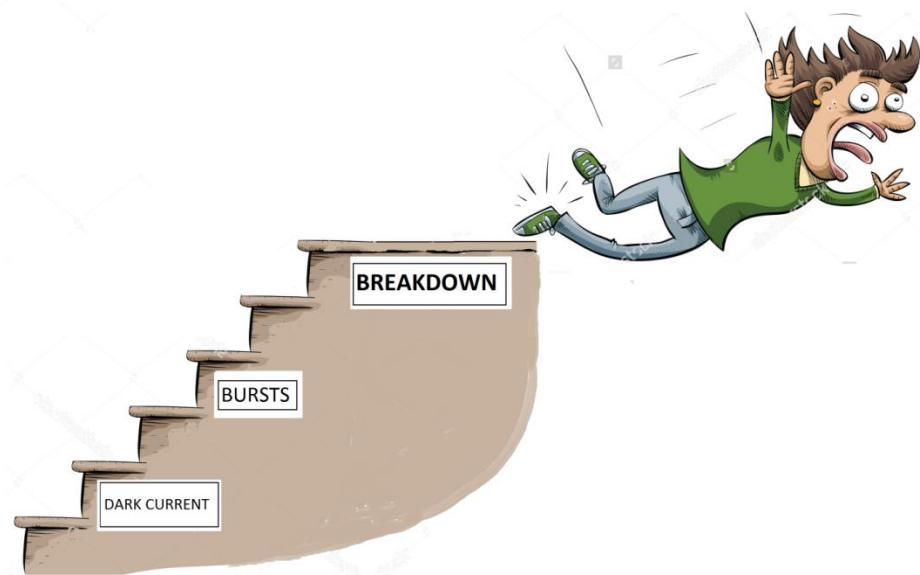


B.I.R.D. model

Breakdown Induced by Rupture of Dielectric layer



This is a three-step process



DARK CURRENT



CONSORZIO RFX
Ricerca Formazione Innovazione

Electron Emission in Intense Electric Fields.

By R. H. FOWLER, F.R.S., and Dr. L. NORDHEIM.

(Received March 31, 1928.)

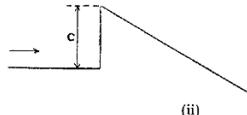
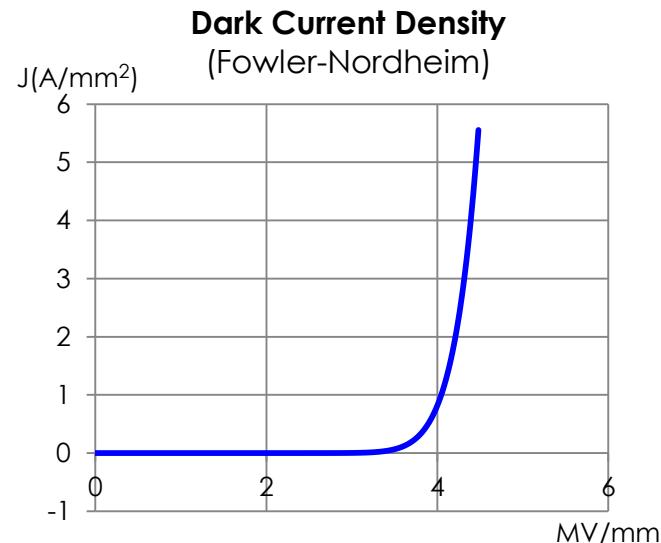


FIG. 1.

$$k_1 = \frac{a}{\Phi} \quad a = \frac{e^3}{8\pi h} = 1.54 \cdot 10^{-6} \frac{A \text{ eV}}{V^2}$$

$$J = k_1 E_0^2 e^{-\frac{k_2}{E_0}}$$

$$k_2 = b \Phi^{\frac{3}{2}} \quad b = \frac{4}{3e} \sqrt{\frac{8\pi^2 m}{h^2}} = 6.82 \cdot 10^9 \frac{V}{m(eV)^{\frac{3}{2}}}$$

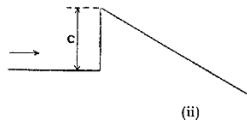


DARK CURRENT



CONSORZIO RFX
Ricerca Formazione Innovazione

Fowler-Nordheim



$$k_1 = \frac{a}{\Phi}$$

$$a = \frac{e^3}{8\pi h} = 1.54 \cdot 10^{-6} \frac{A \text{ eV}}{V^2}$$

$$J = k_1 E_0^2 e^{-\frac{k_2}{E_0}}$$

$$k_2 = b \Phi^{\frac{3}{2}}$$

$$b = \frac{4}{3e} \sqrt{\frac{8\pi^2 m}{h^2}} = 6.82 \cdot 10^9 \frac{V}{m(eV)^{\frac{3}{2}}}$$

