Introduction to Accelerator Physics
Beam Dynamics for „Summer Students“

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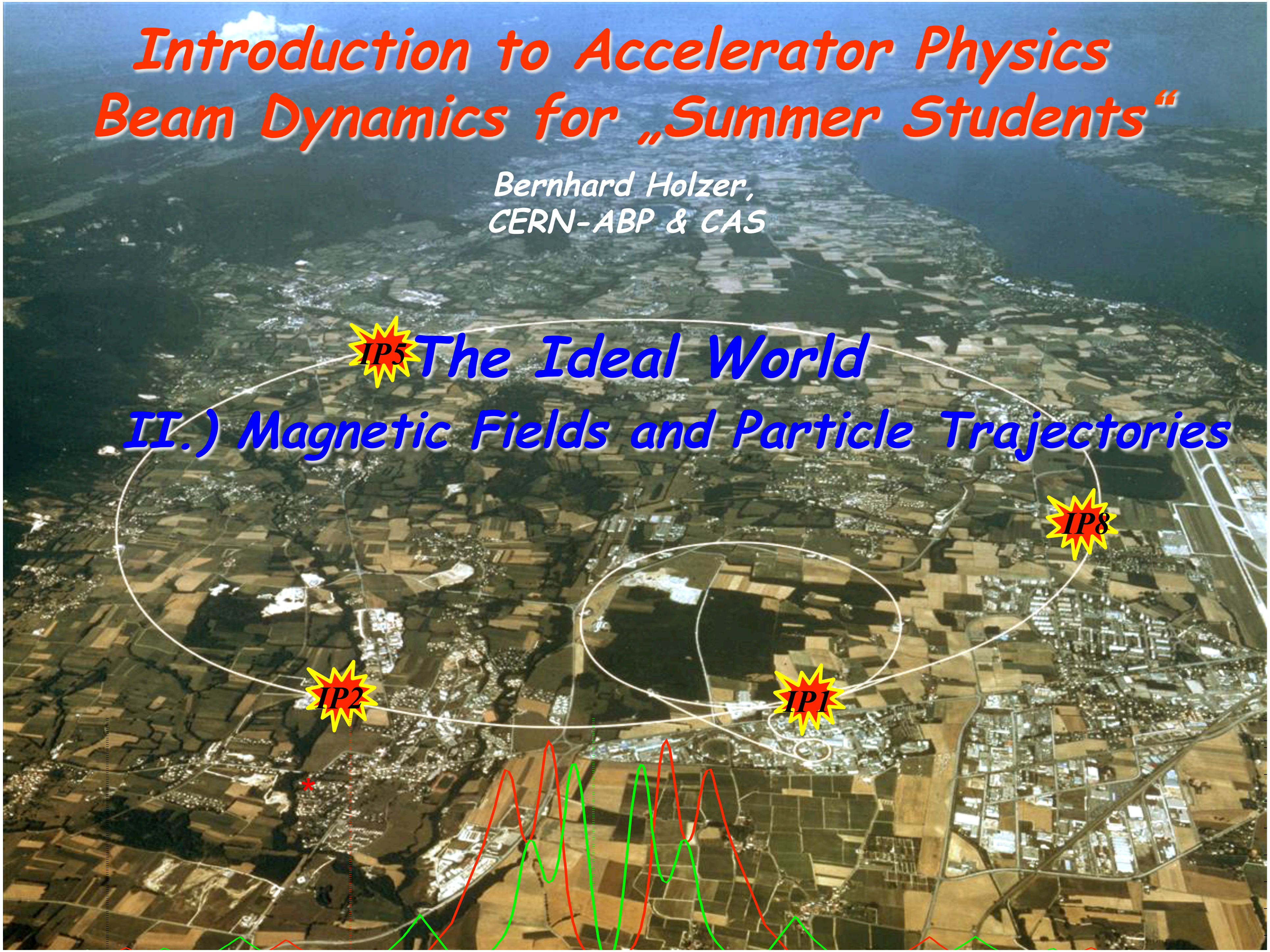
CERN-ABP & CAS

The Ideal World

II.) Magnetic Fields and Particle Trajectories



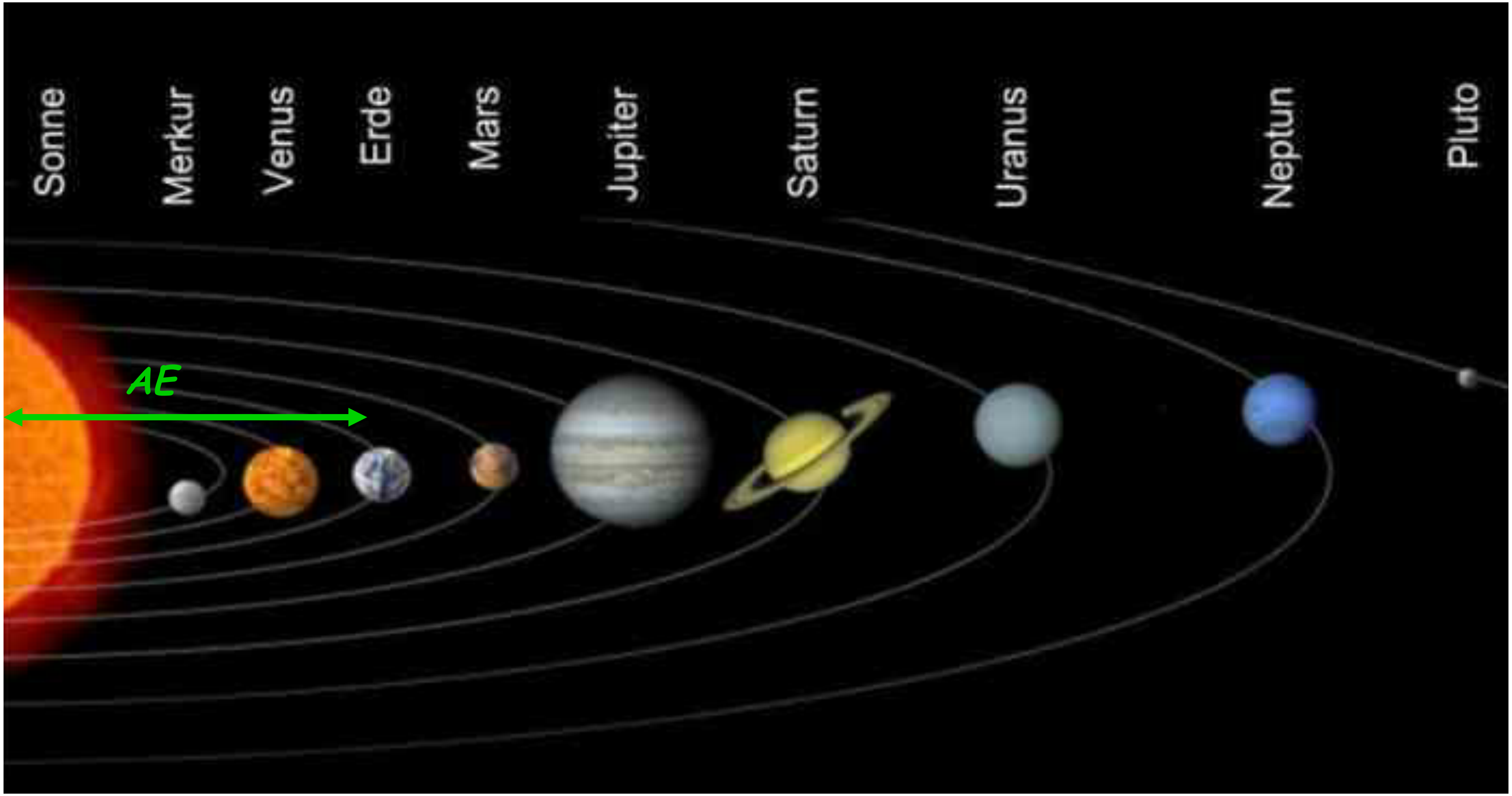
II A Bit of Theory

Largest storage ring: The Solar System

astronomical unit: average distance earth-sun

$1AE \approx 150 * 10^6$ km

Distance Pluto-Sun ≈ 40 AE



Luminosity Run of a typical storage ring:

