UPDATE ON THE STATUS OF FIELD ELECTRON EMISSION THEORY

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Aims of talk

Talk relates primarily to core theory of field electron emission (FE).
It has three main aims:

- **Indicate some theoretical progress made in last ten years or so.**
- **Indicate why it seems desirable to put FE theory onto a better scientific basis.**
- **Indicate what needs doing "soon".**

Talk provides "update" for people not closely linked to modern developments in FE theory.

Structure of talk

1. Introductory issues
2. The definition of mainstream emission theory
3. Transmission and emission current density regimes
4. Core emission theory for the FNFE regime
5. The principal SN barrier function v
6. Some other items of recent progress
7. Some immediately outstanding tasks

Why work on FE theory?

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- **FE is an enduring part of physics, because it is one of the paradigm exemplars of tunnelling, and its basic theory is likely to be of permanent interest.**
- **FE contributes to some enduring technology (e.g., electron microscopes).**
- **FE contributes to some enduring technological problems (e.g., vacuum breakdown).**

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- **FE contributes to some enduring technology**
  (e.g., electron microscopes).
- **FE contributes to some enduring technological problems**
  (e.g., vacuum breakdown).
- **FE theory is needed in order to interpret experimental results and hence characterize emitters.**
- **Accurate FE theory is needed in order to carry out accurate simulations of various kinds [in particular vacuum breakdown effects].**

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- **Many experimentalists do not fully understand existing theory.**
- **There are many defective equations and many spurious "experimental" results in the literature.**
- **Existing theory (and its limitations) are often not well described, and there is significant confusion over terminology.**

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- Many experimentalists do not fully understand existing theory.
- There are many defective equations and many spurious "experimental" results in the literature.
- Existing theory (and its limitations) are often not well described, and there is significant confusion over terminology.
- Existing theory is not well tested against experiment (except as regards the linearity of Fowler-Nordheim plots).
- There are various inadequately discussed fundamental problems (the "field emission elephants").
- There is also a general need to extend existing theory beyond its current limits of applicability.

A strategic approach to FE theory

RGF's strategic approach is:

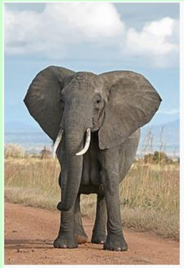
(1) Specify an area of basic theory [so-called "mainstream theory"] that most FE researchers ought to be familiar with.

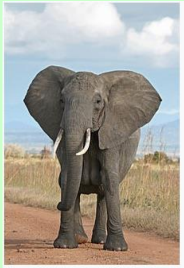
(2) Aim to put mainstream theory onto a "good scientific basis", involving

- clear intellectual structure
- clear terminology
- coherent physical arguments
- all necessary supporting information (e.g. "where is barrier ?)
- transparent proofs
- clear statements of limits of self-consistency/validity
- statements of unsolved fundamental problems
- indications of how theory could be tested,
  and whether this has been done

(3) Later, build outwards and integrate other existing work

A strategic approach to FE theory

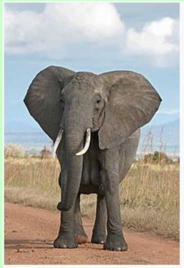


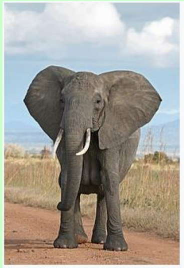


Elephant territory

Mainstream theory

Elephant territory





The definition of "mainstream emission theory"

(in the context of more general FE theory)

Structure of FE theory – I

Main divisions of FE theory are seen to be:

- **Central FE theory ("emission theory")**

- **Supporting theory**

Structure of FE theory – I

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- **Central FE theory ("emission theory")**
  - **Core emission theory [for local emission current density]**
  - **Theory of current-voltage characteristics**
  - **Energy distribution theory**
  - **Field emitter optics**
  - **Theory of field electron microscope images (including resolution)**
- **Supporting theory**

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  - **Theory of field electron microscope images (including resolution)**
- **Supporting theory**
  - **Field emitter electrostatics**
  - **Electroformation theory (thermal-field shape changes)**
  - **Effects due to field emitted vacuum space charge (FEVSC)**
  - **Other breakdown/degradation effects**
  - **Emittance theory for wide FE beams**

Structure of FE theory – II

For each central FE topic [e.g., current-voltage characteristics], one may need to consider

(A) Core emission theory [gives local emission current density]

Theory for ideal devices and situations

Theory for non-ideal devices and situations

Structure of FE theory – II

For each central FE topic [e.g., current-voltage characteristics], one may need to consider

(A) Core emission theory [gives local emission current density]

Theory for ideal devices and situations

Theory for non-ideal devices and situations

(B) Predictive theory

Data interpretation theory

(C) Theory for different materials, possibly with different emitter shapes, band-structures and barrier forms.

Mainstream emission theory

Mainstream FE theory covers the basic topics that ALL researchers working in FE probably ought to have some familiarity with.

Specifically, mainstream EMISSION theory **is the simple basic theory that**

- **Uses a smooth-surface conceptual model that involves**
    - **the Sommerfeld free-electron (-METAL) model**
    - **Fermi-Dirac statistics**
    - **the assumption of a smooth planar surface**
    - **locating the electrical surface at the Sommerfeld-well edge**
    - **locating all induced charge in the electrical surface**
- **Within the framework of the smooth-surface conceptual model, normally evaluates transmission probabilities by using semi-classical quantum mechanics, usually the Kemble or simple-JWKB approximations.**

Mainstream emission theory

Most experimentalists use mainstream emission theory to interpret their results, whatever the material they are working on,

- even though, strictly, mainstream emission theory applies only to metals with flat planar surfaces.

Will now indicate some relatively recent (last ten years) progress in mainstream emission theory.

Transmission regimes
and
Emission-current-density (ECD) regimes
and
related issues

Transmission terminology

We need to be "properly quantum-mechanical" about electron emission fundamentals. In relation to transmission, my nomenclature now is:

Transmission =
Wave-mechanical escape of an entity *across* a potential-energy barrier.

Tunnelling =
Wave-mechanical escape *below* the top of the barrier.

Flyover =
Wave-mechanical escape *above* the top of the barrier.

Classical transmission =
Wave-mechanical escape *greatly above* the top of the barrier, at a level where surface reflection effects are negligibly small, and the transmission probability $D \approx 1$.
[In practice, typically 5 eV or more above the barrier top.]

Energy terminology

Forwards =

Direction normal to and away from emitter surface

[also called "normal direction"]

[Distance in forwards direction is denoted by z and measured from emitter's electrical surface]

Forwards energy E_n =

Total-energy component associated with forwards direction

[relative to any arbitrary but specified energy reference zero]

W = Forwards energy relative to base of Sommerfeld well.

w = Forwards energy relative to top of barrier

Total electron potential energy (total EPE) $U^T(z)$:

measured relative to same energy reference zero as forwards energy

Electron motive energy $M(z)$:

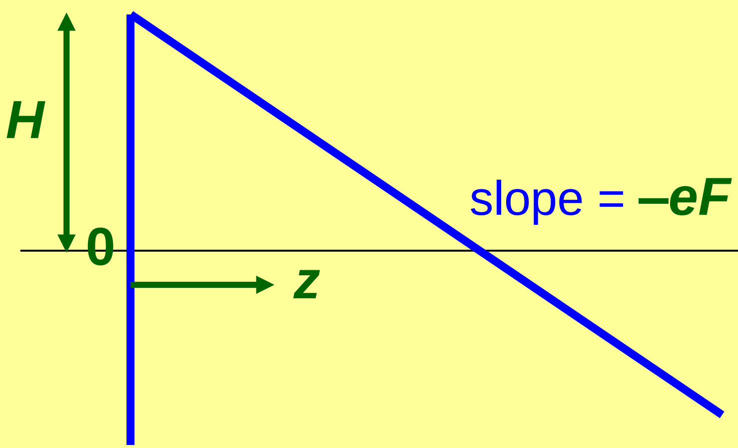
$$M(z) = U^T(z) - E_n$$

[The form of a transmission barrier is determined by the form of $M(z)$.]

Two special barrier forms

Two well-known special barrier forms exist:

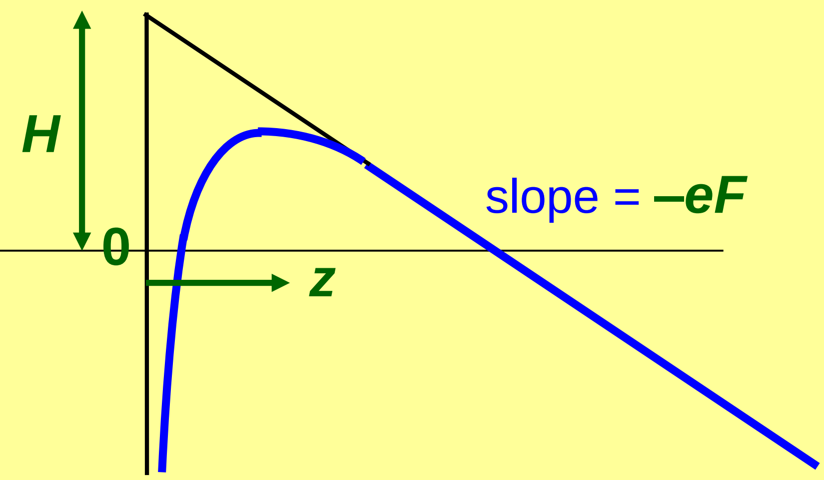
Exactly triangular (ET) barrier



$M(z) = H - eFz$

used by **Fowler & Nordheim**

Schottky-Nordheim (SN) barrier



$M(z) = H - eFz - 1/16\pi\varepsilon_0 z$

used by **Murphy & Good**

Regime terminology

A transmission regime =

Region of parameter-space (typically field and forwards energy) where particular effects determine transmission, or a particular formula for transmission probability *D* is an adequate approximation.

An emission (or emission-current-density [ECD]) regime =

Region of parameter-space (typically field and temperature, for given work-function) where a particular formula for local ECD *J* is an adequate approximation.

A relationship exists between transmission regimes and ECD regimes, but this is not necessarily one-to-one.

Also, the word "adequate" has to be seen in context (more later).

Transmission regimes

OLD VIEW	NEWER RGF VIEW
	High flyover (HF) all high flyover
Over the barrier (Classical)	**Barrier-top regime (BT)** low flyover (LF) + shallow tunnelling (ST)
Through the barrier (WM)	**Deep tunnelling (DT)** all deep tunnelling

Transmission regimes (revised approach)

OLD VIEW

Over the barrier (Classical)

Through the barrier (WM)

NEWER RGF VIEW

High flyover (HF)
all high flyover

Barrier-top regime (BT)
low flyover (LF) +
shallow tunnelling (ST)

Deep tunnelling (DT)
all deep tunnelling

PRESENT RGF VIEW

Classical transmission regime (CT)
all classical transmission ($D \approx 1$)

Surface-reflection regime (SR)
mostly high flyover (HF)

Barrier-top regime (BT)
low flyover (LF) +
shallow tunnelling (ST)

Deep tunnelling (DT)
all deep tunnelling

ET barrier – exact formula for D^{ET}

For the exactly triangular barrier, wave-matching at the emitter surface leads to the exact general formula:

$$D^{\mathrm{ET}} = \frac{1}{\frac{1}{2} + \frac{1}{4}\pi\omega\left(A^2 + B^2\right) + \frac{1}{4}\pi\omega^{-1}\left(A'^2 + B'^2\right)}$$

where *A*, *B* are the values of the Airy functions Ai, Bi, and *A′*, *B′* are the values of the derivatives of Ai, Bi, all evaluated at the emitter surface.