Local field-enhancement factor β

$$E_0 \rightarrow \beta E_0$$

$$I = A k_1 (\beta E_0)^2 e^{-\frac{k_2}{\beta E_0}}$$

$$\beta \approx 10^2 \div 10^3$$

$$A \approx 10^{-16} \div 10^{-12} \text{ m}^2$$

Fowler-Nordheim

Geometrical amplification factor β



Fowler-Nordheim

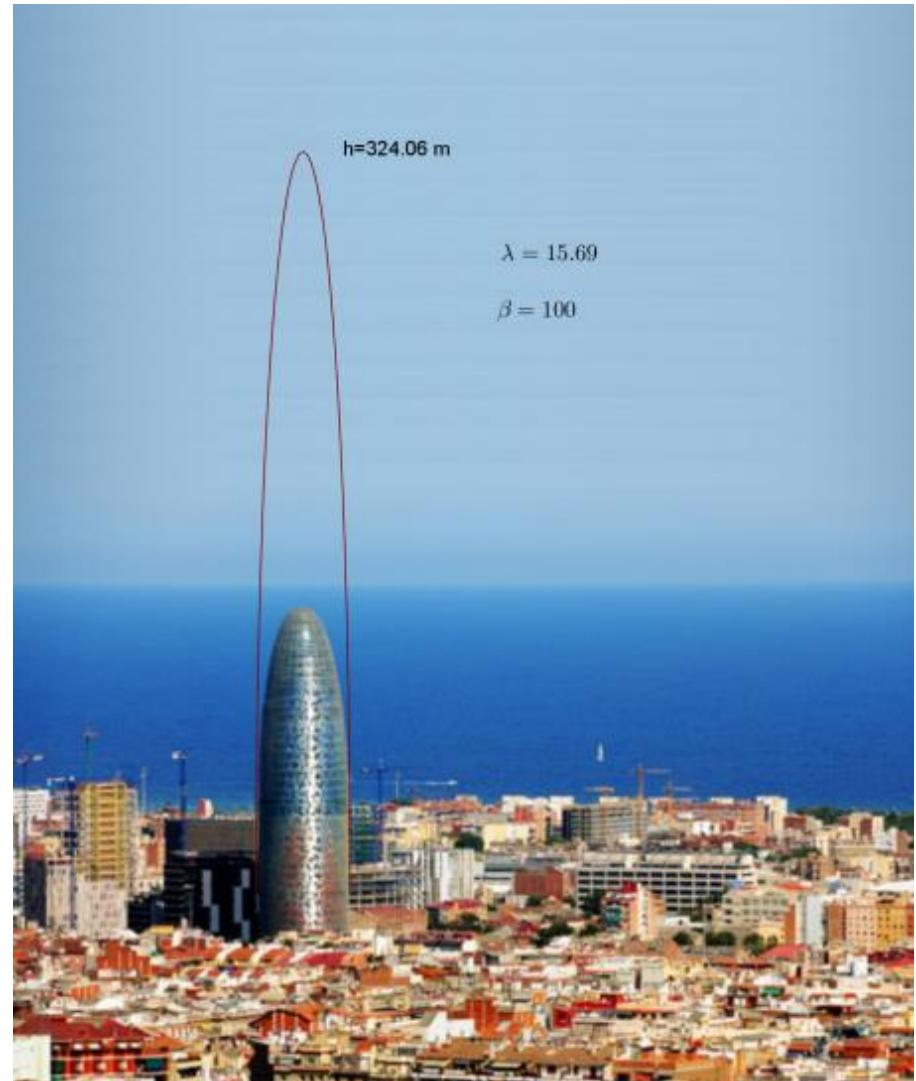
Geometrical amplification factor β



Fowler-Nordheim

Geometrical amplification factor β

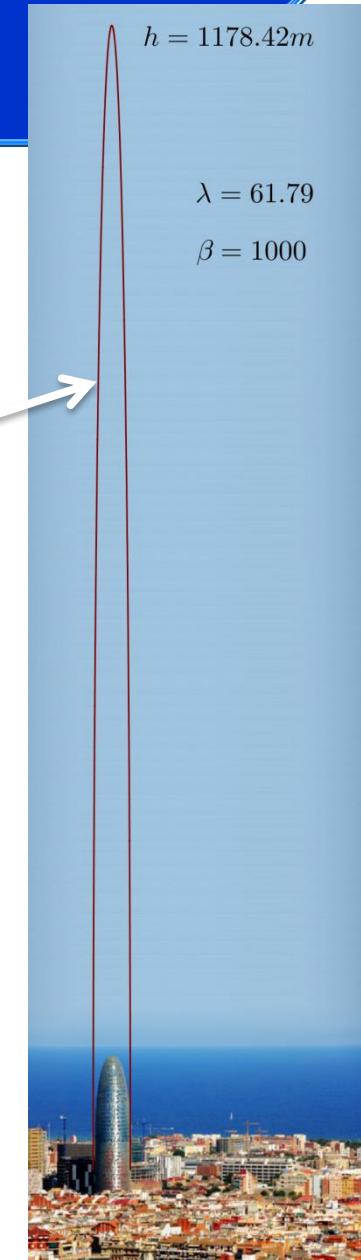
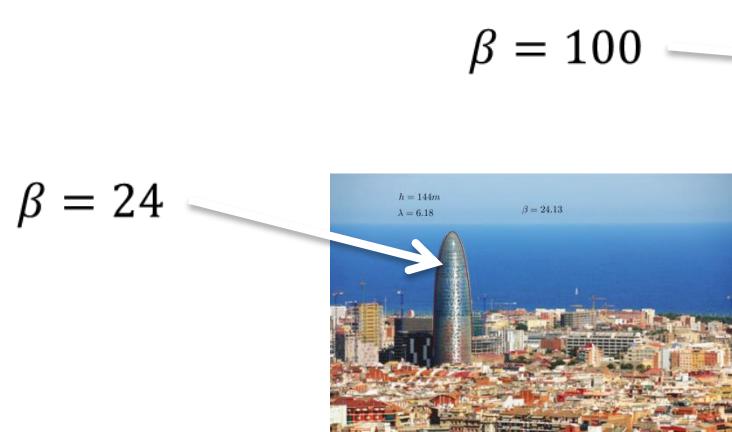
$$\beta = 24$$



DARK CURRENT

Fowler-Nordheim

Geometrical amplification factor β



Fowler-Nordheim

Geometrical amplification factor β

$$\beta = 4$$



DARK CURRENT



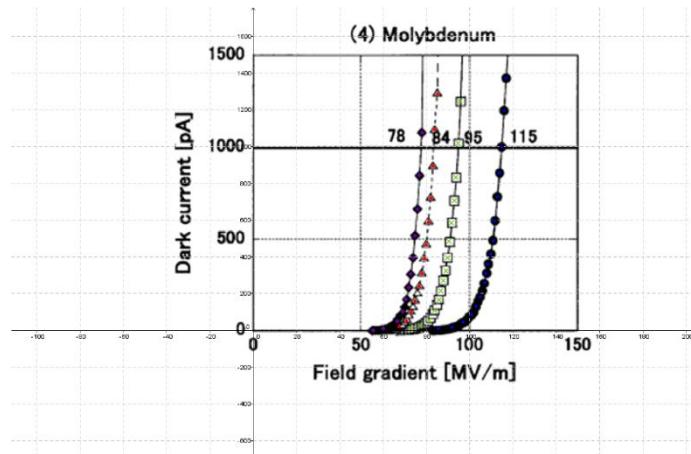
CONSORZIO RFX
Ricerca Formazione Innovazione

Fowler-Nordheim + geometrical amplification

$$E_0 \rightarrow \beta E_0$$

$$I = A k_1 (\beta E_0)^{-\frac{k_2}{\beta E_0}}$$

$h(mm)$	β	$A(m^2)$
0.50	46	4.0E-17
0.75	52	1.2E-16
1.00	58	1.9E-16
1.25	62	1.9E-16



F. Furuta et al. / Nuclear Instruments and Methods in Physics Research A 538 (2005) 33–44

DARK CURRENT



Fowler-Nordheim + different amplification mechanisms

$$I = A k_1 E_0^2 e^{-\frac{k_2}{E_0}}$$

$$k_1 = \frac{a}{\Phi}$$

$$k_2 = b\Phi^{\frac{3}{2}}$$

$$k_1 \rightarrow f_1(V)k_1 \quad \Phi \rightarrow \frac{\Phi}{f_2(V)} \quad I = A k_1 f_1(V) f_2(V) E_0^2 e^{-\frac{k_2}{f_2(V)E_0}}$$

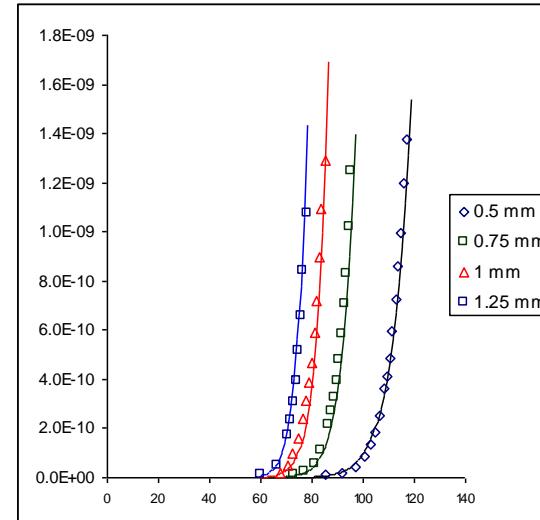
$$f_2(V) = 1 \quad \lim_{V \rightarrow 0} f_1(V) = 1$$

$$f_1(V) = 1 + \alpha V^\gamma$$

γ	$\alpha(kV^{-\gamma})$
7.4	7.2E-12

	F.N.	Exper.
$k_1 (\text{A/V}^2)$	3.31E-07	2.50E-19
$k_2 (\text{V/m})$	6.85E+10	7.45E+08

$$A = 5.20 \text{ mm}^2$$



Fowler-Nordheim law

- Fits well all experimental data (for given β and A)
- Too high β -value and (often) unreliable emitting surfaces
- Dependence of β and A from the electrode gap

Fowler-Nordheim-like law

- Different amplification mechanism must be considered
- k_1 and k_2 differ from F.N. constants

- Breakdown is usually interrupted by externally circuitry
- Breakdown changes the microscopic surfaces
- Breakdown Voltage is a function of distance

