Stochastic Model of Breakdown Nucleation Under Intense Electric Fields

Yinon Ashkenazy   Michael Assaf   Eli Engelberg

Hebrew University of Jerusalem

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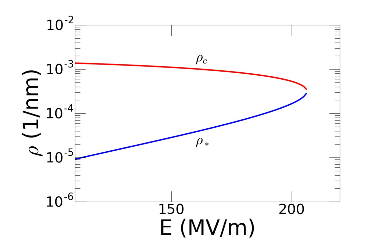
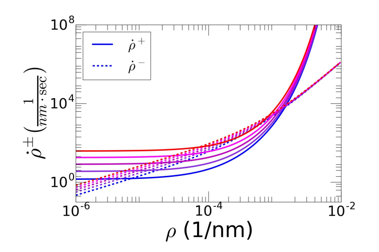
Rates

$$\dot{\rho}^+ = \frac{25\kappa C_t}{G^2 b}(\rho + c)\sigma^2 e^{-\frac{E_a - \Omega\sigma}{k_B T}}$$

$$\dot{\rho}^- = \frac{50\xi C_t}{G}\sigma\rho(c + \rho)$$





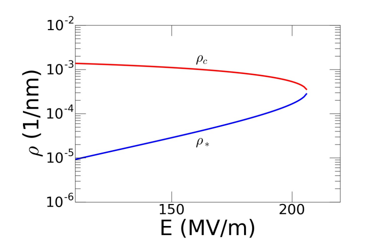
eli.engelberg@mail.huji.ac.il

Rates

$$\dot{\rho}^{+} = B_1(\rho + c)\sigma^2 e^{\alpha\sigma}$$

$$\dot{\rho}^{-} = b_2\sigma\rho(\rho + c)$$

$$\sigma = A_1 + a_2\rho$$



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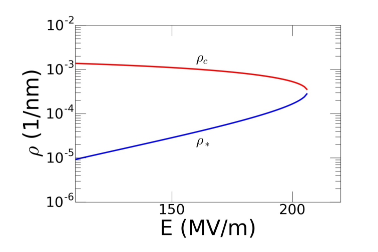

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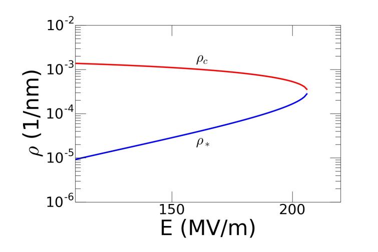

$$\rho_* = \frac{B_1 A_1}{b_2} e^{\alpha A_1}$$

$$\rho_c = \frac{1}{\alpha a_2} \ln \frac{b_2}{B_1 a_2}$$

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Conversion to Steps

$$n \equiv \frac{\rho}{\Delta\rho}$$

$$n_* \equiv \frac{\rho_*}{\Delta\rho}, \quad n_c \equiv \frac{\rho_c}{\Delta\rho}$$

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Conversion to Steps

$$\lambda(n) = B_1(n + n_c C)\sigma^2 e^{\alpha\sigma}$$

$$\mu(n) = B_2 \frac{n}{n_c}(n + n_c C)\sigma$$

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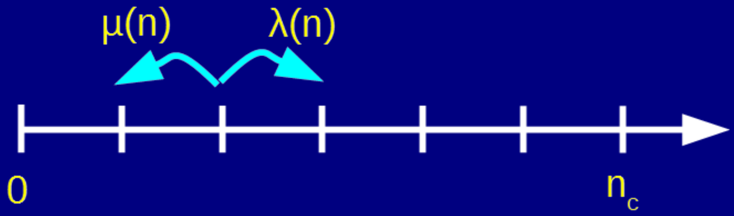