The dimensionless parameter ω is given by an expression of the form (where c_κ is an universal constant):

$$\omega = c_\kappa W^{1/2}/F^{1/3} = [1.723903\ \mathrm{eV}^{-1/2}\,(\mathrm{V/nm})^{1/3}]\ (W^{1/2}/F^{1/3})\ .$$

where *W* is forwards energy measured relative to the base of the conduction band.

ET barrier – approximate formulae for D^{ET}

The transmission regimes and good working formulae (for D^{ET}) are:

1) For deep tunnelling (DT) [H>>0 (w<<0), & W>0]
the Fowler-Nordheim approximate formula:

$$\boxed{D^{\mathrm{ET}} \approx P^{\mathrm{FN}} \exp[-bH^{3/2}/F] = \{4W^{1/2}H^{1/2}/(W+H)\} \exp[-bH^{3/2}/F]}$$

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2) For the barrier top (BT) regime [w ~ 0]

$$D^{\mathrm{ET}} \approx \frac{1}{\frac{1}{2} + c_0 F^{-1/3} W^{1/2} + c_\infty F^{1/3} W^{-1/2} - c_1 F^{-1} \chi W^{-1/2} w}$$

where c_0, c_∞, c_1 are constants with known values.

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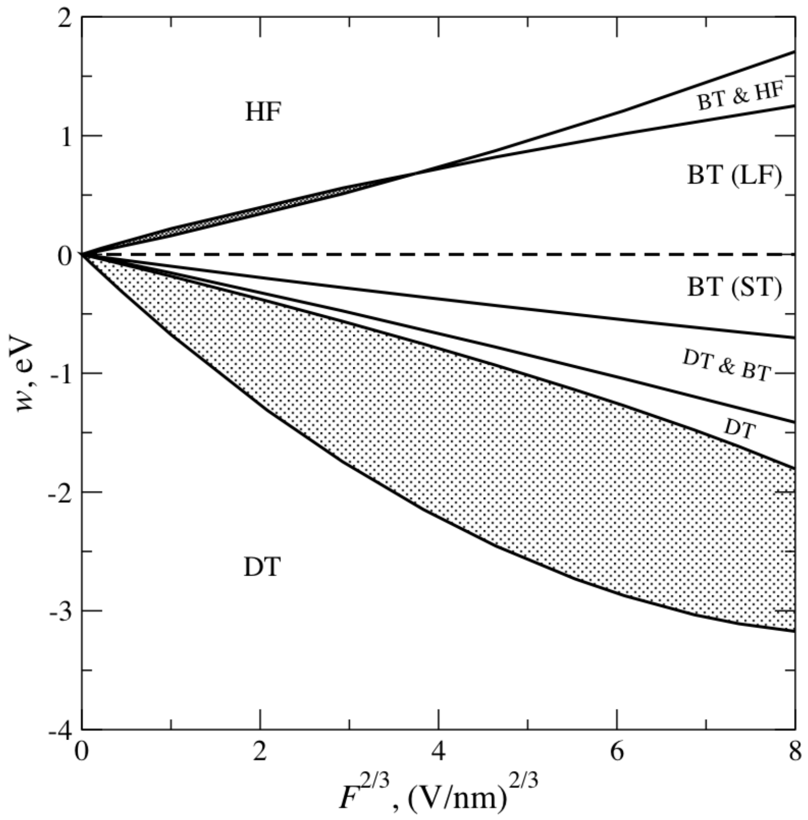
where c_0, c_∞, c_1 are constants with known values.

3) For high flyover (HF) [w>>0]

$$D^{\mathrm{ET}} \approx \frac{4W^{1/2}w^{1/2}}{(W^{1/2} + w^{1/2})^2}$$

[This is also the formula for transmission across a rectangular step.]

ET barrier – transmission regime diagram



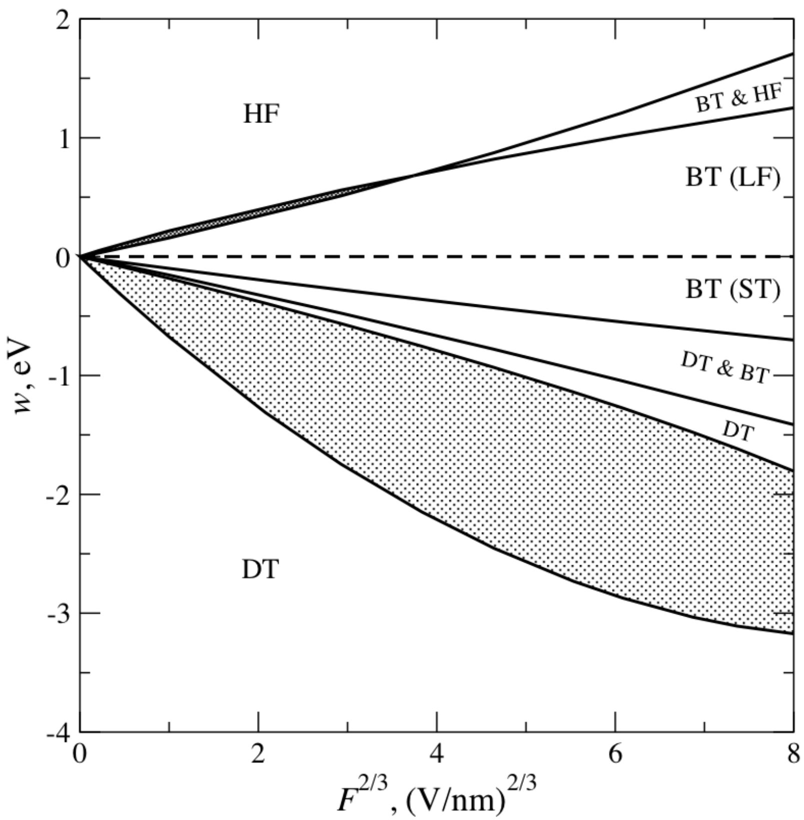
HF = high flyover; LF = low flyover; BT = barrier-top regime; ST = shallow tunnelling; DT = deep tunnelling.

Boundaries represent 10% difference between approximate and exact formulae.

Transmission regimes (revised approach)

CT regime

above 5 eV



Classical transmission regime (CT)
all classical transmission ($D \approx 1$)

Surface-reflection regime (SR)
all high flyover (HF)

Barrier-top regime (BT)
low flyover (LF) + shallow tunnelling (ST)

Deep tunnelling (DT)
all deep tunnelling

Boundaries represent 10% errors.

SN barrier - relating ECD regimes to D-regimes

Transmission regimes	ECD regimes (Swanson/Bell/Forbes)
Classical transmission regime	***High-T (low-F) limit =*** Classical thermal electron emission (CTE)
Surface-reflection regime	Quantum-mechanical thermal electron emission (QMTE)
Barrier-top regime	Barrier-top electron emission (BTE) [or "extended Schottky"]
Deep tunnelling	Fowler-Nordheim FE (FNFE) [or "cold FE" (CFE)]
	Low-*T* limit = Zero-*T* FNFE

Relating practical devices to SN ECD regimes

Transmission regimes	ECD regimes (Swanson/Bell/Forbes)	Commercial items
Classical transmission regime	*High-T (low-F) limit =* Classical thermal electron emission (CTE)	No devices
Surface-reflection regime	Quantum-mechanical thermal electron emission (QMTE)	Thermionic emitter
Barrier-top regime	Barrier-top electron emission (BTE) [or "extended Schottky"]	Schottky emitter
Deep tunnelling	Fowler-Nordheim FE (FNFE) [or "cold FE" (CFE)]	Field emitter
	Low-*T* limit = Zero-*T* FNFE	

Alternative definitions for SN ECD regimes

ECD regimes (Swanson/Bell/Forbes)	ECD regimes (Murphy-Good) (1956)
High-T (low-F) limit = Classical thermal electron emission (CTE)	*High-T (low-F) limit =* Classical thermionic emission
Quantum-mechanical thermal electron emission (QMTE)	Thermionic emission
Barrier-top electron emission (BTE) ["extended Schottky"]	
Fowler-Nord. (FNFE) [or "cold FE" (CFE)]	Field emission
Low-*T* limit = Zero-*T* FNFE	Low-*T* limit = Zero-*T* FE

Murphy-Good ECD-regime diagram (SN barrier)

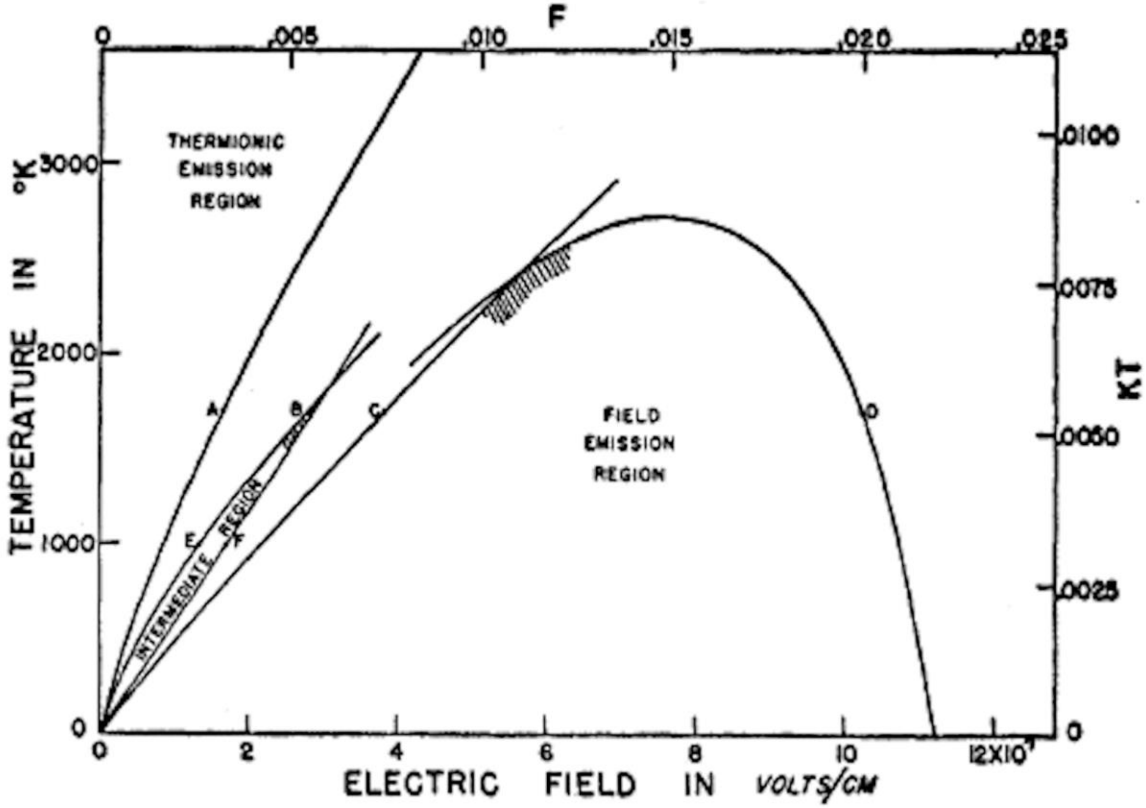
ECD regimes (Murphy-Good) (1956)

High-T (low-F) limit =
Classical thermionic emission

Thermionic emission

Field emission

Low-*T* limit =
Zero-*T* FE



Murphy-Good ECD-regime diagram (SN barrier)

ECD regimes (Murphy-Good) (1956)