TOTAL VOLTAGE EFFECT $V_{BD} \sim h^\alpha$

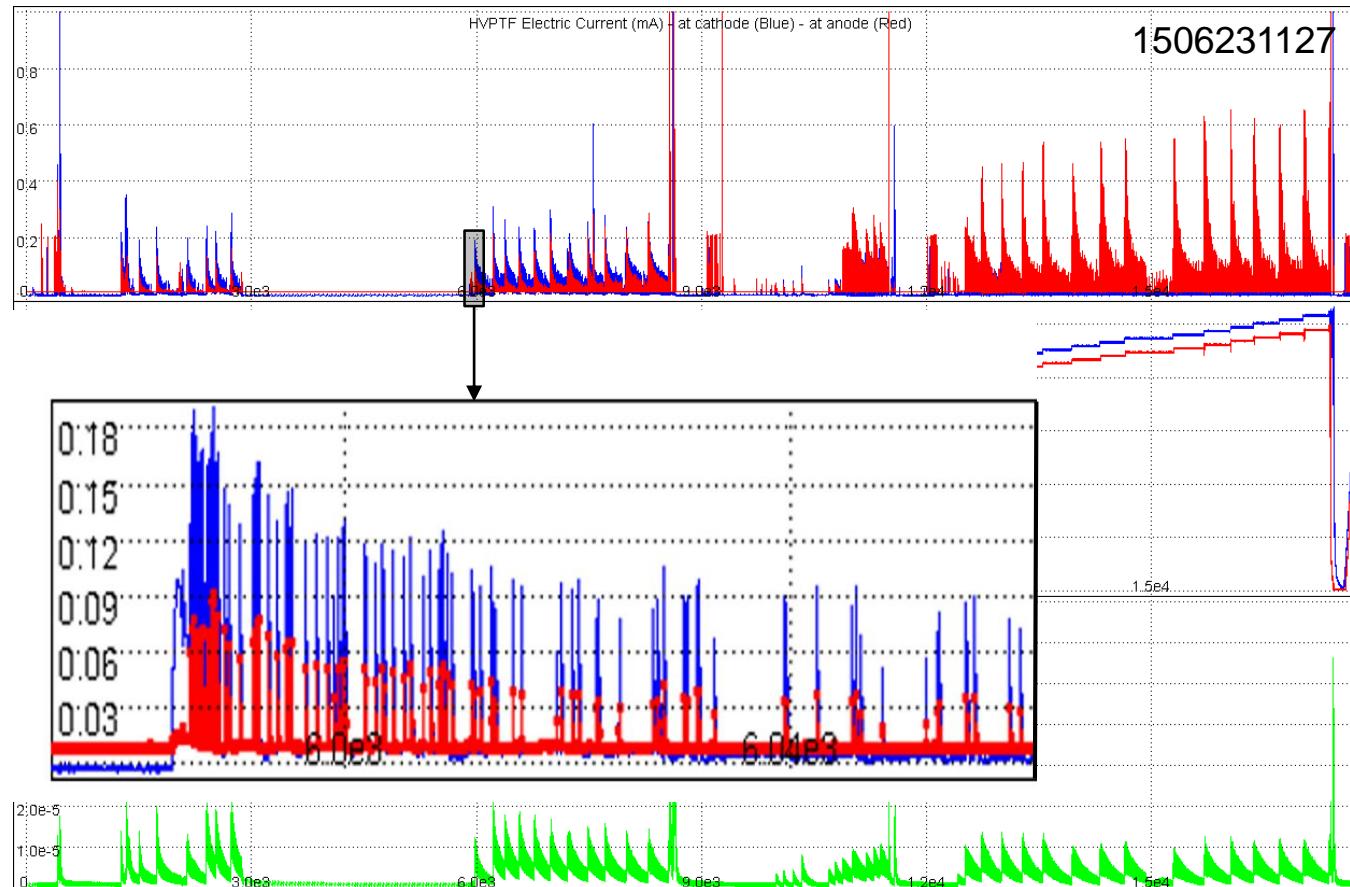
Micro-Current (BURSTS)



CONSORZIO RFX
Ricerca Formazione Innovazione

High Voltage Holding

CURRENT-CONDITIONING



Sphere-Plane configuration – gap=30mm

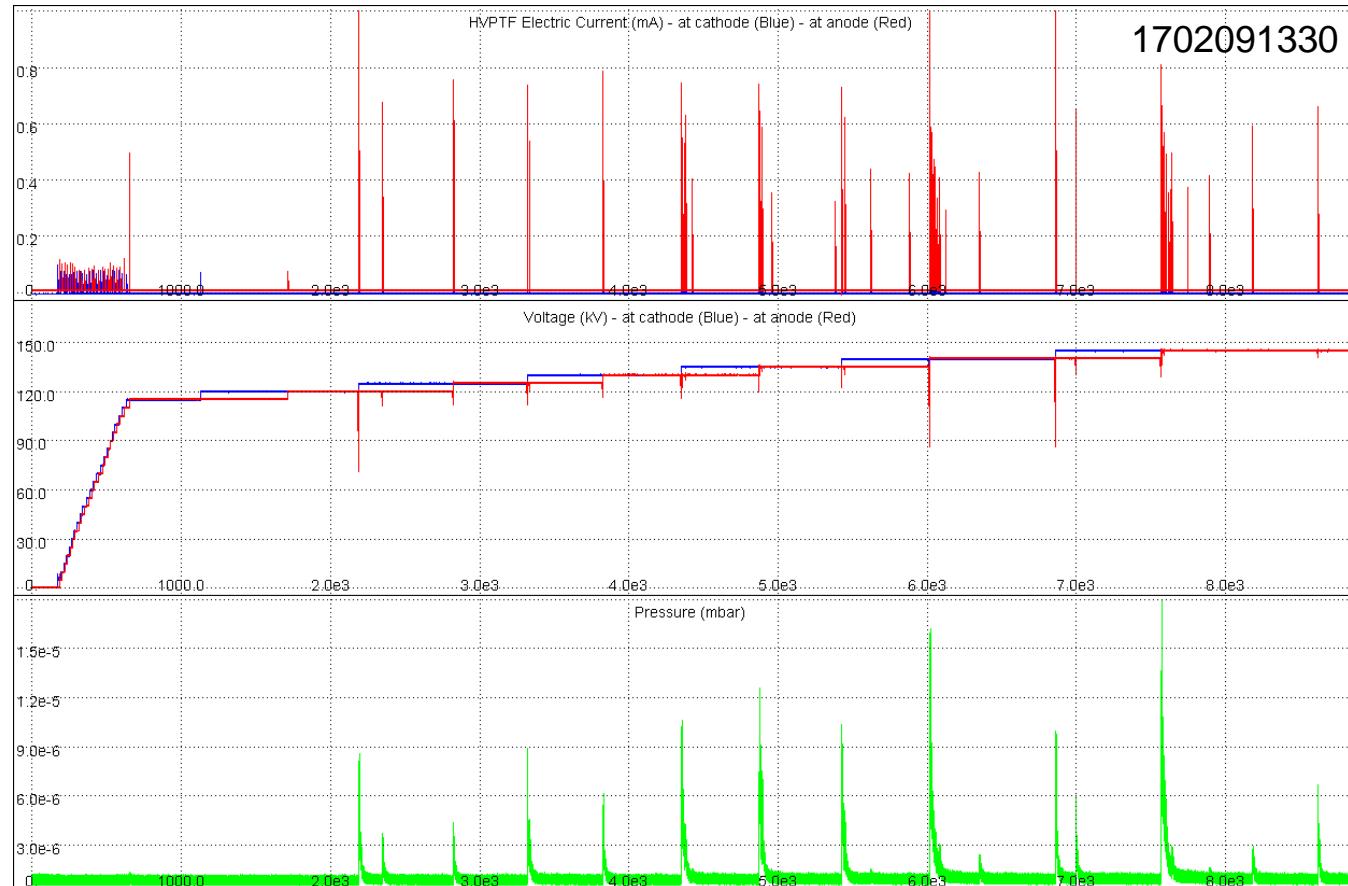
Micro-Current (BURSTS)



