Cosmology I: Measuring and weighing the Universe

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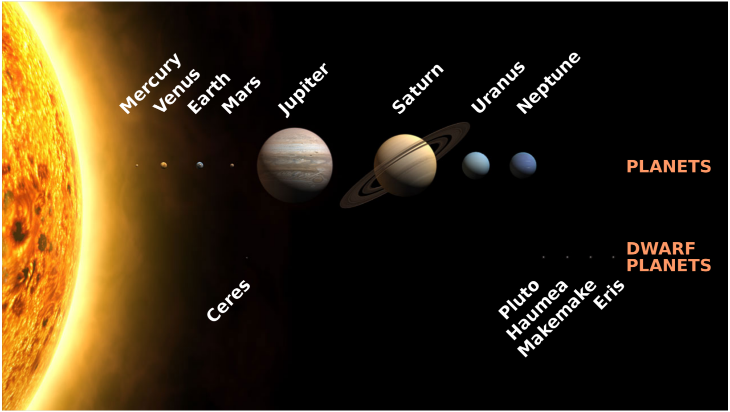
Distances

Radius of the Earth: $\simeq$ 6000km

Distances

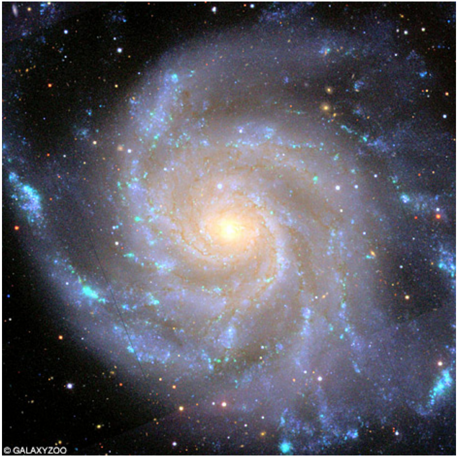
The solar system $\simeq 7 \times 10^9$km ($\simeq$ 50au =50 'astronomical units')
1 au $\simeq 1.5 \times 10^8$km is the average distance between the earth and the sun

(source: wikipedia)



Distances

The Milky Way (visible part)$\simeq 10^{18}$km $\simeq 10^5$ light years $\simeq 30'000$parsec



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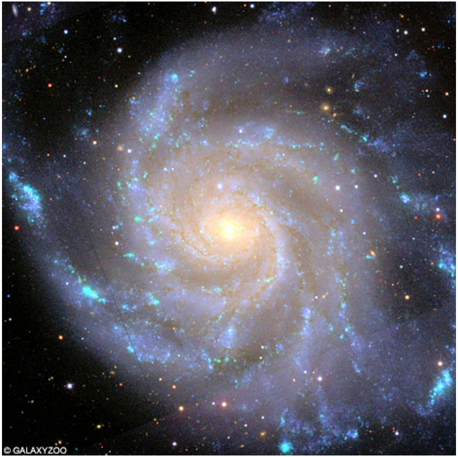
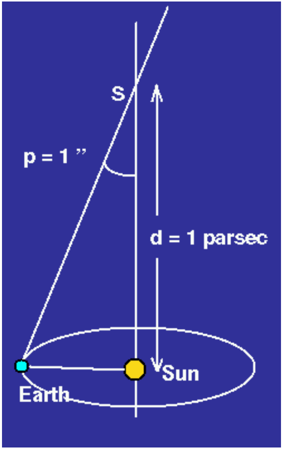


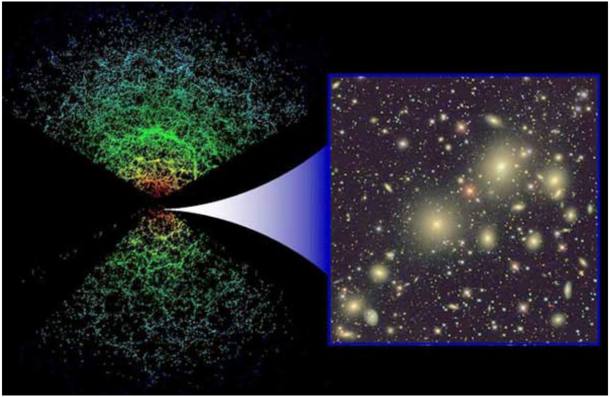


$$1'' = 1^o/3600 = 1\text{arc second}$$

$$1\text{parsec} \simeq 3.26 \text{ light years}$$

Distances

The size of the 'visible Universe' (Hubble scale) $\simeq$ 28000Mpc $= 2.8 \times 10^{10}$parsec
(Contains about 0.5×10^{12} galaxies like the Milky Way with mass of about $10^{12} M_{\odot}$)



Each point represents a galaxy
(Sloan digital sky survey, SDSS)

The Universe is expanding

As you know, Newtonian gravity is an attractive force.
Each mass is attracted by every other mass.

