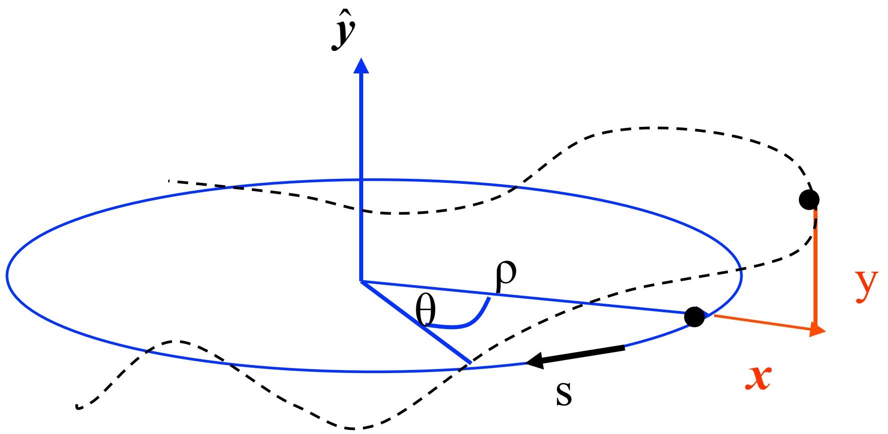
III.) Colliding Beams

Recapitulation:



The Equation of Motion:

* *Equation for the horizontal motion:*

$$x'' + x\,(\frac{1}{\rho^2} + k) = 0$$

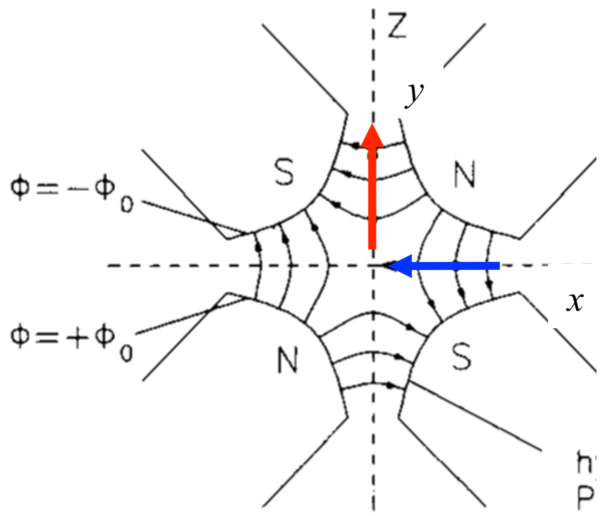
***x* = particle amplitude**
***x'* = angle of particle trajectory (wrt ideal path line)**

* *Equation for the vertical motion:*

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$k \leftrightarrow -k$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



...the story with the matrices !!!

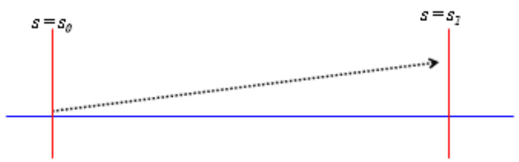
Equation of Motion:

$$x'' + K\,x = 0 \qquad K = 1/\rho^2 - k \;\dots\; \textit{hor. plane:}$$

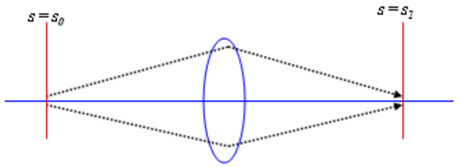
$$K = k \qquad \dots\; \textit{vert. Plane:}$$

Solution of Trajectory Equations

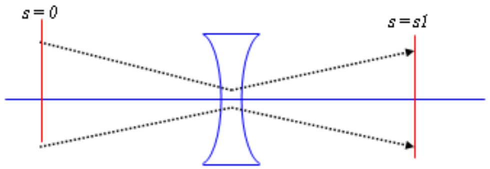
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}l) \\ -\sqrt{|K|}\sin(\sqrt{|K|}l) & \cos(\sqrt{|K|}l) \end{pmatrix}$$