CONSORZIO RFX
Ricerca Formazione Innovazione

High Voltage Holding

CURRENT-CONDITIONING (cleaned electrodes)



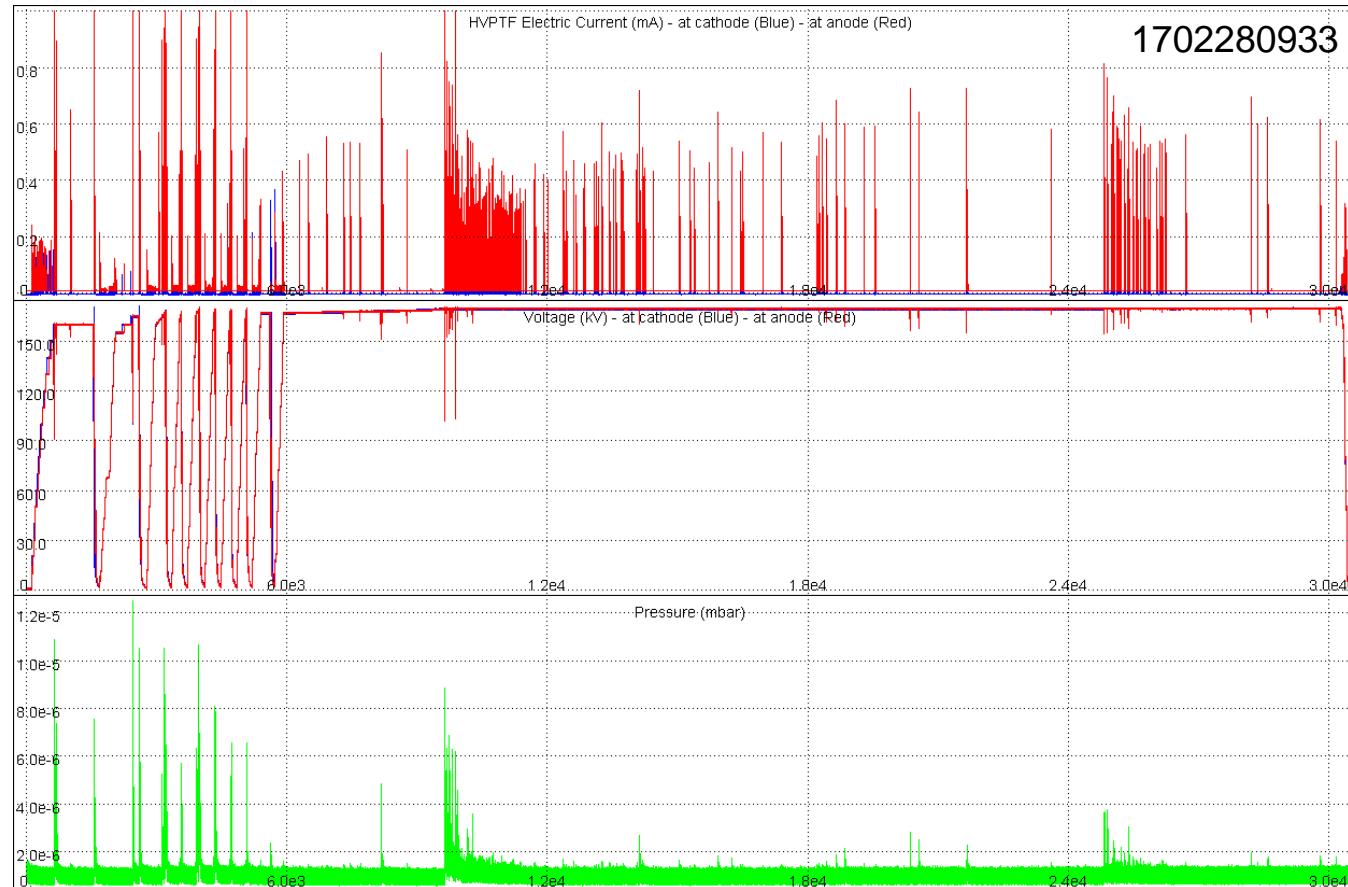
Micro-Current (BURSTS)



CONSORZIO RFX
Ricerca Formazione Innovazione

High Voltage Holding

CURRENT-CONDITIONING (cleaned electrodes)



Sphere-Plane configuration – gap=30mm

High Voltage Holding

CURRENT-CONDITIONING

- Involves Current and Pressure Peaks
- Depends on the surface treatment

CLUMP MODEL (Detachment of material)

[Journal of Applied Physics](#) **23**, 518 (1952)
The Initiation of Electrical Breakdown in Vacuum

Lawrence Cranberg

II. PROPOSED HYPOTHESIS AND SUPPORTING DATA

A hypothesis is proposed herein, the implications of which are briefly elaborated, and a summary of pertinent evidence is presented. The hypothesis is that the initiation of breakdown is due to detachment by electrostatic repulsion of a clump of material loosely adhering to one electrode, but in electrical contact with it;

High Voltage Holding

CURRENT-CONDITIONING

- Involves current and Pressure Peaks
- Depends on the surface treatment

CLUMP MODEL (Detachment of material)

- It seems to be a good answer to experimental observation
- It also gives a good answer to the Total Voltage Effect of Breakdown Phenomena

CLUMP MODEL TAKES INTO ACCOUNT THE ENERGY GAINED BY CHARGE PARTICLES THROUGH THE GAP

A new Model needed

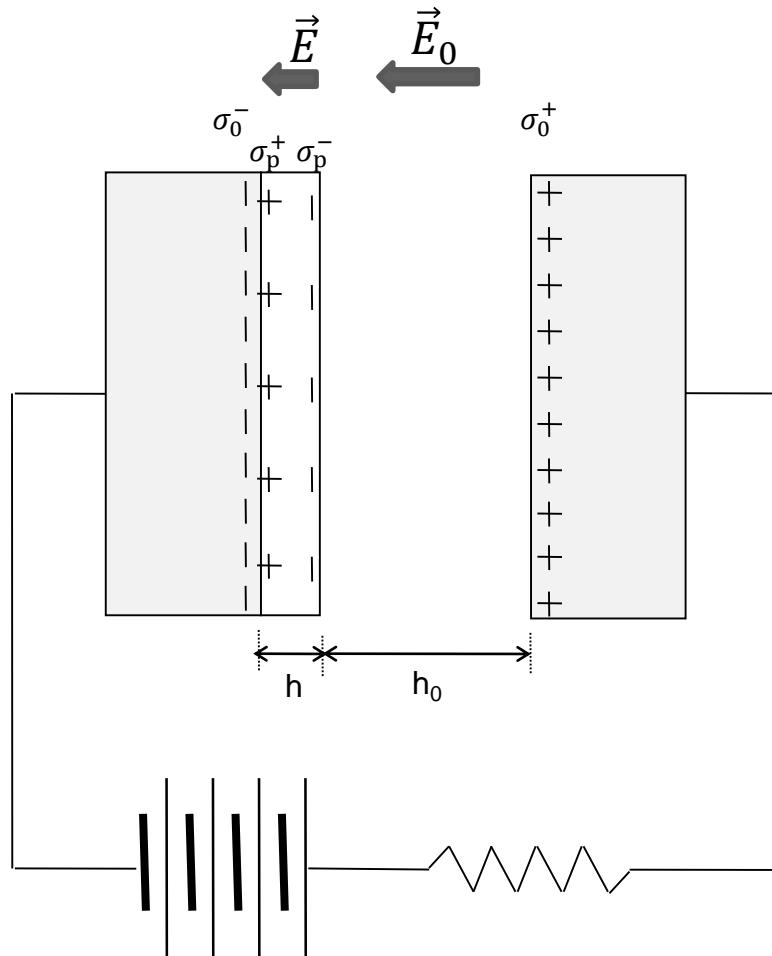
1. **Dark current** seems to be a quantum-tunneling-effect, qualitatively described by Fowler-Nordheim law
2. **Current** (and pressure) random **bursts** are due to detachment of clumps from electrode surfaces
3. **Breakdown** could be thought as a particular intense burst that starts an avalanche-like