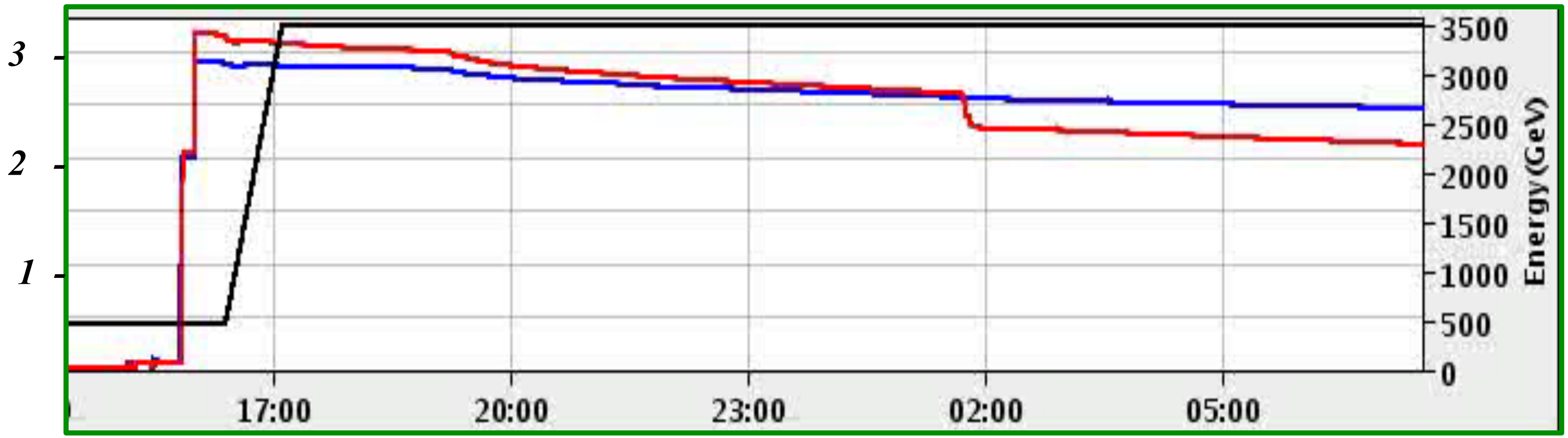
LHC Storage Ring: Protons accelerated and stored for 12 hours

distance of particles travelling at about $v \approx c$

$L = 10^{10}$-10^{11} km

... several times Sun - Pluto and back ♪

intensity (10^{11})



→ *guide the particles on a well defined orbit („design orbit")*

→ *focus the particles to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.*

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine"
→ need transverse deflecting force

Lorentz force $$\vec{F} = q * (\cancel{\vec{E}} + v \times \vec{B})$$

typical velocity in high energy machines: $$v \simeq c \simeq 3 * 10^8 m/s$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * \underbrace{300 \frac{MV}{m}}$$

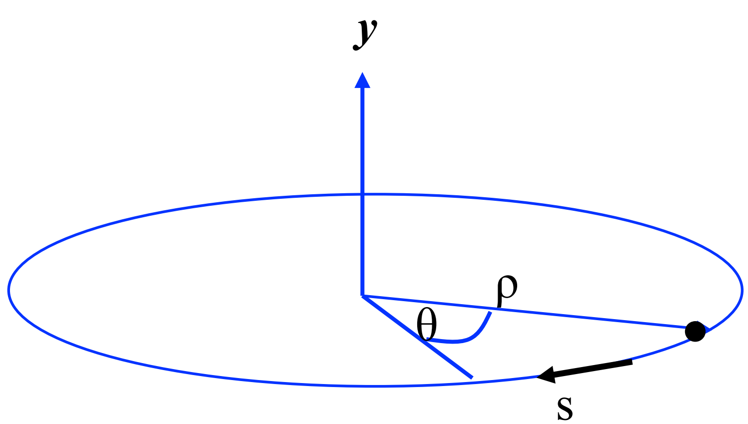
equivalent el. field ... E

technical limit for el. field:

$$E \le 1 \frac{MV}{m}$$

Pearl of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

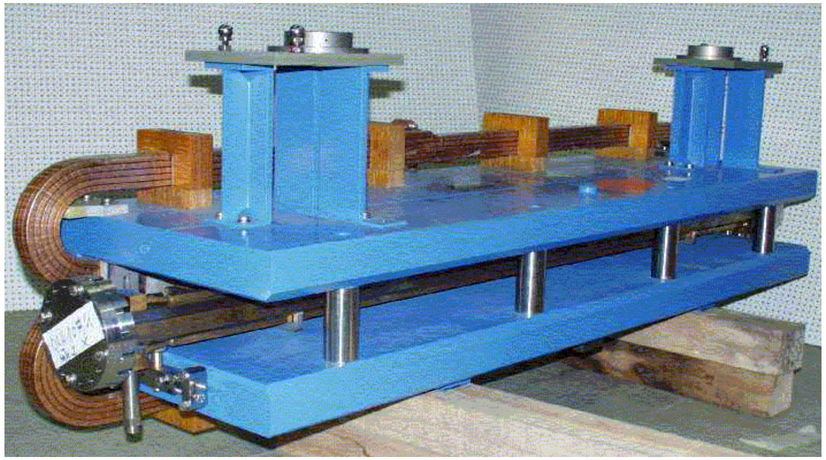
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, \cancel{v^2}}{\rho} = e\,\cancel{v}\,B$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

$B\,\rho$ = ***"beam rigidity"***

2.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

Normalise magnetic field to momentum:

$$\frac{p}{e} = B\rho \longrightarrow \frac{1}{\rho} = \frac{eB}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

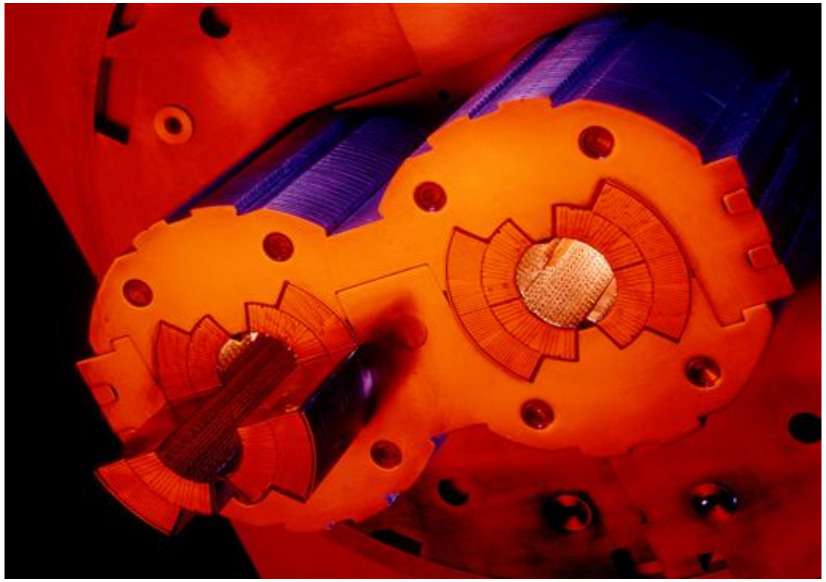
Example LHC:

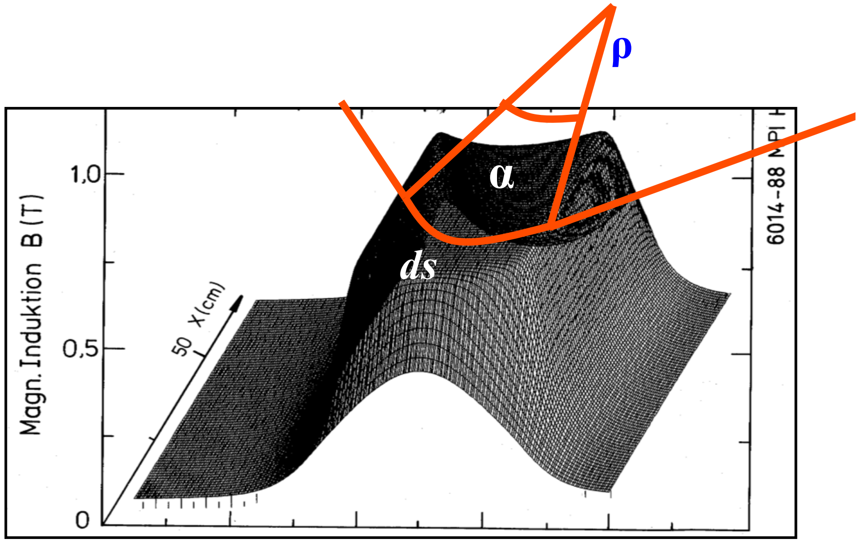
$$\left.\begin{aligned} B &= 8.3\,T \\ p &= 7000\,\frac{GeV}{c} \end{aligned}\right\}$$

$$\frac{1}{\rho} = e\,\frac{8.3\,{}^{Vs}\!/\!_{m^2}}{7000 * 10^9\, {}^{eV}\!/\!_{c}} = \frac{8.3\,s * 3 * 10^8\, {}^{m}\!/\!_{s}}{7000 * 10^9\, m^2}$$

$$\frac{1}{\rho} = 0.333\,\frac{8.3}{7000}\,{}^{1}\!/\!_{m}$$

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.82\ km$ → $\boldsymbol{2\pi\rho = 17.6\ km}$
$\boldsymbol{\approx 66\%}$

$B \approx 1 \ldots 8\ \boldsymbol{T}$

rule of thumb:

$$\boxed{\frac{1}{\rho} \approx 0.3\,\frac{B[T]}{p[GeV/c]}}$$

„normalised bending strength"

The Problem:

LHC Design Magnet current: I=11850 A

and the machine is 27 km long !!!

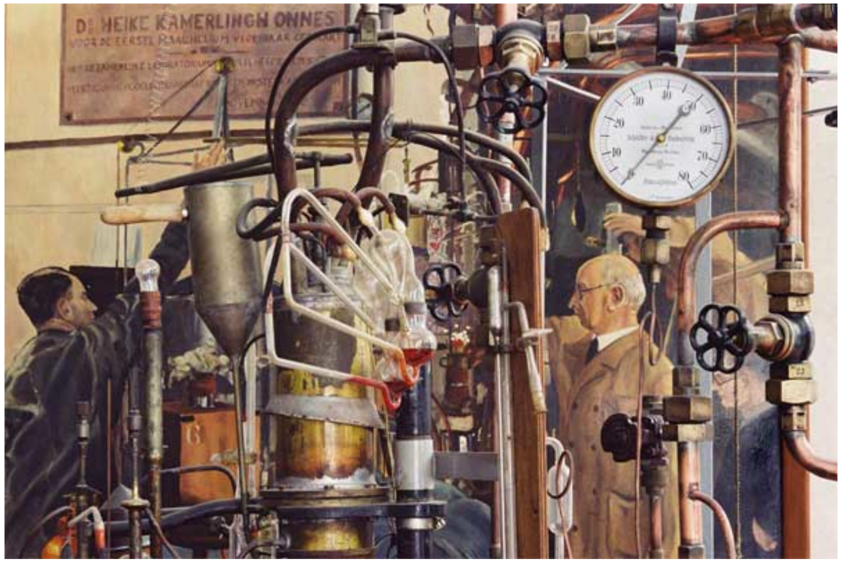
Ohm's law: $U = R * I, \qquad P = R * I^2$

Problem:

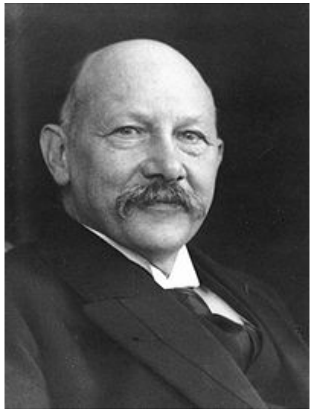
reduce ohmic losses to the absolute minimum

The Solution:

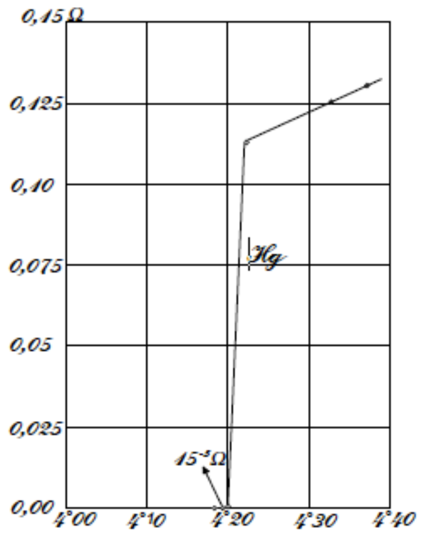
super conductivity

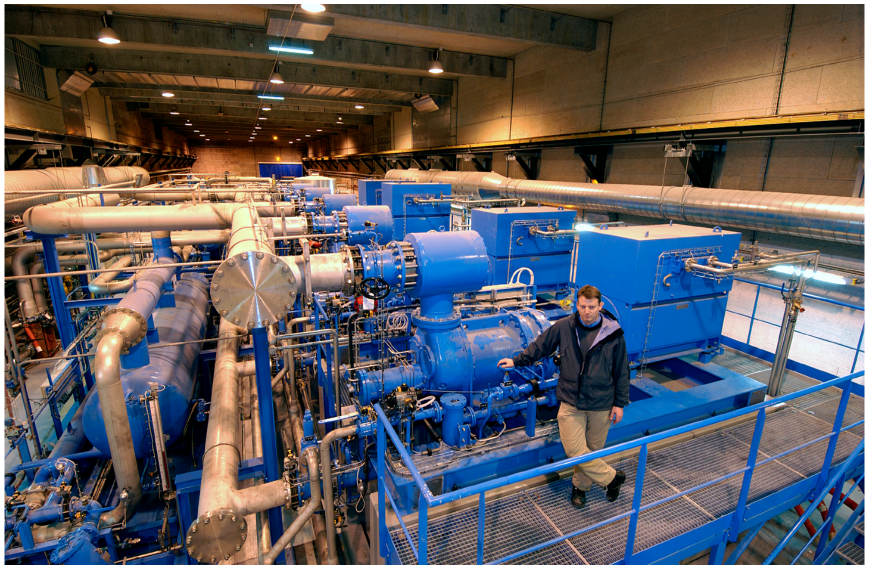


Super Conductivity

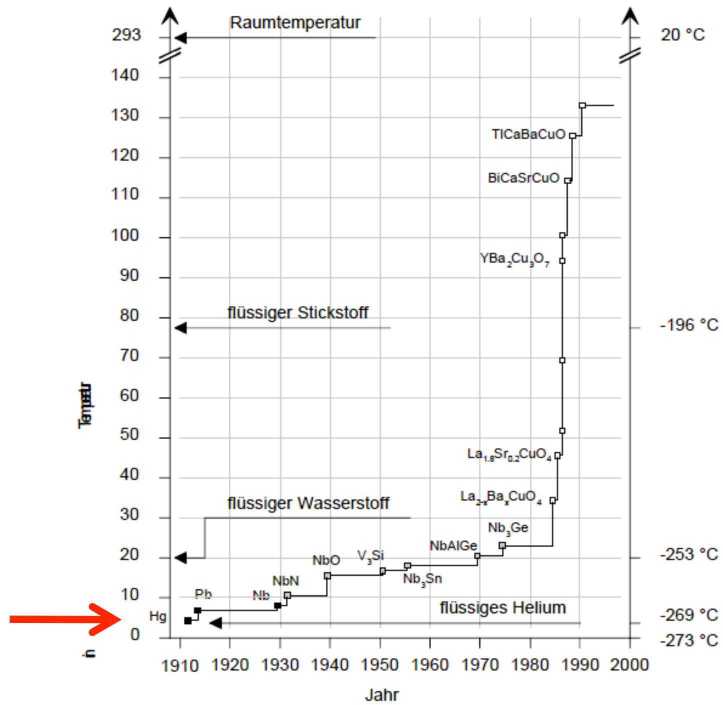


discovery of sc. by
H. Kammerling Onnes,
Leiden 1911



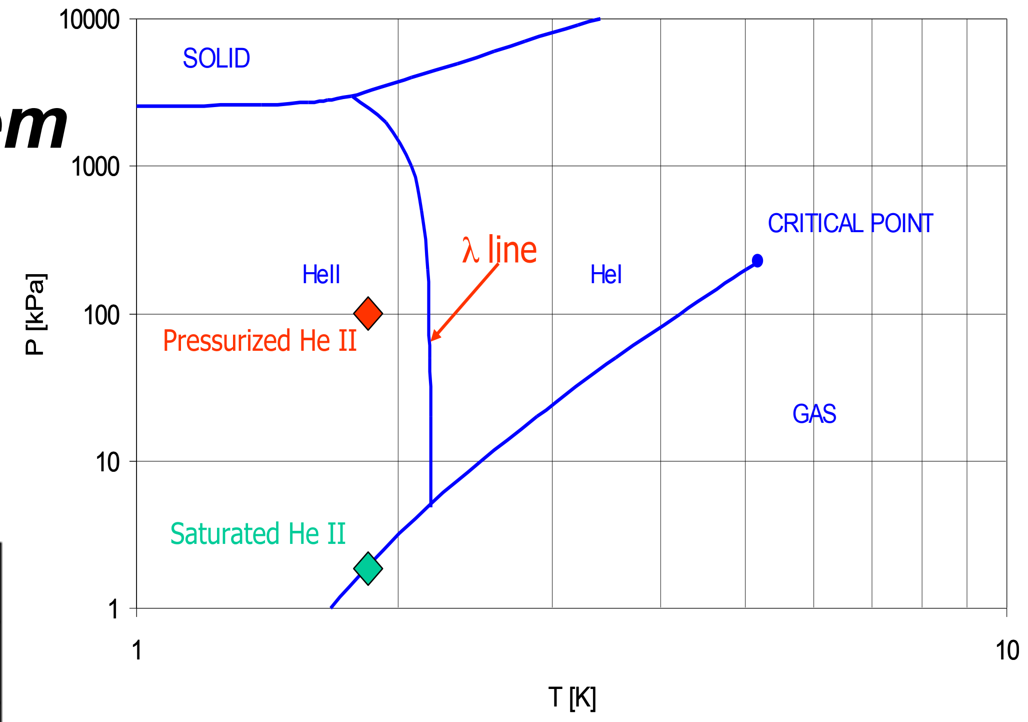


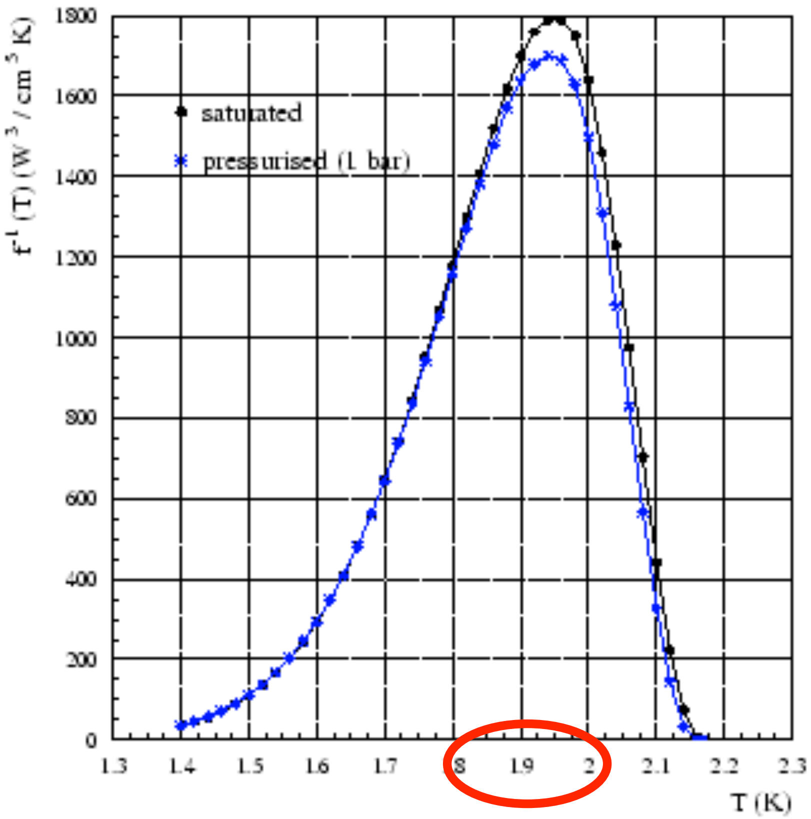
LHC 1.9 K cryo plant



Superfluid helium:
1.9 K cryo system

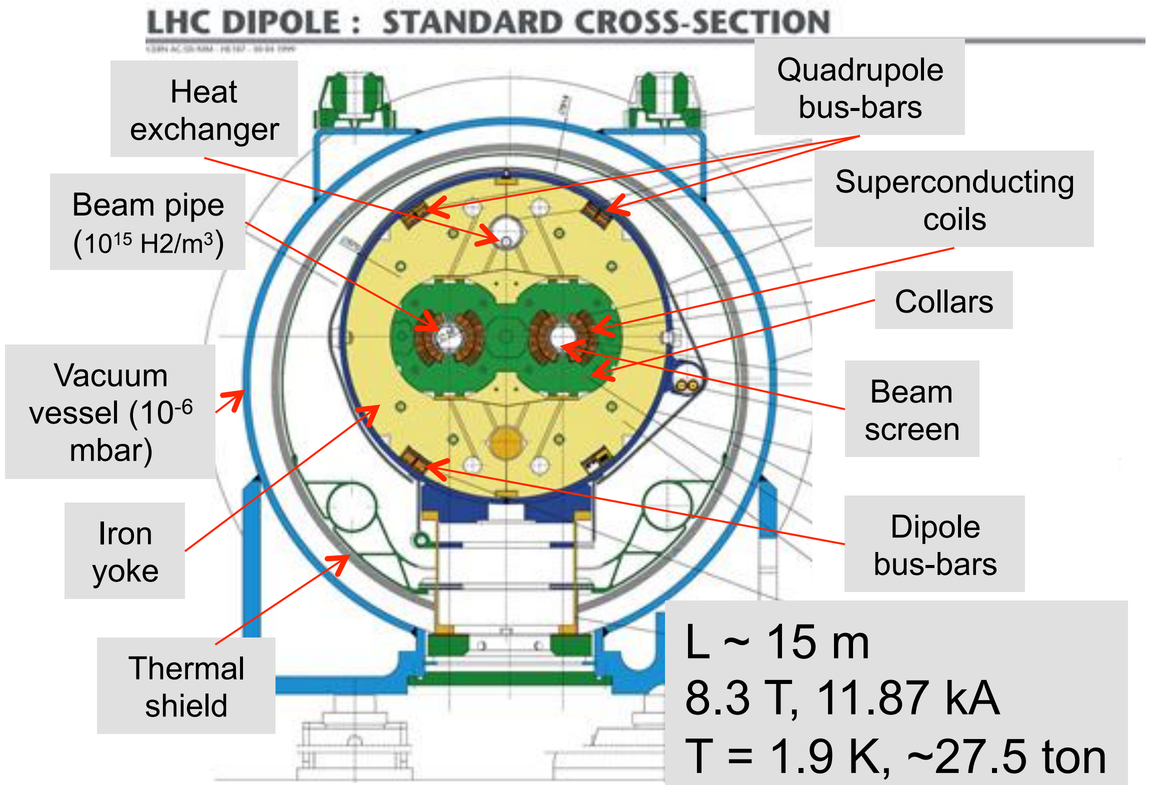
Phase diagramm of Helium



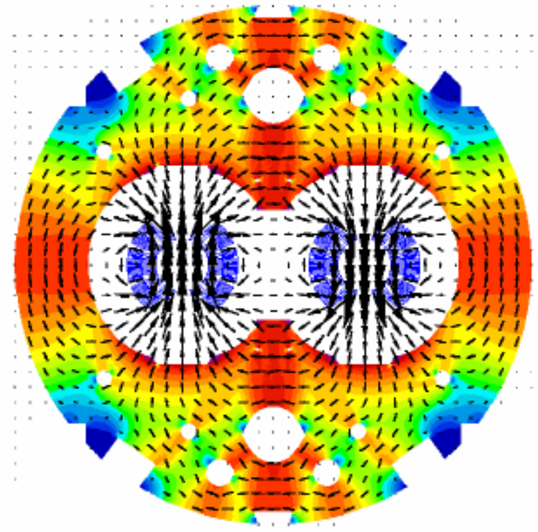


thermal conductivity of fl. Helium in supra fluid state

