CERN, July 2016

# The Standard Model of particle physics

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**The Standard Model Lagrangian is determined by symmetries**

- space-time symmetry: global Poincaré-symmetry
- internal symmetries: local $SU(n)$ gauge symmetries

$$\begin{aligned} \mathcal{L}_{\mathrm{SM}} = \; & -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi} \not{D} \psi && \text{gauge sector} \\ & + |D_\mu H|^2 - V(H) && \text{EWSB sector} \\ & + \psi_i \lambda_{ij} \psi_j H + \mathrm{h.c.} && \text{flavour sector} \end{aligned}$$

. . . requiring renormalizability and ignoring the strong CP-problem.

# Outline

- QED as a gauge theory
- Quantum Chromodynamics
- Breaking gauge symmetries:

  the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism
- Exploring electroweak symmetry breaking at the LHC

# QED as a gauge theory

We start with the Lagrangian for a free Dirac field,

$$\mathcal{L} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi,$$

and observe that it is invariant under a phase transformation:

$$\psi \to e^{-i\omega}\psi, \quad \overline{\psi} \to e^{i\omega}\overline{\psi},$$

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We now require invariance under local $U(1)$ transformations, i.e.

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$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$ is not invariant under local $U(1)$ transformations:

$$\mathcal{L} \to \mathcal{L} + \delta\mathcal{L} = \mathcal{L} \; + \overline{\psi}\gamma^\mu[\partial_\mu \omega(x)]\psi\,,$$

where we consider infinitesimal transformations

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad \overline{\psi} \to \overline{\psi} + \delta\overline{\psi} = \overline{\psi} + i\omega(x)\overline{\psi}\,.$$

We can restore invariance under local $U(1)$ transformations if we introduce a vector field $A_\mu(x)$ with the interaction

$$-e\overline{\psi}\gamma^\mu A_\mu \psi \,,$$

so that the Lagrangian density becomes

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi \,.$$

The new Lagrangian is invariant under local $U(1)$ transformations if we require

$$A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]\,.$$

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We need to add a Lorentz- and gauge invariant kinetic term for the field $A_\mu$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi \,,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,.$$

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where

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A mass term for the new field $\propto m_A^2 A_\mu A^\mu$ is not invariant under gauge transformations,

$$\delta\mathcal{L} = \frac{2m_A^2}{e}A^\mu \partial_\mu \omega(x) \neq 0\,,$$

and thus not allowed.

It is useful to introduce the concept of a "covariant derivative" $D_\mu$ as

$$D_\mu = \partial_\mu + ieA_\mu \, .$$

With

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]$$

one finds

$$D_\mu \psi \to D_\mu \psi + \delta(D_\mu \psi) = D_\mu \psi - i\omega(x) D_\mu \psi$$

so that

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} \, (i\gamma^\mu D_\mu - m) \, \psi$$

is gauge invariant.

One can express $F_{\mu\nu}$ in terms of the covariant derivative:

$$F_{\mu\nu} = -\frac{i}{e}[D_\mu, D_\nu] = \ldots = \partial_\mu A_\nu - \partial_\nu A_\mu \, .$$

# Gauge transformations: Summary

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} \left( i\gamma^\mu D_\mu - m \right) \psi$$

- The Dirac Lagrangian is invariant under local $U(1)$ transformations if we add a vector field $A_\mu$ and an interaction $-e\overline{\psi}\gamma^\mu A_\mu \psi$.
- The interaction is obtained by replacing the derivate $\partial_\mu$ with the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$.
- The gauge-invariant kinetic term for the vector field is $\propto F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} \propto [D_\mu, D_\nu]$.
- The new vector (gauge) field is massless, since a term $\propto A_\mu A^\mu$ is not gauge-invariant.
- The Lagrangian resulting from local $U(1)$ gauge-invariance is identical to that of QED.

# Gauge transformations: non-abelian gauge groups

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We focus on the group $SU(n)$, i.e. the group of special unitary transformations.

To specify an $SU(n)$ matrix, we need $n^2 - 1$ real parameters, so we can write

$$e^{-i\omega^a T^a}$$

where the $\omega^a, a \in \{1, ..., n^2 - 1\}$ are real parameters, and the $T^a$ are called generators of the group. [If you are unfamiliar with the concept of a group generator, you can think of the $T^a$ as traceless, hermitian $n \times n$ matrices.]