In the Fowler-Nordheim, Cramberg-Slivkov frameworks, the main unsolved problems are

1. β and A unrealistic experimental values and their dependence on the gap.
2. Continuous presence of clumps

(Breakdown Induced by Rupture of Dielectric layer)

$E_0 = \text{Vacuum electric Field}$ $E = \text{Electric Field inside the dielectric layer}$



$$E_0 = \frac{\sigma_0^+ + \sigma_0^- - \sigma_p^+ + \sigma_p^-}{2\epsilon_0}$$

$$E = \frac{\sigma_0^+ + \sigma_0^- - \sigma_p^+ - \sigma_p^-}{2\epsilon_0}$$

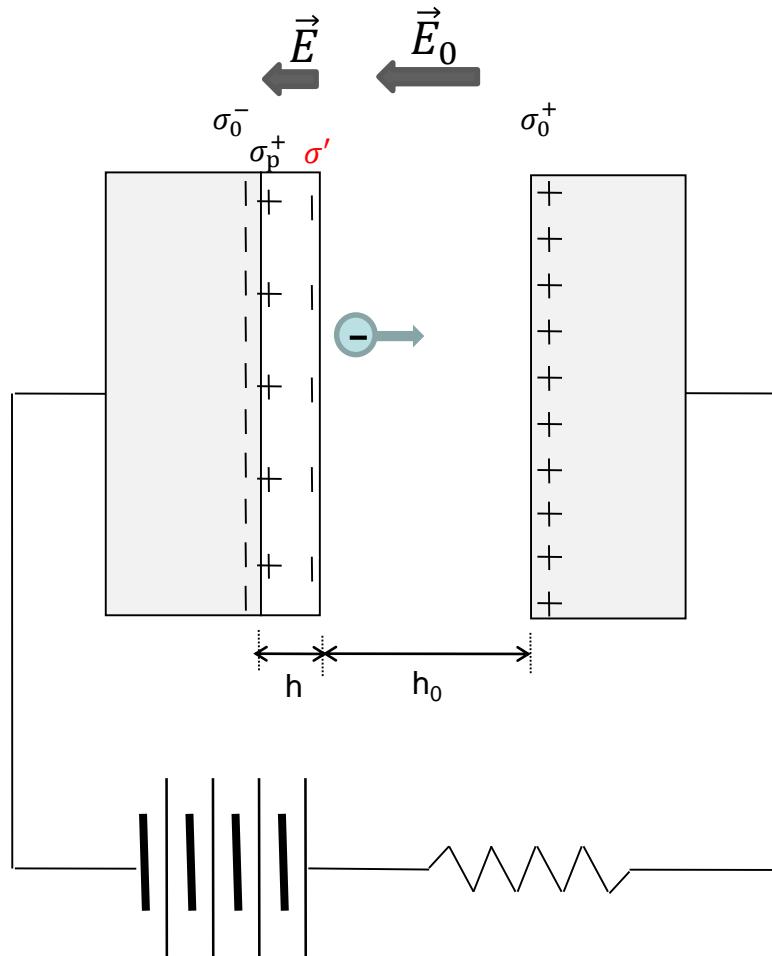
Charge conservation:

$$\sigma_0^+ + \sigma_p^+ = \sigma_0^- + \sigma_p^-$$

$$\sigma_p^+ = \sigma_p^-$$

(Breakdown Induced by Rupture of Dielectric layer)

$E_0 = \text{Vacuum electric Field}$ $E = \text{Electric Field inside the dielectric layer}$



$$\sigma' = \sigma_0^- - \sigma_e$$

$$E_0 = \frac{\sigma_0^+ + \sigma_0^- - \sigma_p^+ + \sigma'}{2\epsilon_0}$$

$$E = \frac{\sigma_0^+ + \sigma_0^- - \sigma_p^+ - \sigma'}{2\epsilon_0}$$

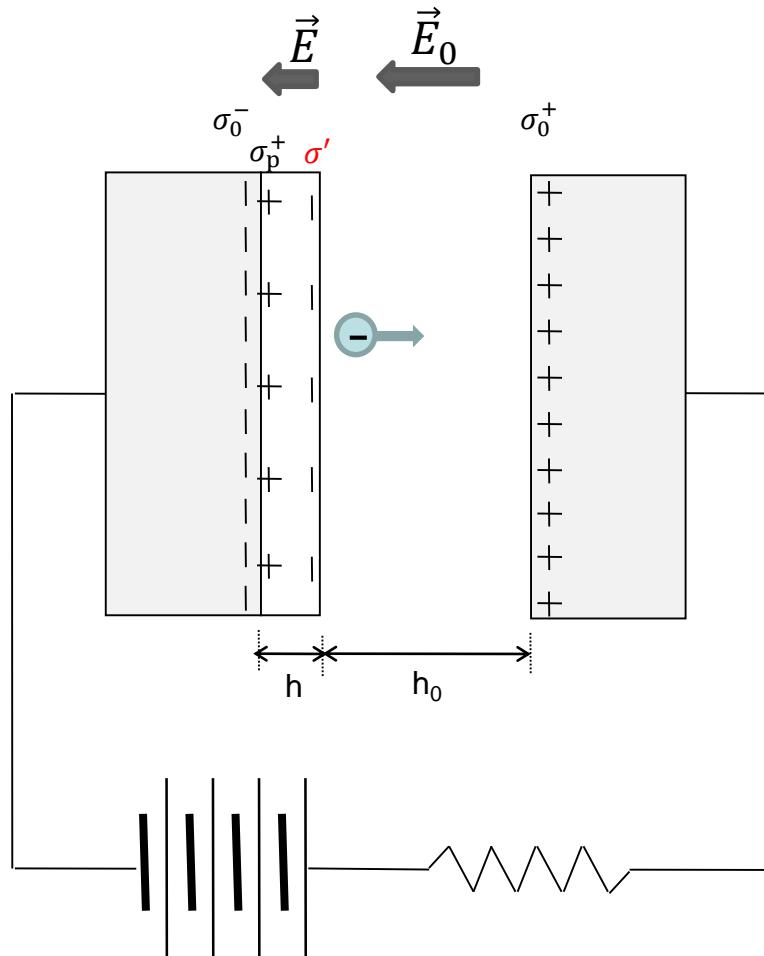
Charge conservation:

$$\sigma_0^+ + \sigma_p^+ = \sigma_0^- + \sigma'$$

$$\sigma_p^+ = \sigma_p^-$$

(Breakdown Induced by Rupture of Dielectric layer)