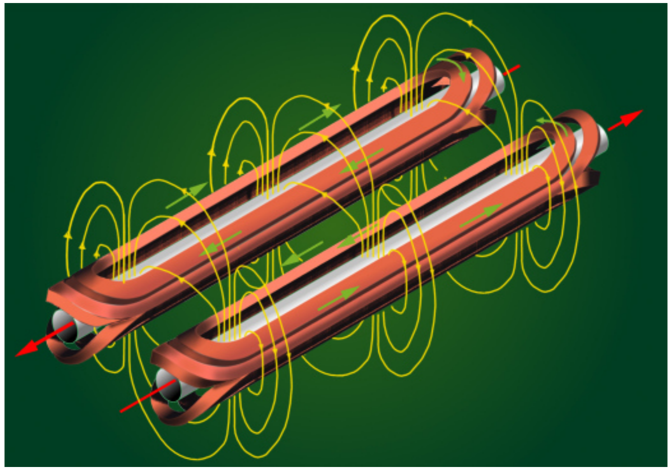
LHC: The -1232- Main Dipole Magnets

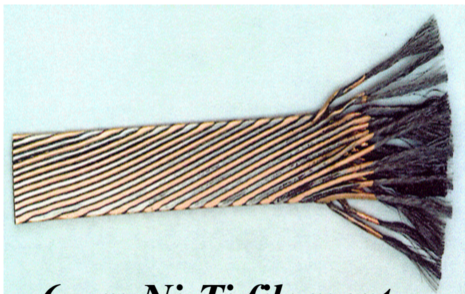


L ~ 15 m
8.3 T, 11.87 kA
T = 1.9 K, ~27.5 ton

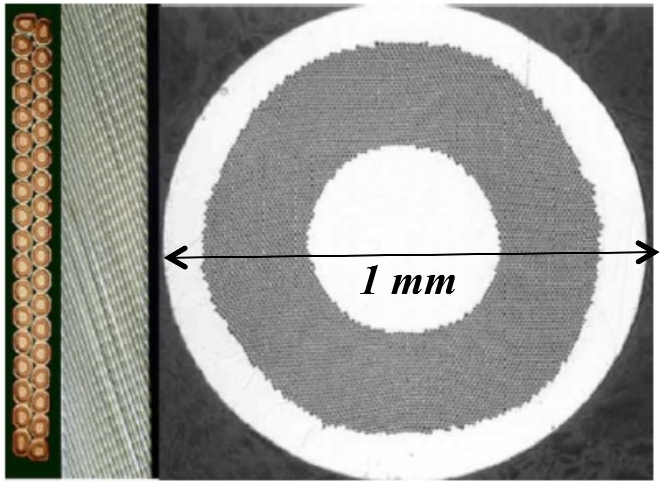


required field quality:
$\Delta B/B = 10^{-4}$



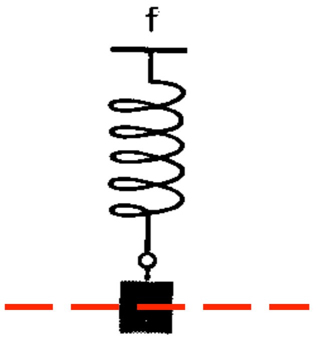


6 μm Ni-Ti filament



2.) Focusing Properties – Transverse Beam Optics

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

Storage Ring: ***we need a Lorentz force that rises as a function of the distance to .... ?***