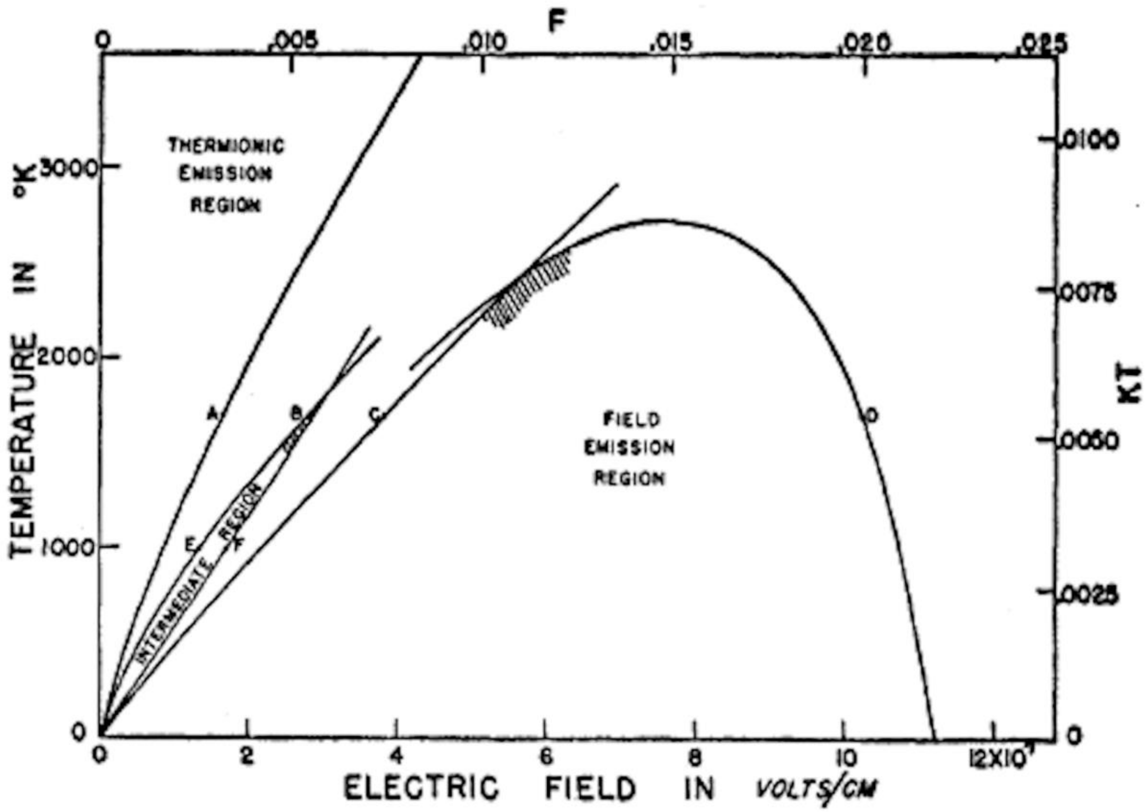
High-T (low-F) limit =
Classical thermionic emission

Thermionic emission

Field emission

Low-*T* limit =
Zero-*T* FE

The Murphy-Good (1956) regime diagram (below), for the Schottky-Nordheim (SN) barrier, badly needs updating.



General thermal-field formulations

ECD regimes (Swanson/Bell/Forbes)	**ECD regimes (Murphy-Good) (1956)**	**General Thermal-Field (Jensen) (2006)**
High-T (low-F) limit = **Classical thermal electron emission (CTE)**	***High-T (low-F) limit =*** **Classical thermionic emission**	***High-T (low-F) limit =*** **Classical TE (CTE)**
Quantum-mechanical thermal electron emission (QMTE)	**Thermionic emission**	**General Thermal-Field (GTF) formulae**
Barrier-top electron emission (BTE) ["extended Schottky"]		
Fowler-Nordheim (FNFE) [or "cold FE" (CFE)]	**Field emission**	**FNFE**
Low-T limit = Zero-T FNFE	**Low-T limit = Zero-T field emission**	**Low-T limit = Zero-T FNFE**

General thermal-field formulations

ECD regimes (Swanson/Bell/Forbes)	General thermal-field (Jensen)	Numerical formulations based on NED integral
High-T (low-F) limit = Classical thermal electron emission (CTE)	*High-T (low-F) limit* = Classical TE (CTE)	Numerical results
Quantum-mechanical thermal electron emission (QMTE)	General Thermal-Field (GTF) formulae	For planar emitters – Jensen (to be published)
Barrier-top electron emission (BTE) ["extended Schottky"]		For earthed spheres – Kyritsakis & colleagues (see his presentation)
Fowler-Nordheim (FNFE) [or "cold FE" (CFE)]	FNFE	
Low-*T* limit = Zero-*T* FNFE	Low-*T* limit = Zero-*T* FNFE	

ECD regimes – notes

#1
The new numerical treatments should enable us to construct proper ECD regime diagrams for the simple Murphy-Good approximate ECD equations, particularly if the numerical treatments are based on exact numerical solutions of the Schrödinger equation, as well as numerical evaluation of integrals.

#2
For the SN barrier, there are also special high-field ECD regimes, for example "explosive emission" and "liquid-metal electron source" regimes. But I do not currently consider these to be part of mainstream theory.

Core emission theory for the FNFE regime:

4a: Classification of core FN-type equations

Fowler-Nordheim-type equations

A Fowler-Nordheim-type (FN-type) equation is any FNFE equation with the mathematical form

$$Y = C_{YX} X^2 \exp[-B_X / X] \ ,$$

where: X is any FNFE independent variable (e.g., a field or a voltage);

Y is any CFE dependent variable (e.g., a current or current density);

B_X is a function related to choice of X and barrier form;

C_{YX} is a function related to other choices, including X, Y, and B_X .

B_X and C_{YX} are NOT constants (except in the most elementary models).

The core theoretical forms of FN-type equation (those derived directly from theory) give local emission current density (ECD) J in terms of local work function ϕ and (the magnitude F of) local barrier electrostatic field.

The elementary $J(F)$-form FN-type equation

The simplest core FN-type equation is the elementary $J(F)$-form equation:

$$J^{\mathrm{el}} = a\phi^{-1}F^{2}\exp[-b\phi^{3/2}/F]\ .$$

where a and b are the FN constants.

This is based on assuming an exactly triangular (ET) barrier, and is a simplification of the original equation derived by FN in 1928.

The equation above is good for undergraduate teaching, but is too simple to describe real situations. Hence, it has to be generalised, in TWO ways.

The concept of "kernel current density"

(1) The elementary equation relates to an ET barrier. BUT:

- **it neglects exchange-and-correlation (XC) effects (usually modelled as image effects);**
- **it is not adequately valid for highly curved emitters.**

We formally include both effects with a barrier form correction factor, here for the general barrier (GB).

Thus, the general barrier of zero-field height ϕ has a correction factor ν_F^{GB} ("*nu*$_F^{GB}$ "), and the resulting equation is

$$J_k^{GB} = a\phi^{-1}F^2 \exp[-\nu_F^{GB} b\phi^{3/2}/F].$$

J_k^{GB} is a mathematical quantity that can be calculated *exactly* for a given barrier form, when values of ϕ and F are given.

I call J_k^{GB} the kernel current density for the chosen barrier form "GB".

Local pre-exponential correction factor

(2) To allow for other corrections, it is necessary to include a **local pre-exponential correction factor** λ^{GB}.

Thus the physical local ECD J_C^{GB} is given by

$$J^{GB} = \lambda^{GB} J_k^{GB} = \lambda^{GB} (a\phi^{-1}F^2) \exp[-\nu_F^{GB} b\phi^{3/2}/F].$$

The factor λ^{GB} allows ***formally*** for corrections due to all of:

- improved tunnelling theory that includes a tunnelling pre-factor;
- more accurate integration over emitter electron states;
- temperature effects;
- effects due to the use of atomic-level wave-functions;
- effects related to non-free-electron band-structure;
- any other operating physical effect not specifically considered;
- any unrecognized theoretical inadequacy.

The equation above is the core general-barrier FN-type equation.

Equation complexity levels

Historically, many different assumptions/models have been used to obtain expressions for ν_F^{GB} and λ^{GB} .

The complexity level of a FN-type equation is decided by the choices of:

(a) barrier form (which determines ν_F); and

(b) what effects/approximations to include in λ_C .

For planar emitters, the main complexity levels used historically and currently are shown in the following table.

I give each of the main complexity levels a specific name.

Most significant planar complexity levels

TABLE 1. Complexity levels of core planar Fowler-Nordheim-type equations.

Name	Date	$/^{\mathrm{GB}} \rightarrow$	Barrier form	$n_{\mathrm{F}}^{\mathrm{GB}} \rightarrow$	Note
Elementary	?	1	ET	1	a
Original	1928	P^{FN}	ET	1	b
Fowler-1936	1936	4	ET	1	
Extended elementary	2015	$/^{\mathrm{ET}}$	ET	1	
Dyke-Dolan	1956	1	SN	v_{F}	c
Murphy-Good (zero temperature)	1956	t_{F}^{-2}	SN	v_{F}	c
Murphy-Good (finite temperature)	1956	$/_{T}t_{\mathrm{F}}^{-2}$	SN	v_{F}	d
Orthodox	2013	$/^{\mathrm{SN0}}$	SN	v_{F}	e
New-standard	2015	$/^{\mathrm{SN}}$	SN	v_{F}	
"Barrier-effects-only"	2013	$/^{\mathrm{GB0}}$	GB	$n_{\mathrm{F}}^{\mathrm{GB}}$	e
General	1999	$/^{\mathrm{GB}}$	GB	$n_{\mathrm{F}}^{\mathrm{GB}}$	

[a]Many earlier imprecise versions exist, but the first clear statement seems to be in 1999.
[b]P^{FN} is the Fowler-Nordheim tunnelling pre-factor.
[c] v_{F} and t_{F}^{-2} are appropriate particular values of the SN barrier functions v and t.
[d]$/_{T}$ is the Murphy-Good temperature corrction factor
[d]The superscript " ^{0} " indicates that the factor is to be treated mathematically as constant.

For details, see: R.G. Forbes et al., "Fowler-Nordheim plot analysis: a progress report", Jordan J. Phys. 8 (2015) 125; arXiv:1504.06134v7 .
Historically, around 15-20 different mathematical approximations have been used for the particular value v_{F} of the principal SN barrier function v.

Core emission theory for the FNFE regime:

4b: The concept of scaled barrier field

Scaled barrier field f

When the Schottky reduction Δ is equal to the work function ϕ, we have

$$\Delta = \phi = c_S F_R^{1/2} = (e^3/4\pi\varepsilon_0)^{1/2} F_R^{1/2} ,$$

where F_R [$= c_S^{-2}\phi^2$] is the reference field needed to reduce to zero a barrier of zero-field height ϕ.

The scaled barrier field f is defined by

$$f = F / F_R = c_S^2 \phi^{-2} F .$$

This dimensionless parameter f plays an important role in modern FE theory.

Relevant numerical values are:

$$c_S = 1.199\,985 \text{ eV (V/nm)}^{-1/2}$$

$$c_S^2 = 1.439\,965 \text{ eV}^2 \text{ (V/nm)}^{-1}$$

[In this presentation, universal constants are given to 7 significant figures.]

Core emission theory for the FNFE regime:

4c: The value of lambda

The value of λ

A major problem in FE science is that the value of λ is unknown for any physically realistic barrier.

For the SN barrier, the value of the λ^{SN} is thought to lie in the range

$$0.005 < \lambda^{SN} < 11 ,$$

but this could be an underestimate of the range of uncertainty.

This is one of the "field emission elephants".