$E_0 = \text{Vacuum electric Field}$ $E = \text{Electric Field inside the dielectric layer}$



$$\sigma' = \sigma_p^+ - \sigma_e$$

$$\sigma_p \stackrel{\text{def}}{=} \sigma_p^+ = \sigma_p^-$$

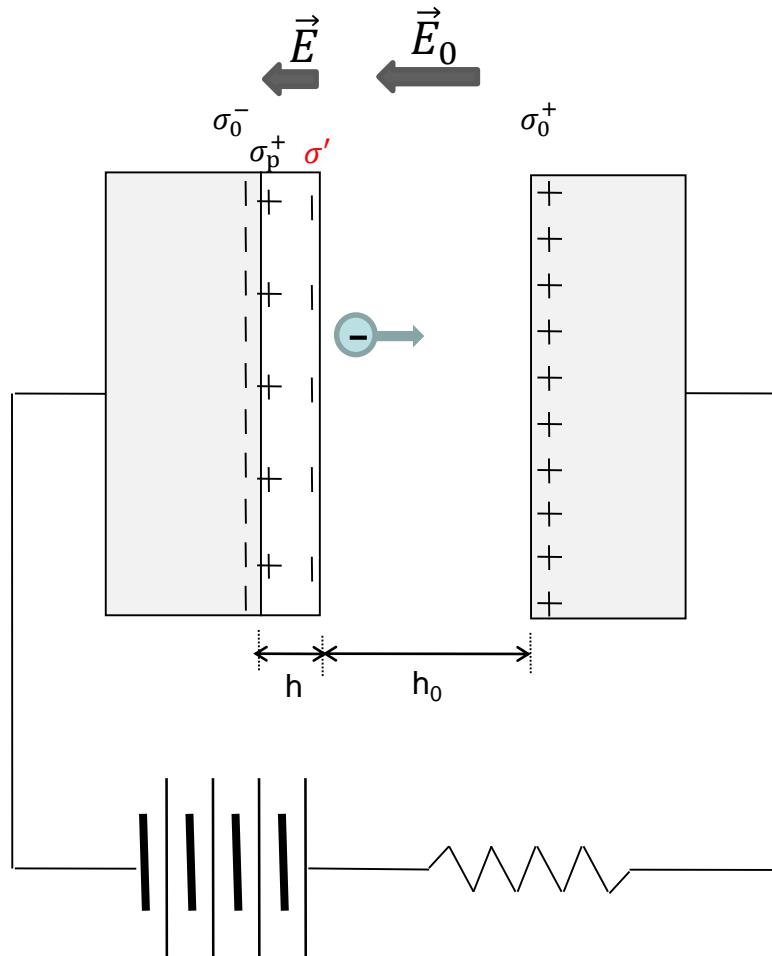
$$\sigma_0 \stackrel{\text{def}}{=} \sigma_0^+$$

$$E_0 = \frac{\sigma_0}{\epsilon_0}$$

$$E = \frac{\sigma_0 - \sigma'}{\epsilon_0}$$

(Breakdown Induced by Rupture of Dielectric layer)

E_0 = Vacuum electric Field E = Electric Field inside the dielectric layer



$$\sigma_p = \epsilon_0 \chi E$$

$$\epsilon_r = 1 + \chi$$

$$\sigma_e = \epsilon_r \epsilon_0 E - \epsilon_0 E_0$$

$$\epsilon_r \epsilon_0 \frac{dE}{dt} = \epsilon_0 \frac{dE_0}{dt} + J_e$$

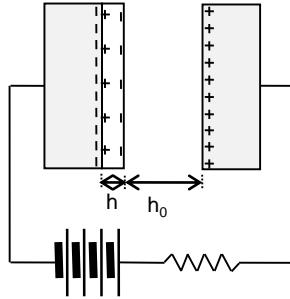
This equation has to be coupled with

$$RI = V_0 - h_0 E_0 - hE$$

(Breakdown Induced by Rupture of Dielectric layer)

E_0 = Vacuum electric Field

E = Electric Field inside the dielectric layer



$$\begin{cases} \epsilon_0 \frac{dE_0}{dt} = \frac{V_0 - h_0 E_0 - hE}{RS} - J_e \\ \epsilon_0 \epsilon_r \frac{dE}{dt} = \frac{V_0 - h_0 E_0 - hE}{RS} \end{cases}$$

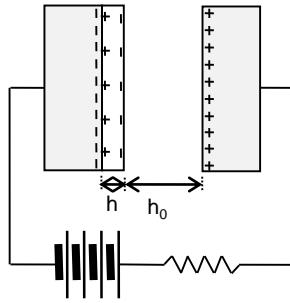
A model for J_e is needed.

It depends on

- charge density $\sigma' = \epsilon_0(E_0 - E)$
- average electric field \bar{E} at surface

B.I.R.D. model

(Breakdown Induced by Rupture of Dielectric layer)



E_0 = Vacuum electric Field

E = Electric Field inside the dielectric layer

$$\begin{cases} \varepsilon_0 \frac{dE_0}{dt} = \frac{V_0 - h_0 E_0 - hE}{RS} - J_e \\ \varepsilon_0 \varepsilon_r \frac{dE}{dt} = \frac{V_0 - h_0 E_0 - hE}{RS} \end{cases}$$

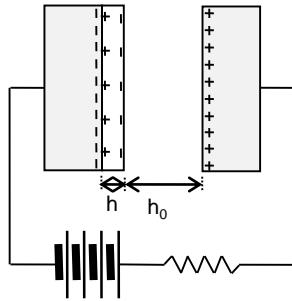
A model for J_e is needed.

$$J_e \sim (E_0 - E)(\bar{E} - E_d) \quad \bar{E} = \frac{E_0 + E}{2}$$

$$J_e = k(E_0 - E) \left(\frac{E_0 + E}{2} - E_d \right) \quad k = 0 \quad \text{for} \quad \bar{E} < E_d$$

B.I.R.D. model

(Breakdown Induced by Rupture of Dielectric layer)



E_0 = Vacuum electric Field

E = Electric Field inside the dielectric layer

$$J_e = k(E_0 - E) \left(\frac{E_0 + E}{2} - E_d \right)$$

$$E(0) = \frac{E_0(0)}{\varepsilon_r}$$

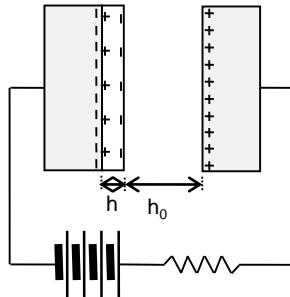
F.N.-like current

$$J_e(0) = k_1 E_0^2 \left(1 - \frac{k_2}{E_0} \right) \approx k_1 E_0^2 e^{-\frac{k_2}{E_0}}$$

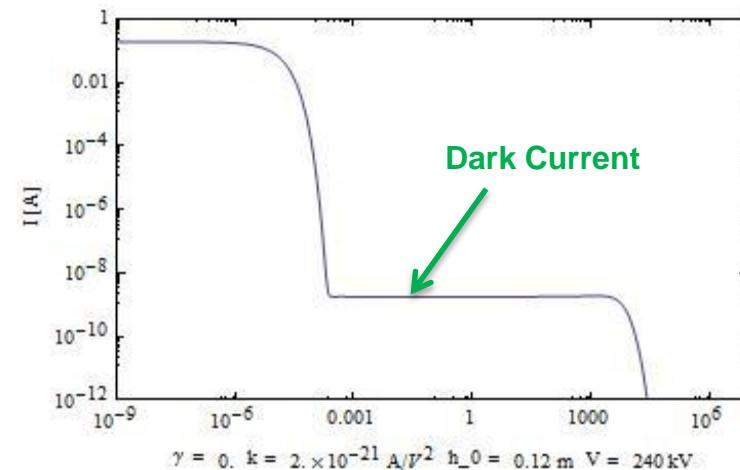
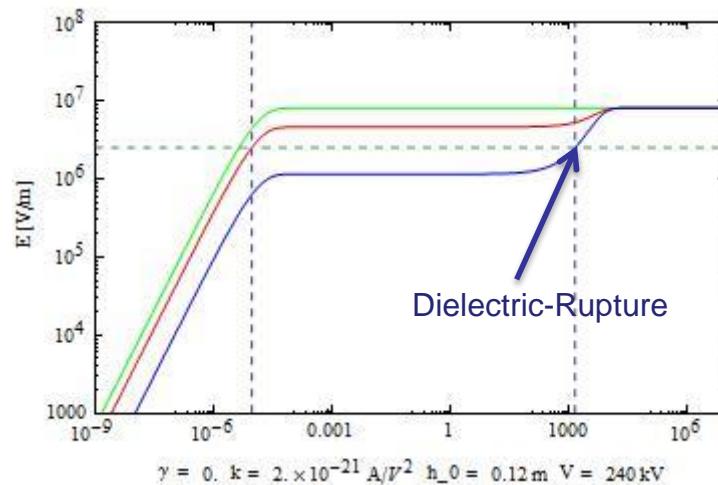
(Breakdown Induced by Rupture of Dielectric layer)