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

required: *focusing forces* *to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

linear increasing magnetic field

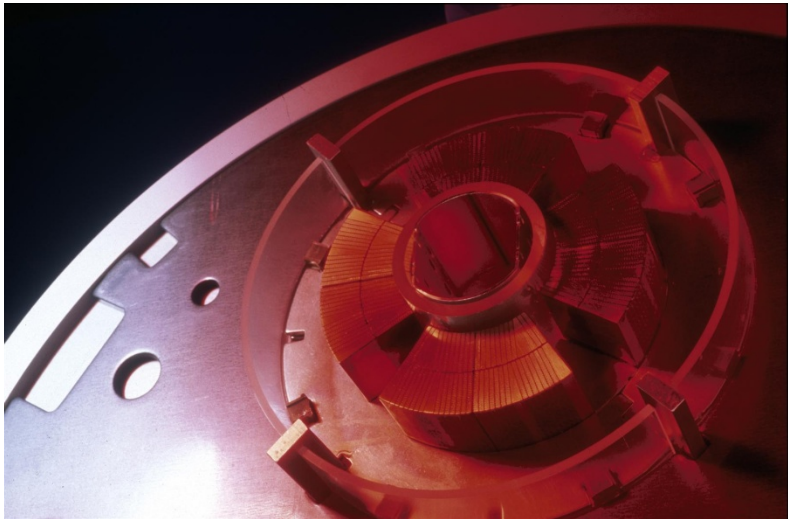
$$B_y = g\,x \qquad B_x = g\,y$$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$

simple rule:

$$\boxed{k = 0.3\,\frac{g(T/m)}{p(GeV/c)}}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220 \;\; T/m$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

Focusing forces and particle trajectories:

normalise magnet fields to momentum
*(remember: **B*ρ = p / q**)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

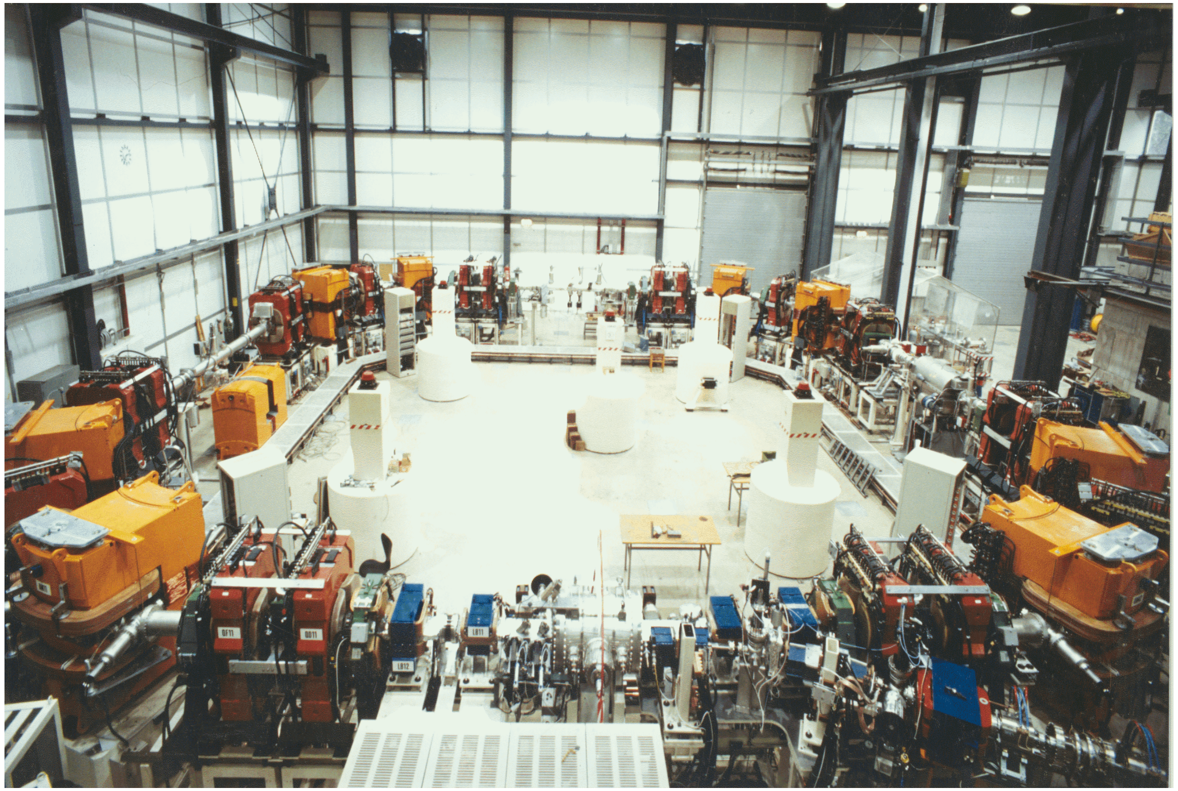
Quadrupole Magnet

$$k := \frac{g}{p/q}$$

3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}m}\,x^2 + \cancel{\frac{1}{3!}n}\,x^3 + \dots$$

only terms linear in x, y taken into account ***dipole fields***
quadrupole fields



Separate Function Machines:

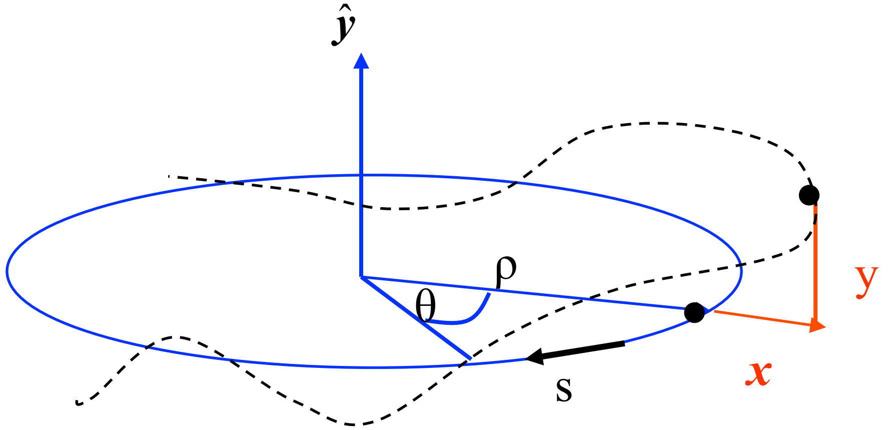
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* ***man sieht nur dipole und quads → linear***

The Equation of Motion:



* *Equation for the horizontal motion:*

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

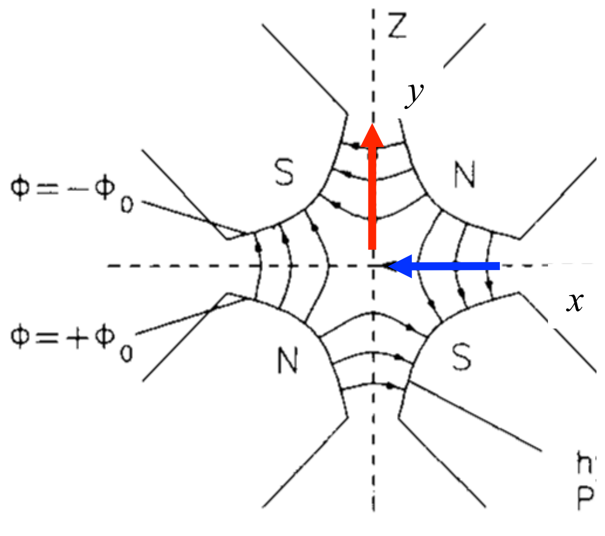
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* *Equation for the vertical motion:*