The origin of the λ-value problem

The origin of the λ-value problem lies in:

- **(a) the use of smooth-surface conceptual models;**
- **(b) the failure (until recently) to formulate a theory of FN-type equations sufficiently general to allow the problem to be discussed;**
- **(c) the prolonged (90-year) failure to satisfactorily test the predictions of the smooth-surface models against experiment.**

Smooth-surface models:

- **disregard the existence of atoms;**
- **disregard the role of atomic wave-functions in transmission theory;**
- **assume the induced surface charge is located in an infinitesimally thin classical surface layer.**

For real field emitters, these assumptions are wildly unrealistic.

Dealing with the λ-value problem

For the λ-value problem, there are two obvious solutions:

(a) prediction of λ, using much-improved atomic-level transmission theory;

(b) experimental measurement of λ.

The accurate prediction of λ would be intensely difficult, probably beyond the existing boundaries of quantum mechanics. It may not be unreasonable to think in terms of a time-scale of another 50-100 years for its full solution.

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Experimental measurements look relatively easier, but would be very far from straightforward.

But at present, no experimentalists seem prepared to carry out the relevant experiments, and funding authorities have shown no inclination to fund such experiments.

Dealing with the λ-value problem

My solution to this situation is

(1) Identify "mainstream emission theory";

(2) Make its presentation "properly scientific", as discussed earlier;

(3) Specify how to test the theory (which may not be straightforward, and measure λ;

(4) Look for people and funding to do part or all of this testing.

This paper is part of (1) and (2) above.

Core emission theory for the FNFE regime:

4d: Area-like quantities

Emission area definitions

To derive expressions for emission current, one needs to identify a characteristic point "C" on the emitter surface. In modelling, "C" is usually taken at the emitter apex. Parameters relating to "C" are subscripted "$_{C}$".

An expression for total emission current i_e is obtained by integrating over the emitter surface and writing result in form

$$i_e = \int J \, dA \equiv A_n J_C = A_n \lambda_C J_{kC} ,$$

where A_n is the notional emission area.

For all emitters, the value of λ_C is uncertain, and for LAFEs the value of A_n is also uncertain. Having two parameters of uncertain value in an equation is unhelpful, so define a new parameter, the formal emission area A_f by

$$A_f \equiv A_n \lambda_C .$$

Area efficiency definitions

Correspondingly, for large-area field electron emitters (LAFEs) the notional area efficiency α_n is defined by

$$\alpha_n = A_n/A_M ,$$

where A_M is the LAFE macroscopic area or "footprint".

And the formal area efficiency α_f is defined by

$$\alpha_f = A_f/A_M = A_n\lambda_C/A_M = \lambda_C\alpha_n .$$

Different roles of formal and notional parameters

We need both formal and notional "theoretical" area-like parameters, because:

- in appropriate circumstances ("where emission is orthodox"), good values of the formal parameters can deduced from experiment, via FN plots;
- but the notional parameters appear in some existing theory.

In principle, the notional parameters are probably closer to "geometrical" area estimates, but (due to present uncertainty in the value of λ_C), accurate values of the notional parameters cannot be deduced from experiment.

It is to be expected that values of formal emission area deduced from FN plots may sometimes look implausibly low.

Consequences of the λ-value problem

In summary, two consequences of our lack of good knowledge of λ are

- **we cannot carry out accurate simulations of FE currents;**
- **we cannot accurately deduce values of emission area from FE experiments.**

[For the SN barrier, the value of the λ^{SN} is thought to lie in the range

$$0.005 < \lambda^{SN} < 11 \, ,$$

but this could be an underestimate of the range of uncertainty.]

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$$0.005 < \lambda^{SN} < 11 ,$$

but this could be an underestimate of the range of uncertainty.]

How should we deal with this uncertainty? Probably, by carrying out simulations for both limiting values and reporting both limiting results.

Improvements in theory of the principal SN-barrier function v

Improvements in the theory of the function v †

As indicated earlier, the core "new-standard" FN-type equation (which is based on the SN barrier) has the form

$$J \quad = \quad \lambda^{\mathrm{SN}}\,(a\phi^{-1}F^{2})\,\exp[-\,\mathrm{v}_{\mathrm{F}}b\phi^{3/2}/F].$$

where v_{F} is the appropriate particular value (determined by the values of ϕ and F) of the principal SN-barrier function v † .

In recent years, useful progress has been made with the mathematics of v .

†Since v, like "cos" and the Airy function "Ai", is a special (i.e., exactly defined) mathematical function, the typesetting conventions of the international system of measurement allow/require it to be typeset "upright", rather than "italic". This convention is used here and does make things marginally clearer.

Progress with the SN barrier function v

Conceptual progress

1. **Need to distinguish between the concept of barrier form correction factor for the SN barrier, which is a physical quantity, and the special mathematical function v called the principal SN barrier function.**

2. **Best to change independent mathematical variable from the Nordheim variable y to the Gauss variable x [= y^2], and write v as v(x).**

3. **Need to distinguish between mathematical (x) and modelling (f) variables.**

4. **Need to recognize that the parameter v_F in FN-type equations based on the SN barries is obtained by setting $x=f$ in an appropriate expression for v(x).**

Progress with the SN barrier function v

Technical mathematical progress

5. Have found simple good approximation for v(x) and hence v(f).
6. Have shown that this is a superior approximation.
7. Have found defining equation for v(x).
8. Have found exact series expansion for v(x) and hence v(f).
9. Have found accurate numerical expressions for v(x) and hence v(f).
10. Have redefined expressions and approximations for other SN barrier functions.
11. Have derived scaled form for SN-barrier kernel current density.

The special mathematical equation for v

Mathematically, the "principal SN barrier function" v is a special mathematical function that is a particular solution of a special equation of mathematical physics identified by Deane and Forbes, namely

$$x(1-x)\mathrm{d}^2W/\mathrm{d}x^2 = (3/16)W .$$

This equation is itself a special case of the Gauss hypergeometric differential equation.

***x* is in origin the independent mathematical variable that appears in the Gauss hypergeometric differential equation. Hence, I call *x* the Gauss variable.**

Notation for the mathematical argument of v

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Earlier, it was known that v can be written in terms of complete elliptic integrals. Thus, it was mathematically appropriate to write v(l'), where the symbol l' has a particular meaning in elliptic-function theory.

However, taking v as a function specified via the hypergeometric equation is mathematically more fundamental; it now seems better to write v(x). This is just a change to a simpler and better-justified notation, and $l' \equiv x$.

Why change from y to x [= y^2] ?

In older literature, the principal SN barrier function is written in the form $v(y)$, where the Nordheim parameter $y = x^{1/2}$. There are good mathematical and physical and reasons for now writing it in terms of the Gauss variable x, as v(x), and for using f rather than y in modeling contexts. These include the following.

- **No terms linear in $x^{1/2}$ appear in the defining equation for v(x).**
- **No terms linear in $x^{1/2}$ appear in the exact series expansion for v(x).**
- **The concept of scaled barrier field f is easier to understand, and easier to generalize to real physical barriers.**
- **f is *linearly* related to the barrier field F *(this is important)*.**
- **For SN-barrier-related FN-type equations it is possible to write simple scaled forms that have f as the only independent variable.**
- **The parameter f is useful in other contexts, e.g. the orthodoxy test.**

A simple good approximation for v(*x*) *or* v(*f*)

The "Forbes-Deane" approximation for v(*x*) is

$$v(x) \approx 1 - x + (1/6)x\ln x \, .$$

Over the range $0 \le x \le 1$, this expression is accurate to better than 0.33% .

Obviously, in terms of *f* this becomes

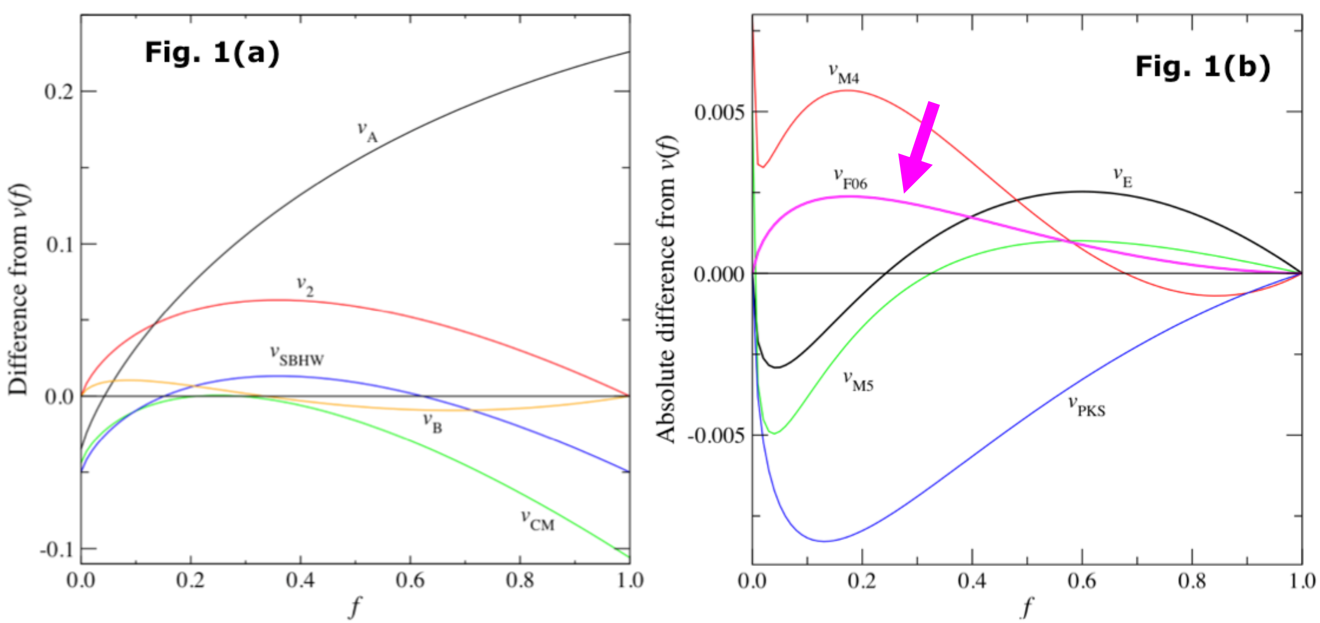
$$v(f) \approx 1 - f + (1/6)f\ln f \, .$$

This approximation is more accurate than historical approximations of equivalent complexity ….

Historical approximations for v(*f*) compared

Table 1: APPROXIMATIONS COMPARED

A1:	v_2	=	$1 - f$,
A2:	v_A	=	$0.965 - 0.739f$,
A3:	v_{CM}	=	$0.956 - 1.062f$,
A4:	v_B	=	$1 - f[1+0.9\sin\{(1-f^{1/2})/2\}]$,
A5:	v_{SBHW}	=	$0.95 - f$
A6:	v_E	=	$1 - f^{\,0.845}$
A7:	v_{M4}	=	$1.008 - 0.118f^{1/2} - 1.14f + 0.25f^{3/2}$
A8:	v_{M5}	=	$1.0050 - 0.1654f^{1/2} - 1.0412f + 0.2320f^{3/2} - 0.0304f^2$
A9:	v_{PKS}	=	$1 - f^{\,0.82758}$
A10:	v_{F06}	=	$1 - f + (1/6)\, f \ln f$ [the new approximation].



The exact series expansion for v(*x*)

The lowest few terms of the exact series expansion for v(*x*) are

$$\mathrm{v}(x) = 1 - \left(\frac{9}{8}\ln 2 + \frac{3}{16}\right)x - \left(\frac{27}{256}\ln 2 - \frac{51}{1024}\right)x^2 - \left(\frac{315}{8192}\ln 2 - \frac{177}{8192}\right)x^3 - \ldots$$
$$+\left(\frac{3}{16} + \frac{9}{512}x + \frac{105}{16384}x^2 + \ldots\right)x\ln x$$

This series is derived from an exact mathematical statement of the functional form of v(*x*). This is too complicated to present here, but may be found in Deane and Forbes, *J. Phys. A: Math. Theor.* 41 (2008) 395301.

