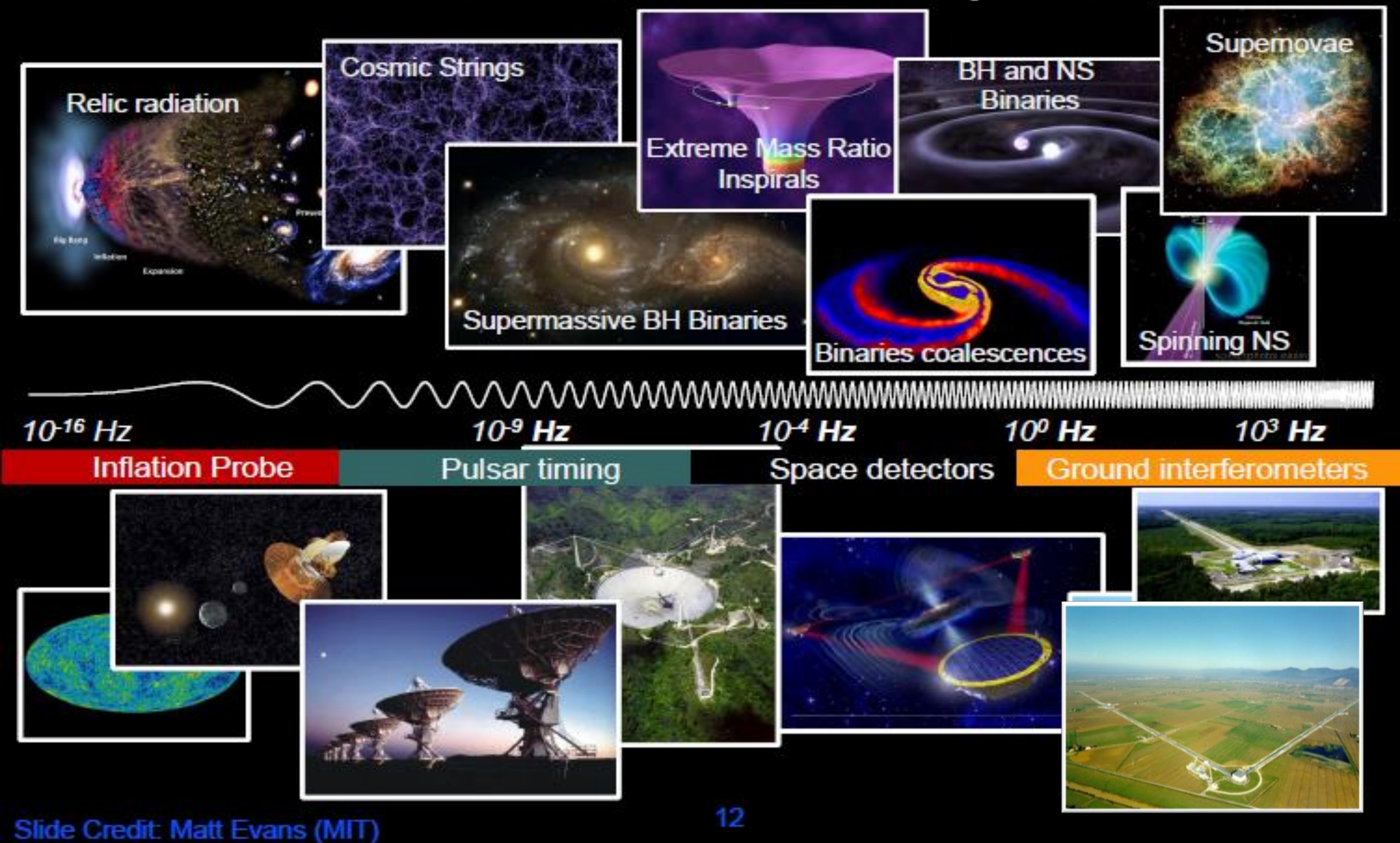
$$h_{\mu\nu} = h_+(t - z/c) + h_x(t - z/c)$$

Emitted from accelerating mass distributions (quadrupole mass moment – no dipole radiation)

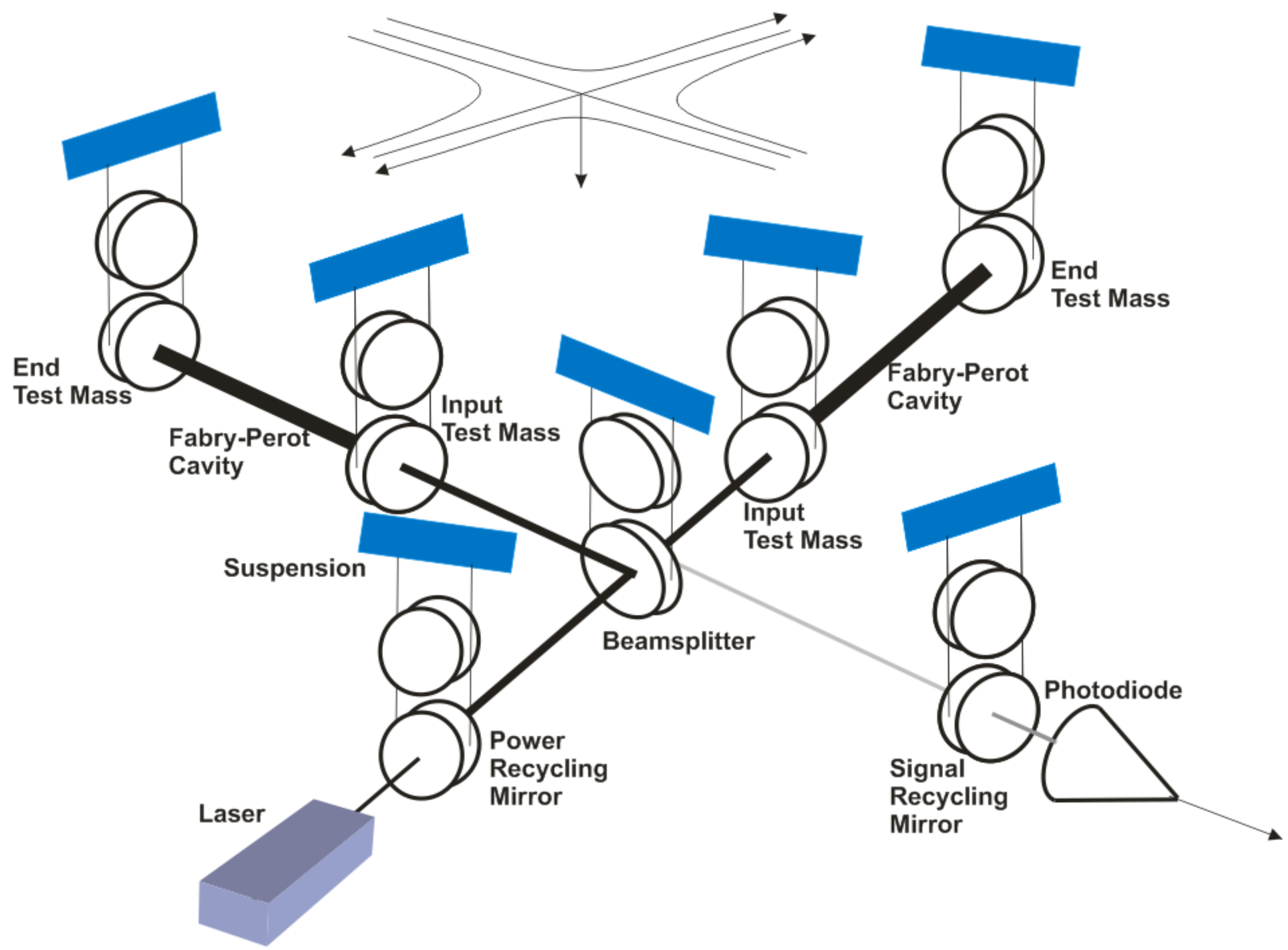
GWs carry *direct* information about the relativistic motion of bulk matter



The Gravitational Wave Spectrum

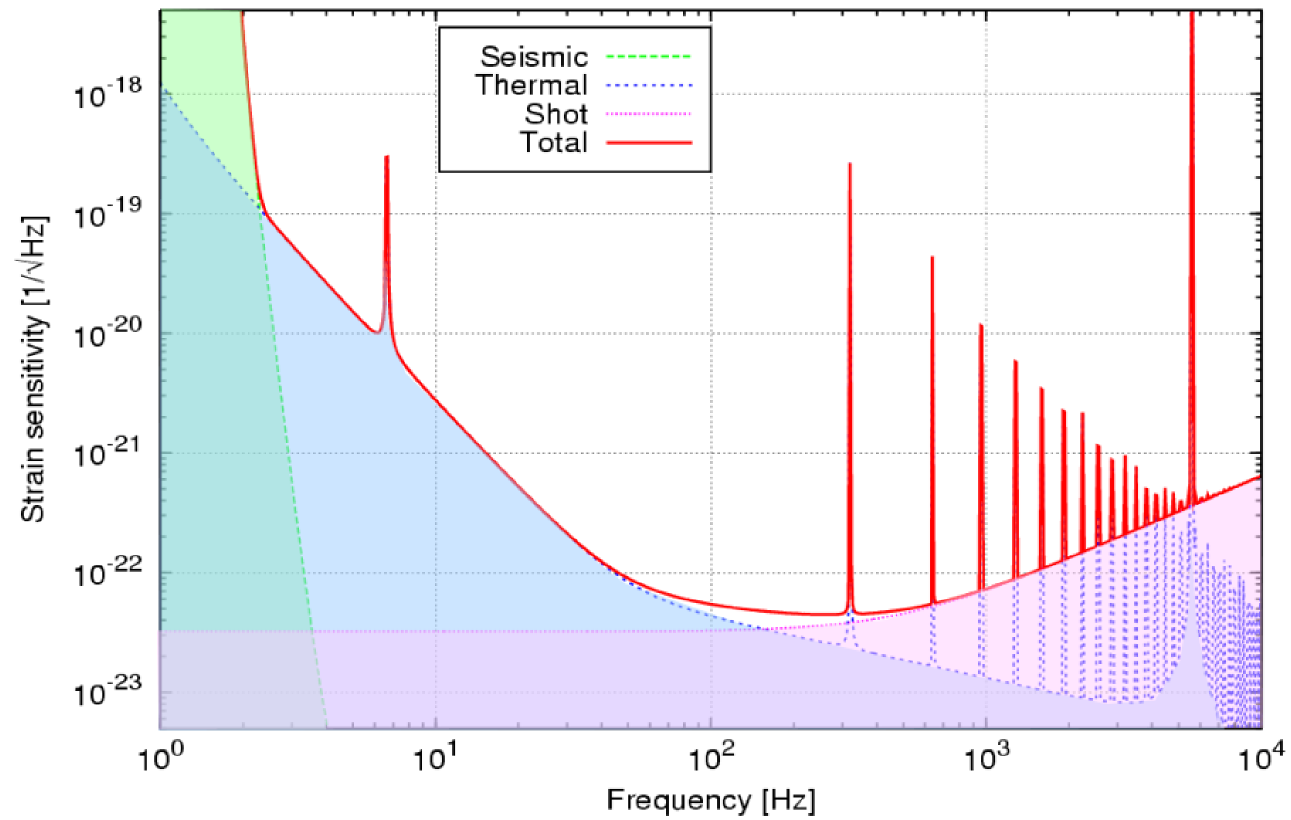


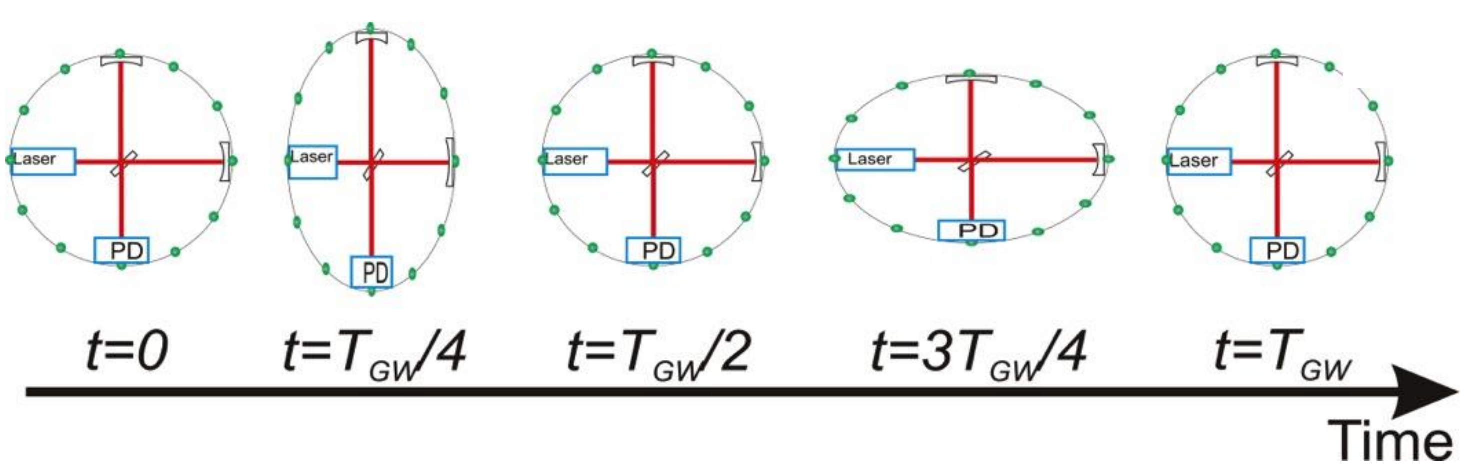
Detector’s working principle



Typically h ~ 10^{-21}

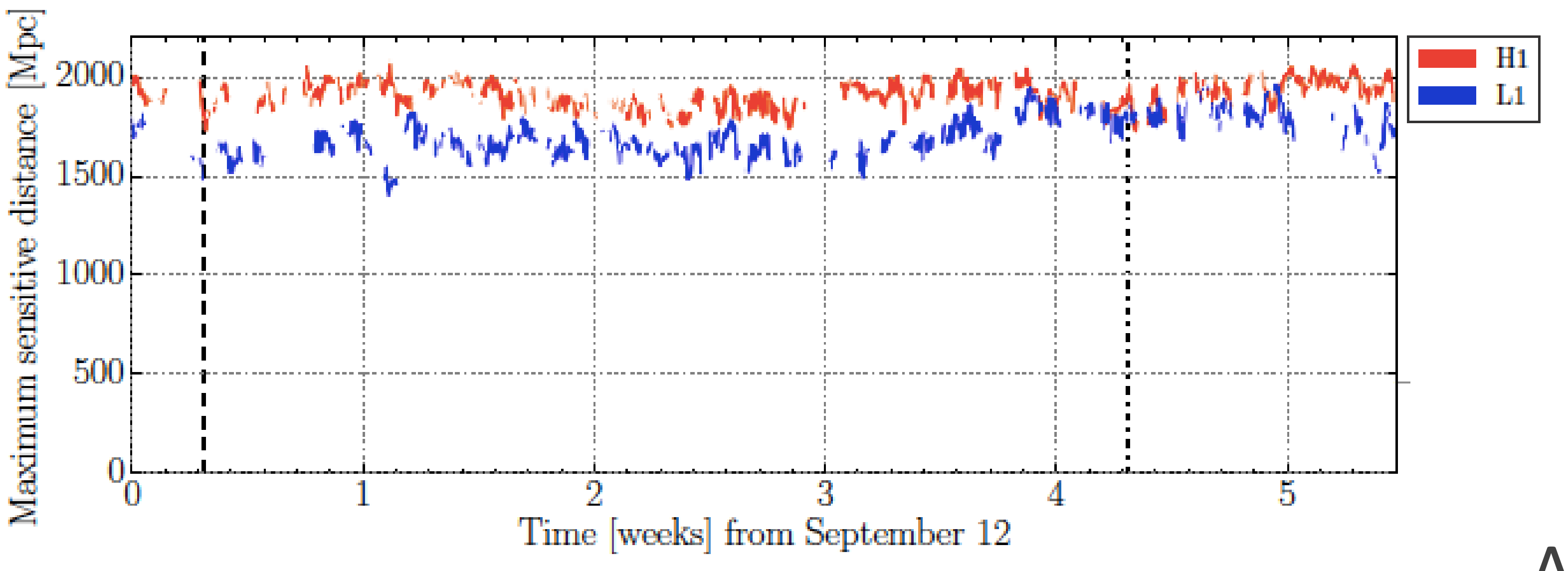
$\Delta L \sim h L \rightarrow$ if L ~ km $\rightarrow \Delta L \sim 10^{-18}$ m

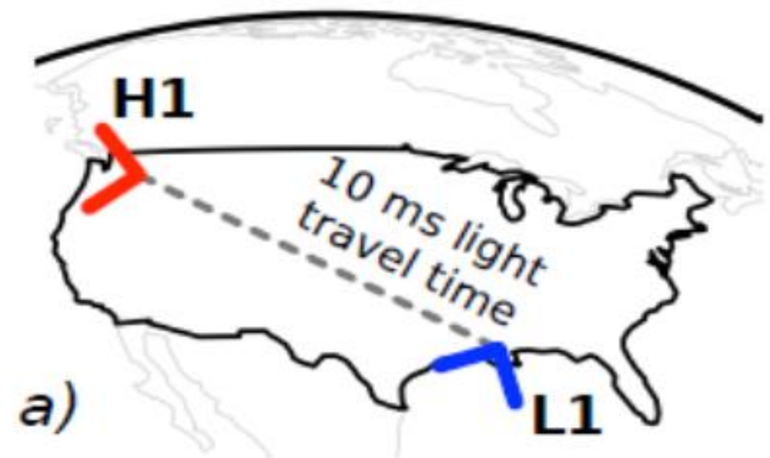




Technical issues - alignment, electronics, acoustics, etc - may limit us before we reach these fundamental noise sources

O1 aLIGO science run





Hanford and Livingstone running with similar sensitivities:

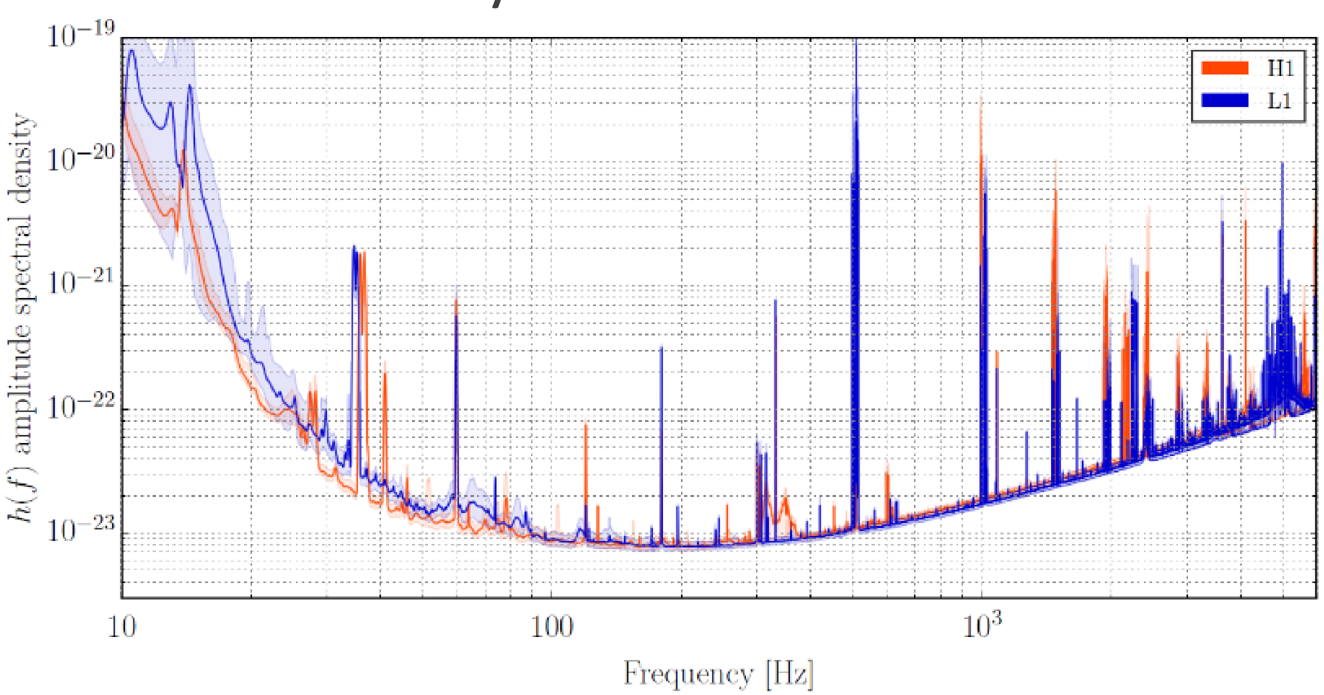
- $10^{-23}/\sqrt{Hz}$ @ 100 Hz
- Improvement by 3-4 times wrt LIGO between 100-300 Hz

O1: from Sept 2015 to Jan 2016

- ER8 before the science run, interferometer configuration frozen since Sept 12th

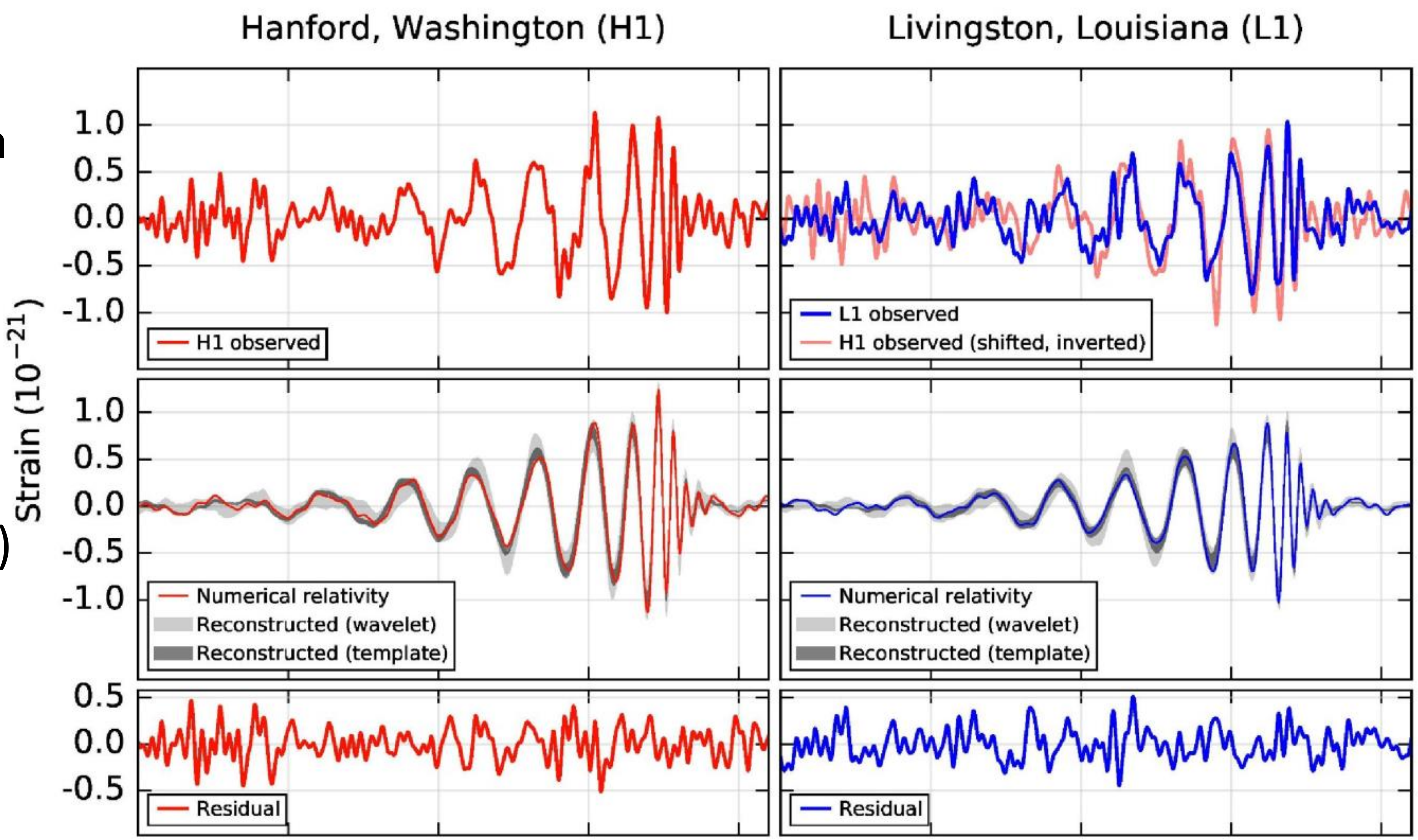
Analyzed data period from Sept 12th to Oct 20th