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$$k \leftrightarrow -k$$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

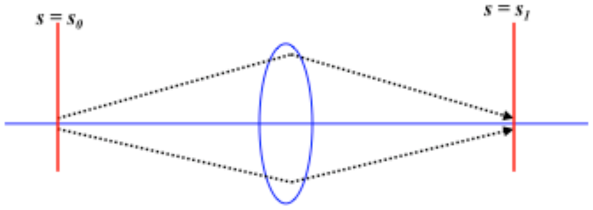
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: ***Hor. Focusing Quadrupole K > 0:***

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



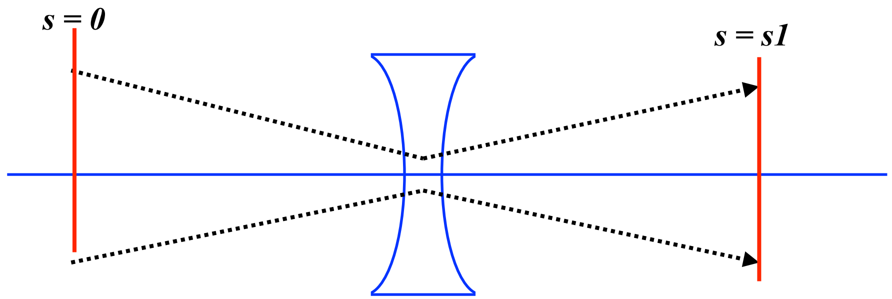
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boldsymbol{M}_{foc} = \begin{pmatrix} \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \frac{1}{\sqrt{|\boldsymbol{K}|}}\sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \\ -\sqrt{|\boldsymbol{K}|}\sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



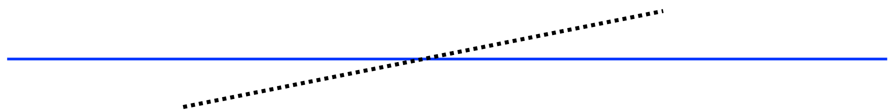
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

K = 0



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Ok … ok ... it's a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...

Weak Focusing / Strong Focusing

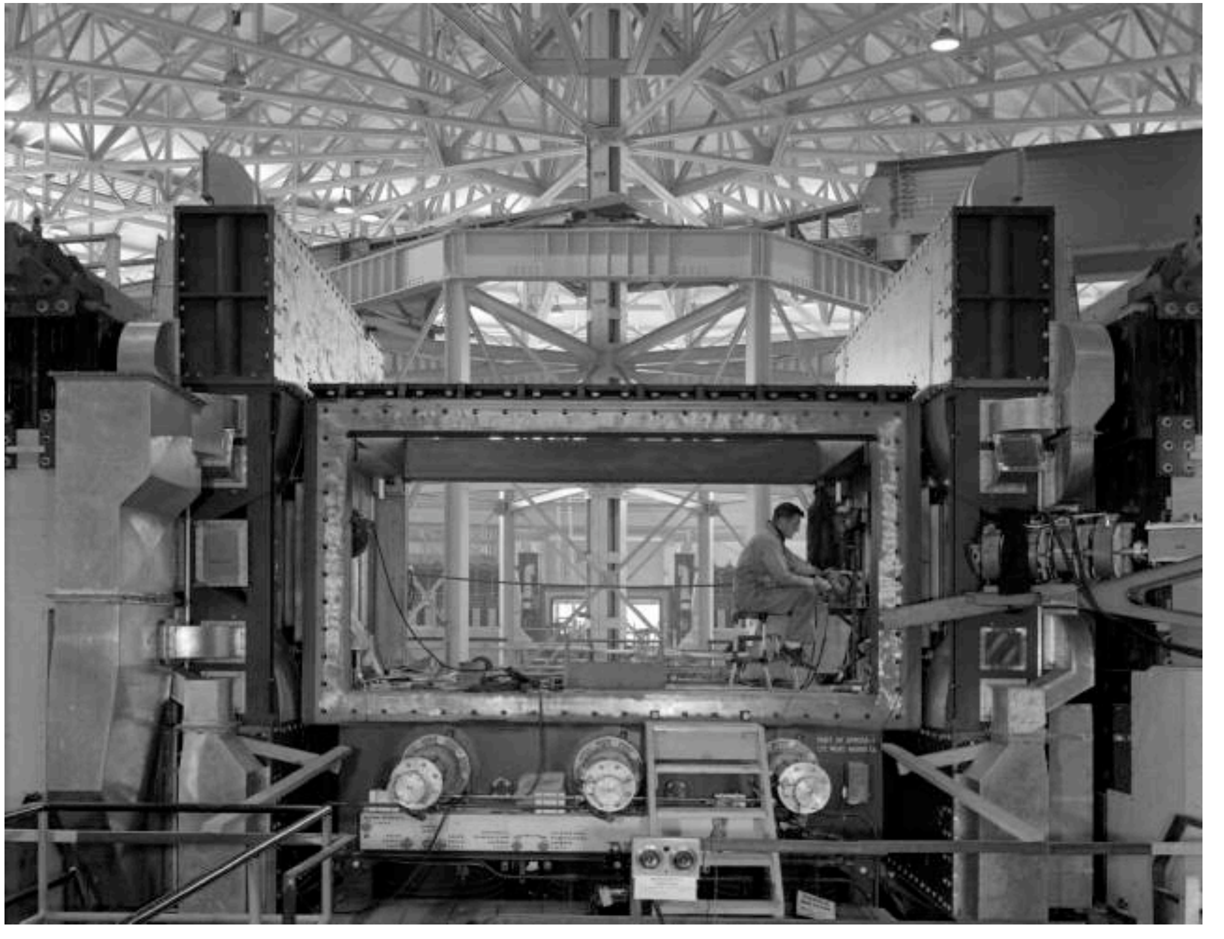
Problem: the higher the energy, the larger the machine

$$\frac{p}{e} = B\,\rho$$

The larger the machine $2\pi\rho$
→ the weaker the focusing
$1/\rho^2$

The weaker the focusing $1/\rho^2$
→ the larger the beam size

The larger the beam size
→ the more expensive the vacuum chamber and the magnets



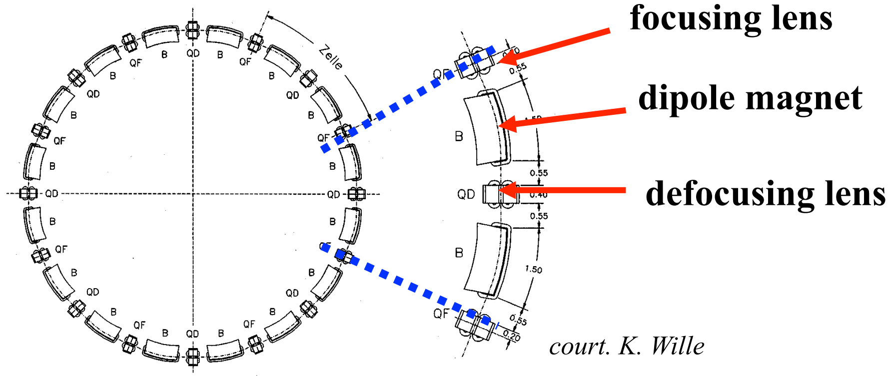
The last weak focusing high energy machine …
BEVATRON