Almost certainly, the reason why the Forbes-Deane approximate formula works well is that it mimics the form of the lowest terms of the exact expansion.

6. Other recent theoretical progress

Other areas of recent progress

In mainstream and closely-related theory, recent progress in reconstruction has included:

A. *In emission theory generally:*

- **Clearer proofs of the original FN result.**
- **Estimation of transmission pre-factor values for SN-barrier.**
- **Jensen's integrated theory of thermal-field effects.**
- **Also, similar theory for earthed spherical emitters.**
- **Exploration of field-fall-off effects for some non-planar emitters.**
- **Exploration of simple cases of quantum-confinement effects.**

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Some areas of recent progress

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B. ***In data-interpretation theory:***

- **Improvements in the concept of an intercept correction factor.**
- **Improvements in understanding of how to deal with emission from large-are field electron emitters (LAFEs).**
- **Some progress in extracting emission area from FN plots, for ideal devices**
- **Better realization of the difference between emission parameters and measured parameters, particularly for non-ideal devices.**
- **The idea of emission orthodoxy, and the introduction of a test for orthodoxy.**
- **The realization that many published field-enhancement-factor values must be spuriously large, and introduction of the idea of “phenomenological adjustment” as a partial remedy for this.**
- **Realization of the need to find ways to extract reliable characterization data in non-orthodox emission situations, including some interesting recent progress.**
- **Understanding the origin of "scaling" of current-voltage characteristics.**

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Other areas of recent progress

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C. *In supporting theory:*

- **Developments in theory of field-emitted vacuum space-charge.**
- **In field emitter electrostatics, developments in electrostatic interaction ("shielding") theory.**
- **Use of DFT theory to help determine field enhancement factors at the atomic level.**
- **Improved understanding of voltage-loss effects.**
- **Much detailed modelling of electroformation effects, increasingly using DFT theory.**

Areas of progress in reconstruction

There has also been significant progress in "non-mainstream" FE science, particularly in the context of FE from carbon nanostructures, and (more recently) in FE from semiconducting nanowire arrays, and in the general area of laser-stimulated FE, but these topics are outside the scope of this talk.

7. Immediately outstanding tasks

Immediately outstanding tasks

The more immediate tasks in developing FE science involve:

a) **Validating mainstream FE science where necessary, filling in gaps where necessary, removing unnecessary causes of confusion, and encouraging a more uniform approach.**

b) **Reaching out from mainstream FE science into adjacent areas, particularly those of topical interest where definitive progress seems possible.**

My list of the immediately outstanding tasks is as follows …..

Immediately outstanding tasks

1. **Encourage all FE work to be presented using exclusively the International System of Quantities (i.e., abandon 1960s-style use of Gaussian system equations).**
2. **Encourage the LAFE experimental community to abandon use of the discredited original 1928 Fowler-Nordheim equation (or simplified versions of it), in favour of a more modern FE equation that takes at least the discoveries of the 1950s (and preferably later improvements) into account. And discourage the use of defective equations.**
3. **Encourage more uniform use of notation, e.g., encourage everyone to denote the principal SN barrier function by the symbol v (or *v*).**
4. **Encourage use of the Gauss variable (and of scaled barrier field), rather than the Nordheim parameter, when discussing the SN barrier.**
5. **Develop a single coherent approach to extracting formal emission area from Fowler-Nordheim plots (when emission is orthodox), to replace the several slightly different methods currently being used.**

Immediately outstanding tasks

6. Develop further the theory of FN-plot analysis for situations where the emission is "non-orthodox".
7. In connection with the "orthodoxy test" and related issues, investigate the extent of the "spurious results" problem in the literature, and the extent to which additional information can be extracted from published papers.
8. Find means of investigating experimentally whether the classical image potential energy is a satisfactory approximate model for the exchange-and-correlation interaction between a departing electron and the emitter.
9. Find means of investigating experimentally what is the actual power of local field in the pre-exponential of Fowler-Nordheim-type equations.
10. Find means of making experimentally-based estimates of the value of the characteristic pre-exponential correction factor λ^{SN}.
11. Investigate further the theory of transmission near the top of a SN barrier, and investigate discrepancies reported some years ago.

Immediately outstanding tasks

12. Integrate into mainstream theory the more general "temperature-field" methods of calculating emission–current density developed by Jensen in recent years.
13. Establish improved methods of defining emission regimes.
14. Investigate further the issue of the validity of JWKB-type methods when the Schrödinger equation does not separate in cartesian coordinates.
15. Investigate further the theory of field electron microscope resolution for very-small-radius emitters, where the apparent experimental ability to "resolve carbon bonds" is incompatible with existing theory.
16. Attempt to relate the theory of FE from carbon nanotubes more closely with mainstream FE theory.

Other FE developments

Obviously, lack of time has prevented discussion of interesting theoretical developments in other parts of FE science, in particular:

- **laser stimulated FE;**
- **forms of FE related to internal FE and emission through films (and within electronic devices in general).**

And there are other aspects of FE (such as explosive FE, liquid metal electron sources, and FE from semiconductors and semiconductor nanowires) that deserve closer theoretical attention.

Thanks for your attention

Thanks for your attention

The general scope of FE science

I argue that FE science should cover all forms of field-assisted electron emission, including the sub-regime (caused by the Schottky effect) that can be described as "field-assisted thermal electron emission".

In this case, it seems logical to include effects that can be described as "field-assisted photoemission" or "photo-assisted field electron emission" or "laser-stimulated field electron emission". [But I regard these topics as "specialized FE"].

Four phases of mainstream theory

I see the development of FE mainstream theory as having taken place in four phases:

1. **Early history (before 1928)**
2. **The Fowler-Nordheim [FN] phase (1928 to mid-1950s)**
3. **The Murphy-Good [MG] phase (mid 1950s to mid 1990s)**
4. **Reconstruction (mid 1990s to present)**

The FN and MG phases involved major theoretical breakthroughs, but the present phase is one of many individual tidy-ups.

Thanks for your attention

Some items of recent progress

Progress item/Proposal 5.1:

To avoid problems over meaning of "β",

use "local conversion length"

rather than "voltage conversion factor"

The concept of local conversion length (LCL)

For a field emitter that is a good conductor, the local barrier field F_L at any point "L" on its surface can be written in the alternative forms

$$F_L = \beta_L V = V / \zeta_L = \zeta_L^{-1} V ,$$

where V is the applied voltage, β_L is the local voltage-to-barrier-field conversion factor (VCF), and ζ_L is the local conversion length (LCL).

Due to the multiple uses of the symbol "β" in current FE literature, I now prefer to use one of the formulae involving the LCL.

The LCL ζ_L is NOT a physical distance (except in special cases).

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Due to the multiple uses of the symbol "β" in current FE literature, I now prefer to use one of the formulae involving the LCL.

The LCL ζ_L is NOT a physical distance (except in special cases).

Obviously, if one chooses "L" to be some point "C" characteristic of the emitter (e.g., the emitter apex), then the characteristic local barrier field F_C can be written

$$F_C = \zeta_C^{-1} V ,$$

where ζ_C is a characteristic LCL. For STFEs, values often lie between 10 and 5000 nm. Gomer's formula (which applies to STFEs but not to LAFEs) is that

$$\zeta_C \approx 5\, r_a ,$$

where r_a is emitter apex radius.

LCLs for parallel-plate geometry

In parallel-plate geometry, one can define a (true) macroscopic field F_M and a (true) macroscopic field enhancement factor (FEF) γ_C by

$$F_M = V / d_{sep}, \quad \gamma_C = F_C / F_M,$$

where d_{sep} is the plate separation. Hence, for a LAFE the LCL is given by

$$\zeta_C = V / F_C = d_{sep} / \gamma_C .$$

In the case of a "hemisphere of radius r_a on a cylindrical post of height h", there is a well-known approximate formula (valid if $h \ll d_{sep}$)

$$\gamma_C \approx 0.7\, h/r_a .$$

Hence, if $h \ll d_{sep}$, an approximate formula for the LCL is:

$$\boxed{\zeta_C \approx 1.4\, (d_{sep}/h)\, r_a .}$$

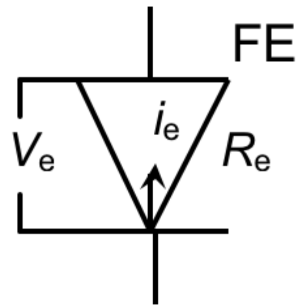
For a LAFE, this replaces Gomer's formula, and shows clearly that LCL-values depend both on the emitter geometry and on system geometry.

With this formula, the shape factor k_a [$\equiv \zeta_C / r_a$] is given by $k_a = 1.4\, (d_{sep}/h)$, and normally has values much greater than 5.

5.2: Distinguish between

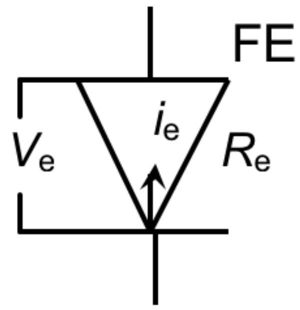
"emission parameters" and "measured parameters"

Emission voltage and emission current



For a diode field electron emitter, the emission current i_e is the current through the device (excluding leakage), and the emission voltage V_e is the voltage between the emitting region at the tip apex and the counter-electrode. [V_e is taken as positive and equal to the magnitude of the difference in Fermi levels].

Emission voltage and emission current



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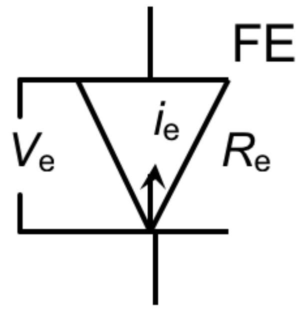
For a general tunnelling barrier, with barrier-form correction factor ν_F^{GB}, the emission current i_e is given by the $i_e(V_e)$-form general-barrier FN-type equation:

$$i_e = A_f^{GB} a\phi^{-1} (\zeta_C^{-1} V_e)^2 \exp[-\nu_F^{GB} b\phi^{3/2} / (\zeta_C^{-1} V_e)] ,$$

where A_f^{GB} is the relevant formal emission area, and a, b and ϕ have their usual meanings.

Emission resistance



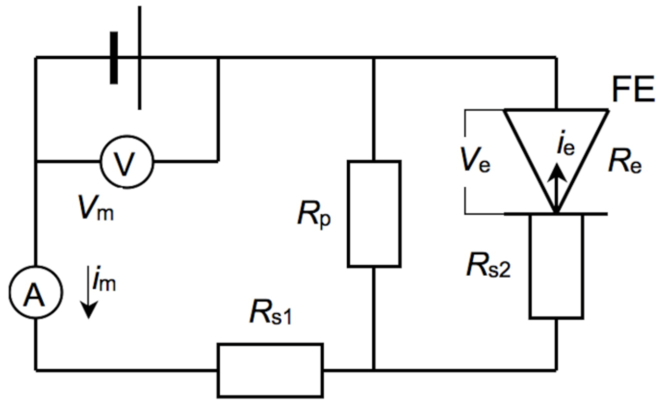
$$i_e = A_f^{GB}\, a\phi^{-1}\, (\zeta_C^{-1} V_e)^2 \exp[-\nu_F^{GB} b\phi^{3/2}/(\zeta_C^{-1} V_e)]$$

For purposes of electrical circuit theory, an FE diode is functionally similar to a p-n junction diode, and has an emission resistance R_e given by

$$R_e = V_e/i_e = (A_f^{GB}\, a\phi^{-1})^{-1}\, \zeta_C^{2}\, V_e^{-1} \exp[+\{\nu_F^{GB} b\phi^{3/2}\zeta_C\}\, V_e^{-1}]\,.$$

R_e is very large at low emission voltages, but decreases as V_e increases.