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The crucial new feature is that the elements of $SU(n)$ do not commute,

$$e^{-i\omega_1^a T^a} e^{-i\omega_2^a T^a} \neq e^{-i\omega_2^a T^a} e^{-i\omega_1^a T^a} ,$$

because the generators do not commute:

$$[T^a, T^b] = if^{abc} T^c \neq 0 .$$

Recall that the $SU(n)$ transformations act on the fermion fields, so $\psi$ carries an index $i$, with $i \in \{1, ..., n\}$:

$$\psi \to \left(e^{-i\omega^a T^a}\right)\psi \quad \text{or} \quad \psi_i \to \left(e^{-i\omega^a T^a}\right)_i^{\ j}\psi_j$$

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Considering infinitesimal transformations

$$\delta\psi_i = -i\omega^a (T^a)_i^j \psi_j \quad \text{and} \quad \delta\overline{\psi}^i = i\omega^a \overline{\psi}^j (T^a)_j^i$$

one finds that the Lagrangian is not invariant under local $SU(n)$ transformations:

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We can restore local $SU(n)$ gauge-invariance by introducing $n^2 - 1$ new vector particles $A_\mu^a$, one for each generator of the group.

They should transform as

$$\delta A_\mu^a(x) = -f^{abc} A_\mu^b(x)\omega^c(x) + \frac{1}{g}\left[\partial_\mu \omega^a(x)\right] .$$

The interaction of the new vector particles and the fermions is obtained by replacing the ordinary derivative with the covariant derivative

$$D_\mu = (\partial_\mu + igT^a A^a_\mu) \,.$$

Note that in this case $D_\mu$ is a $n \times n$ matrix.

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The $SU(n)$ invariant Lagrangian then becomes

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} + \overline{\psi}^i \left(i\gamma^\mu D_\mu - m\right)_i^j \psi_j$$

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The field strength tensor is constructed from

$$F^{\mu\nu} = -\frac{i}{g}[D_\mu, D_\nu]$$

with $F^{\mu\nu} = T^a F^a_{\mu\nu}$.

This gives $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc} A^b_\mu A^c_\nu$.

# Non-abelian gauge transformations: Summary

- A non-abelian gauge theory is a theory in which the Lagrangian is invariant under local transformations of a non-abelian group.
- This invariance is achieved by introducing a gauge boson, $A^a_\mu$, for each generator of the group. The interaction between the gauge bosons and the fermions is obtained by replacing the partial derivative $\partial_\mu$ with the covariant derivative $D_\mu = (\partial_\mu + igT^a A^a_\mu)$.
- The kinetic term for the vector field is $\propto F^a_{\mu\nu} F^{a\,\mu\nu}$, where $F^a_{\mu\nu}$ is constructed from the commutator of the covariant derivative.
- $F^a_{\mu\nu} F^{a\,\mu\nu}$ contains terms which are cubic and quartic in the gauge boson fields, indicating that the gauge bosons interact with each other.
- The gauge bosons are massless, since a term $\propto A^a_\mu A^{a\,\mu}$ is not invariant under local gauge transformations.

# QCD as an SU(3) gauge theory

We start with the Dirac Lagrangian for a free quark $q$,

$$\mathcal{L} = \bar{q}\,(i\gamma^{\mu}\partial_{\mu} - m)\,q\,,$$

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Recall that $SU(3)$ is the group of special unitary transformations, i.e. the group of all $3 \times 3$ unitary matrices with determinant one. To specify an $SU(3)$ transformation, one needs $3^2 - 1 = 8$ real parameters, so we can write

$$e^{-i\omega^a T^a}$$

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$SU(3)$ is a non-abelian group, i.e. its generators and thus its elements do not commute,

$$[T^a, T^b] = if^{abc}T^c \neq 0 \quad \text{and} \quad e^{-i\omega_1^a T^a}e^{-i\omega_2^b T^b} \neq e^{-i\omega_2^b T^b}e^{-i\omega_1^a T^a}\,.$$

The $SU(3)$ transformations act on the quark fields, so $q$ carries an index $i$, with $i \in \{1, ..., 3\}$:

$$q \to \left(e^{-i\omega^a T^a}\right) q \quad \text{or} \quad q_i \to \left(e^{-i\omega^a T^a}\right)_i^{\ j} q_j$$

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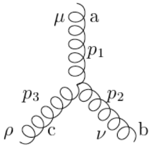
with $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c$.

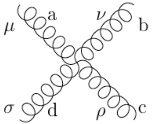
Because of the gauge symmetry, the gluon is massless.