- Coincidence duty cycle ~ 48%
- 16 days of coincidence time



GW150914: the signal

- Top row left – Hanford
- Top row right – Livingston
- Time difference ~ 6.9 ms with Livingston first
- Second row – calculated GW strain using Numerical Relativity** (EOBNR and IMRPhenom) and reconstructed waveforms (shaded)
- Third Row – residuals

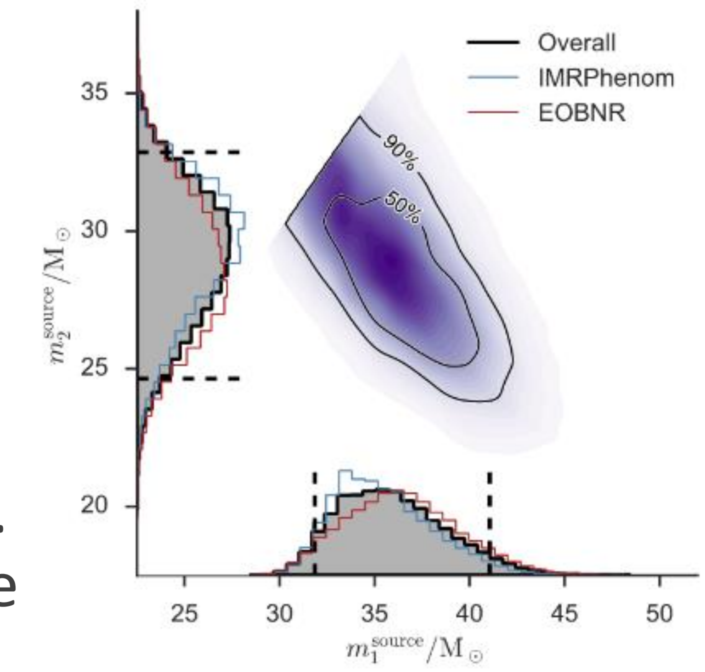


** Talk by A. Nagar, right after this

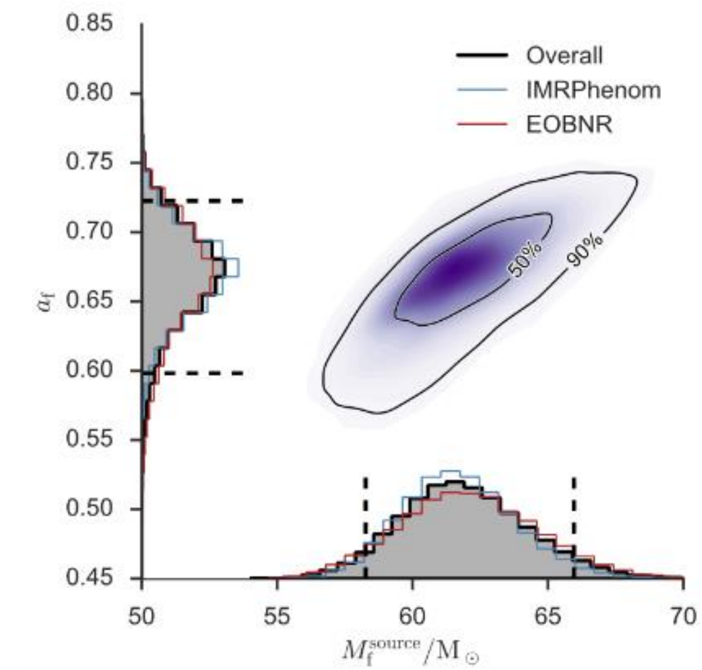
Estimated source parameters

Median values with 90% credible intervals, including statistical errors from averaging the results of different waveform models. Masses are given in the source frame: to convert in the detector frame multiply by (1+z). The source redshift assumes standard cosmology: D_L ➔ z assuming ΛCDM with $H_0 = 67.9$ km s^{-1} Mpc^{-1} and $\Omega_m = 0.306$

Total energy radiated in gravitational waves is $3.0 \pm 0.5\ M_\odot c^2$. The system reached a peak luminosity ~3.6×10^{56} erg, and the spin of the final black hole < 0.7



Primary black hole mass	$36^{+5}_{-4}\ M_\odot$
Secondary black hole mass	$29^{+4}_{-4}\ M_\odot$
Final black hole mass	$62^{+4}_{-4}\ M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift, z	$0.09^{+0.03}_{-0.04}$



GW150914: the source analysis

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

$$\mathcal{M} \simeq 30 M_\odot$$

$$M = m_1 + m_2 \text{ is } \gtrsim 70 M_\odot$$

NS-NS binary excluded

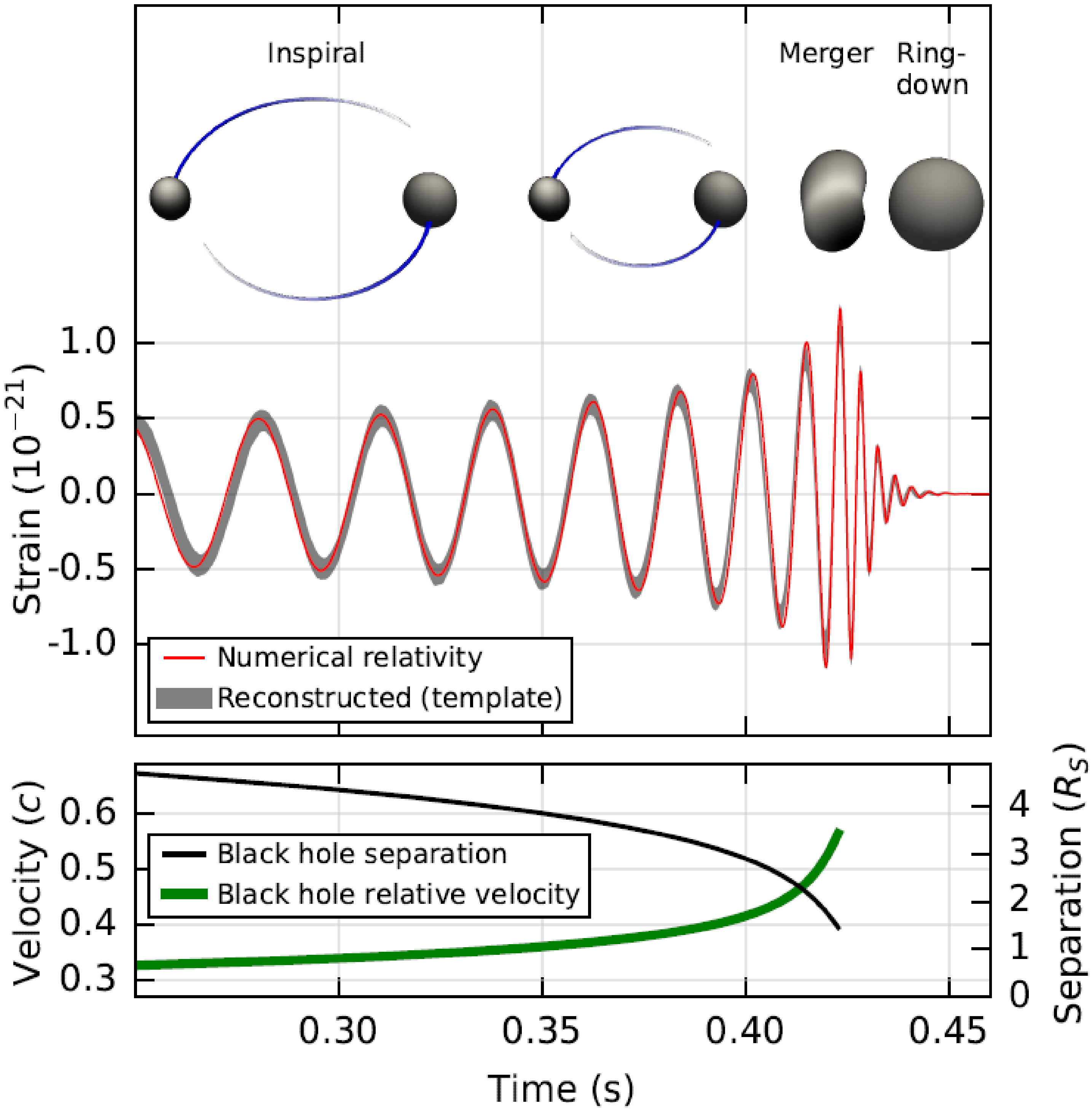
Binary system BH-NS?
If so, M_{BH} very large (~3000 $M_\odot$) ⇒
Coalescence happens at lower frequencies
NS-BH binary excluded

Binary system BH-BH, similar masses;

$$f_{\max} = 150\,\text{Hz} \Rightarrow \omega_{Kepl} = 2\pi \cdot f_{\max}/2 = 2\pi \cdot 75\,\text{Hz}$$

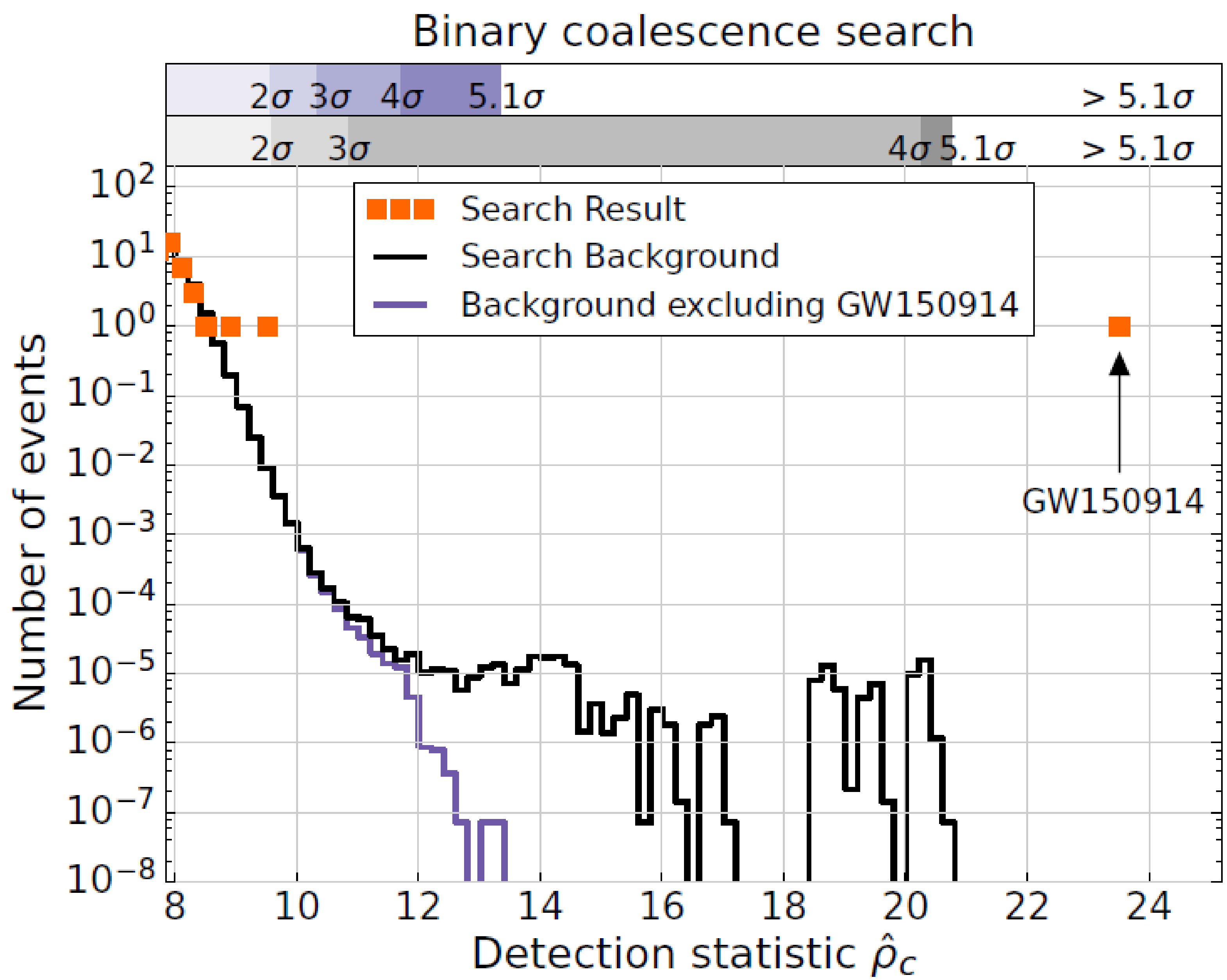
$$R = \left[\frac{GM}{\omega_{Kepl}^2}\right]^{\frac{1}{3}} \approx 350\,\text{km} \quad R_{Schwarz} = \frac{2GM}{c^2} \approx 210\,\text{km}$$

2 BHs (~ 30 $M_\odot$ each) colliding at *c*/2



Assessing the statistical significance

- number of candidate events (orange markers)
- number of background events (black and purple lines)
- significance of an event in Gaussian standard deviations based on the corresponding noise background



- False alarm rate < 1 per 203.000 years,
- Poissonian false alarm probability < 2 x 10^{-7}
- Significance > 5.1 σ

GR Cosmology

GR Cosmology: FRW metric

see Baumann's lectures

Maximally symmetric geometry in comoving coordinates (r,θ,ϕ):

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

FRW METRIC

$a \to \lambda a\ , \quad r \to r/\lambda\ , \quad k \to \lambda^2 k$ rescaling symmetry allows $a(t_0) = 1$

$$r_{\rm phys} = a(t) r \quad \Rightarrow \quad v_{\rm phys} \equiv \frac{dr_{\rm phys}}{dt} = a(t)\frac{dr}{dt} + \frac{da}{dt} r$$

$$k_{\rm phys} = k/a^2(t)$$

$$\equiv \quad v_{\rm pec} \quad + H r_{\rm phys}$$

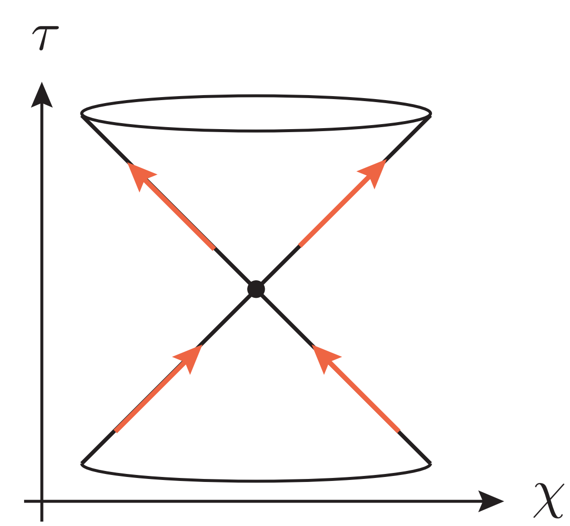
Conformal time: $\tau = \int dt/a(t) \Rightarrow$

$$ds^2 = a^2(\tau)\left[d\tau^2 - \frac{dr^2}{1-kr^2} - r^2 d\Omega^2\right]$$

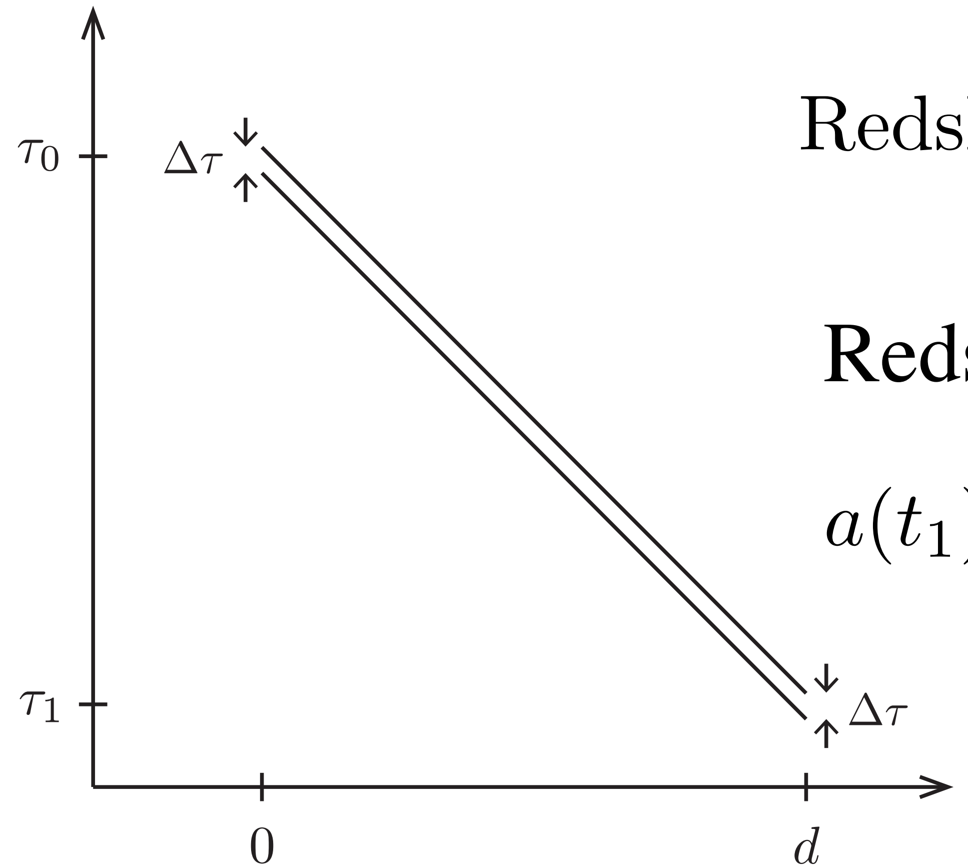
Conformal distance: $\chi = \int dr/\sqrt{1-kr^2}$

$$\Rightarrow \quad ds^2 = a^2(\tau)\left[d\tau^2 - d\chi^2 - \begin{pmatrix} \sinh^2\chi \\ \chi^2 \\ \sin^2\chi \end{pmatrix} d\Omega^2\right] \qquad k = \begin{cases} -1 \\ 0 \\ +1 \end{cases}$$

$$\equiv S_k^2(\chi)$$



Redshift & distance(s)

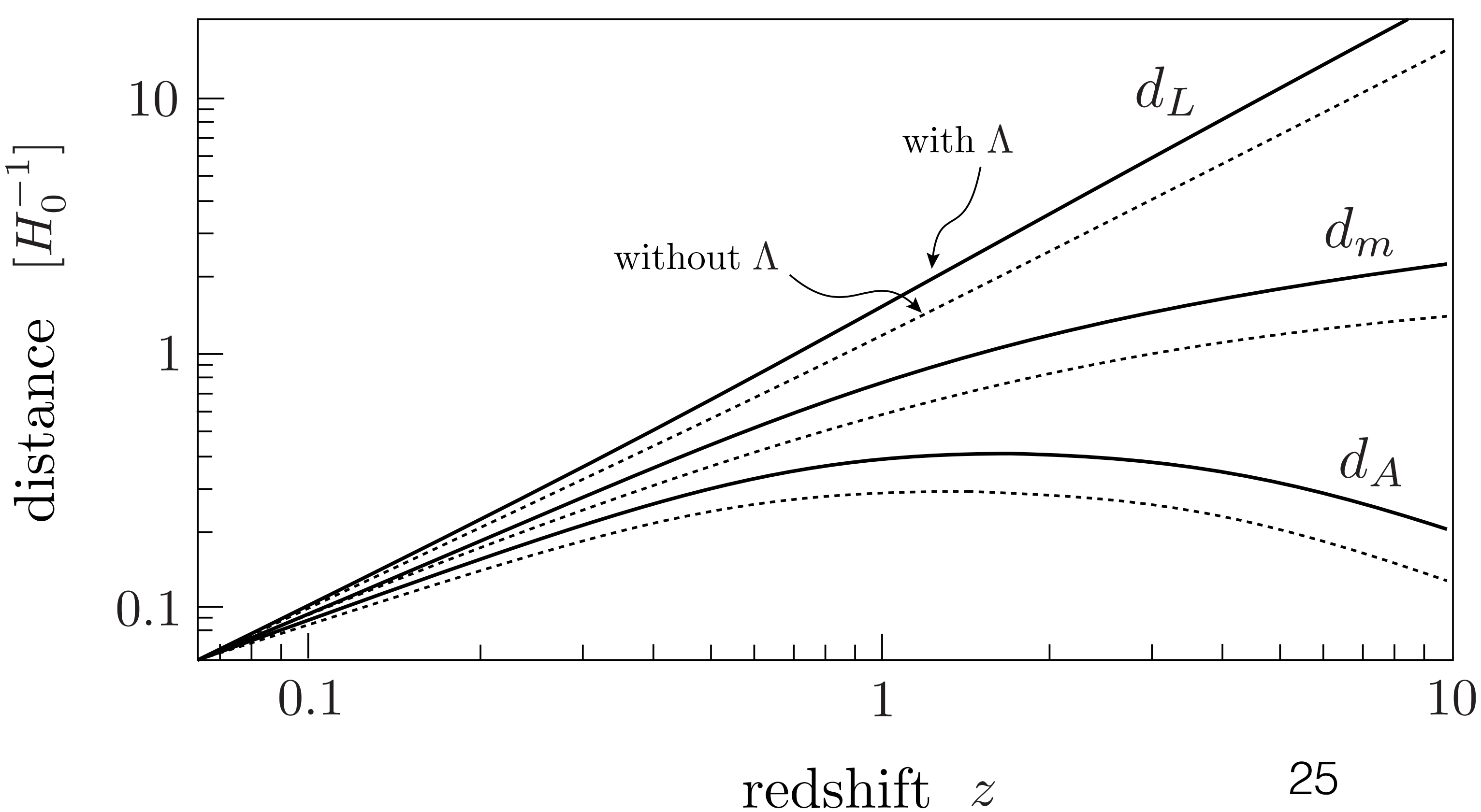


Redshift $z : 1 + z \equiv \frac{\lambda_0}{\lambda_1} = \frac{a(t_0)\Delta\tau}{a(t_1)\Delta\tau}$

Redshift measures (small) distances:

$$a(t_1) = a(t_0)[1 + (t_1 - t_0)H_0 + \cdots] \Rightarrow z \approx H_0 d$$

Which distance?