Emission parameters & measured parameters



Note:
This diagram uses the electron emission sign convention. Fields, currents, and related quantities are treated as positive, even though they would be negative in classical electromagnetism.

A schematic FE measurement circuit is shown above.

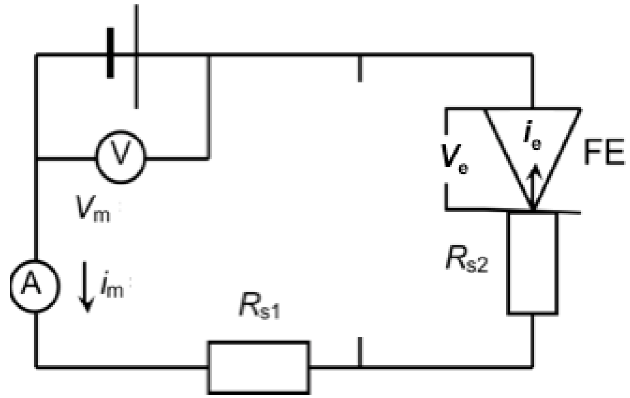
Due to the presence of series and parallel resistance —

the emission current i_e is NOT equal to the measured current i_m , and
the emission voltage V_e is NOT equal to the measured voltage V_m .

In practice, parallel resistance can usually be made effectively infinite by careful system design; but series resistance often cannot be eliminated.

In this case, we get …

Measurement circuit theory



$R_e = V_e / i_e$

$R_s = R_{s1} + R_{s2}$

From the usual elementary law:

$$V_e = \{R_e / (R_e + R_s)\} V_m .$$

Hence define voltage ratio Θ by

$$\Theta = V_e / V_m = R_e / (R_e + R_s) .$$

At low emission voltages R_e is very large, and $\Theta = 1$.
As emission voltage increases, a point may be reached where it is no longer true that $R_s \ll R_e$, and then Θ begins to decrease from unity.

$i_m(F_A)$-form general-barrier FN-type equation

With LAFEs in planar-parallel-plate geometry, it is common literature practice to make FN plots in terms of a mathematical parameter F_A given by

$$F_A = V_m/d_{sep} ,$$

where d_{sep} is the plate separation. I call F_A the apparent macroscopic field.

It can be shown that, in terms of F_A (when $i_m = i_e$), the measured current i_m is given by

$$i_m = A_f^{GB} a\phi^{-1} (\gamma_C \Theta F_A)^2 \exp[-\nu_F^{GB} b\phi^{3/2}/(\gamma_C \Theta F_A)] ,$$

where γ_C is the true macroscopic field enhancement factor.

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where γ_C is the true macroscopic field enhancement factor.

Since the usual procedures for interpreting FN plots only work for the range of V_m where $\Theta \approx 1$, they also only work for the range of F_A where $\Theta \approx 1$. Outside this range, spurious results are generated. This is also true if the macroscopic current density J_M is used to make the plot.

5.3: Classify and name the many different forms of FN-type equation used in past and present literature.

This should help discussion of which form is "best for a given purpose".

Here is my attempt to impose order.

5.3a:

Define "kernel current density" and local pre-exponential correction factor

Thanks for your attention

Improvements in mathematical understanding of the principal SN barrier function v(*x*) and in the presentation of related theory

Auxiliary parameters for the independent variables "X"

To relate characteristic local barrier field F_C to independent variables X, write the auxiliary equation

$$F_C \equiv c_X X ,$$

where c_X is the relevant auxiliary parameter ….

Auxiliary parameters for the independent variables "X"

TABLE 3: Independent variables, and main related auxiliary parameters and equations

Independent variable name and symbol		**links to** (symbol)	**via auxiliary parameter** name and symbol		**Formulae**
Theoretical variables					
Characteristic local barrier field	F_C	-	-	-	-
Scaled barrier field	f	F_C	Reference field	F_R	$F_C = f F_R$
Emission variables					
Emission voltage	V_e	F_C	(True) local voltage-to-barrier-field conversion factor (VCF)[a]	$\beta_{V,C}$	$F_C = \beta_{V,C} V_e$
Emission voltage	V_e	F_C	(True) local conversion length (LCL)	ζ_C	$F_C = V_e / \zeta_C$
True macroscopic field	F_M	V_e	(True) macroscopic conversion length[b]	ζ_M	$F_M = V_e / \zeta_M$
True macroscopic field	F_M	F_C	(True) (electrostatic) macroscopic field enhancement factor (FEF)	γ_C	$F_C = \gamma_C F_M$ $\gamma_C = \zeta_M / \zeta_C$
Measured variables					
Measured voltage	V_m	V_e	Voltage ratio	Θ	$V_e = \Theta V_m$
Measured voltage	V_m	F_C	Measured-voltage-defined LCL[c]	$\zeta_C^{mvd} = \zeta_C / \Theta$	$F_C = V_m / \zeta_C^{mvd}$ $F_C = V_m \Theta / \zeta_C$
Apparent macroscopic field	F_A	V_m	Macroscopic conversion length	ζ_M	$F_A = V_m / \zeta_M$
Apparent macroscopic field	F_A	F_M	Voltage ratio	Θ	$F_M = \Theta F_A$
Apparent macroscopic field	F_A	F_C	Apparent-field-defined FEF[d]	$\gamma_C^{afd} = \gamma_C \Theta$	$F_C = \gamma_C^{afd} F_A$ $F_C = \gamma_C \Theta F_A$

[a] Future use of the parameter $\beta_{V,C}$ is discouraged: use ζ_C and related formulae instead.
[b] In planar-parallel-plate geometry, ζ_M is normally taken as equal to the plate separation d_{sep}.
[c] Use of the parameter ζ_C^{mvd} is discouraged: use the combination (ζ_C / Θ) instead.
[d] Use of the parameter γ_C^{afd} is discouraged: use the combination ($\gamma_C \Theta$) instead.

Expansion of universal-form FN-type equation

The generalized (or "universal") form of FN-type equation given earlier was

$$Y = C_{YX} X^2 \exp[-B_X / X] \, .$$

From above, it can be found that:

$$B_X = \nu_F b \phi^{3/2} / c_X X \, ,$$

$$C_{YX} = c_Y a \phi^{-1} c_X^2 \, .$$

Different choices for the three "pink" parameters lead to different mathematical forms of FN-type equation - in total, there are many hundreds of options.

But, physically, the main distinguishing feature is the equation complexity level.

Thanks for your attention

Motive energy & the Schottky-Nordheim barrier

The one-dimensional Schrödinger equation (along z) can be written:

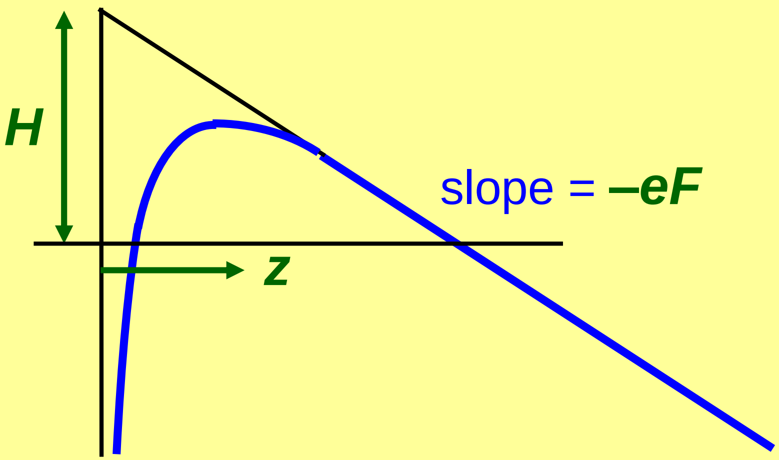
$$(\hbar^2/2m)\mathrm{d}^2\Psi_z/\mathrm{d}z^2 = [U(z)-E_z]\Psi_z = M(z)\Psi_z\,,$$

where $M(z)$ [$\equiv U(z)-E_z$] can be called the motive energy, and the other symbols have their usual meanings.

A range of z where $M(z)\geq 0$ constitutes a tunnelling barrier. The mathematical form of $M(z)$ defines the barrier form.

In FE theory, there is particular interest in the Schottky-Nordheim (SN) barrier, shown alongside, as this is the simplest realistic barrier form.

Schottky-Nordheim (SN) barrier



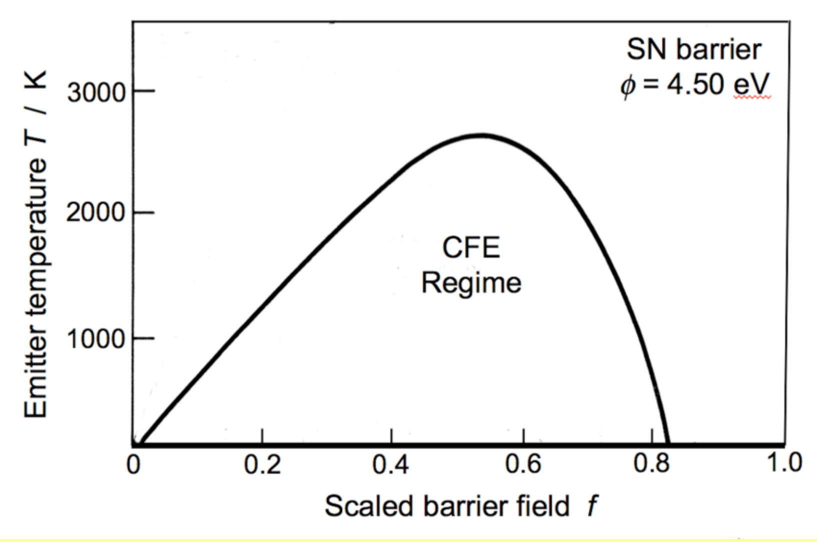
$$M(z) = H - eFz - 1/16\pi\varepsilon_0 z$$

used by **Murphy & Good**

Emission-current-density regimes for CFE

As already indicated, an emission-current-density is a region of parameter space (either $\{F,T\,|\phi\}$ or $\{f,T\,|\phi\}$) where a particular expression for emission current density is approximately valid.

Thus, for a free-electron metal, the CFE regime is where FN-type equations are approximately valid:



Drawing these emission regime diagrams with f rather than F on the horizontal axis gives a slightly more general picture. Note that the CFE regime extends to quite high temperature.

Thanks for your attention

5.3e:

Introduced the concepts of

scaled barrier field,

and the scaled form for J_{kC}^{SN}

The concept of scaled barrier field

The reference barrier field F_R that reduces to zero the height of an SN barrier of zero-field height equal to the local work function ϕ is

$$F_R = c^{-2}\phi^2 \approx (0.694\ 4616\ \mathrm{eV^{-2}\ V\ nm^{-1}})\ \phi^2\ ,$$

where c is the Schottky constant.

The *scaled barrier field* f is related to the characteristic local barrier field F_C by

$$f = F_C/F_R\ .$$

Derivation of scaled form for J_k^{SN}

Define ϕ-dependent parameters $\eta(\phi)$ and $\theta(\phi)$ by

$$\eta(\phi) = b\phi^{3/2}/F_R = bc^2\phi^{-1/2} ,$$

$$\theta(\phi) = a\phi^{-1}F_R^{\ 2} = ac^{-4}\phi^3 ,$$

where bc^2 [≈ 9.836239 eV$^{1/2}$] and ac^{-4} [≈ 7.433979×10^{11} A m^{-2} eV^{-3}] are universal constants.