$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}l) \\ \sqrt{|K|}\sinh(\sqrt{|K|}l) & \cosh(\sqrt{|K|}l) \end{pmatrix}$$

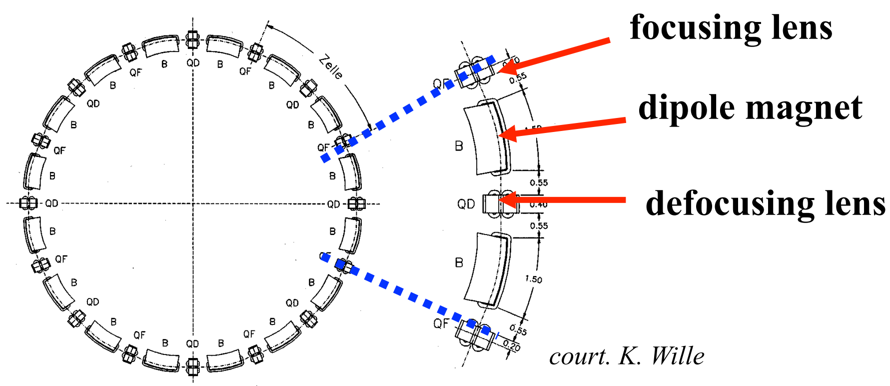
$$M_{total} = M_{QF} * M_D * M_B * M_D * M_{QD} * M_D * \quad \dots$$

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

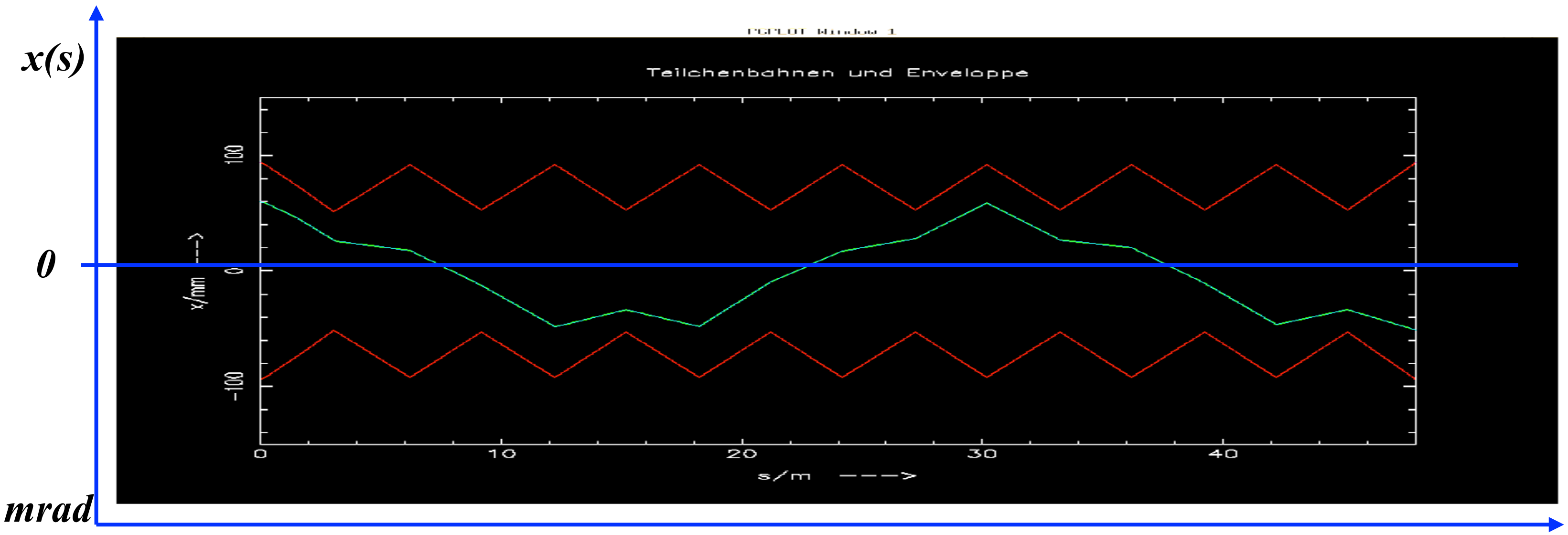
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „

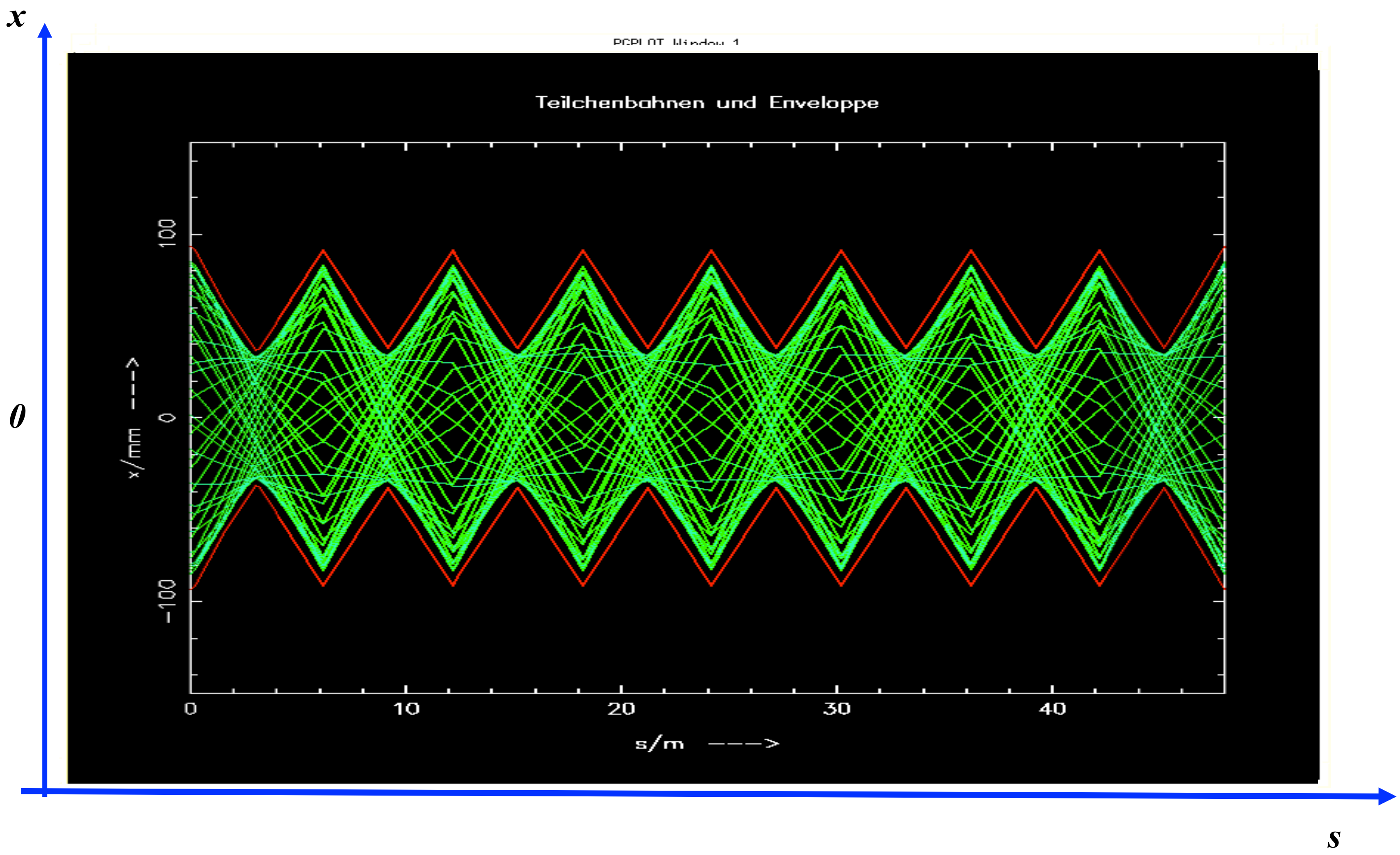


typical values
in a strong
foc. machine:

$x \approx mm$, $x' \leq mrad$

Question: ***what will happen, if the particle performs a second turn ?***

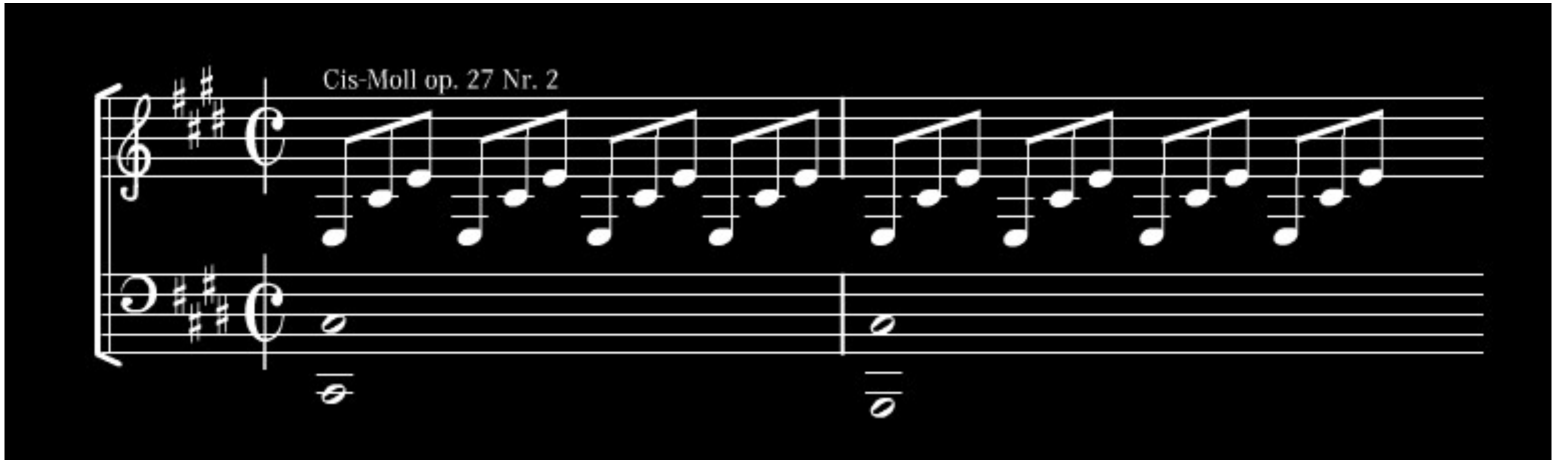
... or a third one or ... 10^{10} turns



19th century:

Ludwig van Beethoven: „Mondschein Sonate“

Sonate Nr. 14 in cis-Moll (op. 27/II, 1801)



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill 's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

6.) The Beta Function

„it is convenient to see"

... after some beer ... general solution of Mr Hill can be written in the form:

Ansatz:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s + L) = \beta(s)$$

ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!

Ψ(s) = „phase advance" of the oscillation between point „0" and „s" in the lattice.
For one complete revolution: number of oscillations per turn „Tune"