- **m**etric distance: (sphere area)

$$d_m = S_k(\chi)$$

- apparent **L**uminosity:

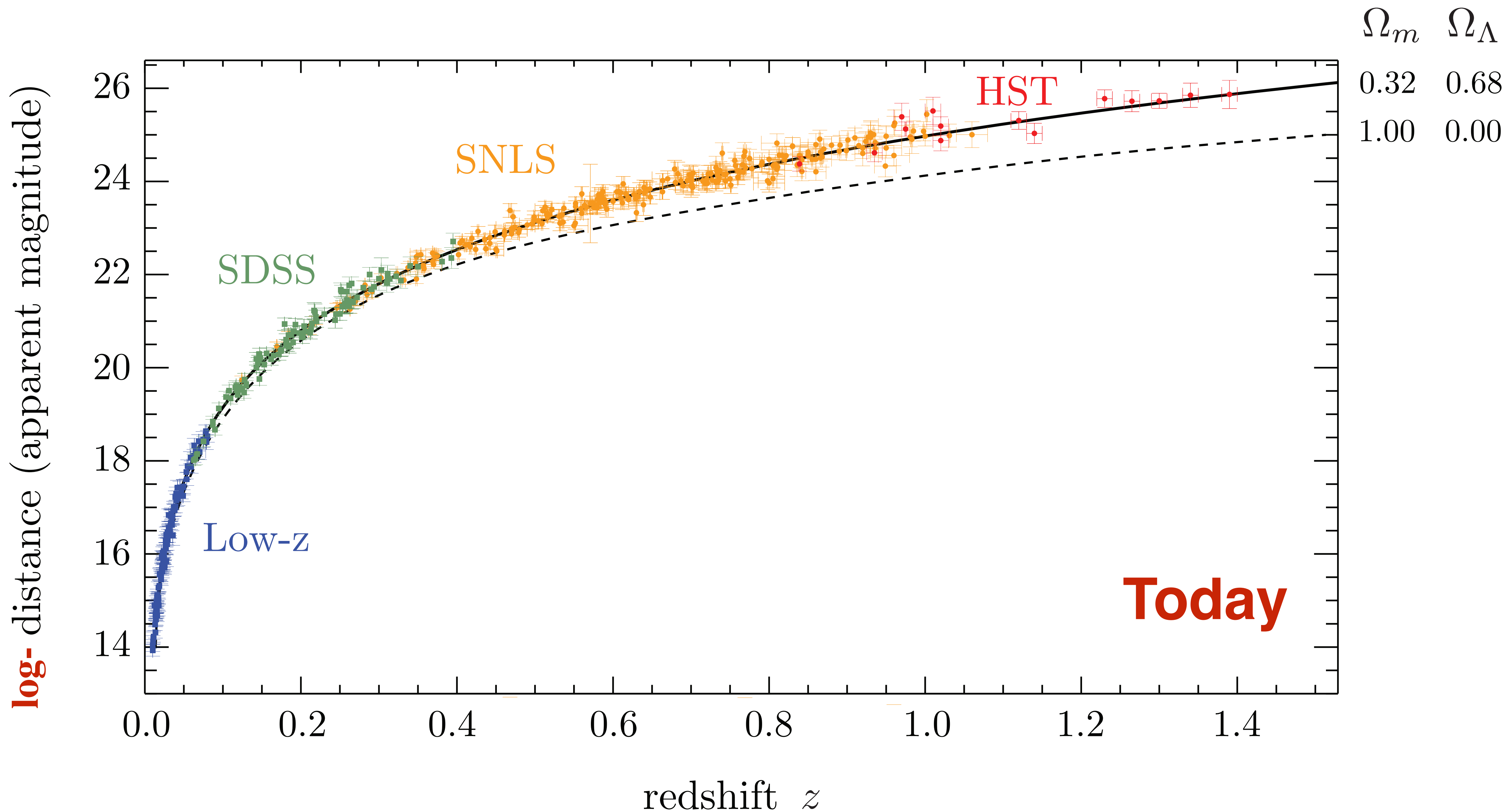
$$d_L = d_m(1 + z)$$

$$\sim \sqrt{\text{Abs.lumi/Flux}}$$

- **A**ngular diameter:

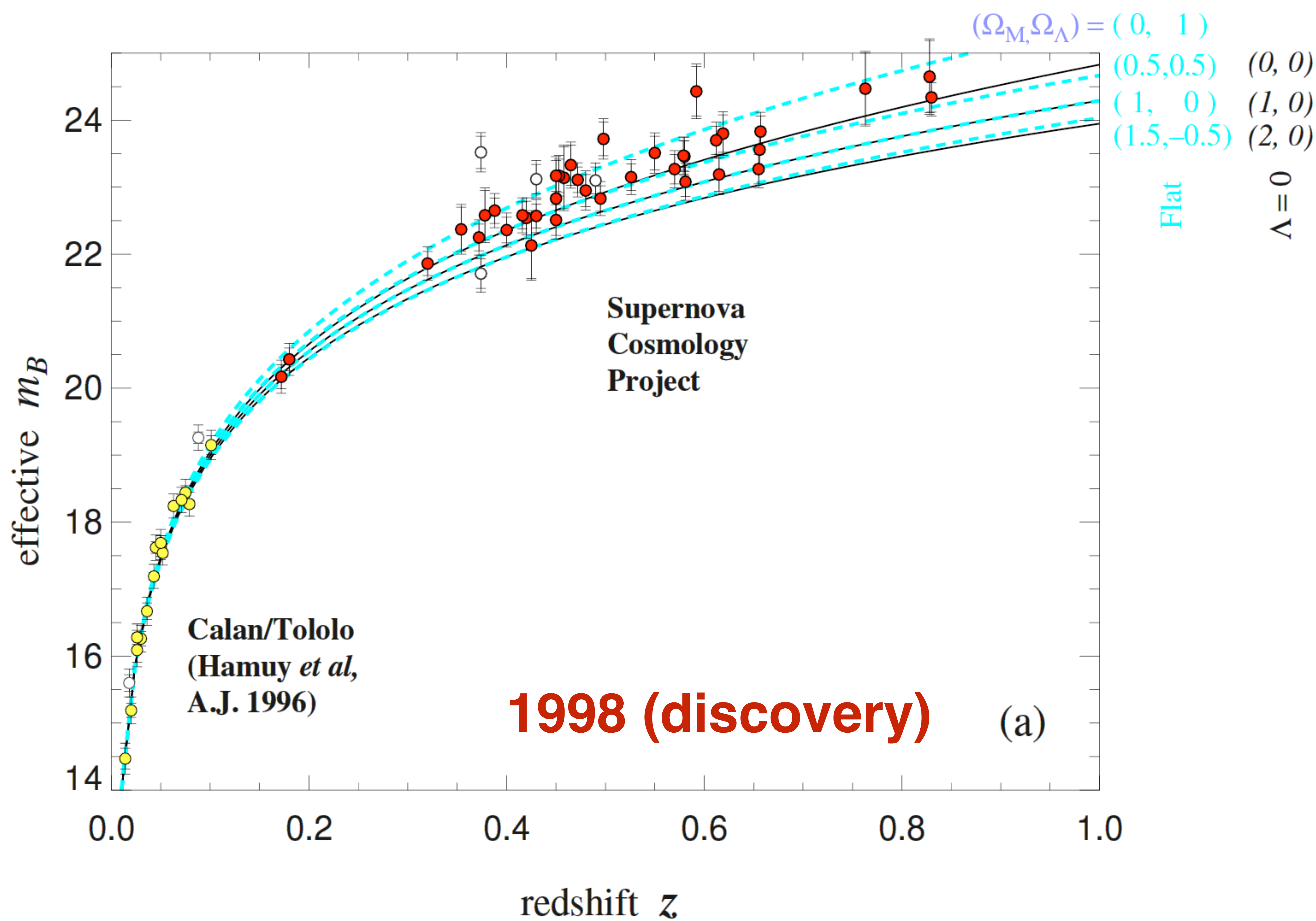
$$d_A = \frac{d_m}{1 + z}$$

Supernovae & The Accelerating Universe



Supernovae are very bright (~galaxy!) & distant probes, with good absolute luminosity → probe $a(t)$ beyond linear

Supernovae & The Accelerating Universe (history)



GR Cosmo: from Einstein to Friedmann eqns

$$\underbrace{G_{\mu\nu}[a(t)]}_{\text{"CURVATURE"}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{"MATTER"}}$$

$$\boxed{T^{\mu}{}_{\nu} = (\rho + P)U^{\mu}U_{\nu} - P\delta^{\mu}_{\nu}}$$

ρ : energy density
P : pressure

$U^{\mu} = (1,0,0,0)$ for observer at rest in fluid

$\boxed{\nabla_{\mu}T^{\mu}{}_{\nu} = 0}$ Energy conservation $\Rightarrow$ $\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0}$ "$\mathrm{d}U = -P\mathrm{d}V$"

FRIEDMANN EQUATIONS

1st eqn $$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

2d eqn $$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)} \quad \Leftrightarrow \quad \boxed{\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P)}$$

Exercise: show that if $w \equiv p/\rho = \text{const}$ $\boxed{\rho \propto a^{-3(1+w)}}$

Various fluids in the Universe

	Name	w	ρ	Examples		
m	MATTER	0	a^{-3}	*non-relativistic* particles	Cold Dark Matter (CDM)	c
					Baryons (nuclei + electrons!)	b
r	RADIATION	$\frac{1}{3}$	a^{-4}	*relativistic* particles	Photons	γ
					Neutrinos	ν
					Gravitons	g
Λ	DARK ENERGY	-1	a^0	"What the hell!?"	Vacuum Energy	Λ
					Modified Gravity	

Notice: $\varrho \propto T^4$
so $T \propto 1/a$

Exercise: find an explanation, and a proof why $\varrho_r \sim a^{-1/4}$ and what is the source of energy produced to keep ϱ_Λ cte, despite expansion

Cosmological constant: origin

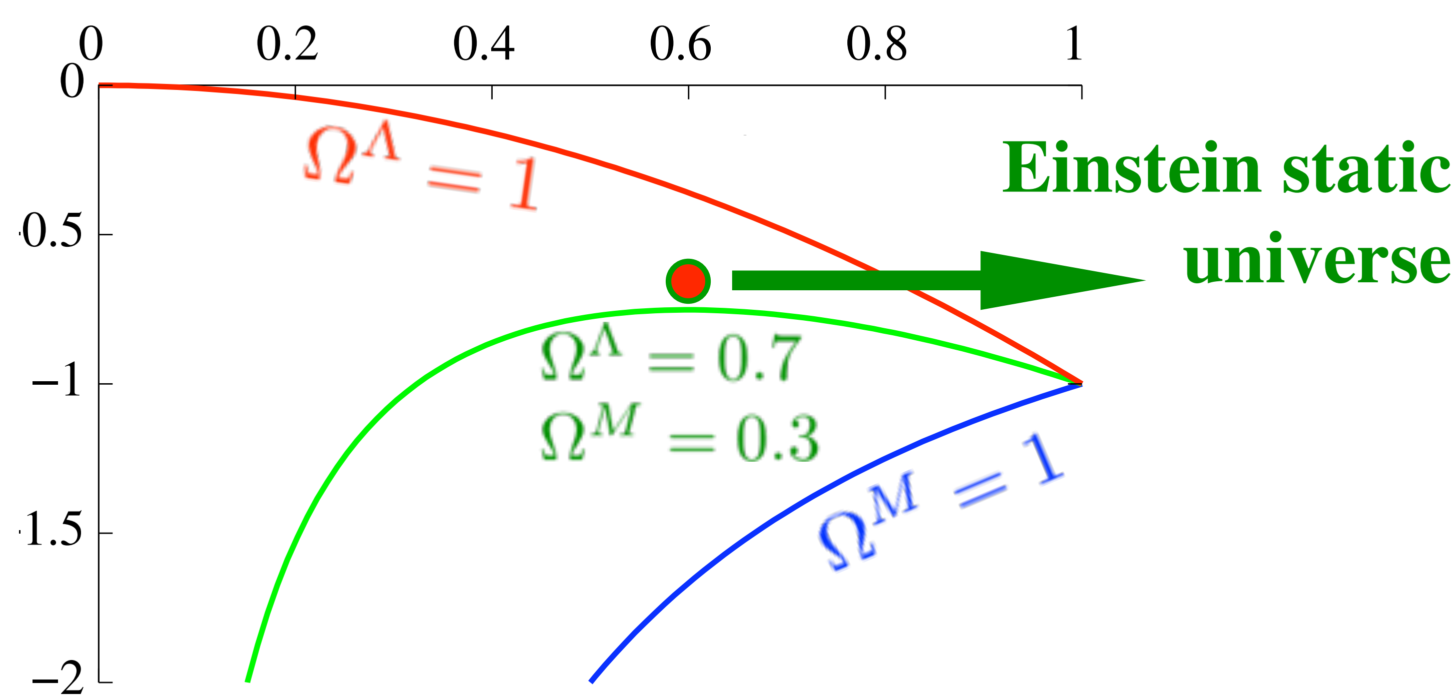
Combining all components

$$\rho \equiv \underbrace{\rho_\gamma + \rho_\nu}_{\rho_r} + \underbrace{\rho_c + \rho_b}_{\rho_m} + \rho_\Lambda$$

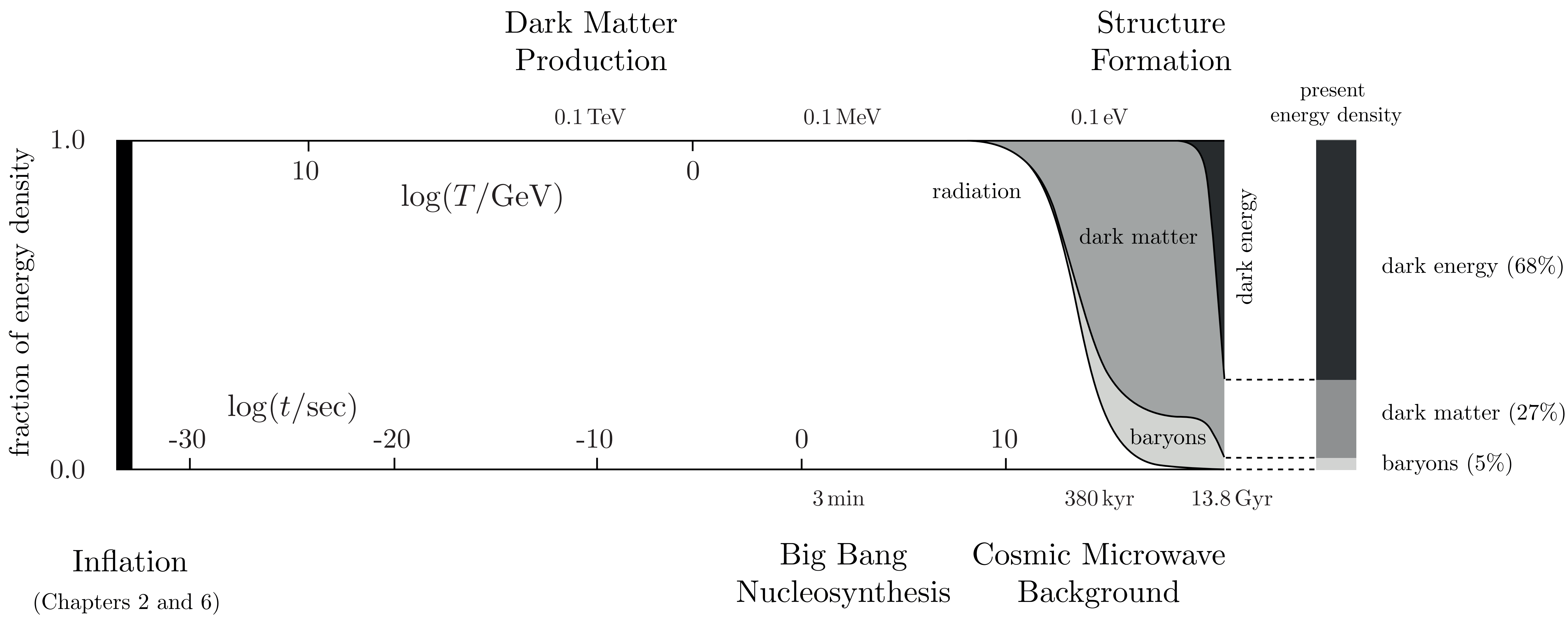
$$H^2 = H_0^2 [\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{(1 - \sum \Omega_i)}{a^2}]$$
$$= -V_{eff}(a)/a^2$$

(with 0 energy: k is in V)

Example:
matter + cosmo.const.
there is a flat region
in V_{eff} with $a \neq 0$
This was Einstein's motivation
to introduce Λ (?!)

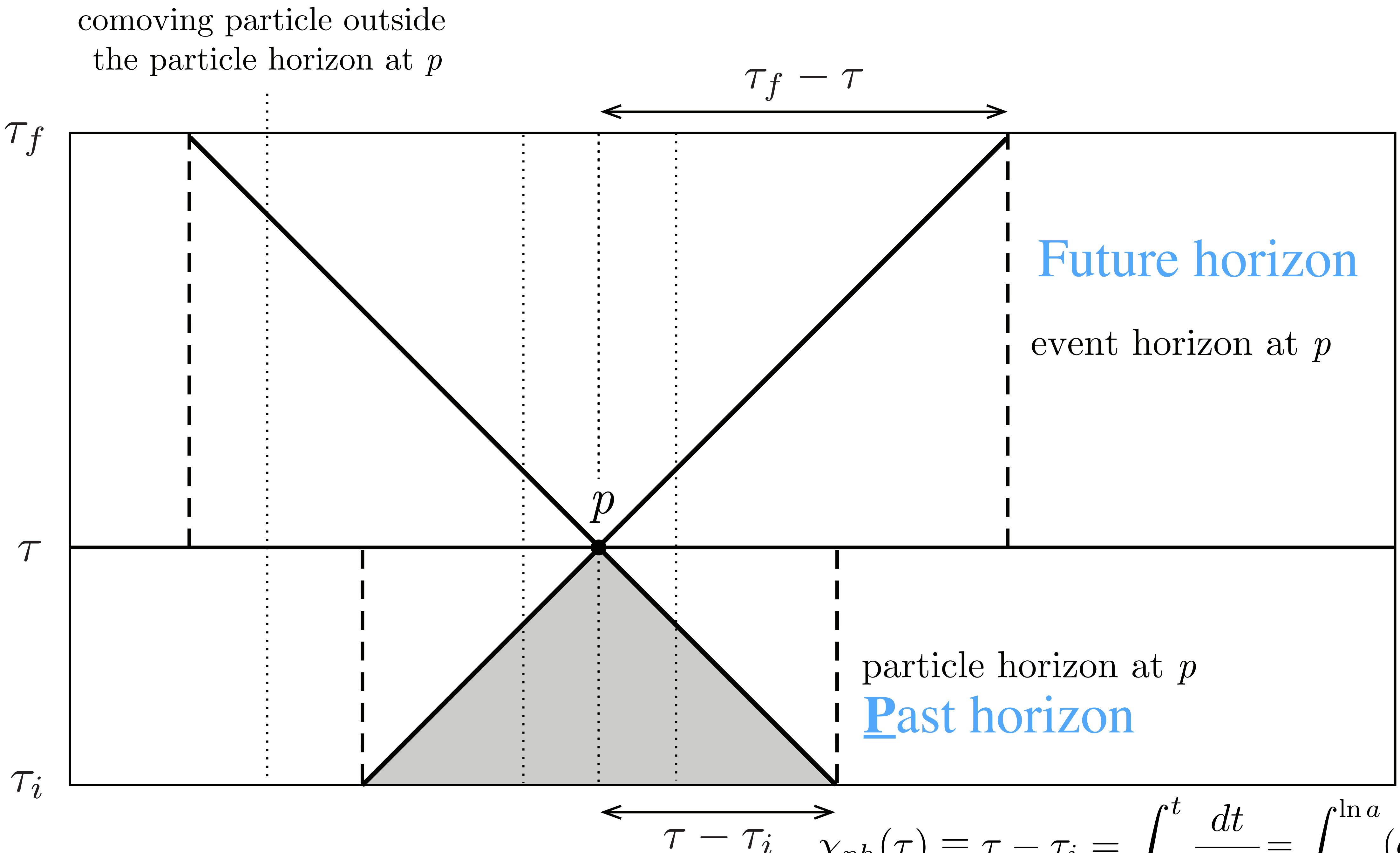


Universe Composition in Time



Horizons & Inflation

Horizons & causality

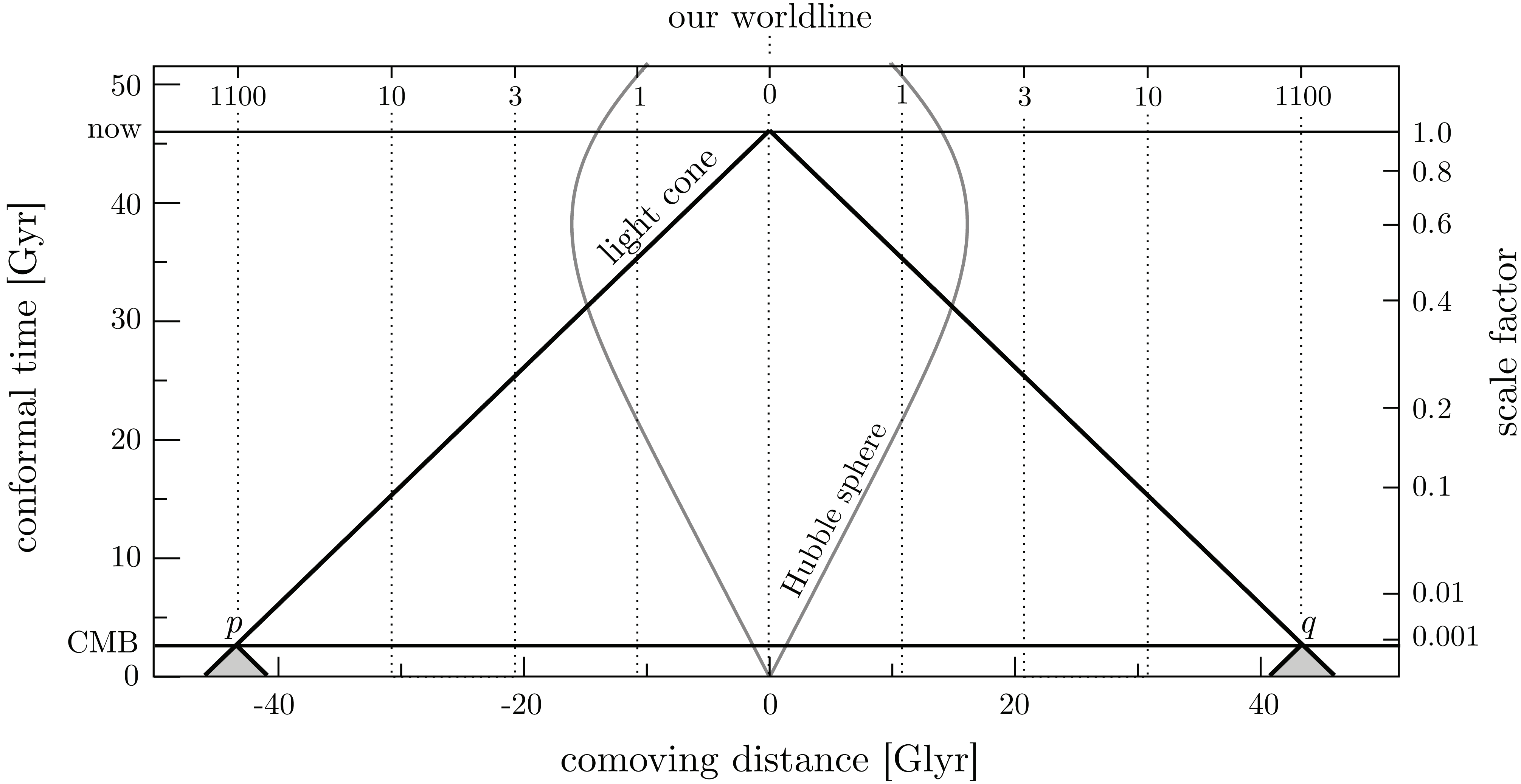


$$\chi_{ph}(\tau) \equiv \tau - \tau_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} (aH)^{-1} \, \mathrm{d} \ln a$$

Exercise: compare the particle horizon and Hubble radius $(aH)^{-1}$ at time t for a single fluid (w>-1/3). What is the value of τ_i?