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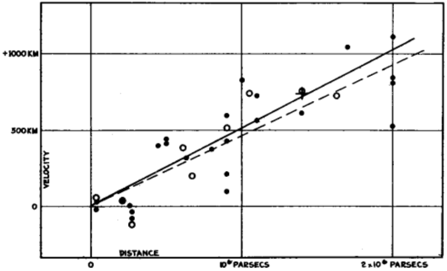
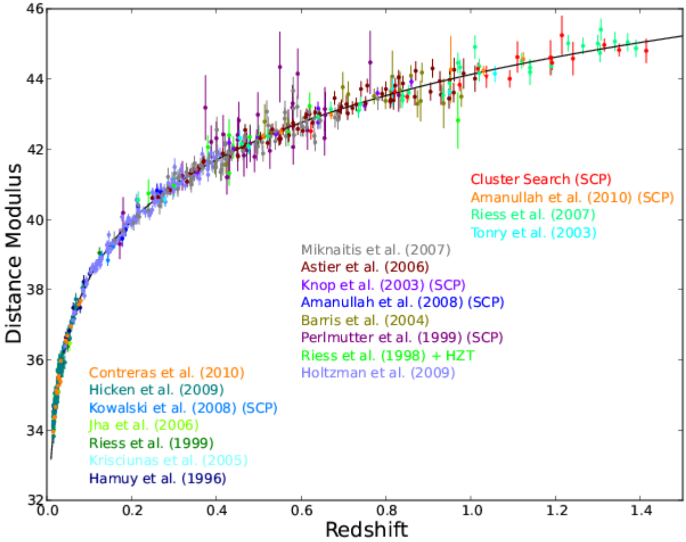
Despite this fact, observations show that the Universe is expanding.
Galaxies recede from each other with a speed proportional to their distance,

$$v = \dot{R} = H_0 \cdot R \qquad \text{(Hubble's law, } H_0 \simeq 70\text{km/s/Mpc)}$$



(Hubble 1932)



(The Supernova Cosmology Project)

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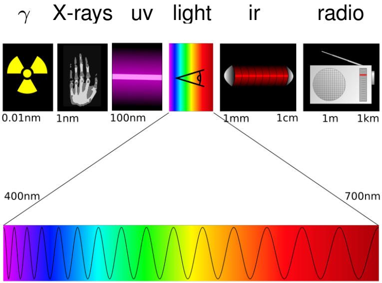
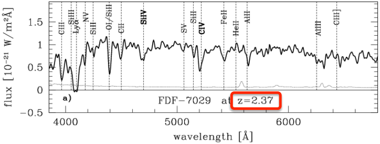
- To mesure the speed we mesure the redshift. This is the Doppler effect for light:

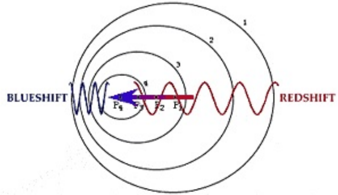
$$z = \frac{\lambda - \lambda_e}{\lambda_e} \simeq v/c \,, \qquad \text{if } \; z \ll 1 \qquad \left(z = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \right) .$$

Astronomical observations can be made in different wavelengths bands of the electromagnetic spectrum. In the optical band specific spectral lines (atomic transitions) are at fixed wavelength





In a source moving away from us these spectral lines are shifted towards the red, 'redshifted'



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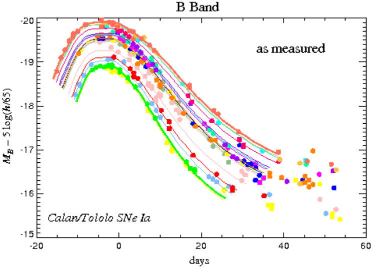
- A moment in the past can be characterized by its redshift z.
- The present expansion rate of the Universe is $H_0 \simeq 70$km/s/Mpc. In the past it has been different. We want to determine the expansion rate as function of the redshift, $H(z)$. For this we have to measure the redshift z and the distance d of far away galaxies.

The most powerful standard candles are supernovae of type Ia.

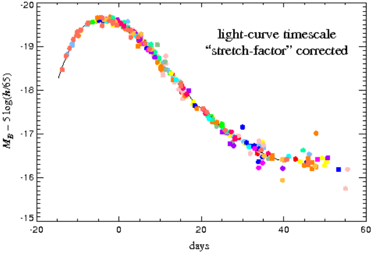
(SN1994D)

SNIa light curve

After application of a 'stretch factor' the maximum of the light curve, i.e. the maximum of the luminosity is nearly the same for all supernovae Ia.