Transformation through a system of lattice elements

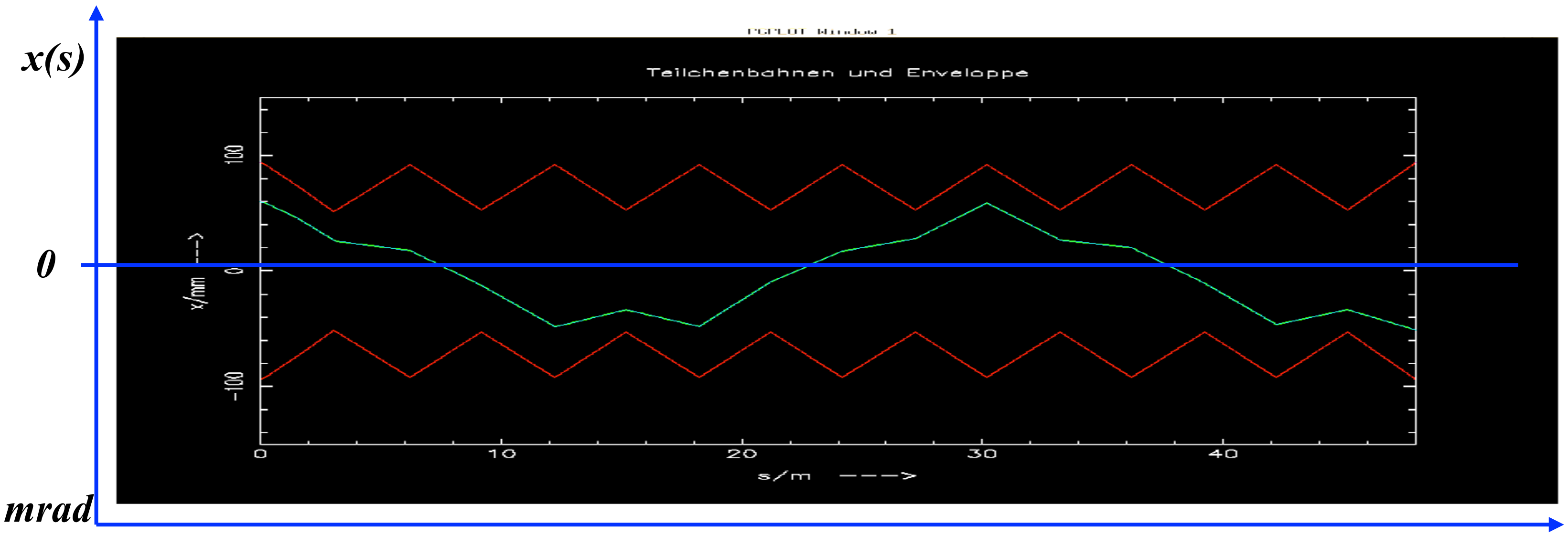
combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



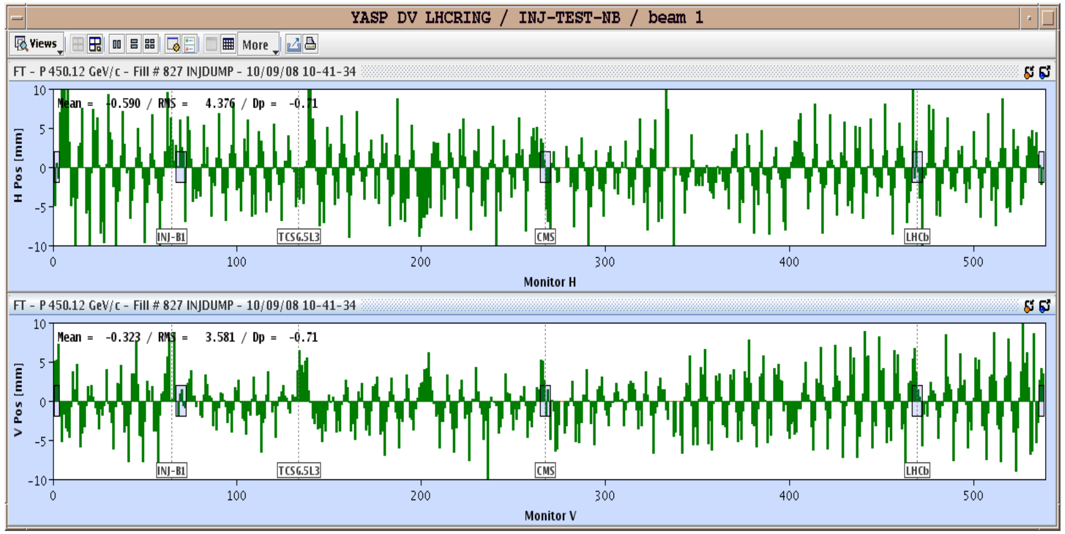
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine:

$x \approx mm$, $x' \leq mrad$

5.) Orbit & Tune:



Tune: number of oscillations per turn

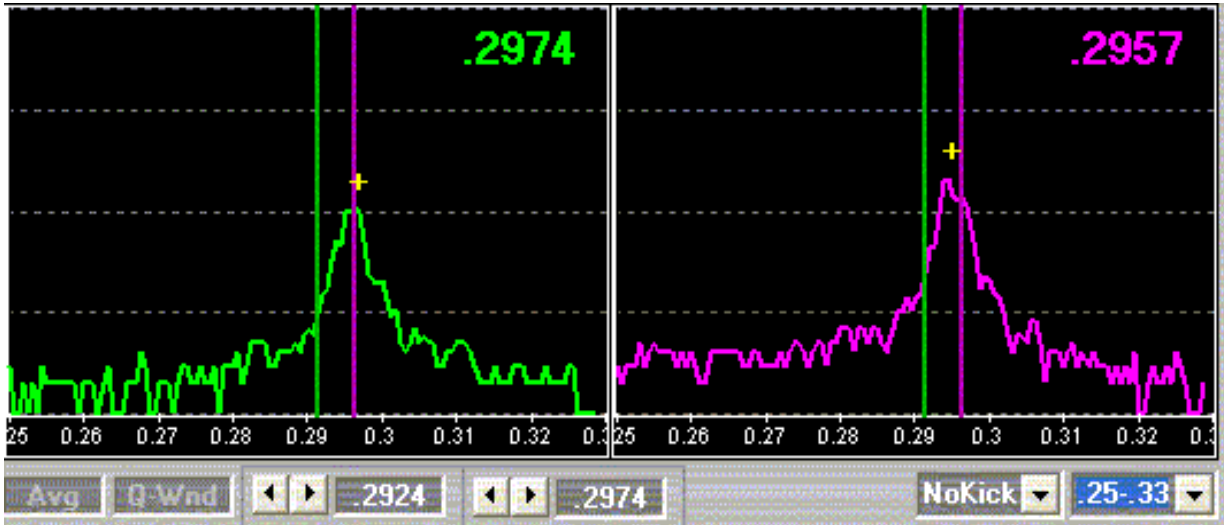
64.31
59.32

Relevant ***for beam stability:***

non integer part

LHC revolution frequency: ***11.3 kHz***

$$0.31 * 11.3 = 3.5 kHz$$



Once more unto the breach, dear friends, once more
(W. Shakespeare, Henry 5)

“Fallen die Dinger eigentlich runter ?”
“do they actually drop ?”

Antwort: No.