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Solution Methods

- Exact
- Metastable approximation
- Simulation
- Experiment

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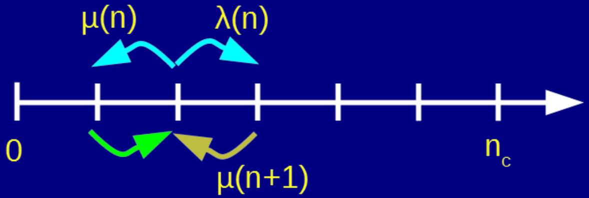
Exact Solution: Master Equation

$$\frac{dP(n,t)}{dt} = \lambda(n-1)P(n-1,t) + \mu(n+1)P(n+1,t) - [\lambda(n)+\mu(n)]P(n,t)$$



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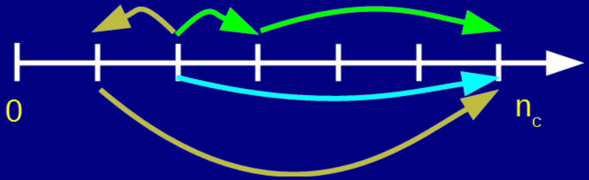
Exact Solution: Mean First Passage Time

$$T(n) = \lambda(n)\Delta t \cdot T(n+1) + \mu(n)\Delta t \cdot T(n-1) + [1 - \lambda(n)\Delta t - \mu(n)\Delta t]\, T(n) + \Delta t$$



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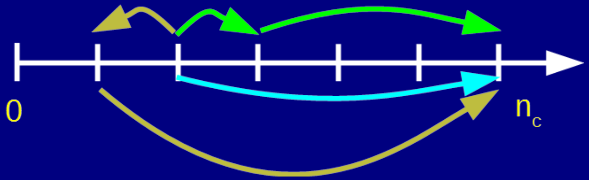

$$\lambda(n)[T(n+1) - T(n)] + \mu(n)[T(n-1) - T(n)] = -1$$

Exact Solution: MBT

$$\tau = \phi(n_c)\left(\sum_{n=0}^{n_c}\frac{1}{\lambda(n)\phi(n)}\right)$$

$$\phi(n) = \prod_{m=1}^{n}\frac{\mu(m)}{\lambda(m)}$$

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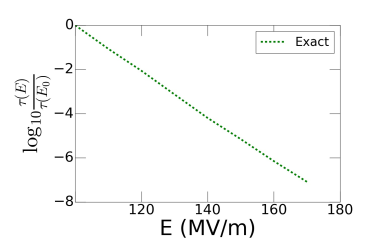

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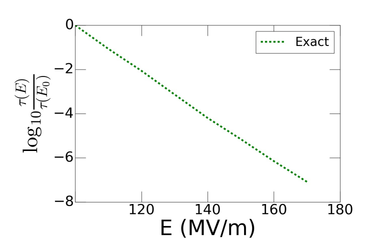

Approximation: The state $n = n_c$ is fully absorbing.

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Metastable Approximation

- Exact
- Metastable approximation
- Simulation
- Experiment

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Metastable Approximation: QSD

The metastability construct: $P(n,t) = \pi(n)e^{-\frac{t}{\tau}}$

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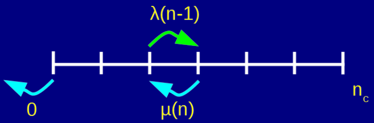

$$\lambda(n-1)\pi(n-1) = \mu(n)\pi(n)$$

$$\Rightarrow \pi(n) = \pi(0)\prod_{n=1}^{n}\frac{\lambda(n-1)}{\mu(n)}$$

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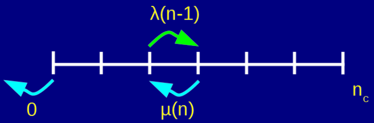