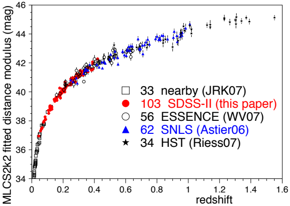
Without correction.



Kim, *et al.* (1997)

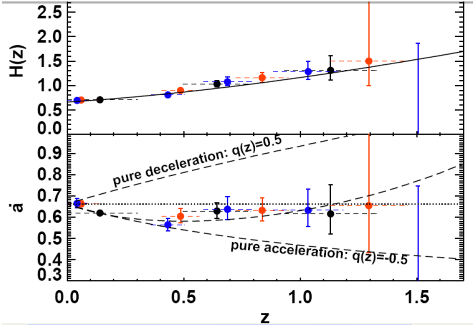
After correction.

The Universe is in *accelerated* expansion



(Kessler et al. 2009)

Distance modulus
$= \log(\text{apparent luminosity}) +$ constant
$= \log(1/d_L^2) +$ constant



(Riess et al. 2007)

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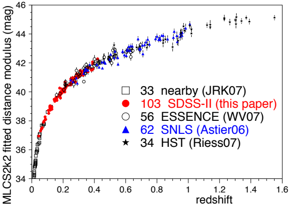

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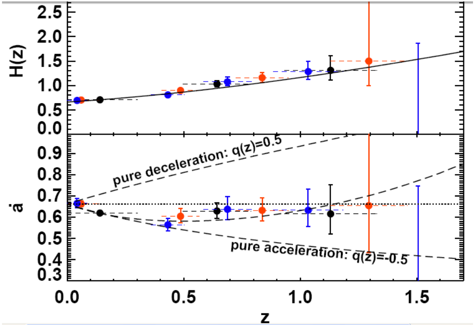
$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$



(Riess et al. 2007)

$$H = \frac{\dot R}{R} = (1+z)\dot a(z)$$

$$a = R/R_0$$

$\dot R(z) > 0\,,\quad \ddot R(z) > 0$ for $z < 0.5$.

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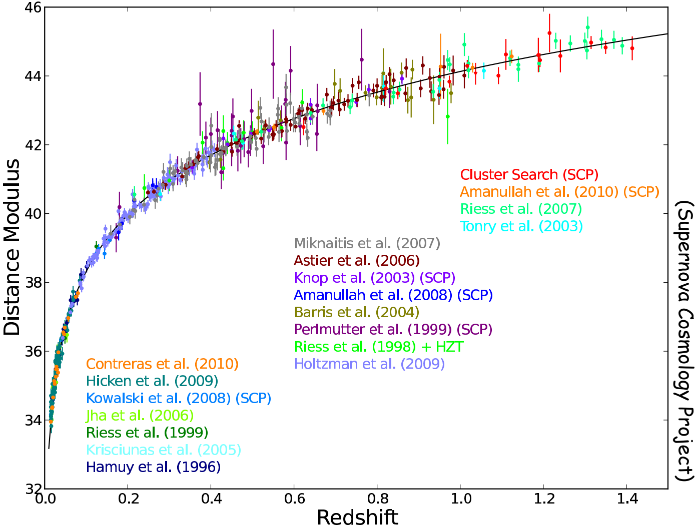


Latest Hubble diagram

Nobel Prize in Physics 2011

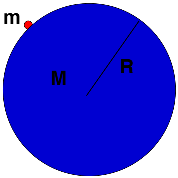
Adam G. Riess

Brian P. Schmidt

Saul Perlmutter

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

We consider a test mass m at the border of a homogeneous sphere of density ρ, which is expanding with velocity $v = \dot{R}$.



$M = (4\pi/3)R^3\rho$

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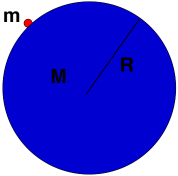

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Its energy is

$$E = \frac{m}{2}v^2 + U = \frac{m}{2}v^2 - \frac{GmM}{R} = \frac{m}{2}v^2 - \frac{4\pi G}{3}m\rho R^2$$

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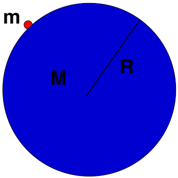

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As energy is conserved, $2E/m =: -K = \text{constant} = \dot{R}^2 - 8\pi G\rho R^2/3$. With $H^2 = \left(\frac{\dot{R}}{R}\right)^2$ we obtain

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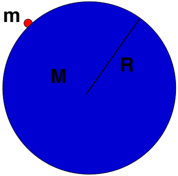

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This is the Friedmann equation (1922).