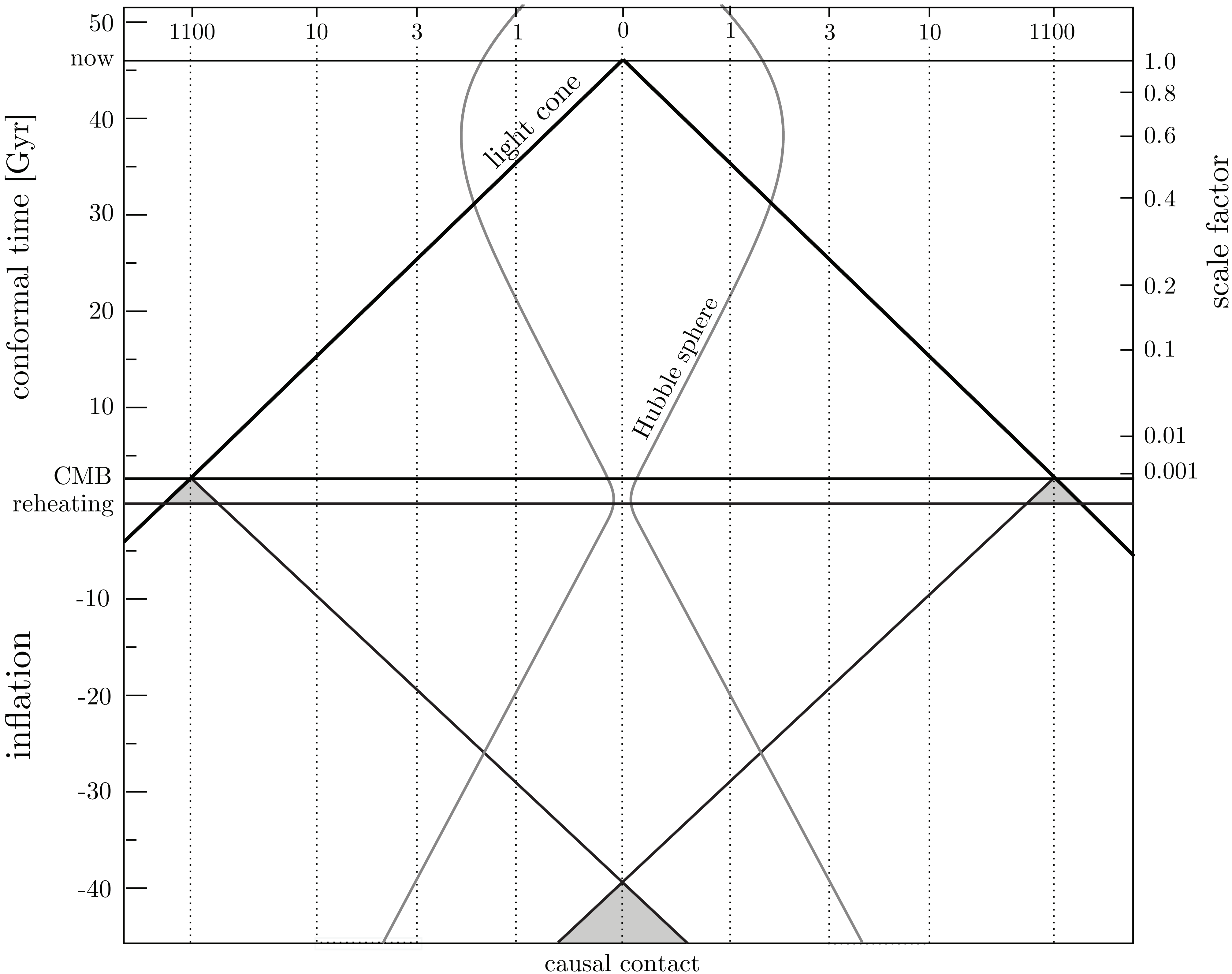
$$\chi_{\rm ph}(t) = \frac{2H_0^{-1}}{(1+3w)} a(t)^{\frac{1}{2}(1+3w)} = \frac{2}{(1+3w)} (aH)^{-1}$$

Horizon problem

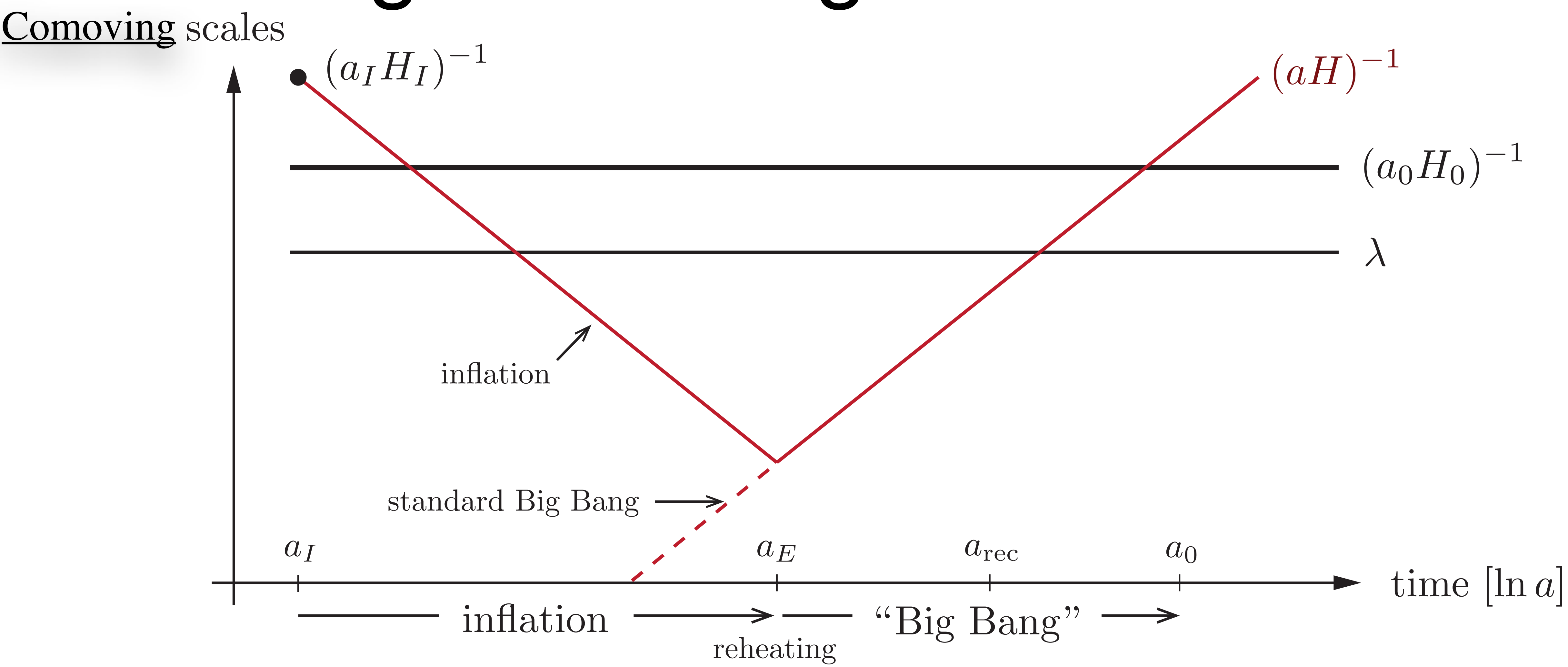


- Q: How can points p and q (at opposite directions on the CMB sky) have equal temperatures (with precision 10^{-5}) ???
- A: by giving them more time to talk, with a shrinking Hubble radius:
  $(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}$ → want w<-1/3, e.g. inflation (w=-1, H=ct)

Inflation solution



Exiting & entering the Hubble radius



Exercise: how many inflation e-folds (N=ln(a_E/a_I) are min. needed to fit the recombination Hubble radius $(a_{rec}H_{rec})^{-1}$ inside a Hubble radius before inflation $(a_I H_I)^{-1}$, if

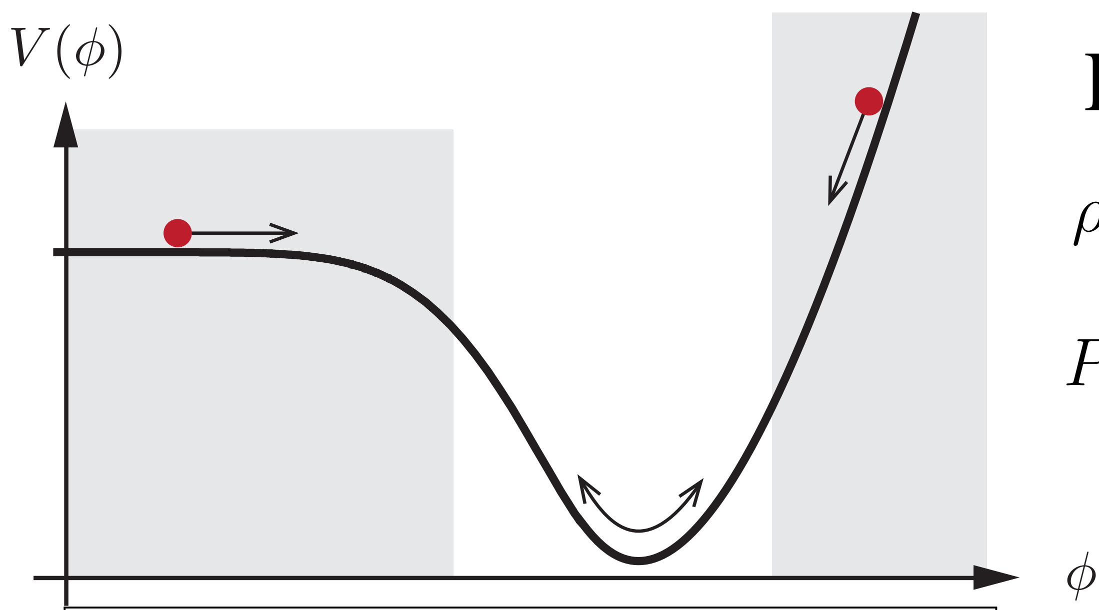
- after inflation, the universe is reheated to $T_E \approx E_{\rm GUT} \approx 10^{15}$ GeV
- assume a radiation domination ($H \propto a^{-2}$) up to $T_{rec} \approx 10^{-1}$ eV

A model: slow roll of «inflaton»

Conditions:

Inflation *occurs*: $\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1,$ **Slow Roll (SR)**

Inflation *lasts*: $\eta = \frac{d\ln\varepsilon}{dN} = \frac{\dot{\varepsilon}}{H\varepsilon} < 1$ **Stays Slow (SS)**



$$H^2 = \frac{\rho_\phi}{3M_{\rm pl}^2} = \frac{1}{3M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V\right]$$

$$\dot{H} = -\frac{\rho_\phi + P_\phi}{2M_{\rm pl}^2} = -\frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2}$$

For scalar « inflaton » field in potential:

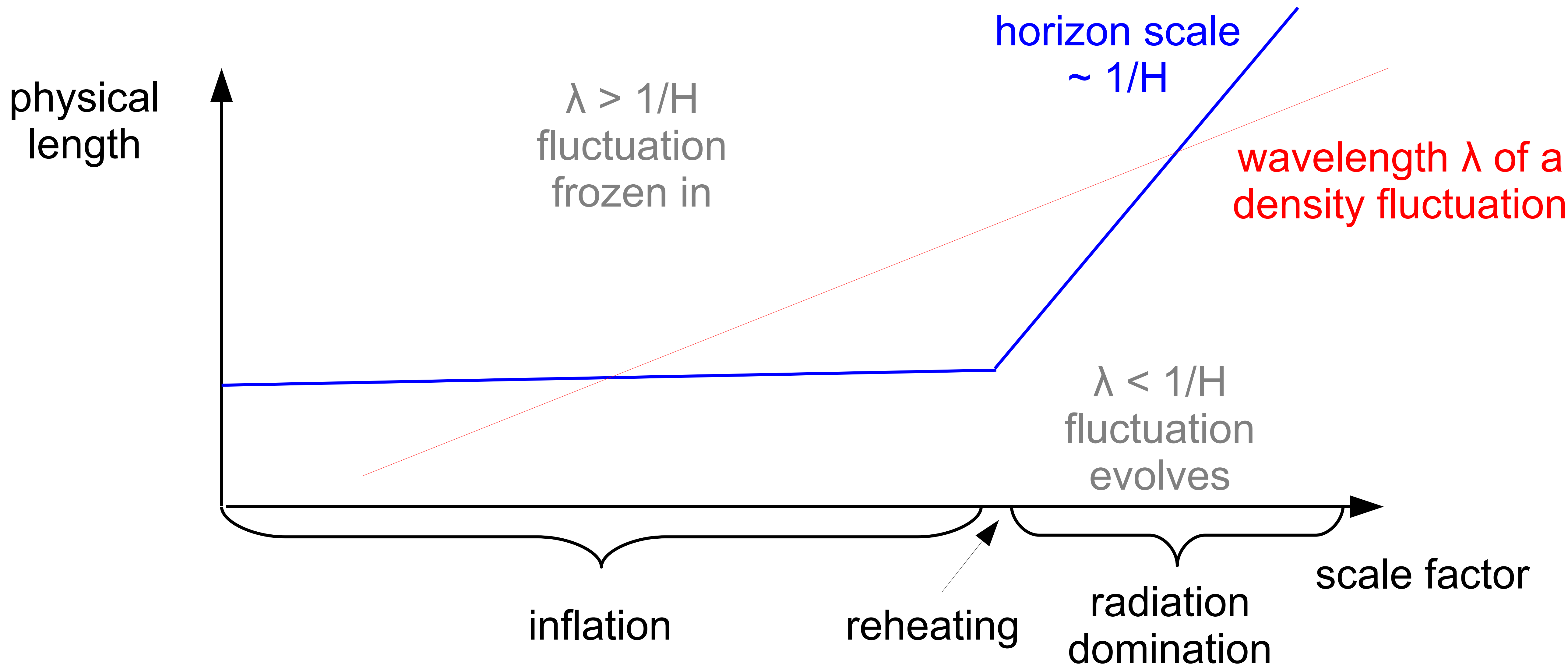
$$\rho_\phi \equiv T^0{}_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (= \mathrm{KE} + \mathrm{PE})$$

$$P_\phi \equiv -\frac{1}{3}T^i{}_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (= \mathrm{KE} - \mathrm{PE})$$

$$\varepsilon = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\rm pl}^2 H^2} \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \equiv \epsilon_{\rm v}$$ <<1 **(SR)**

$$\varepsilon + \delta \approx M_{\rm pl}^2\frac{V''}{V} \equiv \eta_{\rm v}$$ <<1 **(SS)**

The origin of the primordial perturbations: inflation



Quantum fluctuations of ϕ are stretched beyond the horizon and freeze in

Hamann, Moriond'14

Inflationary perturbations

Scalar (curvature) perturbations

$$\mathcal{P}_{\mathcal{R}}(k) \propto \left.\frac{V}{\epsilon}\right|_{k=aH} \approx A_{\rm s} \left(\frac{k}{k_*}\right)^{n_{\rm s}-1+\ldots}$$

$\epsilon \propto \left(\frac{V'}{V}\right)^2$

scalar/tensor amplitude

scalar/tensor spectral index

Tensor perturbations (gravitational waves)

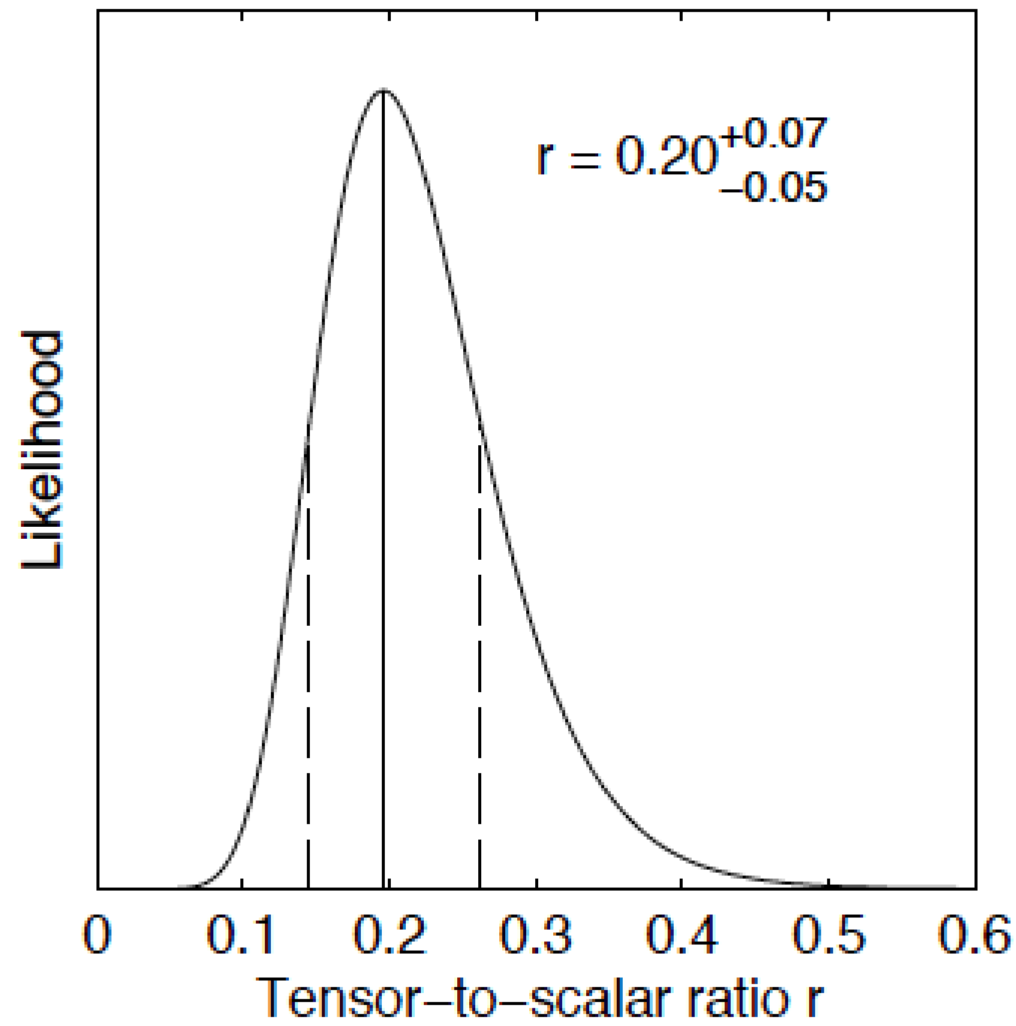
$$\mathcal{P}_t(k) \propto V|_{k=aH} \approx A_{\rm t} \left(\frac{k}{k_*}\right)^{n_{\rm t}+\ldots}$$

Tensor-to-Scalar ratio

$$r \equiv \left.\frac{\mathcal{P}_{\rm t}}{\mathcal{P}_{\mathcal{R}}}\right|_{k=0.002\ {\rm Mpc}^{-1}}$$

Hamann, Moriond'14

Implications of BICEP2 results



[BICEP2 2014]

Energy scale of inflation:

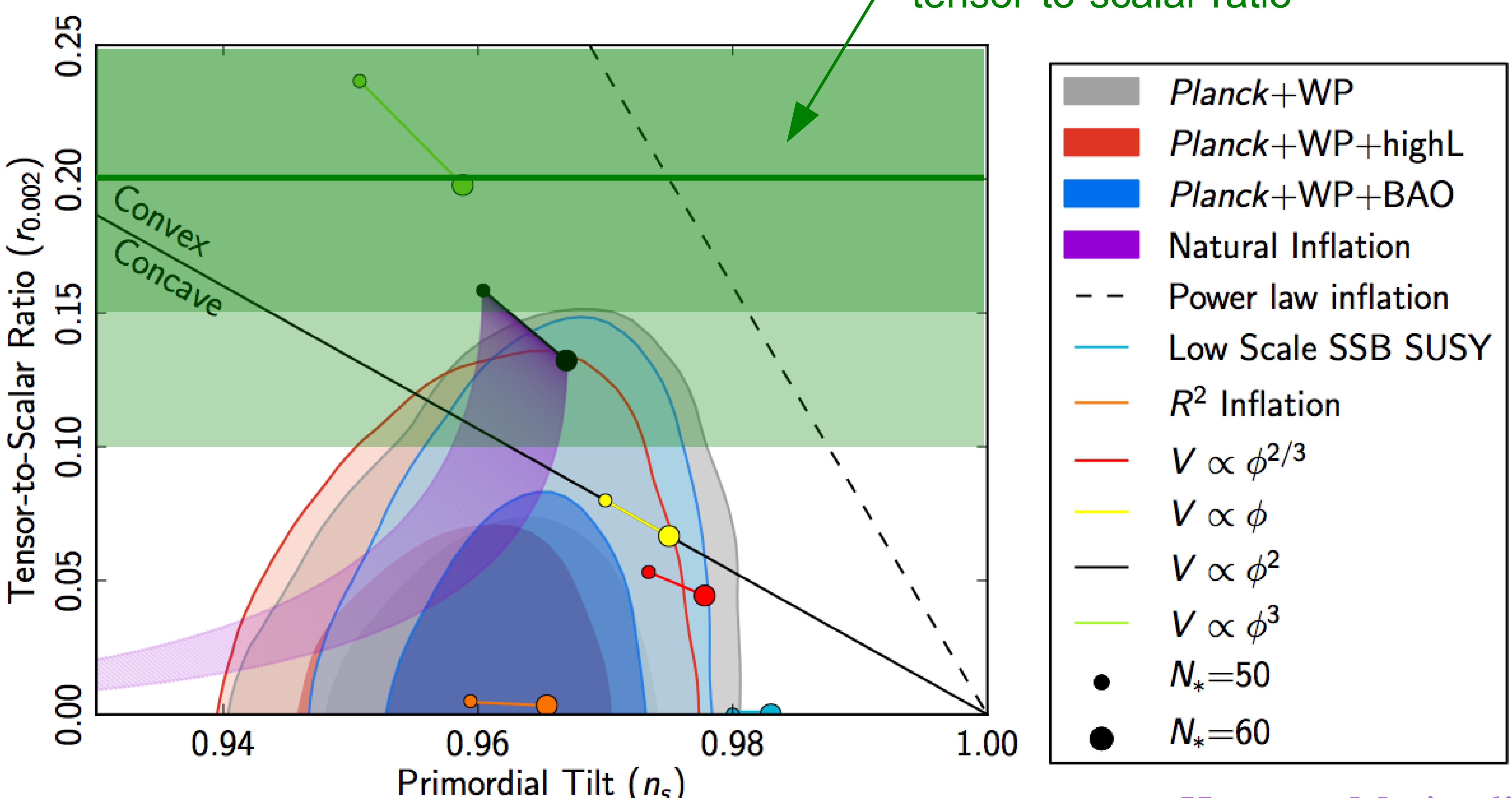
$$V_{\rm inf}^{1/4} \approx 2.2 \cdot 10^{16} \left(\frac{r}{0.2}\right)^{1/4} \ \mathrm{GeV}$$

Hamann, Moriond'14

(This could in principle have been as low as O(10) MeV, we are incredibly lucky!)

Inflation model constraints (post BICEP2) (if taken seriously!)

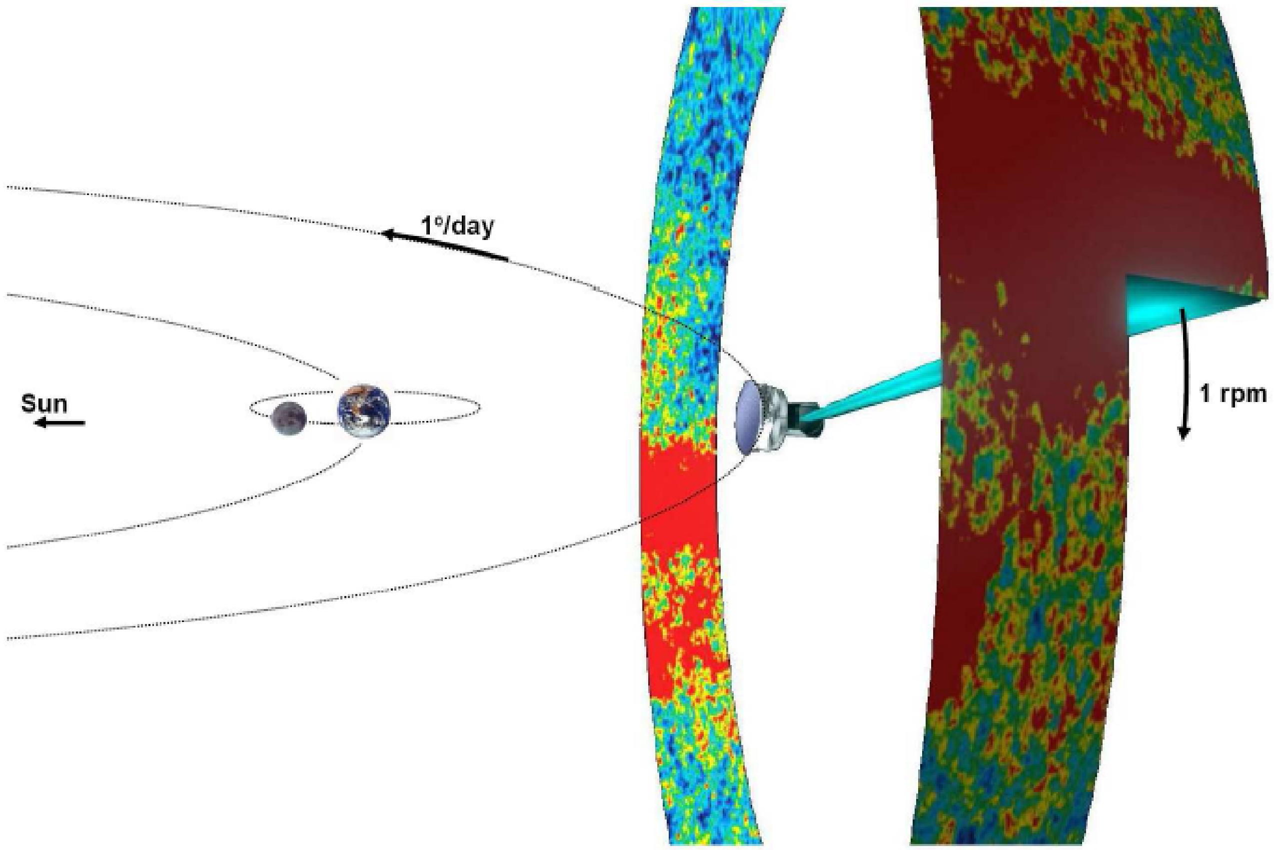
BICEP2 constraint on tensor-to-scalar ratio