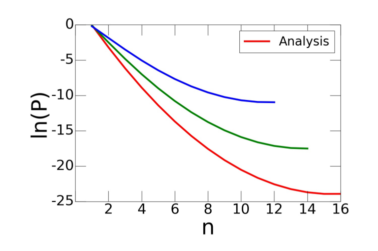

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$$\tau = Ae^{-n_c \Delta S}$$

$$\Delta S = \alpha A_1 (1 + \frac{1}{2\eta}) + (\eta + 1) \ln(1 + \frac{1}{\eta}) + \ln \frac{A_1 B_1}{B_2}$$

$$A = \sqrt{\frac{2\pi}{n_c}} \frac{e^{-\alpha A_1 (1 + \frac{1}{2\eta})}}{A_1^2 B_1 C} \left(1 + \frac{1}{\eta}\right)^{-\frac{1}{2}}$$

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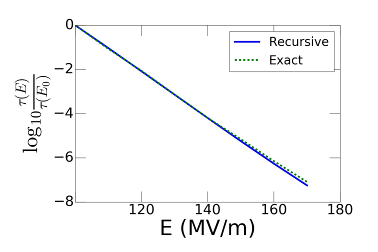

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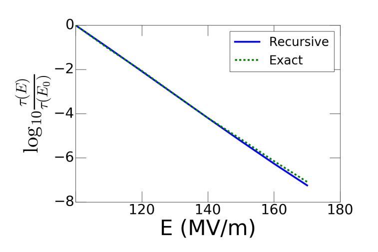

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Simulation

- Get random number.
- Calculate time spent in step (exponential CDF).
- Get random number.
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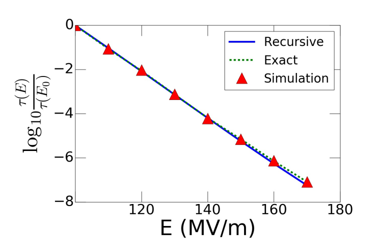

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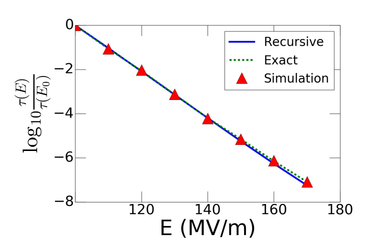

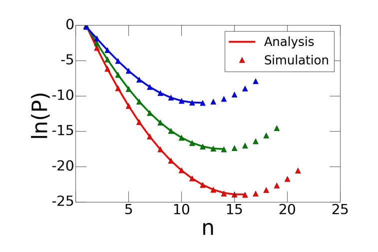

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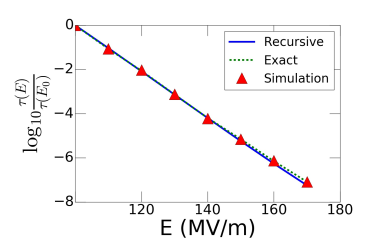

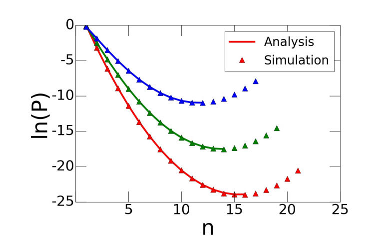

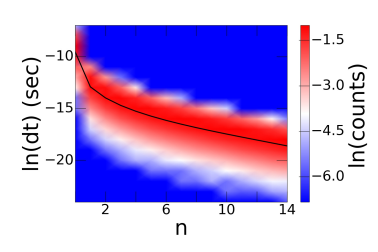

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Choice of Parameters

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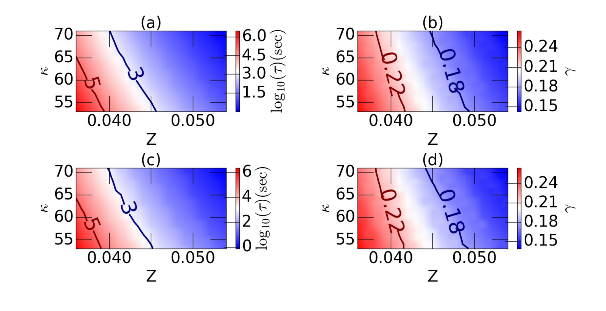