$E_0 = \text{Vacuum electric Field}$

$E = \text{Electric Field inside the dielectric layer}$



NUMERICAL SOLUTIONS



Rupture Time: Time needed for a burst to happen is given by:

$$\tau = \frac{\varepsilon_r \varepsilon_0}{2k} \frac{1}{E_0 - E_{diel}} \ln \frac{(\varepsilon_r - 1)E_0}{(\varepsilon_r + 1)E_0 - 2\varepsilon_r E_{diel}} \quad \frac{2\varepsilon_r}{\varepsilon_r + 1} E_{diel} < E_0 \leq \varepsilon_r E_{diel}$$

$$E_0 = \frac{\beta V}{h_0} \quad V_{\max} = \varepsilon_r E_d h_0 \quad \beta_{\text{sup}} = \frac{V_{\max}}{V} \quad \beta_{\text{inf}} = \frac{2}{\varepsilon_r + 1} \beta_{\text{sup}}$$

$$\tau(\beta) = \left(\frac{\varepsilon_0 h}{2k} \right) \frac{\varepsilon_r^2}{V(\varepsilon_r \beta - \beta_{\text{sup}})} \ln \frac{(\varepsilon_r - 1)\beta}{(\varepsilon_r + 1)(\beta - \beta_{\text{inf}})} \quad \beta_{\text{inf}} < \beta \leq \beta_{\text{sup}}$$

Rupture Time: Time needed for a burst to happen is given by:

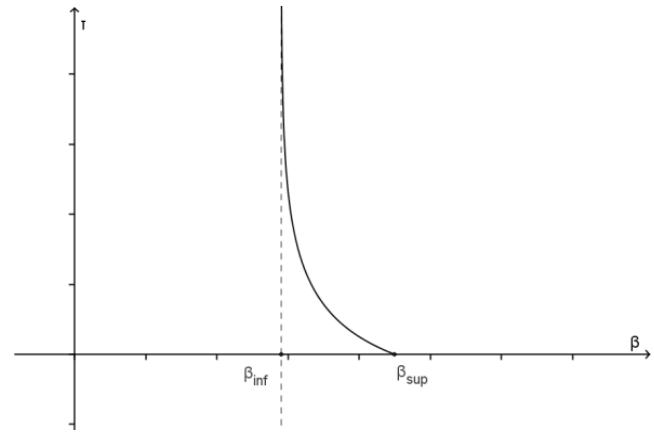
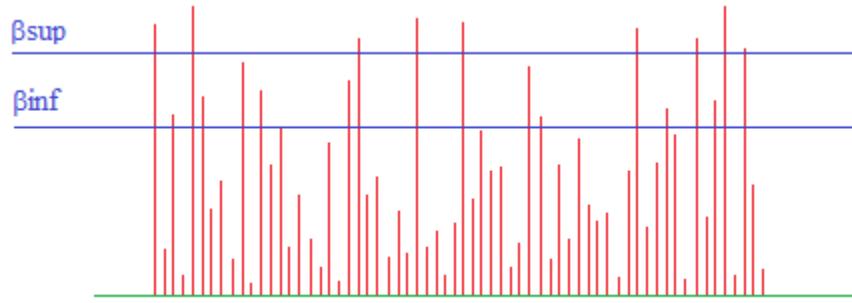
$$\tau = \frac{\varepsilon_r \varepsilon_0}{2k} \frac{1}{E_0 - E_{diel}} \ln \frac{(\varepsilon_r - 1)E_0}{(\varepsilon_r + 1)E_0 - 2\varepsilon_r E_{diel}}$$

$$\frac{2\varepsilon_r}{\varepsilon_r + 1} E_{diel} < E_0 \leq \varepsilon_r E_{diel}$$

$$\tau(\beta) = \left(\frac{\varepsilon_0 h}{2k} \right) \frac{\varepsilon_r^2}{V(\varepsilon_r \beta - \beta_{\text{sup}})} \ln \frac{(\varepsilon_r - 1)\beta}{(\varepsilon_r + 1)(\beta - \beta_{\text{inf}})}$$

$$\beta_{\text{inf}} < \beta \leq \beta_{\text{sup}}$$

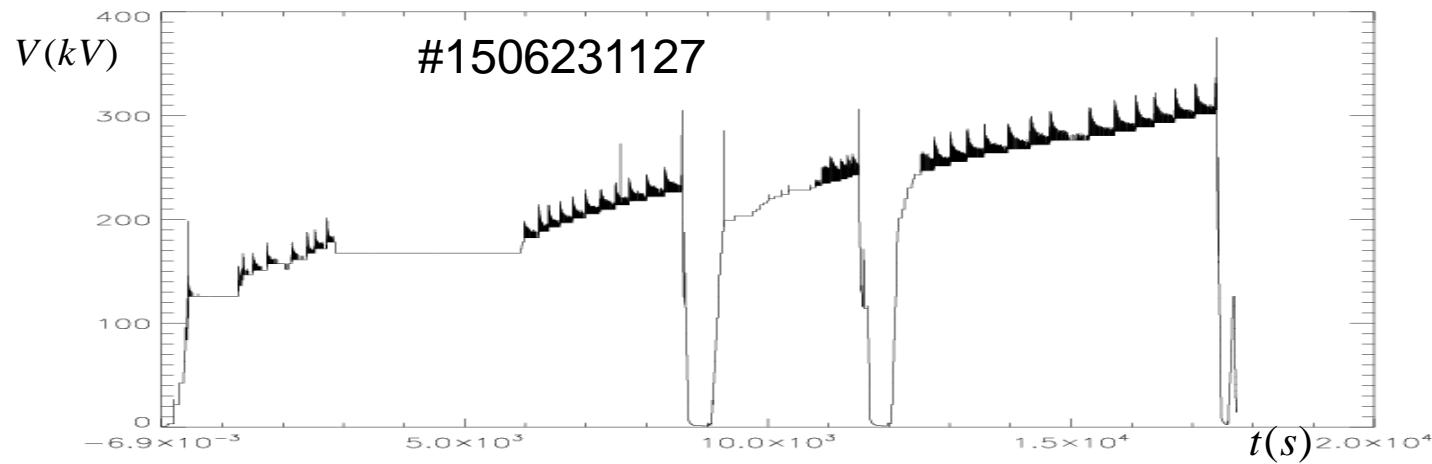
Each voltage V explores a given range of β values