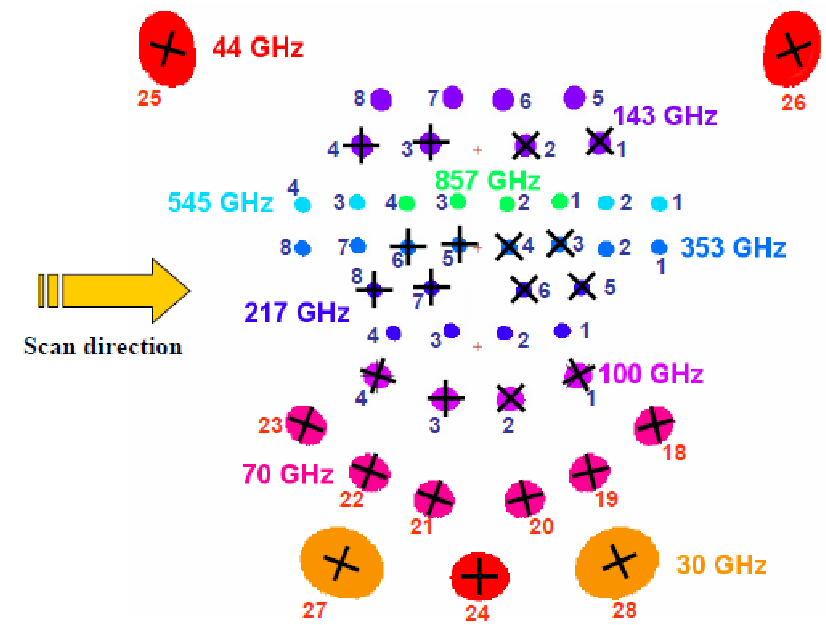


Hamann, Moriond'14

CMB observables

Planck at L2

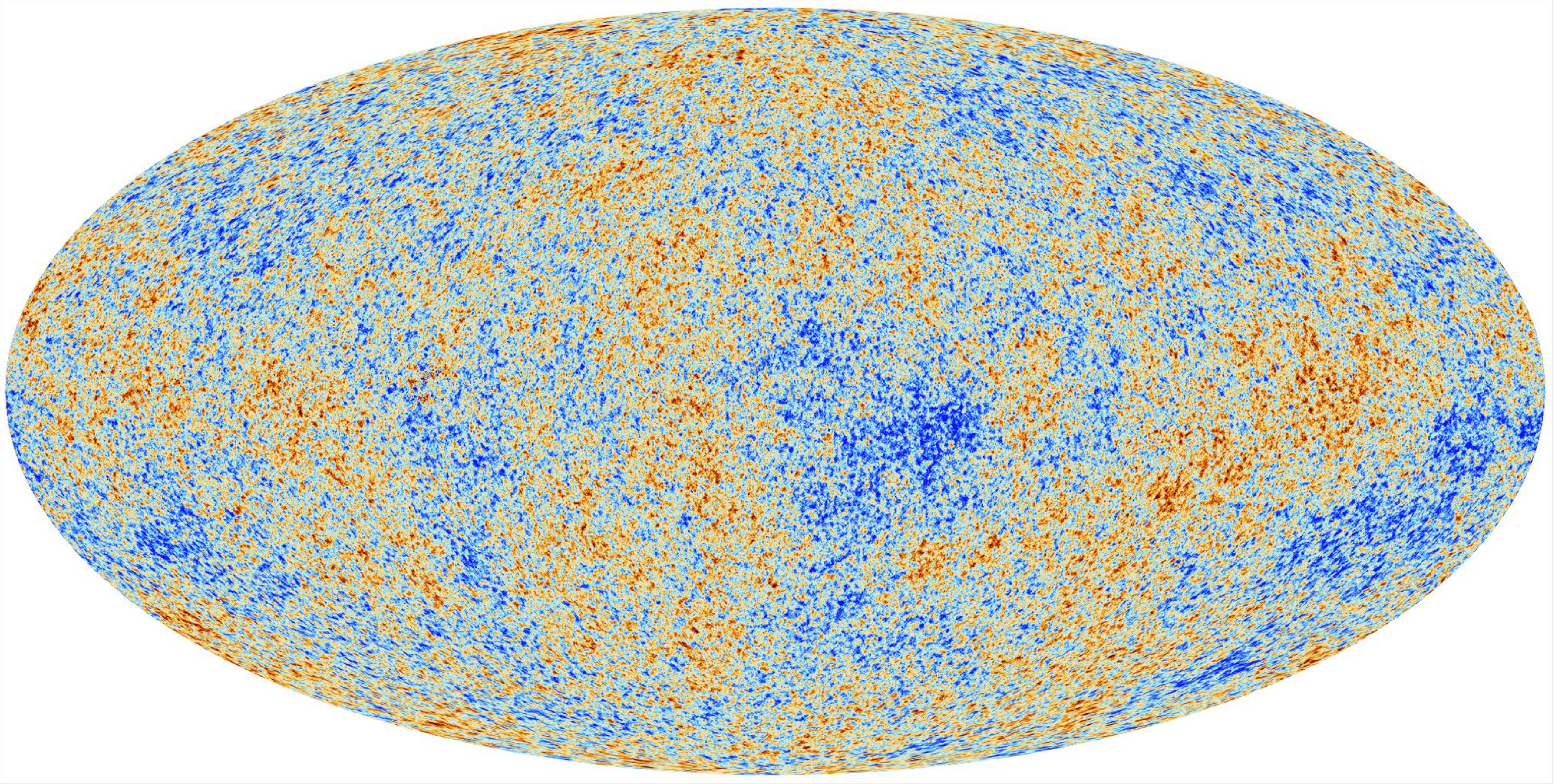




$\sim 8x8$ degree field

Continuous observations (7 months → all sky)
redundancies on different timescales (systematics)
Calibration accuracies .5% → 10% , beams ∼ 5 → 30 *arcmin*

Planck 2013 CMB temperature anisotropies map



4 methods compared in : Planck 2013 results. XII. Component separation

Cosmological parameter analysis in a nutshell

- Spherical harmonic decomposition ($\ell \sim 1/angle$) :

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell} \sum_{m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

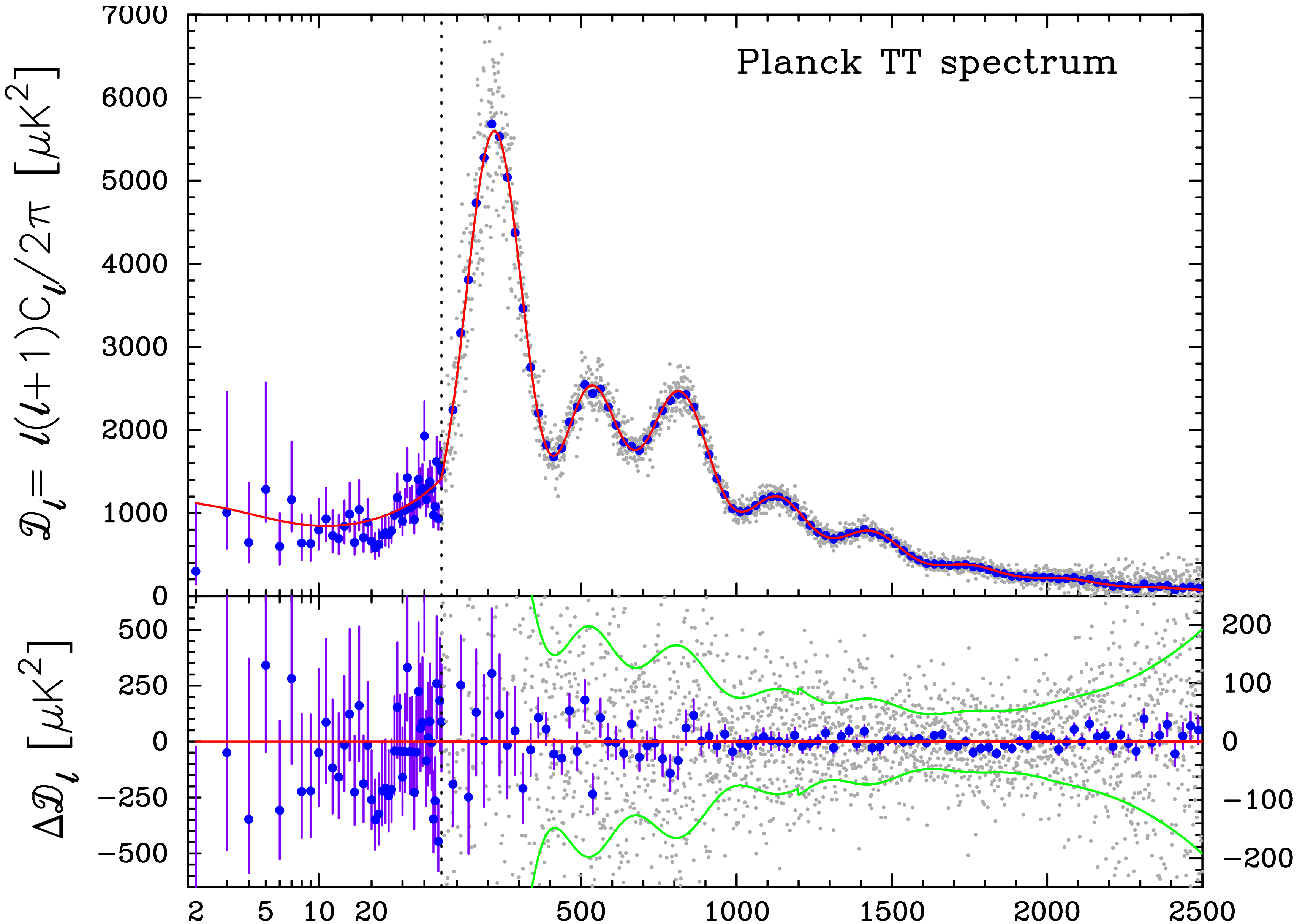
- general assumption $\Rightarrow a_{\ell m}$ are random variables (gaussian p.d.f.) ; $\langle a_{\ell m} \rangle_m = 0$; all information contained in their variance

$$C_\ell = \frac{1}{2\ell + 1} \sum_m a_{\ell m} a^\dagger_{\ell m}$$

predicted by our model

- only one realization is observable → intrinsic dispersion wrt model ("cosmic variance")
- Planck 2013 analysis : 100, 143 and 217 GHz maps cross spectra (suppression of instrumental noise) with masks ($\Rightarrow$ low foregrounds contamination) (high ℓ) ; CMB map ML (low ℓ)
- fit cosmological parameters using a likelihood function (accounting for CMB, residual foregrounds, instrumental nuisance parameters - ~ 20 parameters)

CMB TT power spectrum (Planck 2013)



output of Planck likelihood - foregrounds subtracted

Hybrid method : map based ML (low ℓ) / pseudo-spectra (high ℓ) of masked raw maps

CMB polarization anisotropies

- CMB is (weakly) polarized
- polarization = vector field ⇒ use Stockes parameters Q and U
- decompose $Q + iU$ in the (spinned) spherical harmonics basis

$$Q + iU = \sum {}_{\pm 2}a_{lm} \; {}_{\pm 2}Y_{lm}(\theta, \phi)$$

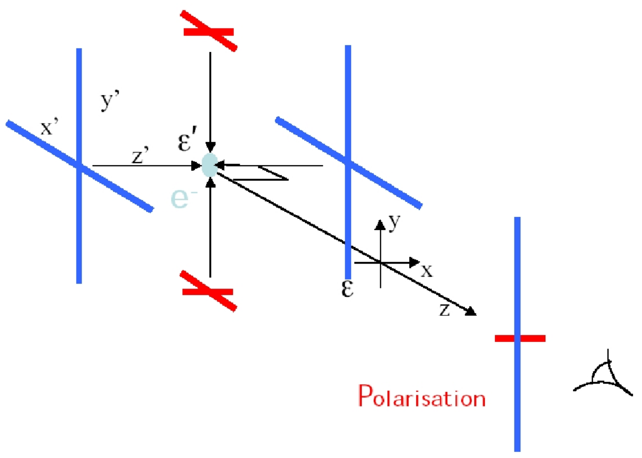
- transform into parity even (E) and odd (B) components :

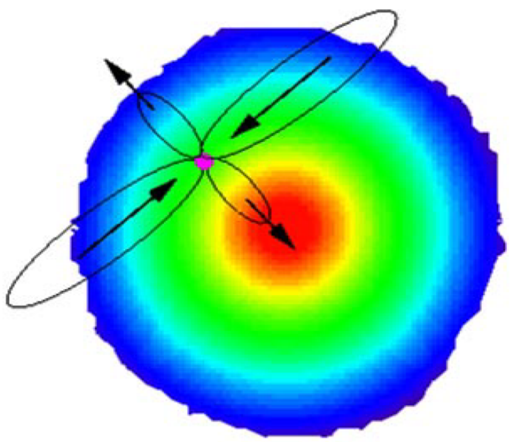
$${}_{\pm 2}a_{lm} = a^E_{lm} \pm i a^B_{lm}$$

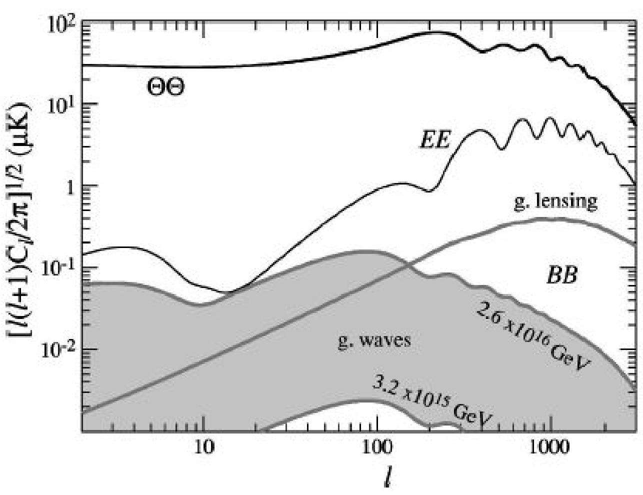
- As for temperature, all information contained in variances C^{XY}_ℓ (X,Y = T,E,B)
- in general 6 power spectra but symetries ⇒ $C^{TB}_\ell = C^{EB}_\ell = 0$

CMB polarization

- Mecanism : temperature quadrupolar anisotropies + Thomson scattering on *e*
- Origins :
  - primordial tensor modes (GW) → B modes
  - plasma dynamics (correlation with temp. anisotropies) → E modes
  - late time re-ionisation ($z \sim 10$) → E modes (low ℓ)
  - gravitational lensing transforms (part of) E into B modes
- very low amplitude signals ($\sim 10^{-2} - 10^{-4}$ temperature)
- amplitude of primordial B modes power spectrum measures $r = A_t/A_s$ ($\propto$ inflation energy scale)

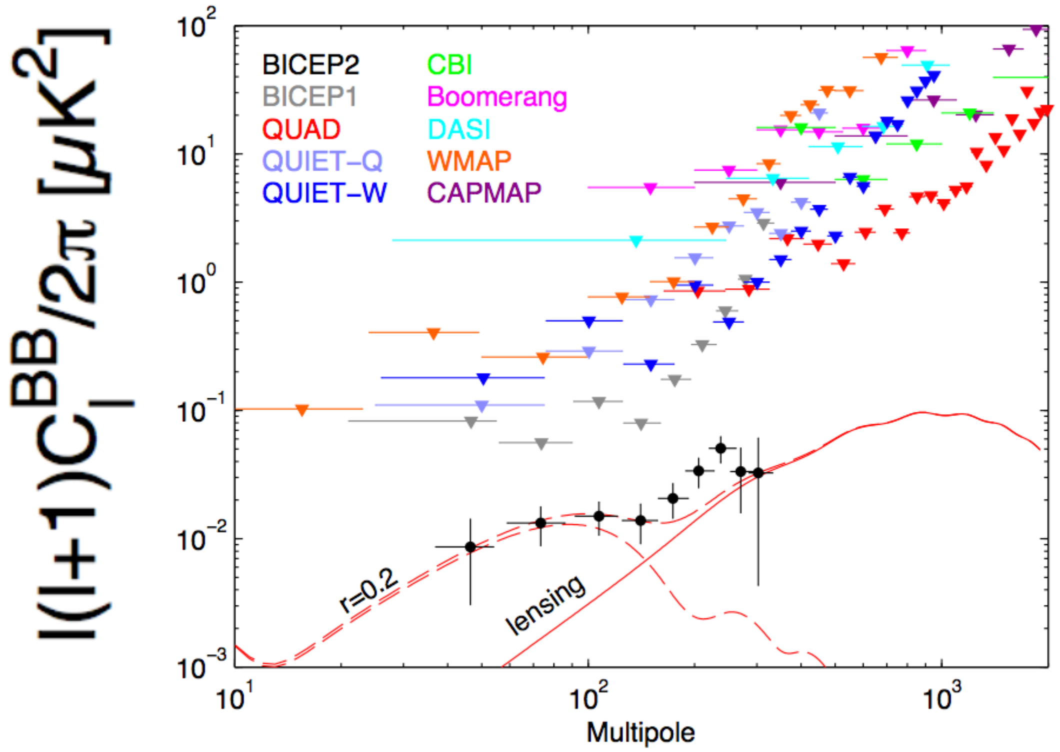




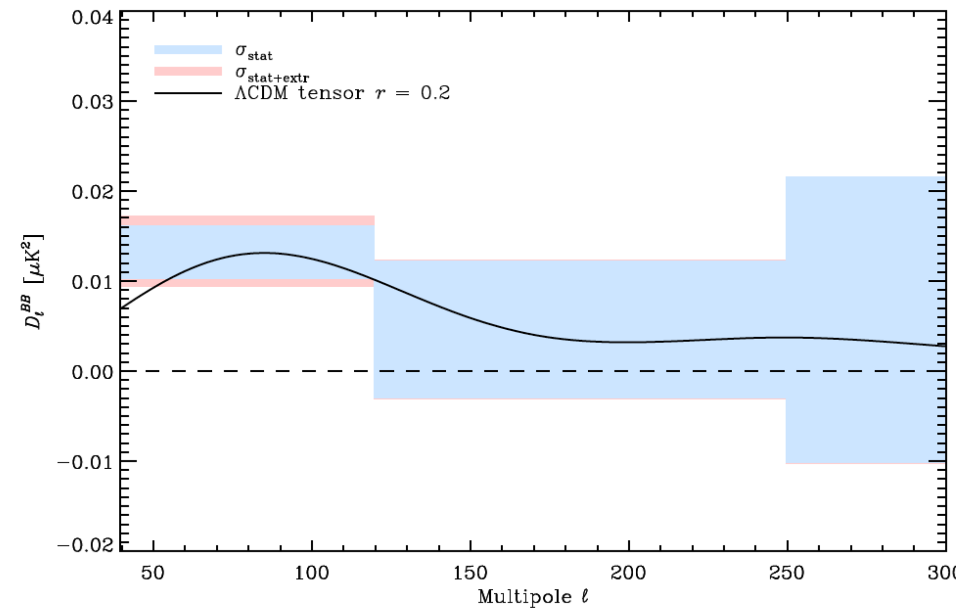


Primordial gravitational waves ?

March 2014…Bicep2/Keck Array

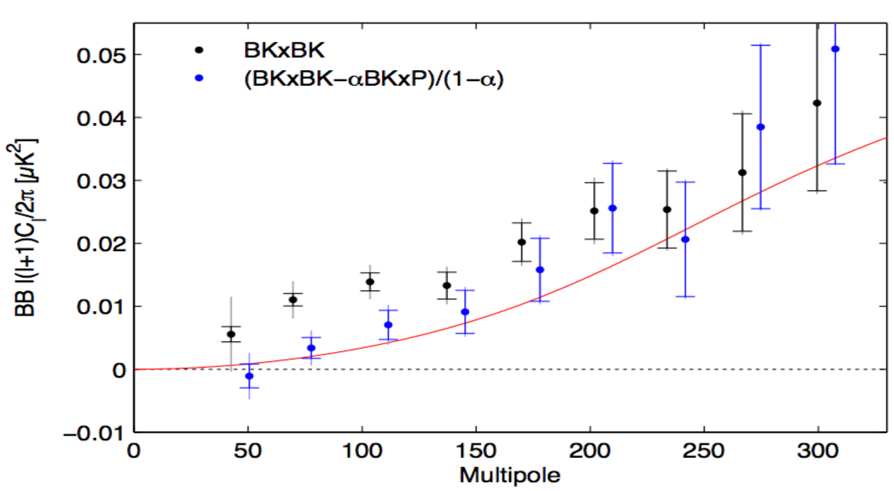


September 2014…answer from Planck



=> The polarized dust contamination cannot be neglected

December 2014… joint Bicep2/Keck Array/Planck analysis



the B-mode excess seen by BICEP2 is consistent with Galactic dust emission, and no significant evidence for primordial gravitational waves is found.
⇒ Upper limit r<0.12 @95%CL
(r is the tensor over scalar ratio)

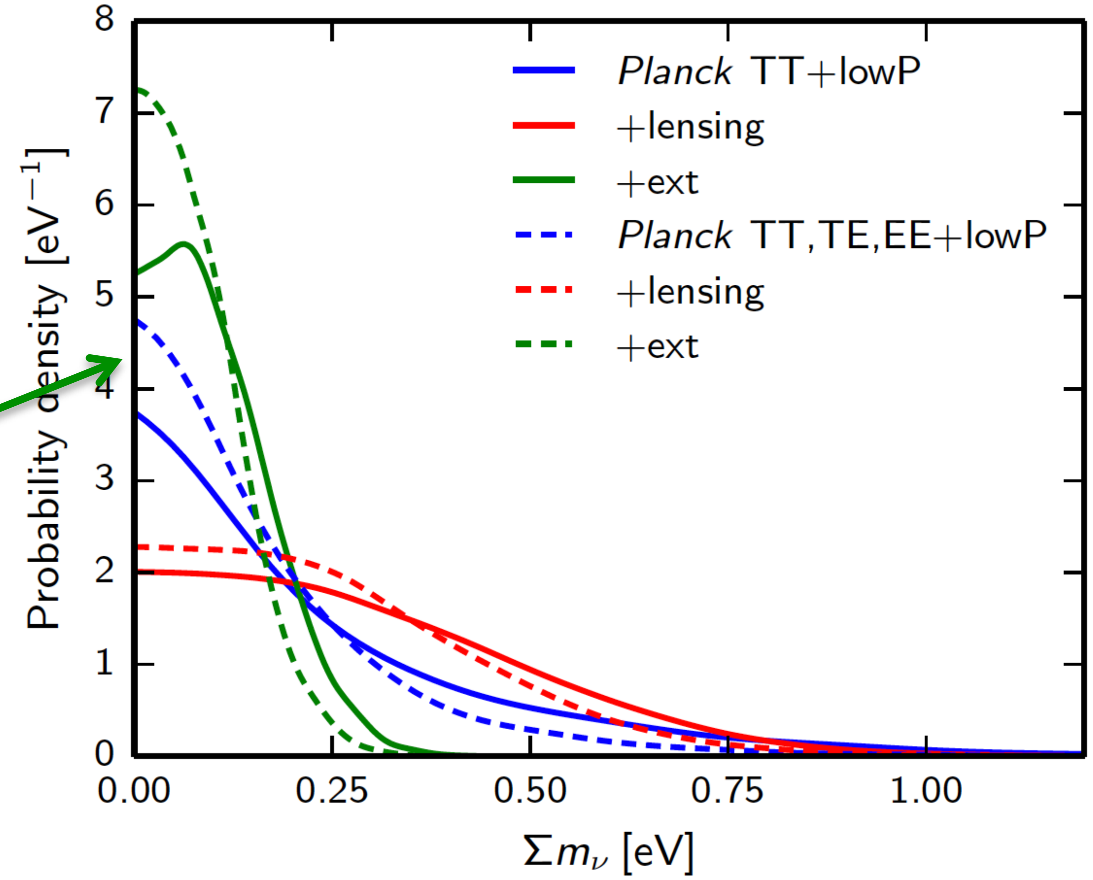
« A Joint Analysis of BICEP2/Keck Array and Planck Data » arXiv:1502.00612

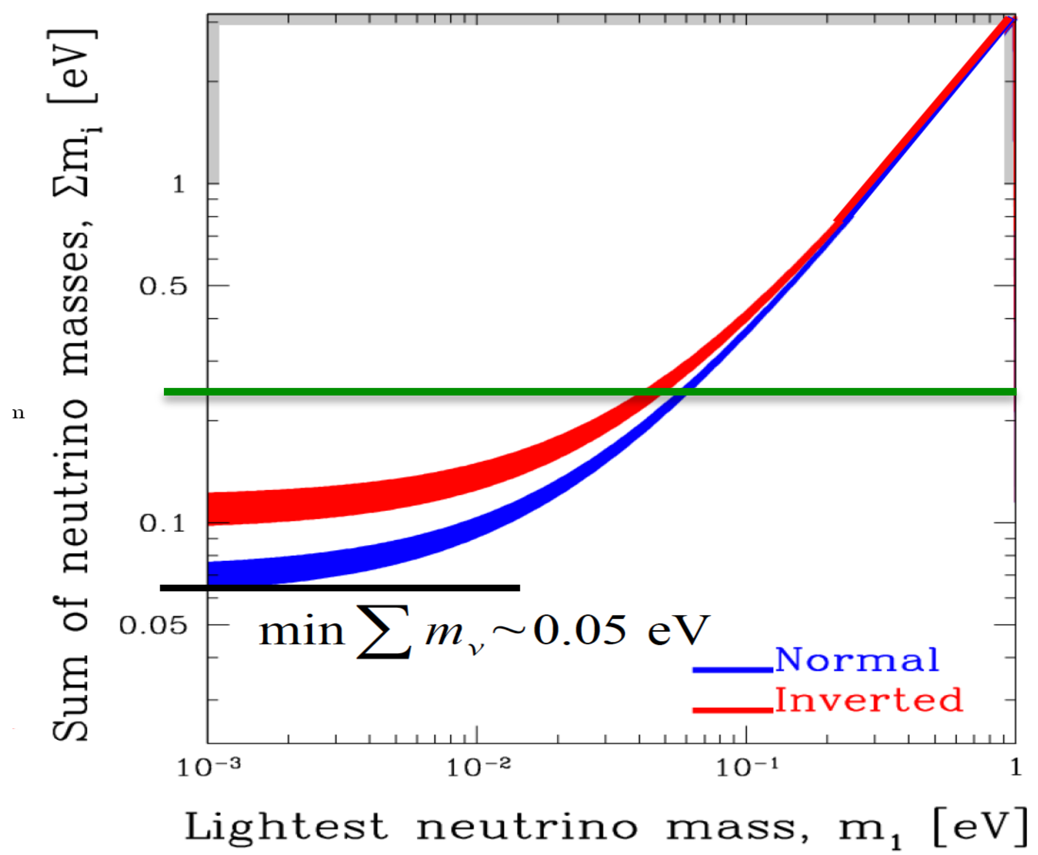
Sum of the Neutrino Masses

⇒ Impact on the first acoustic peak
⇒ + small scales

PlanckTT+LowP+Lensing+BAO+SN/JLA+H0

$\Sigma(m_\nu)<0.23$eV (95%CL limit)





Combined with oscillations measurements
⇒ Starting to test the hierarchy soon ?!?