In scaled form, the equation for J_k^{SN} becomes

$$J_k^{SN} = \theta f^2 \exp[-\eta\, v(f)/f] .$$

For example, for ϕ= 4.50 eV, then F_R≈ 14.1 V/nm, η≈ 4.64, θ≈ 6.77×10^{13} A/m^2.

This scaled equation contains only one field-like variable (f), and (as discussed below) a good simple approximation exists for $v(f)$. Hence J_k^{SN} is well approximated by

$$J_k^{SN} \approx \theta f^2 \exp[\eta\cdot\{1 - (1/6)\ln f - 1/f\}].$$

This formula is useful because it can be evaluated on a spreadsheet.

Thanks for your attention

5.6 – 5.9

The idea of emission orthodoxy

The orthodoxy test

Phenomenological adjustment

Data extraction for non-orthodox emitters

Presented elsewhere recently,

[see: http://dx.doi.org/10.13140/RG.2.1.2536.1361]

so will not be repeated here

Thanks for your attention

5.10: Improvement in the extraction of formal emission area A_f

Procedure if orthodox: area-parameter extraction

If the emission situation is orthodox, then

(a) a reliable field enhancement factor (FEF) value can be extracted;

(b) valid estimates of area-like parameters can be extracted.

There are two main ways of extracting area, as follows.

A) <u>BY FORMULA</u>

Let S^{fit} be the slope, and $\ln\{I^{\mathrm{fit}}\}$ the intercept, of a line fitted to a FN plot made using natural logarithms.

For a current-based plot, the formal emission area A_{f}, or for a macroscopic-current-density-based plot, the formal area efficiency α_{f}, is given by the formula

$$A_{\mathrm{f}} \text{ or } \alpha_{\mathrm{f}} = \Lambda \cdot I^{\mathrm{fit}} (S^{\mathrm{fit}})^2 .$$

The value of Λ depends on the choice of barrier form:

for the exactly triangular (ET) barrier: $\Lambda^{\mathrm{ET}} \sim 687\ \mathrm{nm^2/A}$;

for the Schottky-Nordheim (SN) barrier: $\Lambda^{\mathrm{SN}} \sim 6.1\ \mathrm{nm^2/A}$.

Procedure if orthodox: area-parameter extraction

(B) BY AREA-DEPENDENCE PLOTS

For any given barrier, under orthodox emission conditions (which need Θ=1), the current i_e [= i_m] and macroscopic current density J_M are related to the kernel current density J_{kC} for the barrier by

$$i_e = A_f J_{kC} ,$$

$$J_M = \alpha_f J_{kC} ,$$

$$J_{kC}(F_M) = a\phi^{-1}(\gamma_C F_M)^2 \exp[-\nu_F b\phi^{3/2}/(\gamma_C F_M)] ,$$

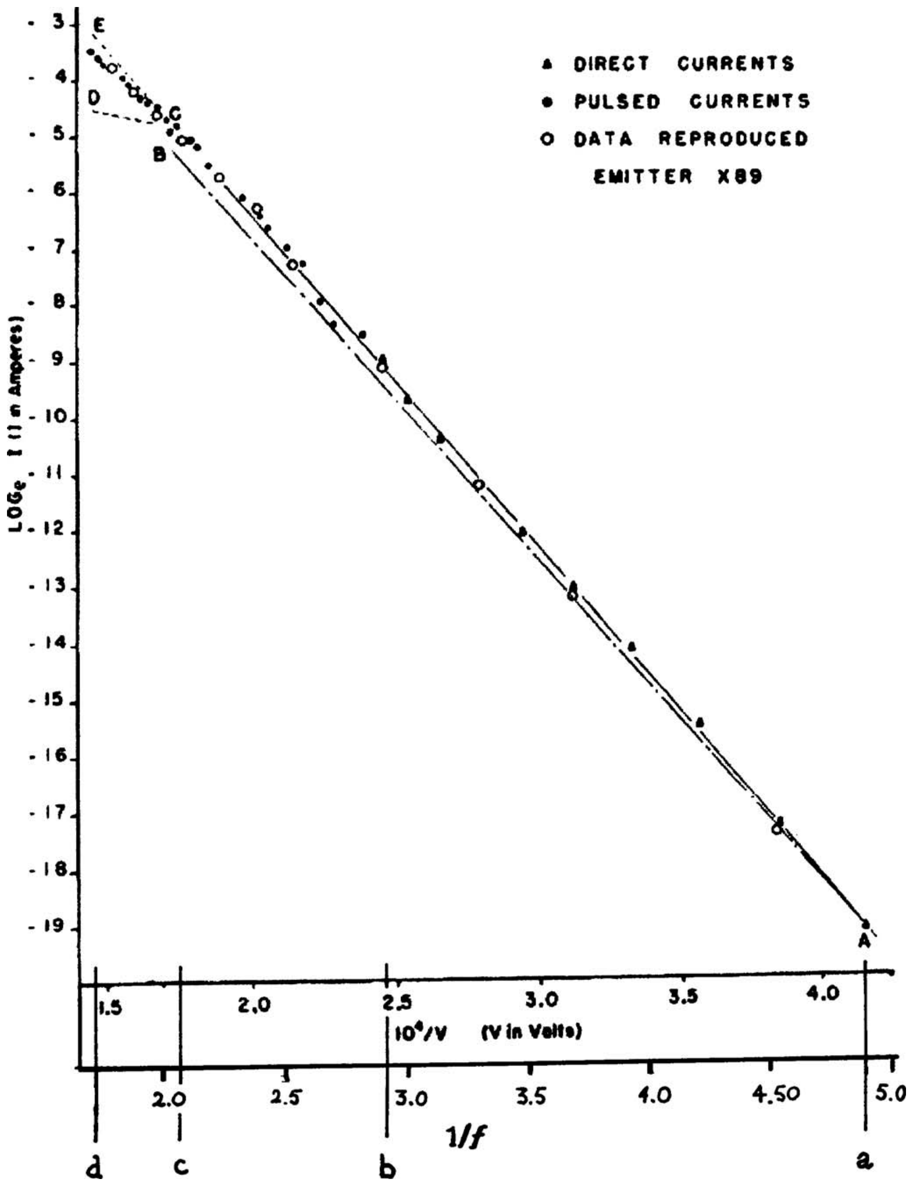
where ν_F is the barrier-form correction factor for the chosen barrier.

For both the ET and SN barriers, accurate estimates of $J_{kC}(F_M)$ can be obtained from the slope of the FN plot, and the known values of ϕ and F_M .

Since $i_e(F_M)$ [or, alternatively, $J_M(F_M)$] is known "from experiment", each point on the FN plot will yield (for given choice of barrier form) an "experimental" estimate of A_f [or, alternatively, α_f].

A similar procedure works if plots are made as a function of voltage.

Procedure if orthodox: area-parameter extraction

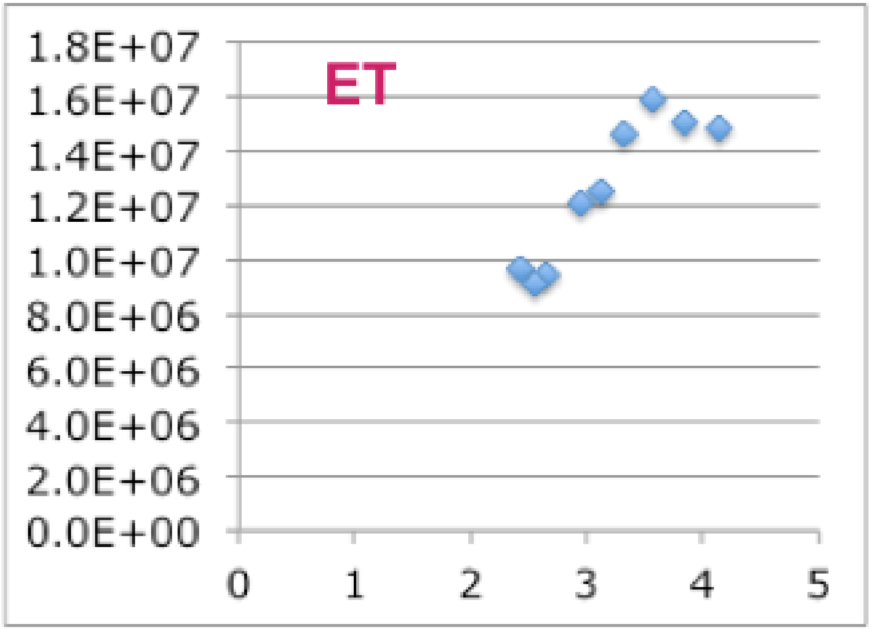


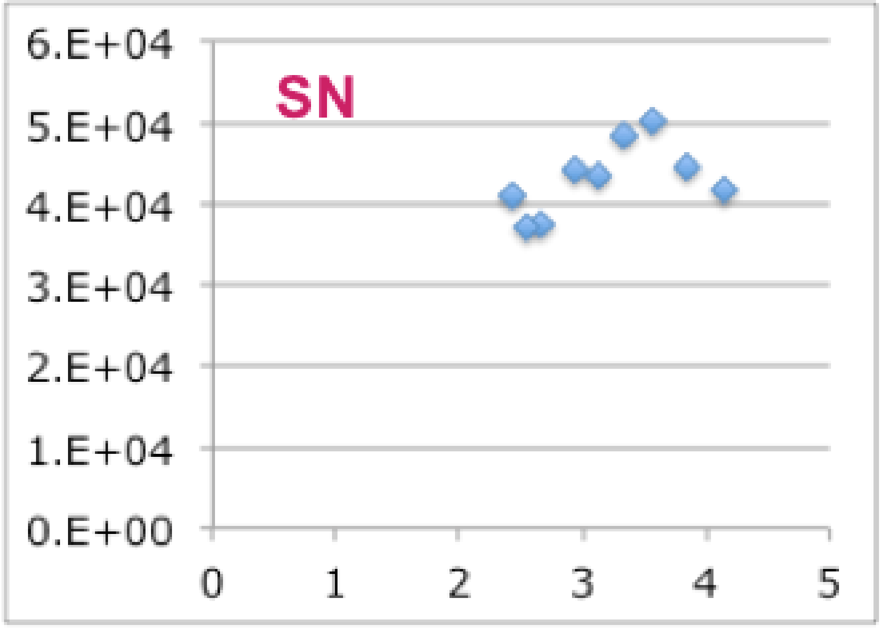
These procedures have been applied to results taken from Dyke and Trolan (1953) (DT) emitter X89.

Independent assessments of emission area, made by electron microscopy (EM), exist for this emitter.

FIG: Current-voltage characteristic for combined pulse and direct current operation of Dyke and Trolan 1953 (DT) emitter X-89. All lines except the continuous line drawn through the data points may be disregarded. Reproduced from Fig. 3 in DT, except that a 1/*f* axis and four markers have been added.

$A_f(V_m^{-1})$ plots for ET & SN barriers (for X89)





Horizontal scales: $\times 10^{-4}$ volts^{-1}

Vertical scales: nm^2

Results for emission area, for X89

TABLE 1. Estimates of emission area for emitter X89				
Method	**Using ET barrier**		**Using SN barrier**	
	[a] Point A	**[a] Point C**	**[a] Point A**	**[a] Point C**
	All areas shown below are in units of 10^4 nm^2			
Extraction parameter method	400		3.6	
$A_f(V_m^{-1})$ plot	1500	970	4.2	4.1
Electron microscopy	7.6	12	7.6	12

[a] Points A (at low voltage) and C (at high voltage) are as defined in DT.