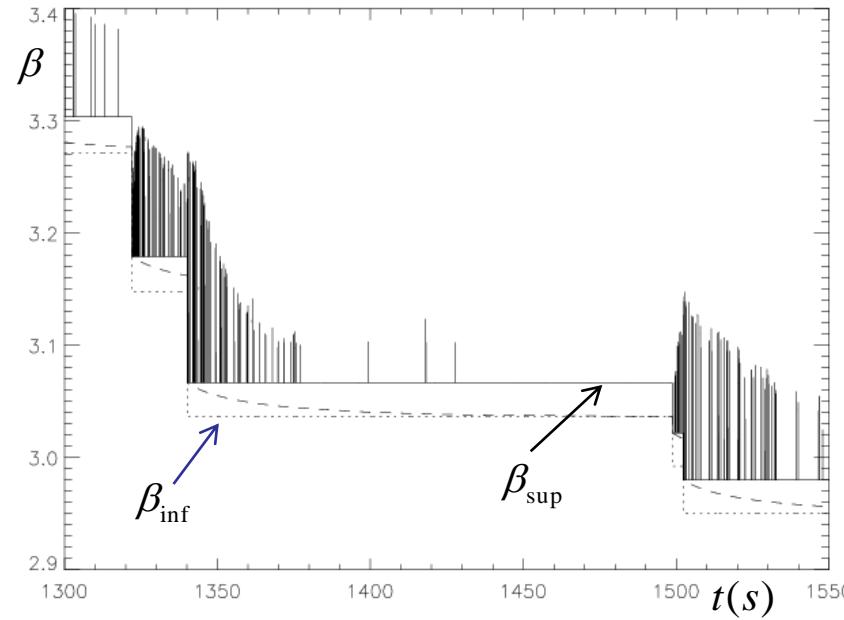
B.I.R.D. model



CONSORZIO RFX
Ricerca Formazione Innovazione



$V_{\max} = 450kV$

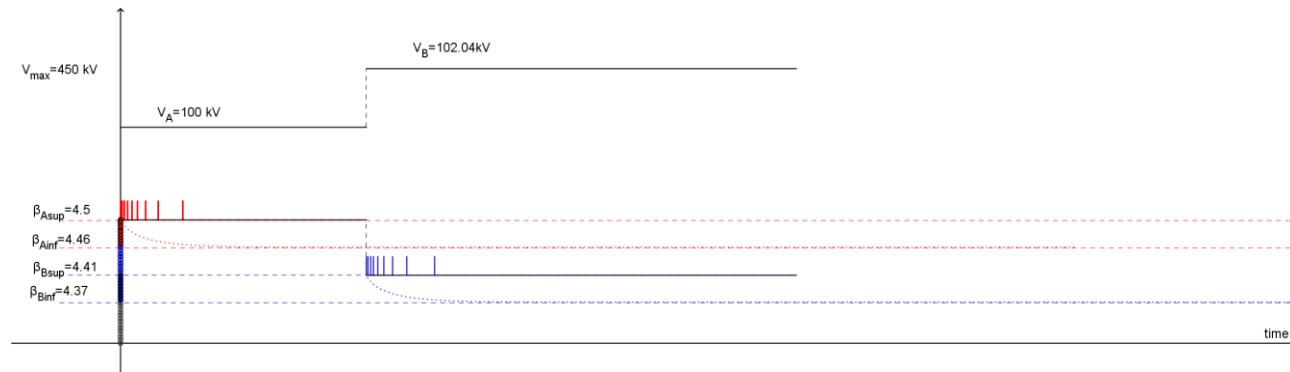


B.I.R.D. model



CONSORZIO RFX
Ricerca Formazione Innovazione

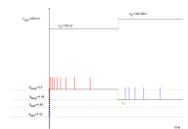
How can you verify BIRD model?



Overlapping consecutive ranges \rightarrow Delay Time τ_D

$$\tau_D = \frac{\varepsilon_0 \varepsilon_r (\varepsilon_r + 1)}{2kE_d} \frac{1}{2\varepsilon_r x - (\varepsilon_r + 1)} \ln \frac{(\varepsilon_r - 1)x}{(\varepsilon_r + 1)(x - 1)}$$

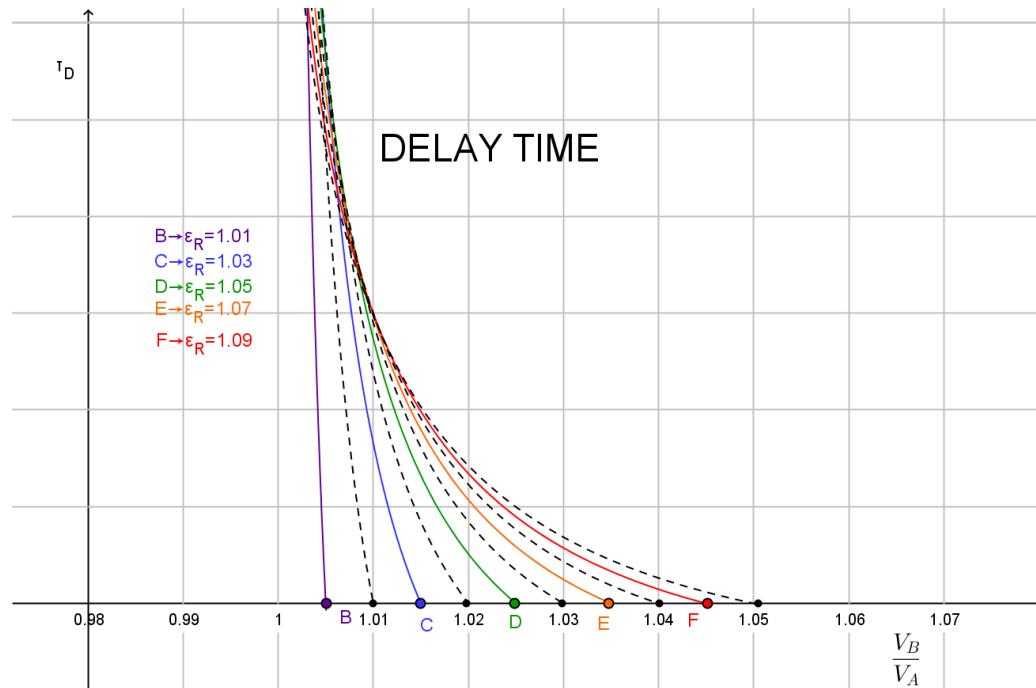
$$x = \frac{V_B}{V_A} \quad 1 < x < \frac{\varepsilon_r + 1}{2}$$



B.I.R.D. model



CONSORZIO RFX
Ricerca Formazione Innovazione



Impact of Time Dependent Dielectric Breakdown and Stress-Induced Leakage Current on the Reliability of High Dielectric Constant (Ba, Sr)TiO₃ Thin-Film Capacitors for Gbit-Scale DRAM's

Shintaro Yamamichi, Akiko Yamamichi, Donggun Park,
Tsu-Jae King, *Member, IEEE*, and Chenming Hu, *Fellow, IEEE*

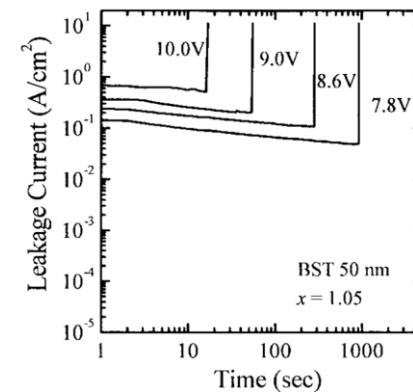
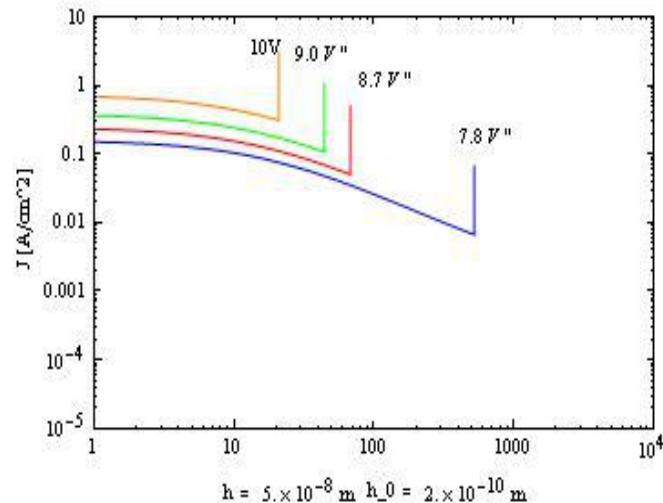


Fig. 2. Leakage current versus time at high stress voltage for 50-nm BST films. Hard breakdown is observed for each voltage.

Qualitatively reproduced
by (modified) B.I.R.D. model





Thank you for the attention