Due to the expansion, the density decreases,

$$\rho = \frac{M}{\frac{4\pi}{3} R^3}\,, \qquad \dot{\rho} = -3\rho\frac{\dot{R}}{R}$$

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$$\frac{d}{dt}\left[\left(\frac{\dot R}{R}\right)^2 + \frac{K}{R^2}\right] = 2\left[\frac{\ddot R}{R} \underbrace{-\left(\frac{\dot R}{R}\right)^2 - \frac{K}{R^2}}_{-8\pi G\rho/3}\right]\frac{\dot R}{R} = \frac{8\pi G}{3}\dot\rho = -8\pi G\rho\frac{\dot R}{R}$$

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This is the 2nd Friedmann equation (1922). It requires that the expansion decelerates!

The expansion of the Universe within General relativity

Including general relativity these equations are modified:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3c^2}\rho_E + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho_E + 3P) + \frac{\Lambda}{3}$$

P is the pressure and Λ is the cosmological constant,
ρ_E is the energy density. For ordinary matter $\rho_E = c^2\rho$, and c is the speed of light.
K now has a new interpretation. It is the curvature of space.

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Introducing the 'density' parameters

$$\Omega_m = \frac{8\pi G\rho_E}{3c^2H^2}\,, \qquad \Omega_K = -\frac{K}{R^2H^2}\,, \qquad \Omega_\Lambda = \frac{\Lambda}{3H^2}\,,$$

the first Friedmann eqn. becomes

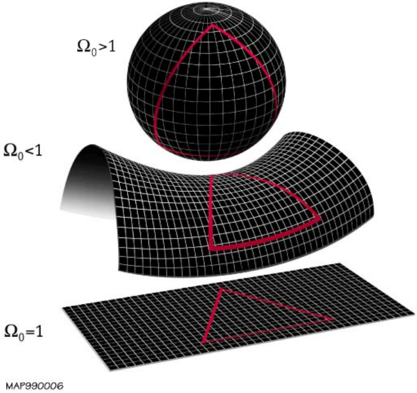
$$\Omega_m + \Omega_\Lambda + \Omega_K = 1\,.$$

Curvature

$K > 0$ ($\Omega_K < 0$): spherical space,

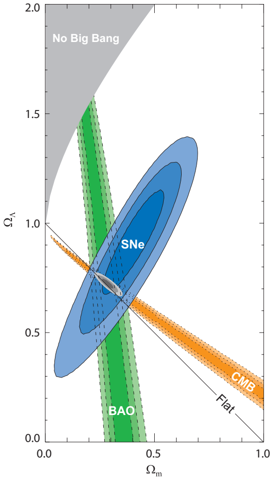
$K < 0$ ($\Omega_K > 0$): pseudo-spherical space (saddle),

$K = 0$ ($\Omega_K = 0$): flat space.

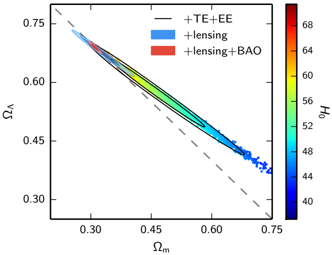


The Universe is in *accelerated* expansion

Matter, Ω_m, and cosmological constant, Ω_Λ (dark energy).



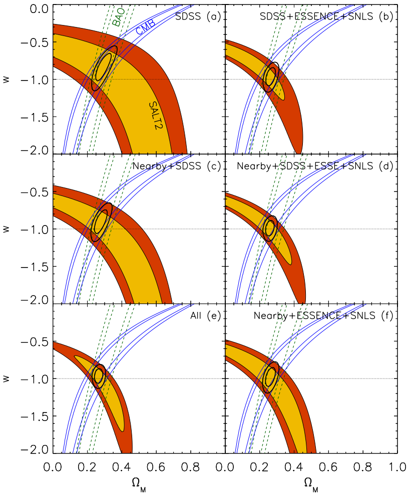
Supernova Cosmology Project, Suzuki et al. 2011



Planck 2015

The Universe is in *accelerated* expansion

If pressure is negative, $P = w\rho_E$ with $w < -1/3$ we can have accelerated expansion ($\ddot{R} > 0$) without a cosmological constant. Such a component is called dark energy. A cosmological constant corresponds to a dark energy component with $w = -1$.



The matter fraction and the parameter w of dark energy
(Kessler et al. '09).

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- Recent observations have shown that the expansion of the Universe is accelerated. Understanding this within general relativity requires 'dark energy'.