## The interactions of QCD follow from gauge invariance:

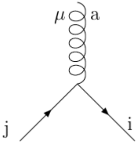
$$\mathcal{L}_{\text{interaction}} = g\, A_\mu^a\, \bar{q}\gamma^\mu T^a q - g\, f^{abc}(\partial_\mu A_\nu^a) A^{b\,\mu} A^{c\,\nu} - g^2 f^{abc} f^{ade} A_\mu^b A_\nu^c A^{d\,\mu} A^{e\,\nu} \; :$$




$$-g\, f^{abc} \left( g_{\mu\nu}\,(p_1 - p_2)_\rho + g_{\nu\rho}\,(p_2 - p_3)_\mu + g_{\rho\mu}\,(p_3 - p_1)_\nu \right)$$




$$-i\, g^2 f^{eab} f^{ecd} \,(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$$
$$-i\, g^2 f^{eac} f^{ebd} \,(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho})$$
$$-i\, g^2 f^{ead} f^{ebc} \,(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma})$$

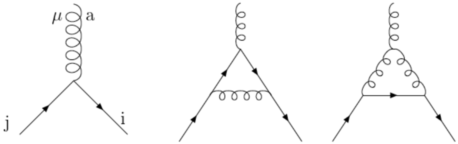



$$-i\, g\, \gamma^\mu\, (T^a)_{ij}$$

# The QCD coupling

Consider a dimensionless physical observable $R$, e.g. the ratio of two cross sections, evaluated at some large energy scale $Q$. If $Q \gg m$, one can set $m \to 0$, and dimensional analysis implies that $R$ should be independent of $Q$.

This is not true in quantum field theory. Quantum fluctuations change the value of the effective coupling:



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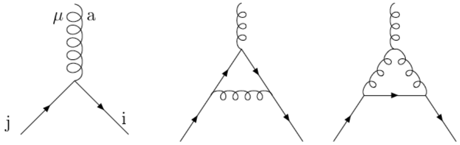


The calculation of $R$ as a perturbation series in the coupling $\alpha_{\rm s} \equiv g/4\pi$ requires renormalization to remove ultraviolet contributions. This introduces a second mass scale $\mu$ – the point at which the UV contributions are subtracted. Thus

$$R = R(Q^2/\mu^2, \alpha_{\rm s}(\mu^2))\,.$$

However, a physical observable must not depend on the scale $\mu$, i.e.

$$\mu^2 \frac{d}{d\mu^2} R(Q^2/\mu^2, \alpha_s(\mu^2)) = \left[\mu^2 \frac{\partial}{\partial\mu^2} + \mu^2 \frac{\partial\alpha_s}{\partial\mu^2} \frac{\partial}{\partial\alpha_s}\right] R = 0 \,.$$

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Introducing

$$t = \ln\left(\frac{Q^2}{\mu^2}\right), \quad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

we have

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This renormalization group equation is solved by defining a running coupling $\alpha_s(Q^2)$:

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The $\beta$-function has a perturbative expansion and can be extracted from an explicit calculation of higher-order loop-corrections to propagators and vertices.

The running of the coupling at one-loop is thus determined from

$$\frac{\partial \alpha_s(Q^2)}{\partial t} = \beta(\alpha_s(Q^2)) \quad \text{and} \quad \beta(\alpha_s) = -b\alpha_s^2$$

which yields

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\, b \ln(Q^2/\mu^2)} \quad \text{with} \quad b = \frac{33 - 2n_f}{12\pi}\,.$$

For $n_f \leq 16$ the QCD coupling decreases with increasing $Q^2$. This is the famous property of asymptotic freedom.

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Note that in QED one finds $b = -1/3$ so that the QED coupling

$$\alpha_{\mathrm{QED}}(Q^2) = \frac{\alpha_{\mathrm{QED}}(\mu^2)}{1 - \frac{\alpha_{\mathrm{QED}}(\mu^2)}{3\pi} \ln(Q^2/\mu^2)}$$

increases with increasing $Q^2$.

# The running QCD coupling

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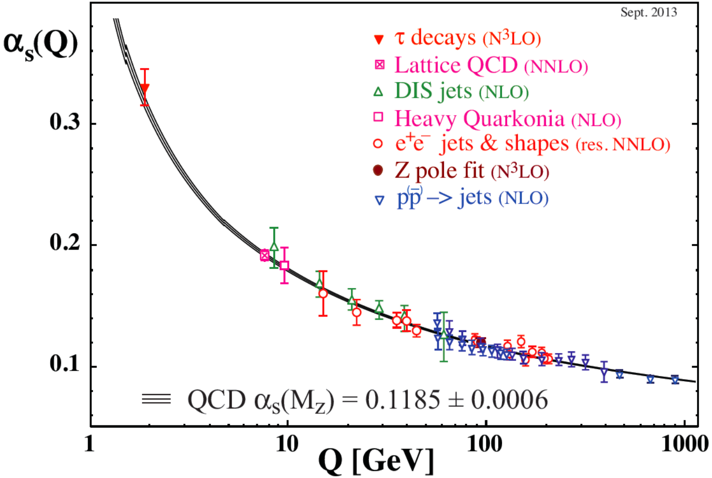


non-perturbative $\longleftrightarrow$ perturbative