A. Descoeudres, T. Ramsvik, S. Calatroni, M. Taborelli, and W. Wuensch, PRST-AB 12, 032001 (2009)

K. Nordlund and F. Djurabekova, PRST-AB 15, 071002 (2012)

Experiment

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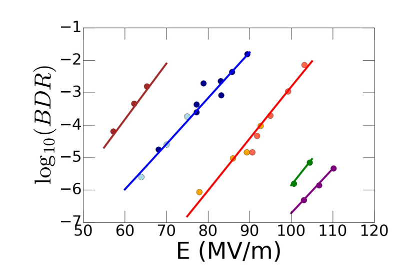
Experiment: Dependence on Field



A. Grudiev, S. Calatroni, and W. Wuensch, PRST-AB 12, 102001 (2009)

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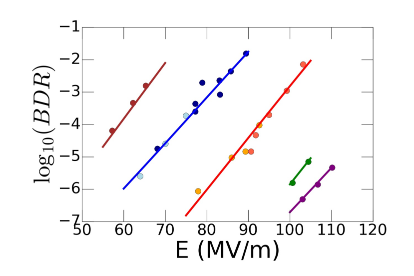

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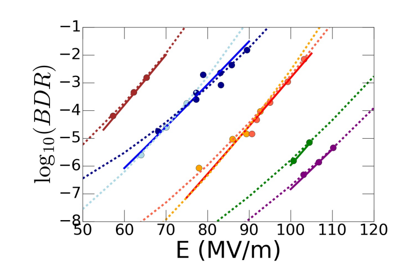

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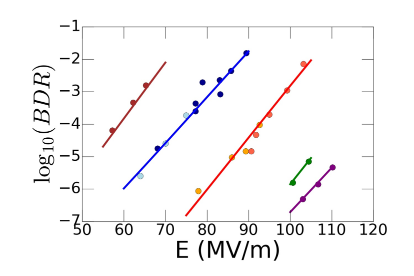

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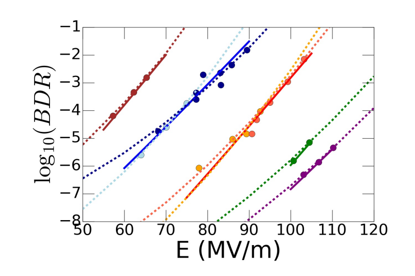
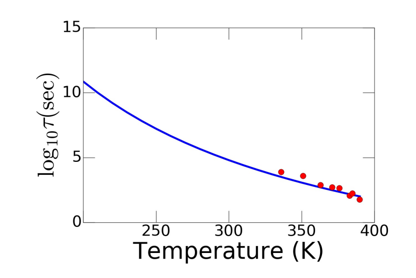


Good fits, consistent with the previously proposed power law!

$$\tau \sim E^{\nu}, \quad 25 < \nu < 30$$

Experiment: Dependence on Temperature



Data from K. Nordlund and F. Djurabekova, PRST-AB 14, 071002 (2012)

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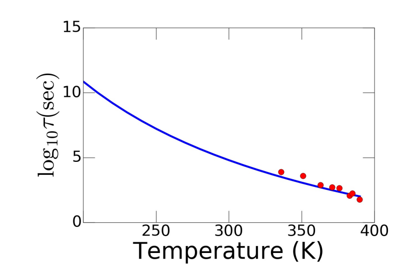

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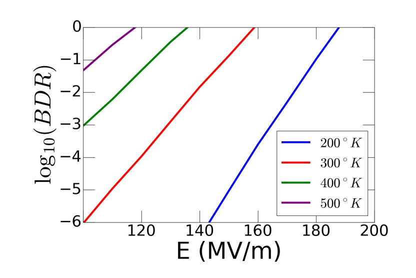

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Conclusions

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- Simple model with a critical transition.
- Stochastic, rather than deterministic, mechanism for BD nucleation.
- Exact analysis, metastable approximation and simulation all agree
- and give good fits to experimental results.
- Can be used to predict BDRs in future configurations.

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