(cf. yesterday's talk : E. lisi, S. Choubey, T. Johnson,...)

Neff

Neff is the effective number of relativistic degrees of freedom

Under the assumption that ONLY photons and standard light neutrinos contribute to the radiation:

⇒ Neff is the effective number of neutrinos and ≈ 3.046

Any deviation from this value can be attributed to sterile neutrinos, axions, lepton number violation (cf. yesteday J. Heeck's talk) primordial gravitational waves (GW)...

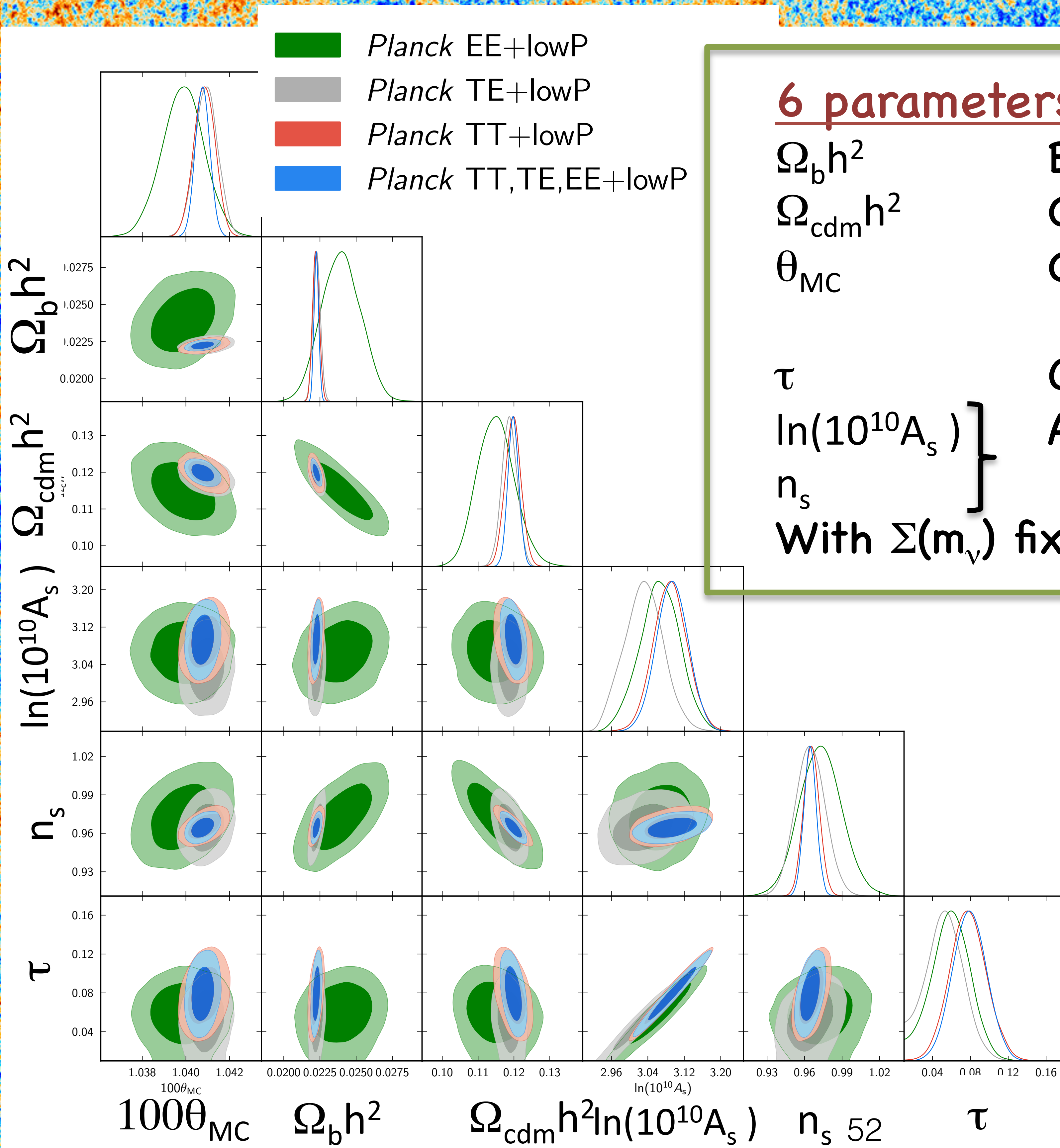
$$N_{\rm eff} = 3.13 \pm 0.32 \quad \textit{Planck}\ \text{TT+lowP};$$
$$N_{\rm eff} = 3.15 \pm 0.23 \quad \textit{Planck}\ \text{TT+lowP+BAO};$$

No convincing evidence for extra relativistic component

Accuracy with Polarization:

$$N_{\rm eff} = 2.99 \pm 0.20 \quad \textit{Planck}\ \text{TT,TE,EE+lowP};$$
$$N_{\rm eff} = 3.04 \pm 0.18 \quad \textit{Planck}\ \text{TT,TE,EE+lowP+BAO}.$$

The "base" ΛCDM Model



6 parameters:

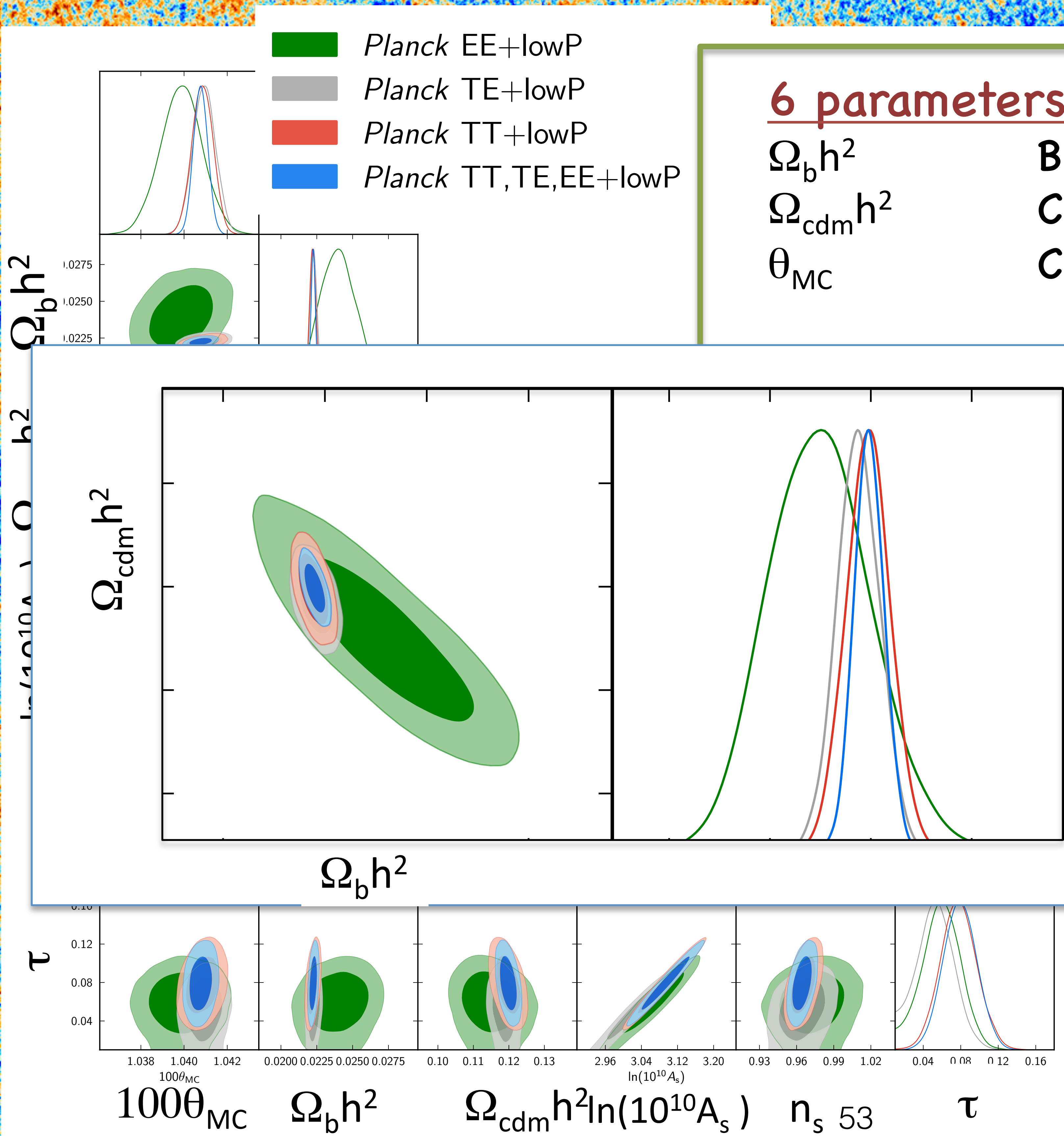
$\Omega_b h^2$	Baryon density
$\Omega_{cdm} h^2$	Cold Dark Matter density
θ_{MC}	Characteristic angular size of the CMB fluctuations
τ	Optical depth to reionization
$\ln(10^{10}A_s)$, n_s	Amplitude and index of primordial fluctuations

With $\Sigma(m_\nu)$ fixed to 0.06eV

Very good agreement between temperature and polarization results !

Parameter	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02230 ± 0.00014
$\Omega_c h^2$	0.1188 ± 0.0010
$100\theta_{MC}$	1.04093 ± 0.00030
τ	0.066 ± 0.012
$\ln(10^{10}A_s)$	3.064 ± 0.023
n_s	0.9667 ± 0.0040

The "base" ΛCDM Model



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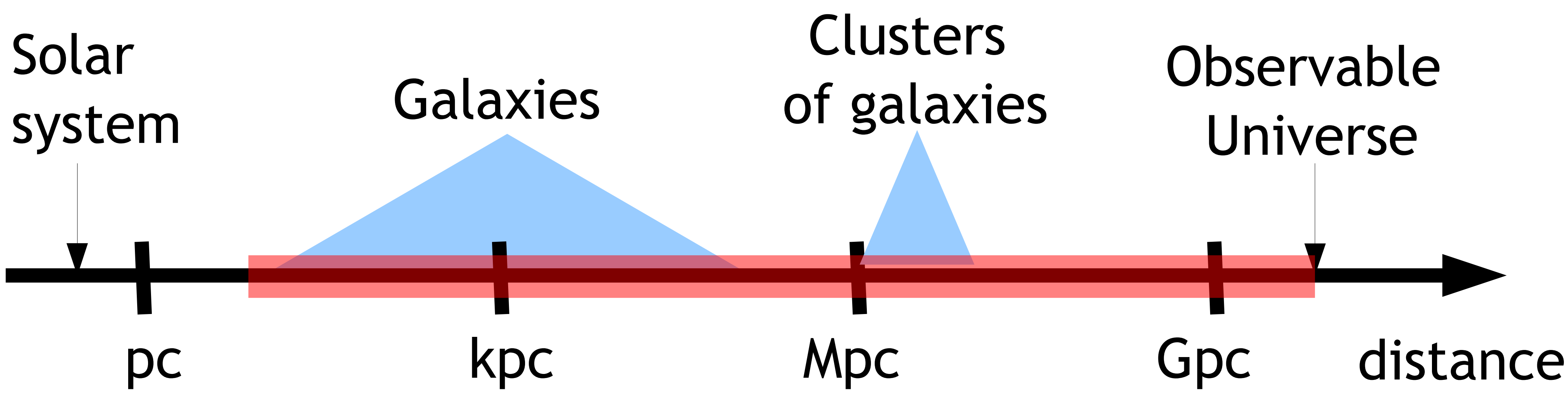
n_s	0.9667 ± 0.0040

Dark Matter

Credits to Ibarra, Cargese School 2014

Dark matter needed!

There is evidence for dark matter in a wide range of distance scales



THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 86 OCTOBER 1937 NUMBER 3

ON THE MASSES OF NEBULAE AND OF CLUSTERS OF NEBULAE

F. ZWICKY

1- Apply the virial theorem to determine the total mass of the Coma Cluster

For an isolated self-gravitating system,

$$2K + U = 0$$

$$K = \frac{1}{2} M \langle v^2 \rangle \qquad U = -\frac{\alpha G M^2}{\mathcal{R}}$$

$$M = \frac{\langle v^2 \rangle \mathcal{R}}{\alpha G}$$

$$\mathcal{M} > 9 \times 10^{46} \text{gr}$$

2- Count the number of galaxies (~1000) and calculate the average mass

$$\overline{M} > 9 \times 10^{43} \text{ gr} = 4.5 \times 10^{10} M_\odot$$

Inasmuch as we have introduced at every step of our argument inequalities which tend to depress the final value of the mass $\mathcal{M}$, the foregoing value (36) should be considered as the lowest estimate for the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about 8.5×10^7 suns. According to (36), the conversion factor γ from luminosity to mass for nebulae in the Coma cluster would be of the order

$$\gamma = 500 , \tag{37}$$

Galaxy

ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS*

VERA C. RUBIN† AND W. KENT FORD, JR.†
Department of Terrestrial Magnetism, Carnegie Institution of Washington and Lowell Observatory, and Kitt Peak National Observatory‡
Received 1969 July 7; revised 1969 August 21

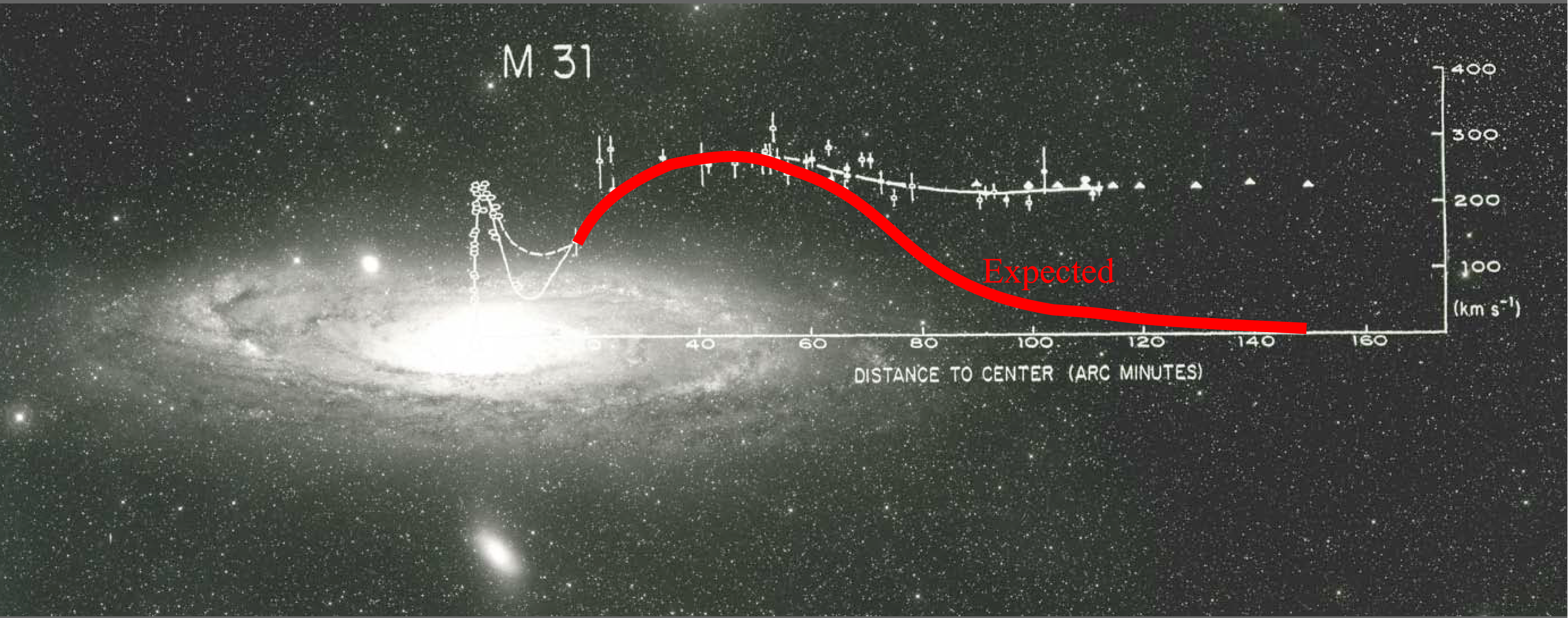
ABSTRACT

Spectra of sixty-seven H II regions from 3 to 24 kpc from the nucleus of M31 have been obtained with the DTM image-tube spectrograph at a dispersion of 135 Å mm^{-1}. Radial velocities, principally from Hα, have been determined with an accuracy of ± 10 km sec^{-1} for most regions. Rotational velocities have been calculated under the assumption of circular motions only.

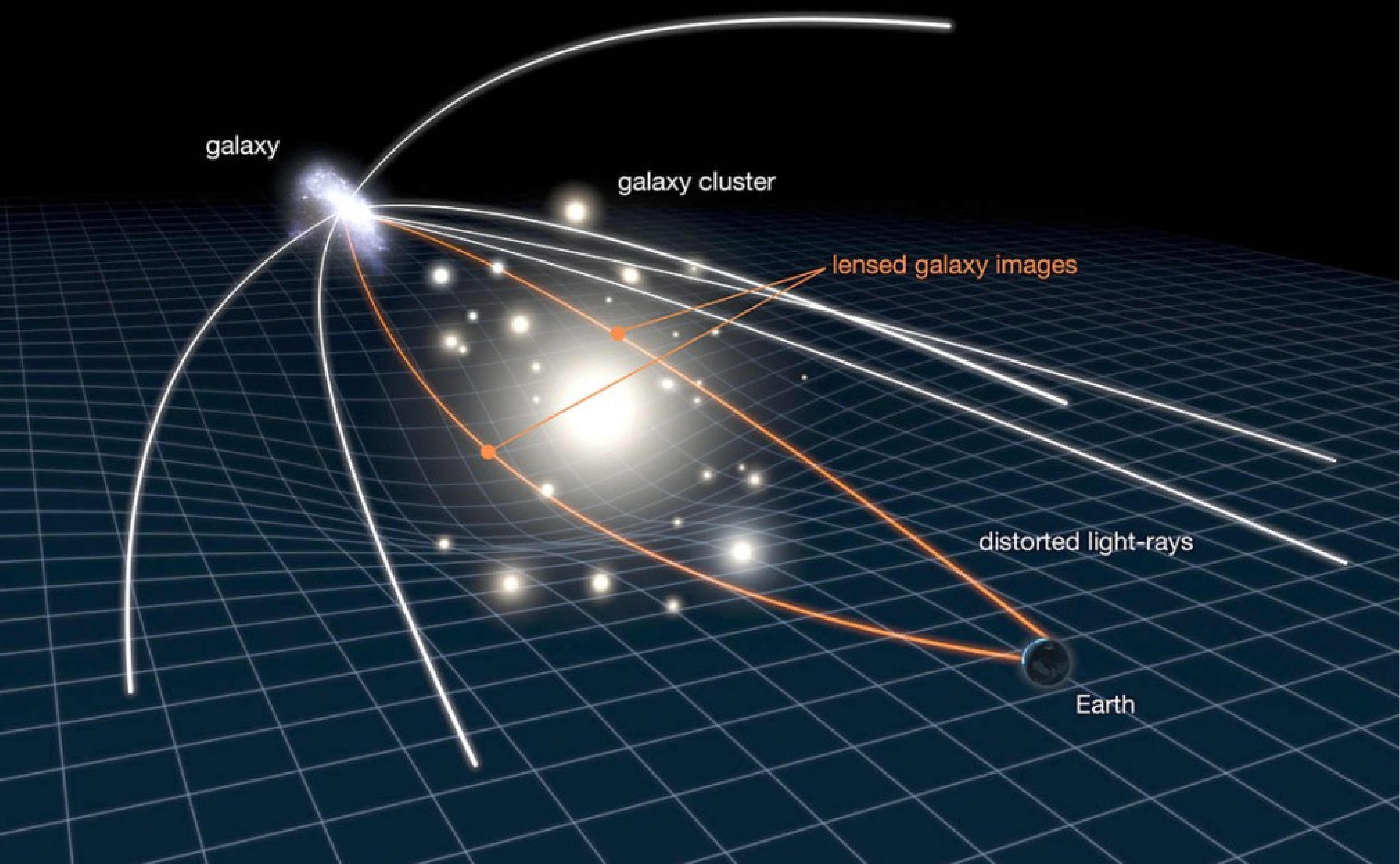
For the region interior to 3 kpc where no emission regions have been identified, a narrow [N II] λ6583 emission line is observed. Velocities from this line indicate a rapid rotation in the nucleus, rising to a maximum circular velocity of $V = 225$ km sec^{-1} at $R = 400$ pc, and falling to a deep minimum near $R = 2$ kpc.

From the rotation curve for $R \leq 24$ kpc, the following disk model of M31 results. There is a dense, rapidly rotating nucleus of mass $M = (6 \pm 1) \times 10^9 M_\odot$. Near $R = 2$ kpc, the density is very low and the rotational motions are very small. In the region from 500 to 1.4 kpc (most notably on the southeast minor axis), gas is observed leaving the nucleus. Beyond $R = 4$ kpc the total mass of the galaxy increases approximately linearly to $R = 14$ kpc, and more slowly thereafter. The total mass to $R = 24$ kpc is $M = (1.85 \pm 0.1) \times 10^{11} M_\odot$; one-half of it is located in the disk interior to $R = 9$ kpc. In many respects this model resembles the model of the disk of our Galaxy. Outside the nuclear region, there is no evidence for noncircular motions.