ET barrier: **Results do NOT agree well with each other, and are massively different from electron microscopy results.**

SN barrier: **Results for both extraction methods agree adequately with each other, and with electron microscopy (EM).**

[For SN barrier, residual discrepancies between FN and EM results are not important *in this test*, because formal area is not an exact measure of geometrical emission area.]

Results for emission area, for X89

TABLE 1. Estimates of emission area for emitter X89

Method	Using ET barrier		Using SN barrier	
	[a] **Point A**	[a] **Point C**	[a] **Point A**	[a] **Point C**
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Extraction parameter method	400		3.6	
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Electron microscopy	7.6	12	7.6	12

[a] Points A (at low voltage) and C (at high voltage) are as defined in DT.

These results are probably as close as we shall get to "experimental proof" that (for metals) the SN barrier is a better model than the ET barrier.

Data analyses should use the SN barrier as a starting point.

Thanks for your attention

6. Outstanding problems in mainstream FE science – overview

Outstanding problems of mainstream FE science

The outstanding problems of mainstream FE science tend to be of the following kinds.

Scientific issues

(1) In some places, physical understanding and/or appropriate physical models are still lacking.

(2) In some places, mathematical or numerical analysis, and/or simulations, have not been carried out, and would be useful.

(3) In some places, experiments are needed in order to resolve theoretical issues that are either contentious or unresolved.

(4) In places, there are "tidy-up" or "theory-detail" issues that could usefully be dealt with.

Formulation and Presentation issues

(5) In places, improvements in notation, terminology and presentation would almost certainly be helpful.

(6) In places, more-transparent derivations are needed for existing results.

Outstanding problems of mainstream FE science

Data-extraction and device characterization issues

(7) There is an urgent need for better approaches to interpreting experimental results and characterizing technological devices, particularly for large-area field emitters (LAFEs).

Community issues

(8) The literature contains many defective formulae and many spurious results, in particular spuriously large values of field enhancement factor (FEF).

(9) Many authors, reviewers and editors do not have adequate understanding of field-emission-related theory.

(10) There would be merit in trying to encourage greater uniformity in the use of notation and terminology.

Outstanding problems of mainstream FE science

At present, the most important problems seem to be the community problems. Will briefly illustrate one of these: the problem of defective equations.

7. Example: The need to replace defective equations

Problem 1: Ambiguous meanings of F and J

Equation typically found in LAFE literature is:

$$J \quad = \quad a\phi^{-1}\,(\beta E)^2\,\exp[-b\phi^{3/2}/(\beta E)]$$

In RGF preferred notation for fields and FEFs (not a problem), this becomes:

$$J \quad = \quad a\phi^{-1}\,(\gamma_C F)^2\,\exp[-b\phi^{3/2}/(\gamma_C F)]$$

Problem 1: Ambiguous meanings of *F* and *J*

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In RGF preferred notation for fields and FEFs (not a problem), this becomes:

$$J \;=\; a\phi^{-1}(\gamma_C F)^2 \exp[-b\phi^{3/2}/(\gamma_C F)]$$

Problem 1: **Meanings of *F* and *J* are usually not well defined.**

Solution 1: **Assume authors usually mean true macroscopic field F_M and macroscopic (i.e., average) current density J_M .**

So change equation to read

$$J_M \;=\; a\phi^{-1}(\gamma_C F_M)^2 \exp[-b\phi^{3/2}/(\gamma_C F_M)].$$

Problem 2: Wrong barrier form

Solution 1: Change equation to read

$$J_M \;=\; a\phi^{-1}(\gamma_C F_M)^2 \exp[-b\phi^{3/2}/(\gamma_C F_M)]\ .$$

Problem 2: This equation assumes an exactly triangular (ET) barrier. It is known that this underpredicts current densities by a factor typically between 200-500, and gets area estimates incorrect. The Schottky-Nordheim (SN) barrier is considered to be a much better model.

Solution 2: Insert the barrier-form correction factor for the SN barrier, which is (an appropriate value v_F of) the principal SN barrier function v. So equation becomes

$$J_M \;=\; a\phi^{-1}(\gamma_C F_M)^2 \exp[-v_F b\phi^{3/2}/(\gamma_C F_M)]\ .$$

Problem 3: Area efficiency omitted

Solution 2: Equation becomes

$$J_M \;=\; a\phi^{-1}(\gamma_C F_M)^2 \exp[-v_F b\phi^{3/2}/(\gamma_C F_M)]\ .$$

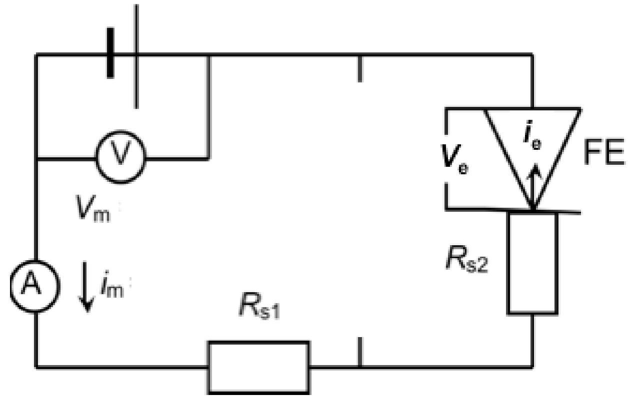
Problem 3: This equation assumes uniform emission across the whole LAFE area. But LAFEs consist of many individual emitters, and only a small fraction of the total area emits. This fraction is not well known, but is thought to be in the range 10^{-12} to 10^{-4} .

Solution 3: Insert a term α_f^{SN} called the formal area efficiency (for the SN barrier). So equation becomes:

$$J_M \;=\; \alpha_f^{SN} a\phi^{-1}(\gamma_C F_M)^2 \exp[-v_F b\phi^{3/2}/(\gamma_C F_M)]\ .$$

[The "SN" is added, because the value you deduce "from experiment" depends on the choice of barrier form.]

Measurement circuit theory



$$R_e = V_e / i_e$$

$$R_s = R_{s1} + R_{s2}$$

REMEMBER (from earlier): When series resistance is present, the emission voltage V_e is not equal to the measured voltage V_m. These are related by the voltage ratio Θ given by

$$\Theta = V_e / V_m = R_e / (R_e + R_s) .$$

Hence, there are two different types of macroscopic field:

- **True macroscopic field F_M:** $F_M = V_e / d_{sep}$
- **Apparent macroscopic field F_A:** $F_A = V_m / d_{sep} = \Theta^{-1} F_M$

In LAFE literature, the quantity actually used to plot FN plots is usually the apparent macroscopic field. Hence we get …

Problem 4: Series resistance ignored

Solution 3: Equation becomes

$$J_M = \alpha_f^{SN} a\phi^{-1} (\gamma_C F_M)^2 \exp[-v_F b\phi^{3/2}/(\gamma_C F_M)] \,.$$

Problem 4: Wrong type of macroscopic field often used. Dividing measured voltage by plate-separation gives apparent macroscopic field F_A .

Solution 4: Replace F_M by ΘF_A . So equation becomes:

$$J_M = \alpha_f^{SN} a\phi^{-1} (\gamma_C \Theta F_A)^2 \exp[-v_F b\phi^{3/2}/(\gamma_C \Theta F_A)]$$

Problem 5: True FEF ≠ Apparent FEF

Problem 5: Most of the literature fails to distinguish between a "true-FEF" and an "apparent FEF" (or pseudo-FEF).

Solution 5a: Introduce idea of slope characterization parameter. Here, this is an "apparent FEF" β^{app} defined by

$$\beta^{app} = - b\phi^{3/2} / S^{fit} ,$$

where S^{fit} is the slope of the line fitted to the FN plot.

Solution 5b: Introduce equation containing a slope correction factor σ_t . Here, this is an equation for the true-FEF γ_C (i.e., β^{true}), namely

$$\gamma_C \text{ (i.e., } \beta^{true}) = \sigma_t \cdot \beta^{app} = - \sigma_t b\phi^{3/2} / S^{fit} .$$

Problem 6: Is FEF spuriously large ?

Solution 5b: Introduce equation containing a slope correction factor σ_t .

$$\gamma_C = \sigma_t \cdot \beta^{app} = -\sigma_t b\phi^{3/2} / S^{fit} .$$

Problem 6: If emission is orthodox, then $\sigma_t \approx 0.95$.

If emission is non-orthodox, then:

(a) σ_t may be <<1 ; and

(b) published β^{app} values are unreliable as estimates of γ_C (i.e., β^{true}), and may be spuriously large.

In many or most cases, nobody knows which FEF-value estimates published in the last 20 years are reliable.

Solution 6: This is the problem that my orthodoxy test was designed to solve [see Proc. R. Soc. Lond A 469 (2013) 20130271]. The test will indicate whether an emission situation is "orthodox", and hence whether the extracted FEF value is likely to be reliable.

Thanks for your attention

Scaled barrier field f

When the Schottky reduction Δ is equal to the work function ϕ, we have

$$\Delta = \phi = c_S F_R^{1/2} = (e^3/4\pi\varepsilon_0)^{1/2} F_R^{1/2} ,$$

where F_R [$= c_S^{-2}\phi^2$] is the reference field needed to reduce to zero a barrier of zero-field height ϕ.

The scaled barrier field f is defined by

$$f = F / F_R = c_S^2 \phi^{-2} F .$$

This dimensionless parameter f plays an important role in modern FE theory.

Relevant numerical values are:

$$c_S = 1.199\,985 \text{ eV (V/nm)}^{-1/2}$$

$$c_S^2 = 1.439\,965 \text{ eV}^2 \text{ (V/nm)}^{-1}$$

[In these tutorials, all universal constants are given to 7 significant figures.]

Electron motive energy $M(z)$

For the ET barrier, an exact approach to calculating D is possible.

In general, for most rounded barriers, an approximate approach (for planar geometry) proceeds via the following steps.

- **Separate the Schrödinger equation into Cartesian coordinates, and let z the direction normal to the emitter surface.**
- **Let Ψ_z be the electron wave-function component for the z-direction.**
- **Define the related kinetic-energy operator by $K_z \equiv -(\hbar^2/2m_e)\, \partial^2 \Psi_z/\partial z^2$, where $\hbar$ is Planck's constant divided by 2π.**
- **Write the related Schrödinger-equation component as**

$$[K_z + U^{tot} - E_z]\ \Psi_z \ \equiv\ [K_z + M(z)]\ \Psi_z \ =\ 0\ ,$$

where, as before, U^{tot} is the (total) electron potential energy, and E_z [$\equiv E_n$] is the electron forwards energy.

The parameter $M(z)$ [$= U^{tot} - E_z$] is termed the electron motive energy, and characterizes the transmission barrier seen by an electron with forwards energy E_z .

Thanks for your attention

Scaled barrier field for the SN barrier

Consider a SN barrier of zero-field height ϕ .

The reference field F_R that reduces the barrier height to zero is

$$F_R = c^{-2}\phi^2 \approx (0.694 \text{ eV}^{-2} \text{ V nm}^{-1})\, \phi^2 ,$$

where c is the Schottky constant.

The *scaled barrier field* f is related to the barrier field F_C by

$$f = F_C/F_R .$$

The kernel current density for the SN barrier, J_k^{SN}, can be written

$$J_k^{SN} = a\phi^{-1}F_C^{2} \exp[-v(f)\, b\phi^{3/2}/F_C] ,$$

where $v(f)$ is the principal SN barrier function, expressed as a function of f.