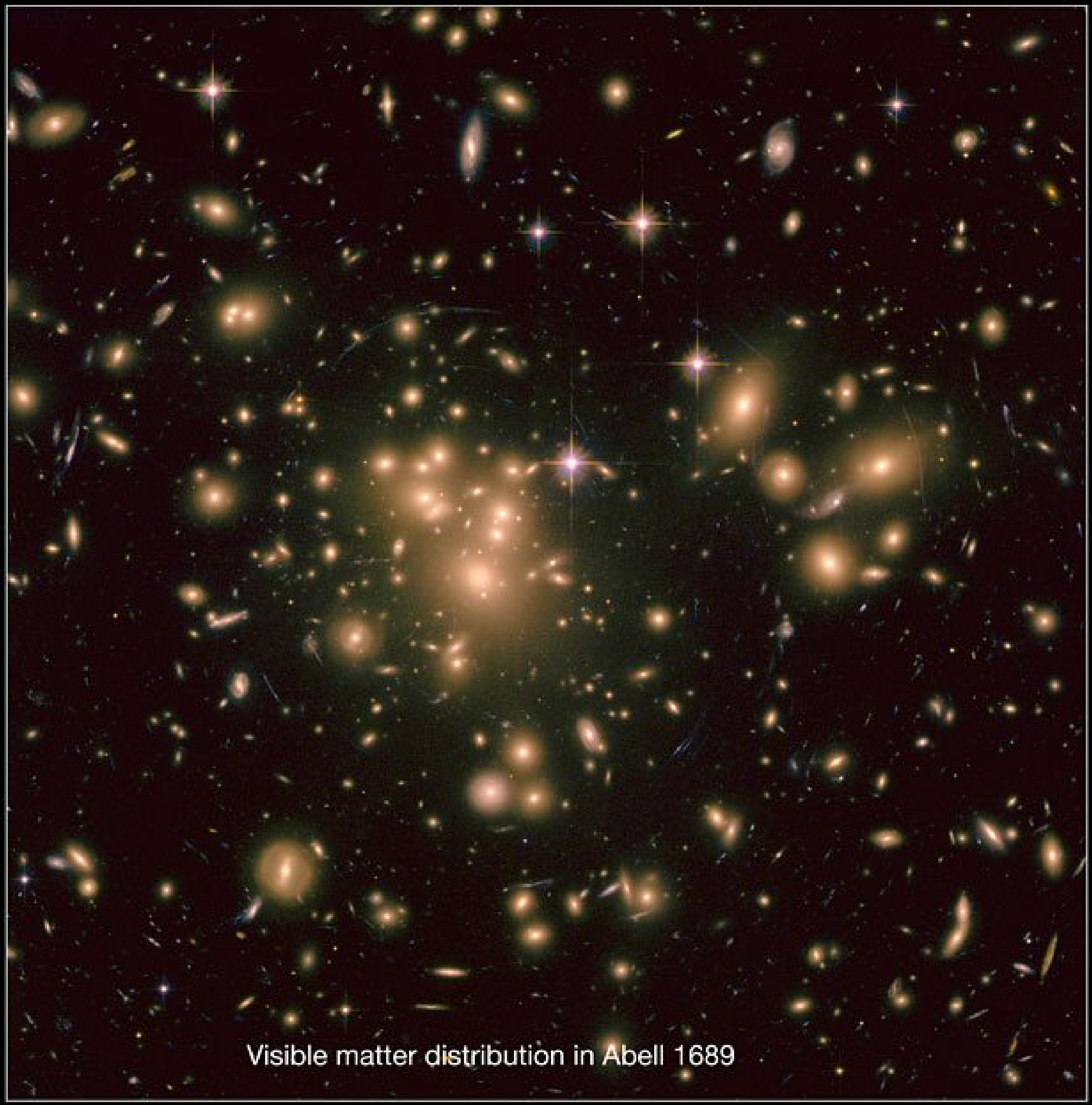
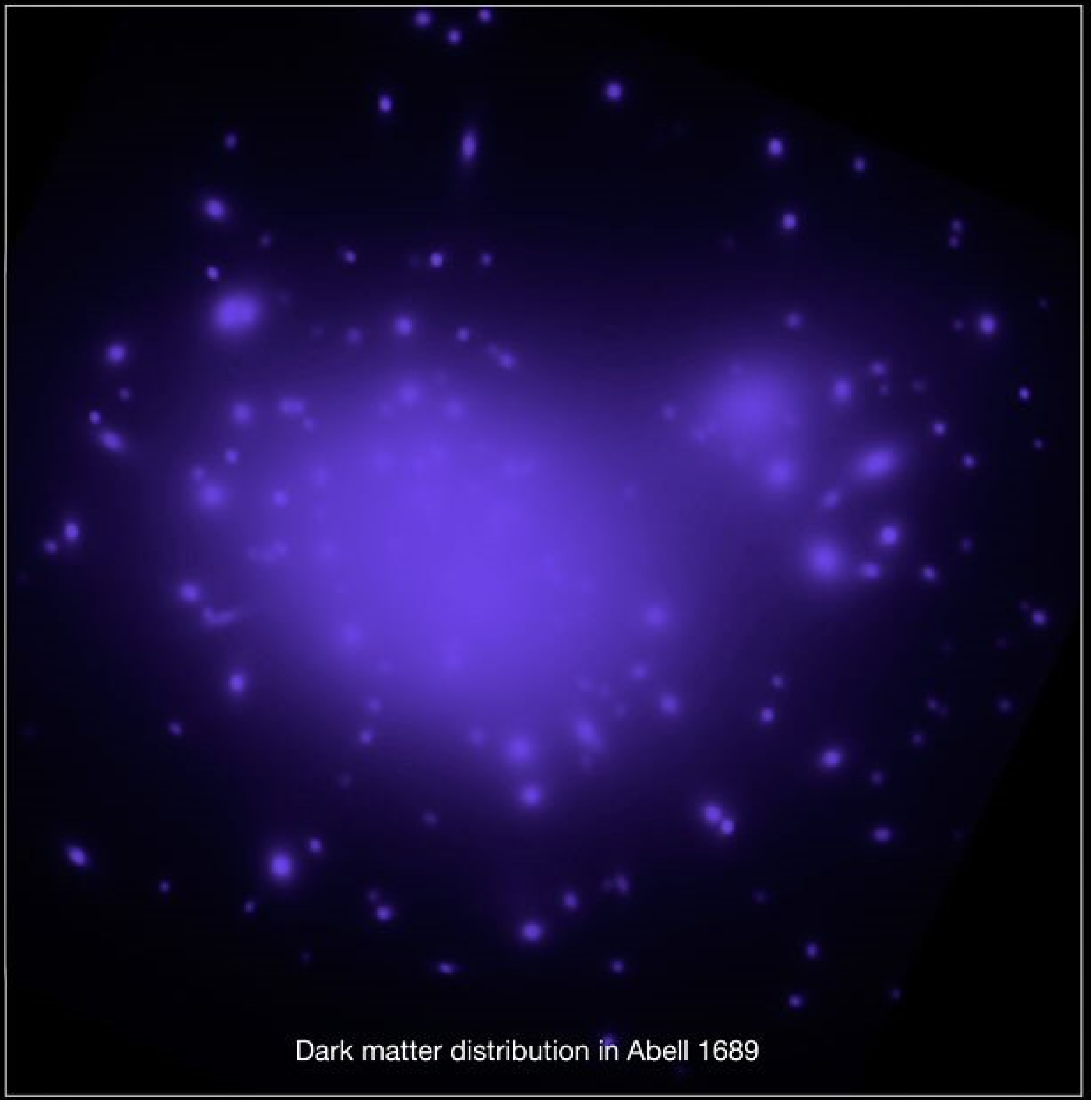
The optical velocities, $R > 3$ kpc, agree with the 21-cm observations, although the maximum rotational velocity, $V = 270 \pm 10$ km sec^{-1}, is slightly higher than that obtained from 21-cm observations.


M 31
400
300
200
100
(km s^{-1})
40
60
80
100
120
140
160
DISTANCE TO CENTER (ARC MINUTES)
Expected

A modern technique: gravitational lensing

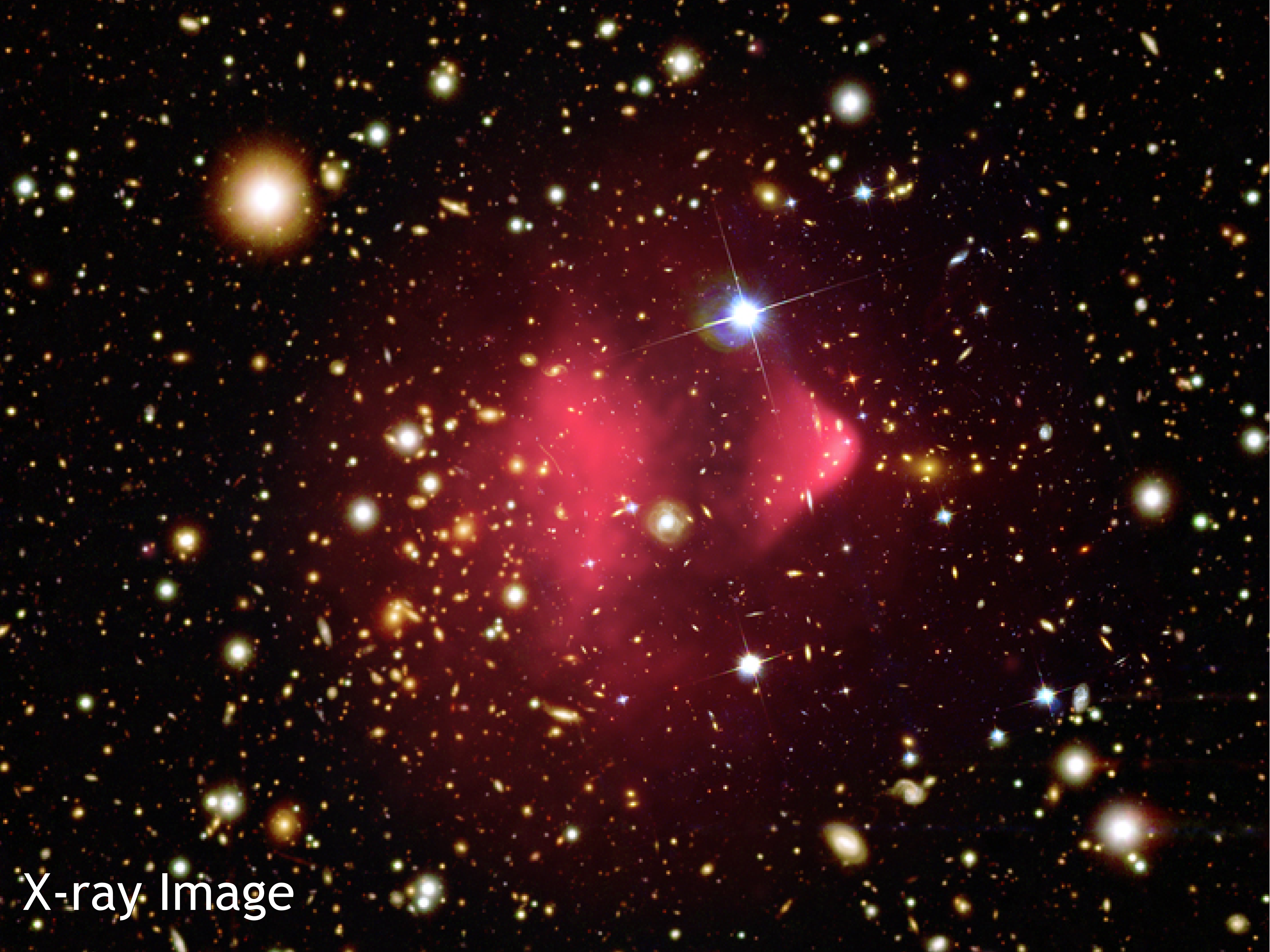


Abell 1689


Visible matter distribution in Abell 1689

Dark matter distribution in Abell 1689

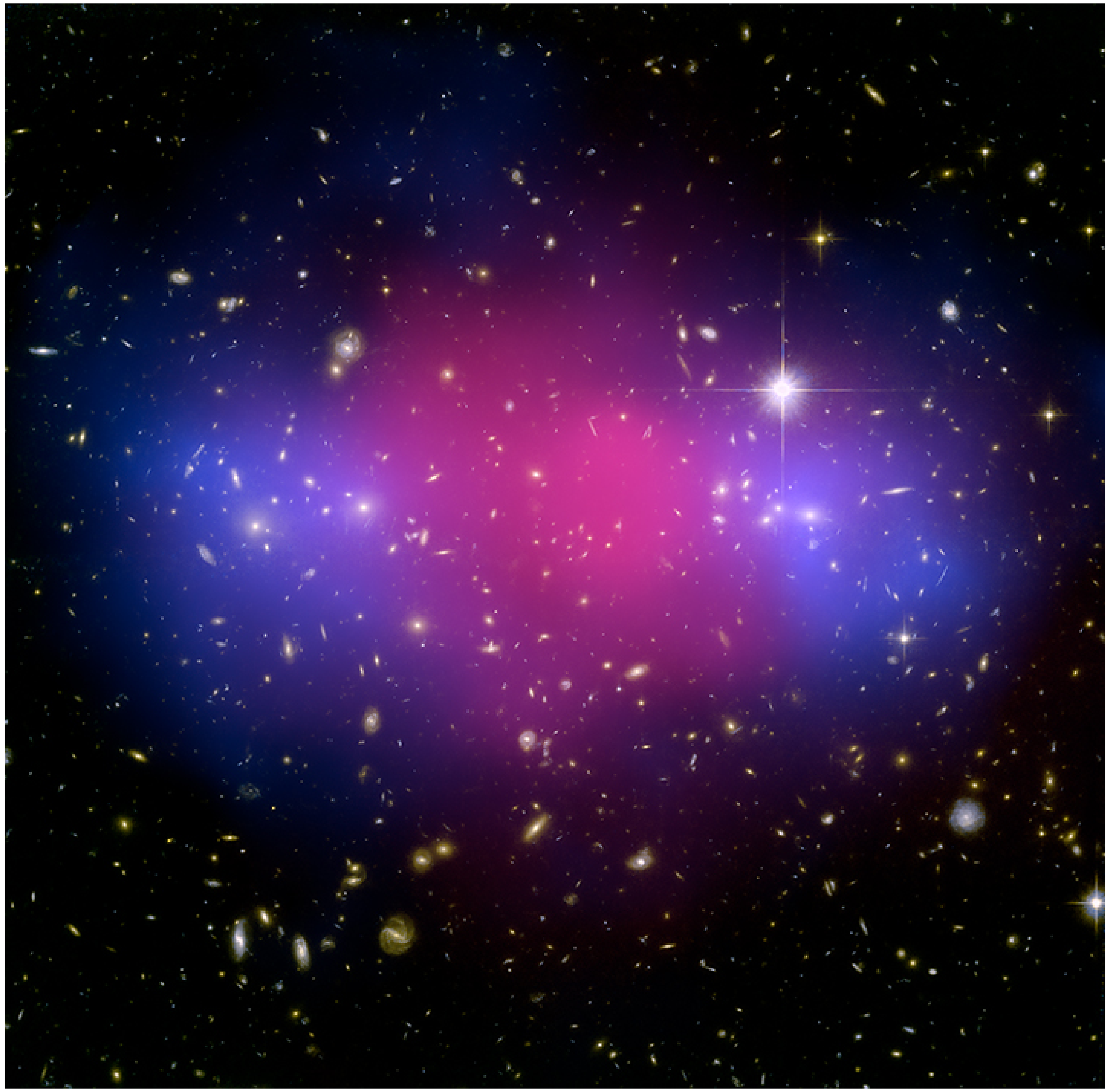
Abell 1689


"A direct empirical proof of the existence of dark matter
Clowe, *et al.*, Astrophys.J.648:L109-L113,2006.
Optical Image
Bullet Cluster (1E 0657-56)

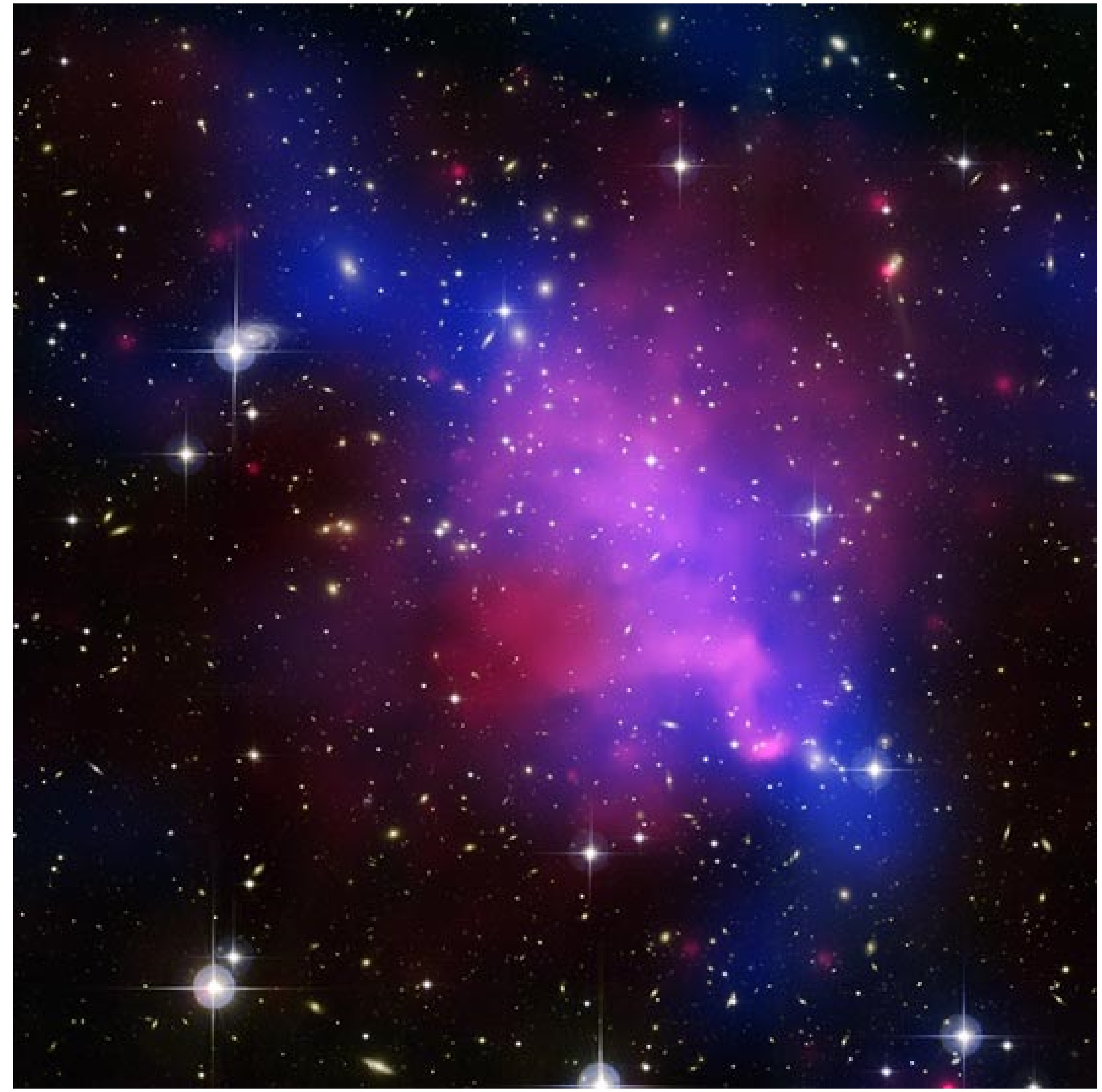

X-ray Image


Weak lensing Image


Composite Image



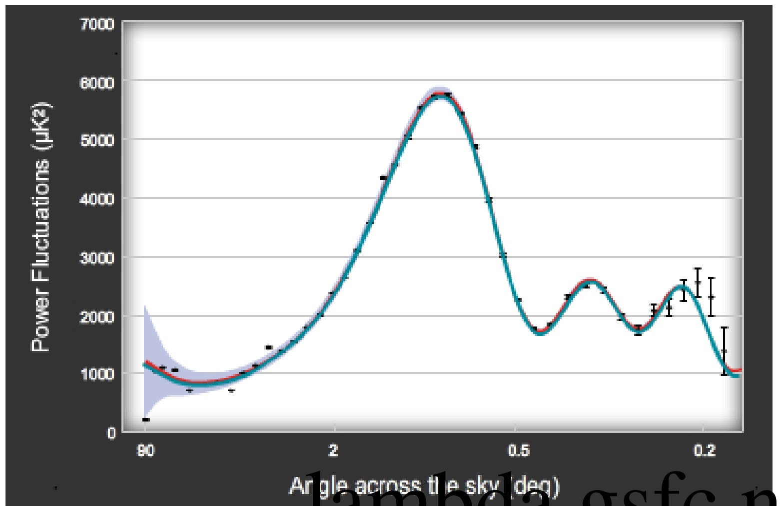
MACS J0025.4-1222

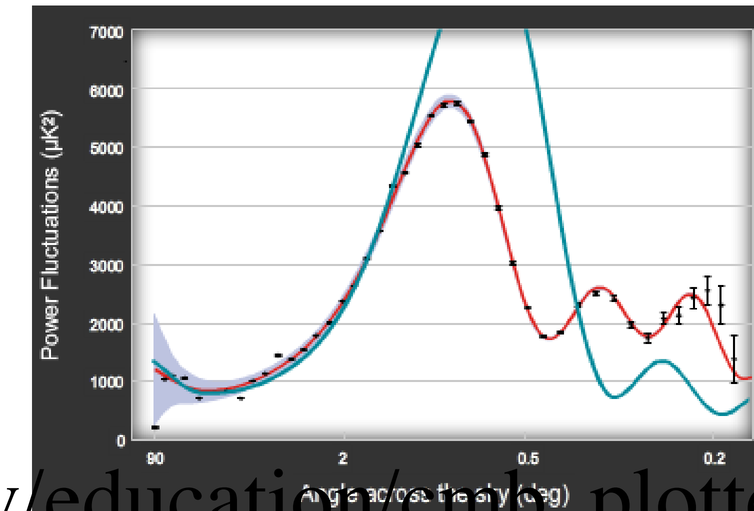


Abell 520

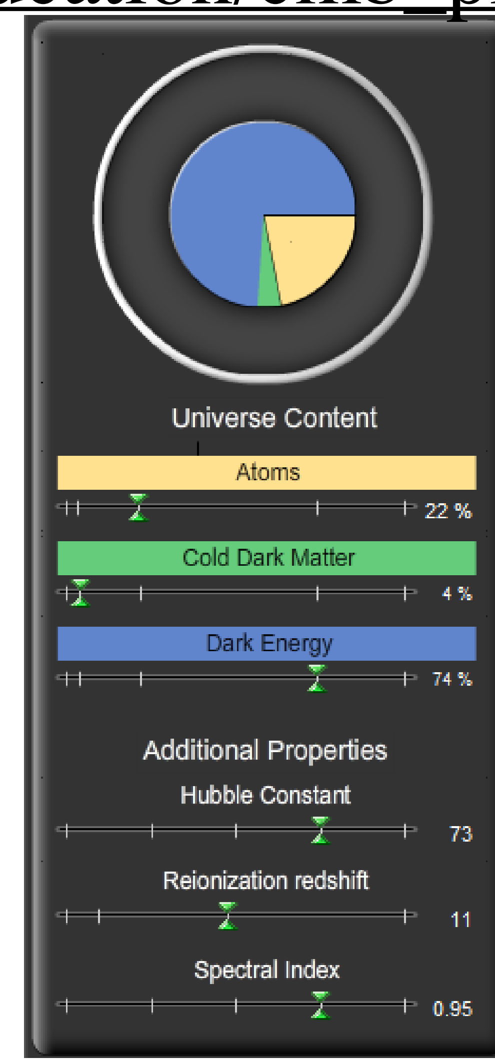
From Planck/CMB

lambda.gsfc.nasa.gov/education/cmb_plotter/





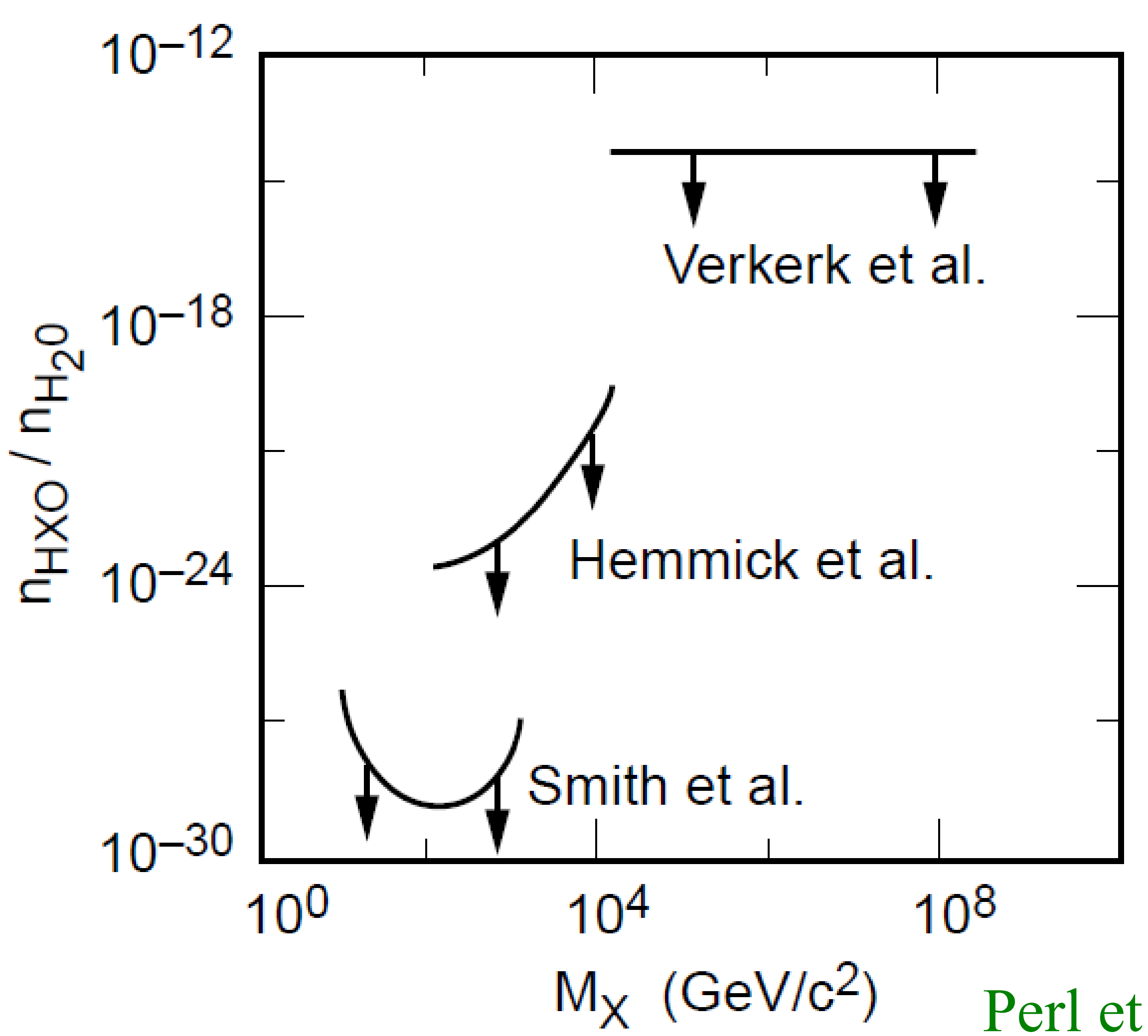
lambda.gsfc.nasa.gov/education/cmb_plotter/

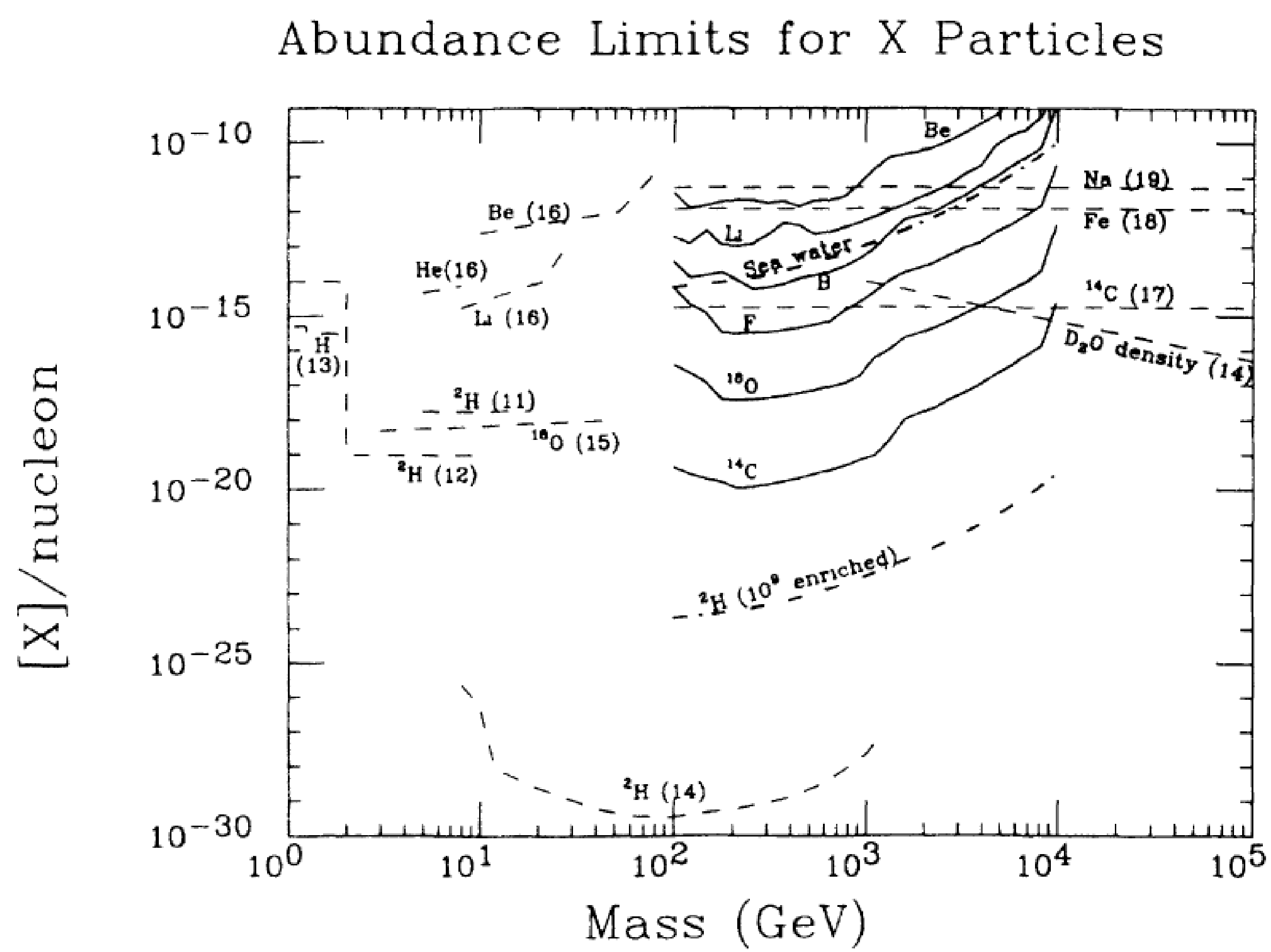


What do we know about dark matter?

1) It is dark. No electric charge.

- If it has positive charge, it can form a bound state X^+e^-, an "anomalously heavy hydrogen atom".
- If it has negative charge, it can bind to nuclei, forming "anomalously heavy isotopes".

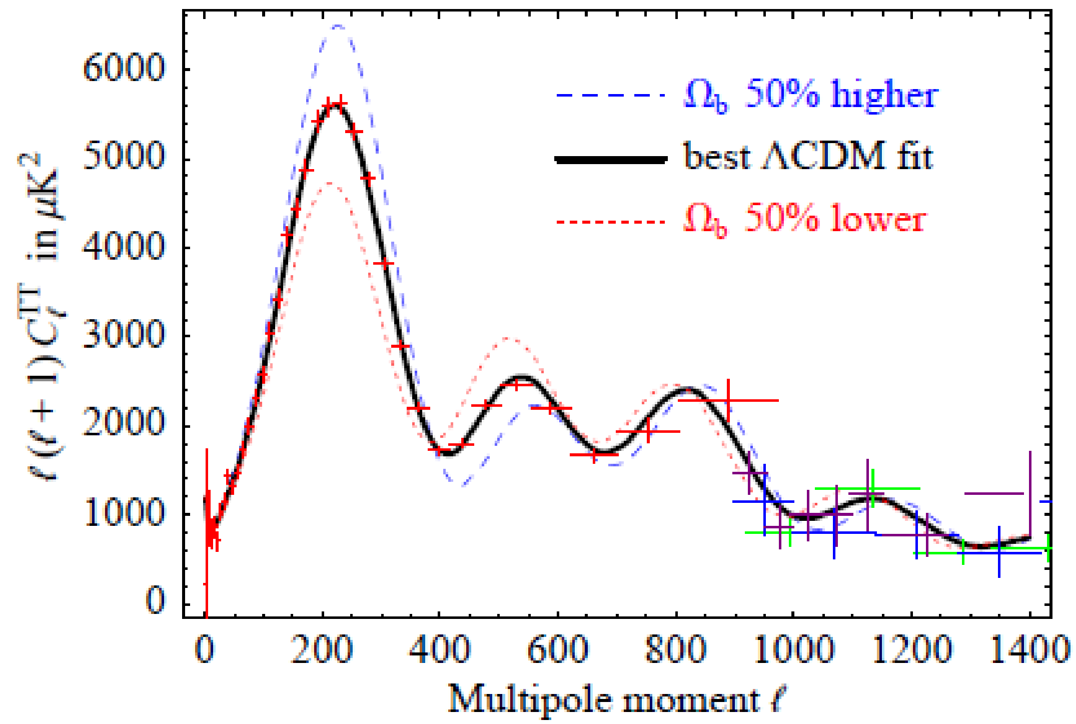




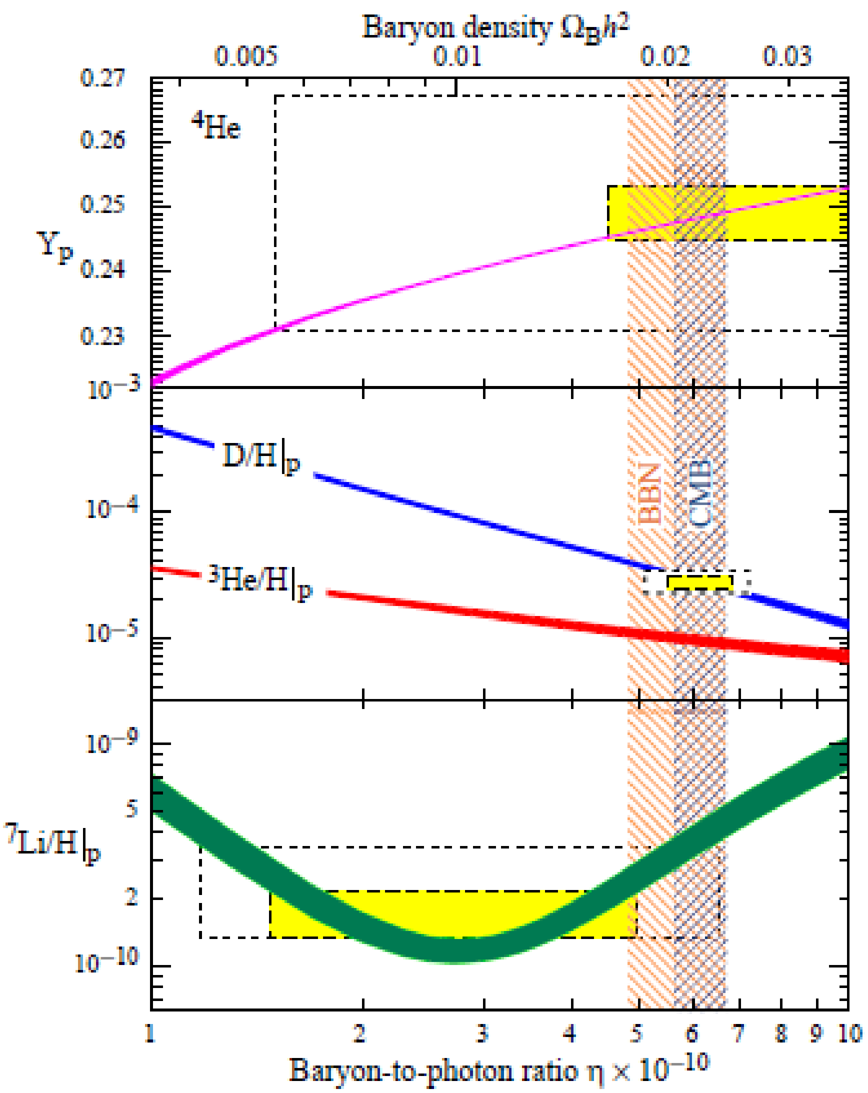
Perl et al.

2) It is not made of baryons.

Cosmic Microwave Background radiation

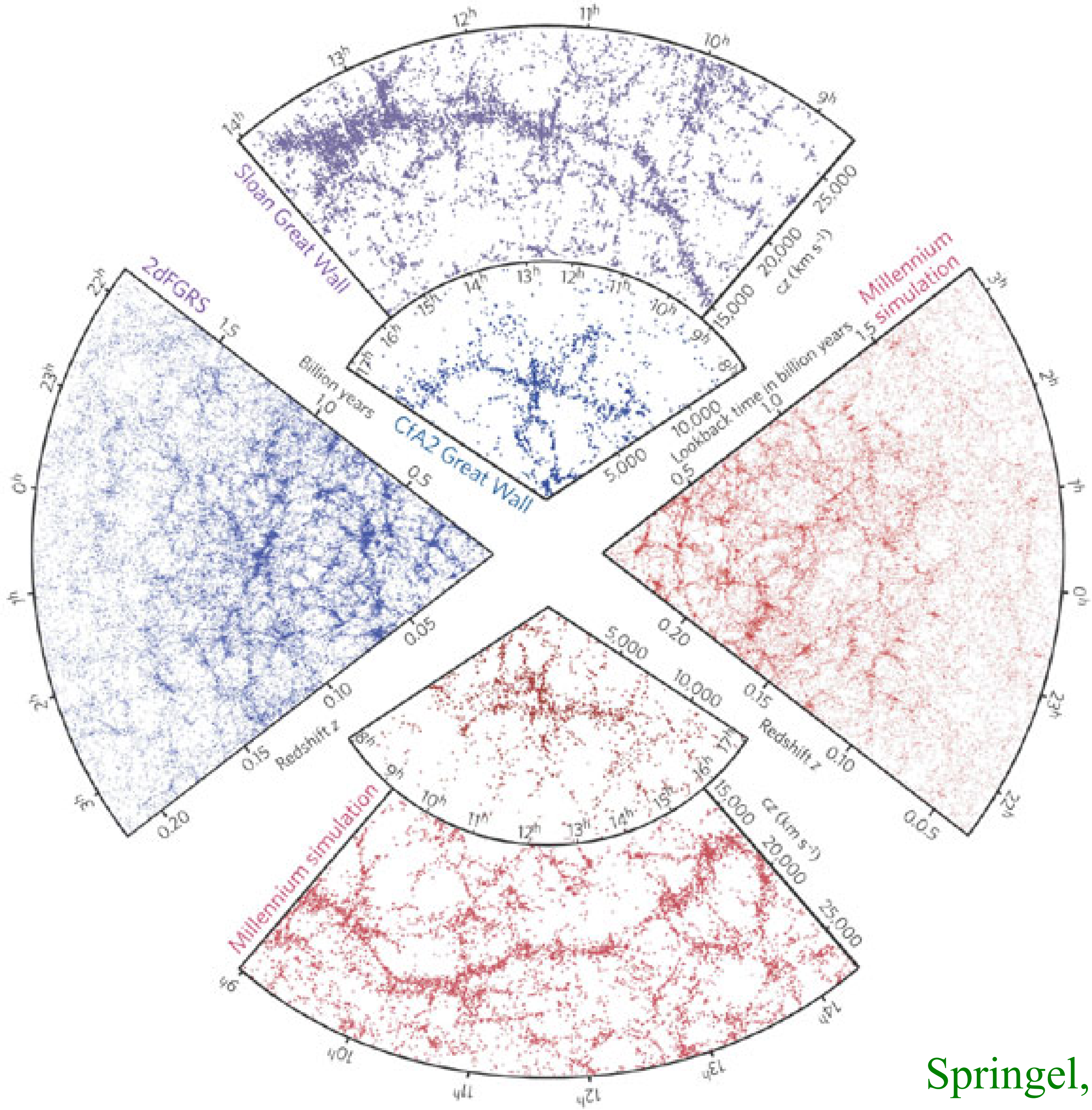


Primordial nucleosynthesis



MACHOs (planets, brown dwarfs, etc.) are excluded as the dominant component of dark matter.

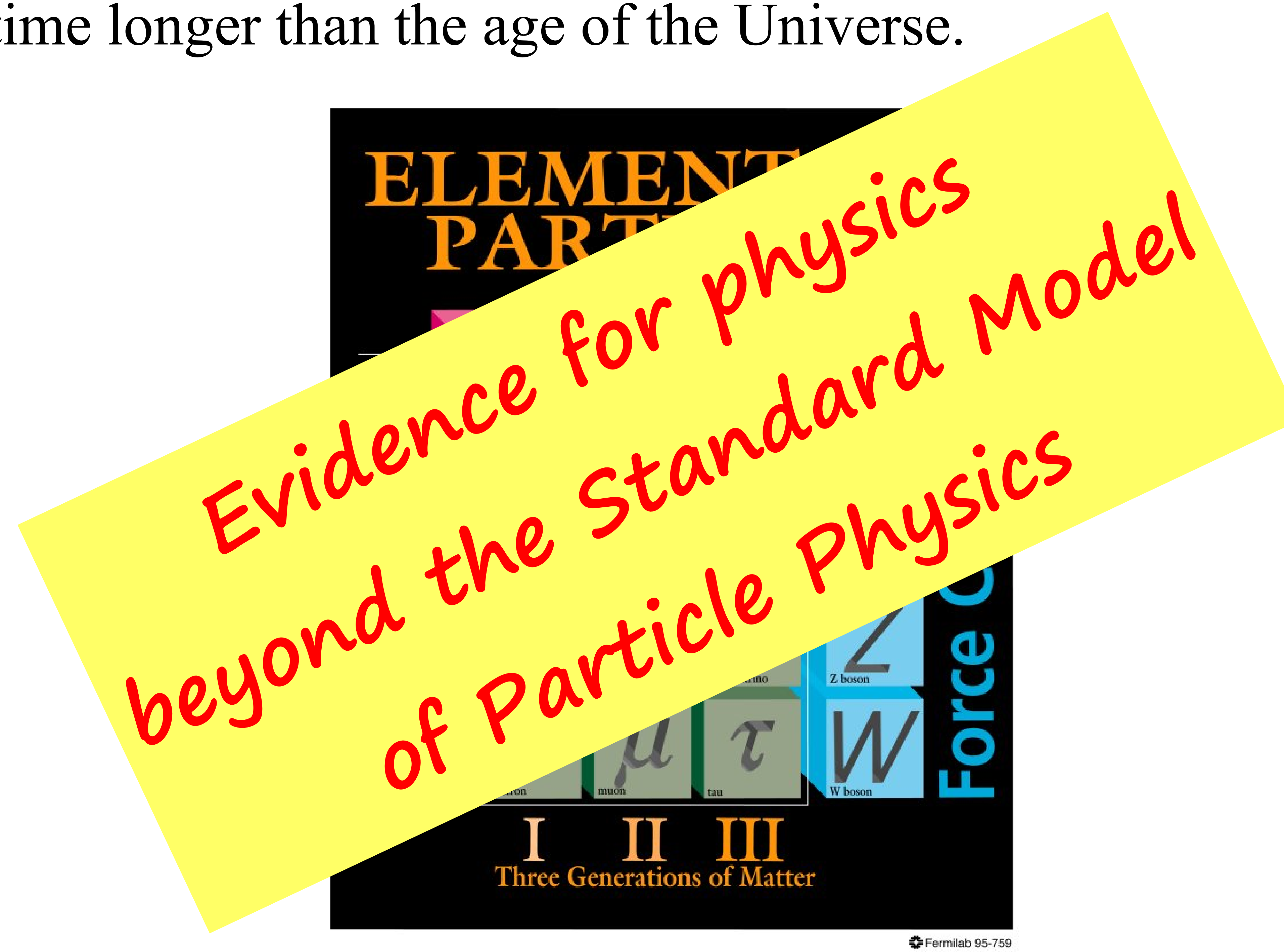
3) It was "slow" at the time of the formation of the first structures.



Springel, Frenk, White

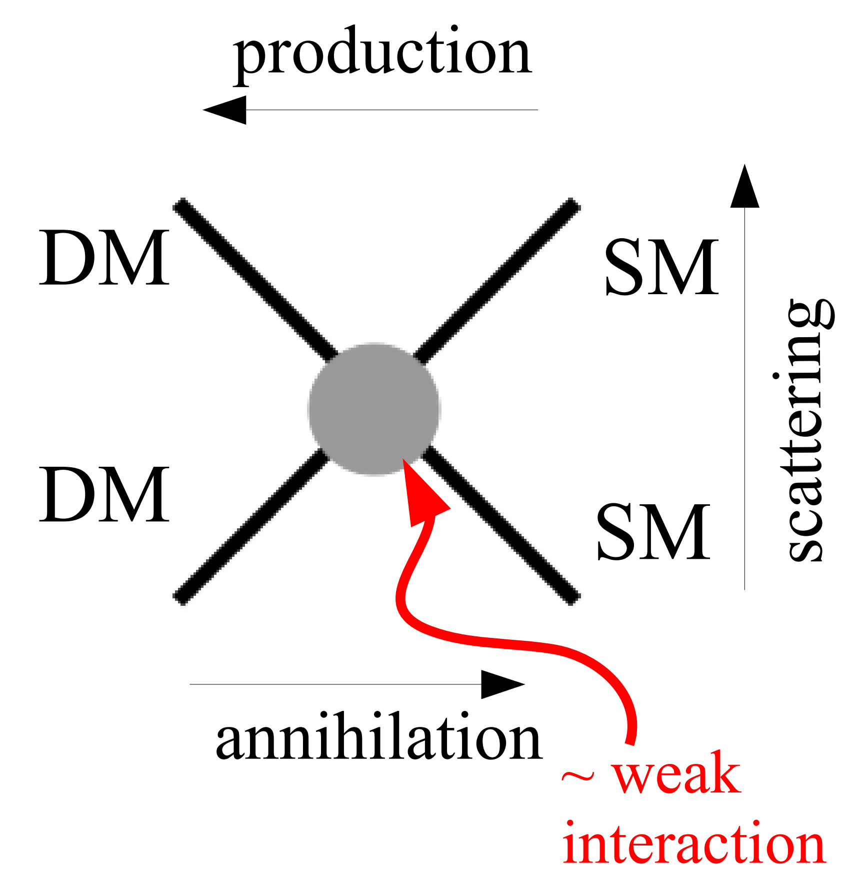
To summarize, observations indicate that the dark matter is constituted by particles which have:

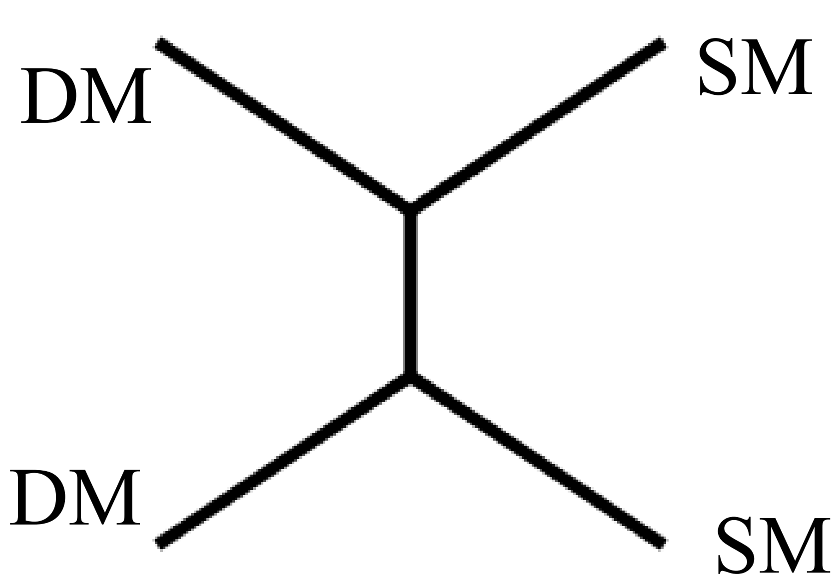
- No electric charge, no color.
- No baryon number.
- Low velocity at the time of structure formation.
- Lifetime longer than the age of the Universe.



Annihilation of DM

WIMP dark matter





Relic abundance of DM particles

$$\Omega h^2 \simeq \frac{3 \times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma v \rangle}$$

Correct relic density if

$$\langle \sigma v \rangle \simeq 3 \times 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1} = 1\,\mathrm{pb} \cdot c$$

$$\sigma \sim \frac{g^4}{m_{\mathrm{DM}}^2} = 1\,\mathrm{pb}$$

$$m_{\mathrm{DM}} \sim 10\,\mathrm{GeV} - 1\,\mathrm{TeV}$$

(provided $g \sim g_{\mathrm{weak}} \sim 0.1$)

Notes

Sean Carroll: Lecture Notes on GR

Baumann cosmology course

Ibarra lectures on Dark Matter @ Cargese 2014

Moriond Talks:

Rocchi'16: 1st observation of Grav. Waves

Nagar'16: th. predictions of merger GW signals

Saviano'15: neutrinos in cosmology (N_eff)

Billard'15: neutrino bkgd for DM DD

Henrot-Versillé'15: Planck results

Kusenko'15: baryogenesis alternative

Branchina'15: EW stability

Salvio'15: scales & inflation

LUX'14: DM best limits

Hamann'14: nice inflation course

Perdereau'14: good intro

Perdereau onBICEP'14: polarisation