$$Q_y = \frac{1}{2\pi} \cdot \int \frac{ds}{\beta(s)}$$

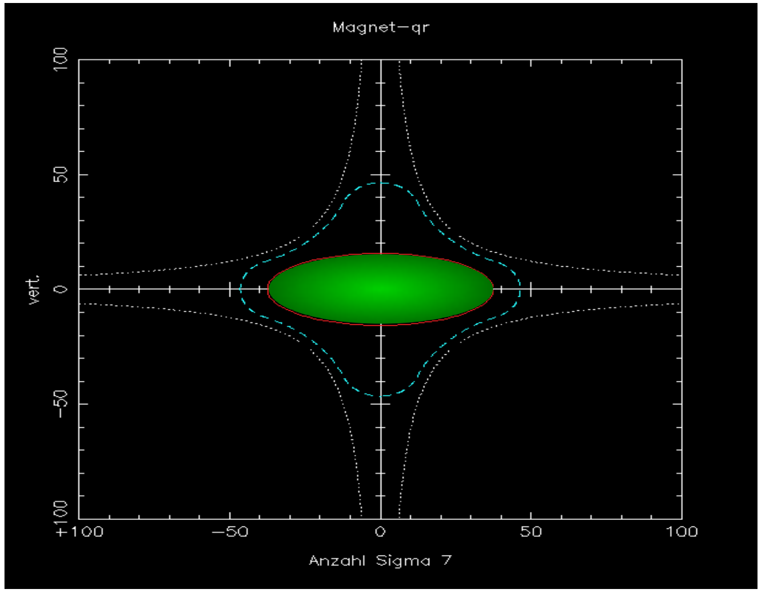
6.) The Beta Function

Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

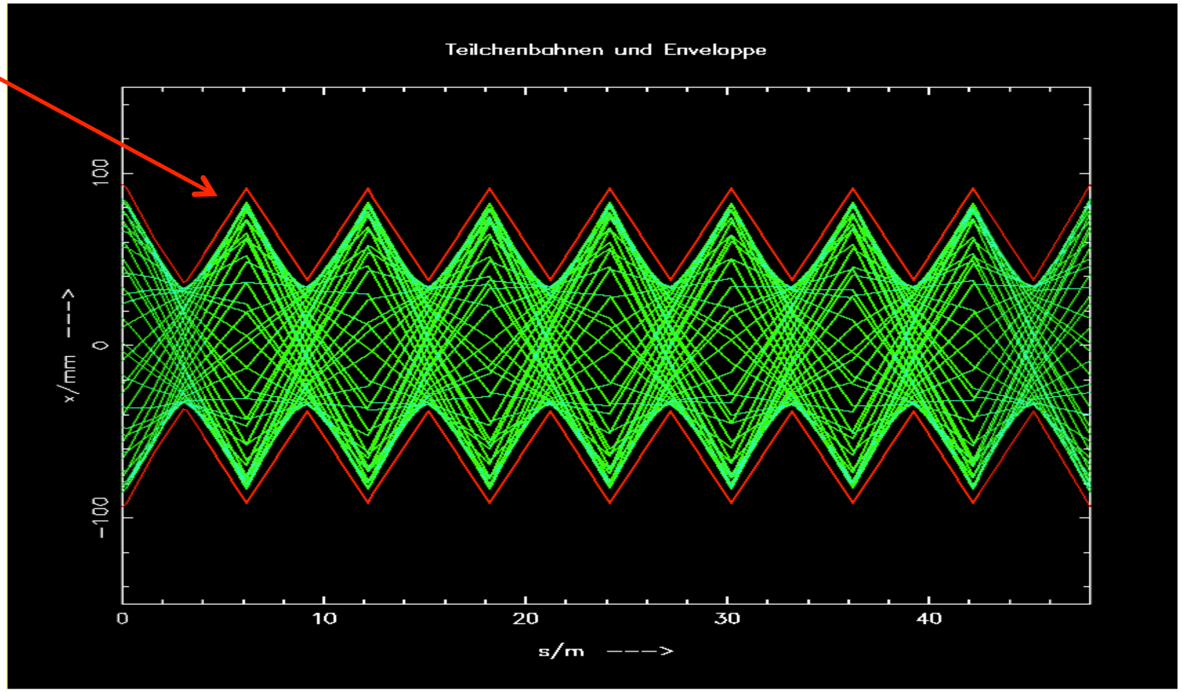
Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



β determines the beam size
(... the envelope of all particle trajectories at a given position "s" in the storage ring.

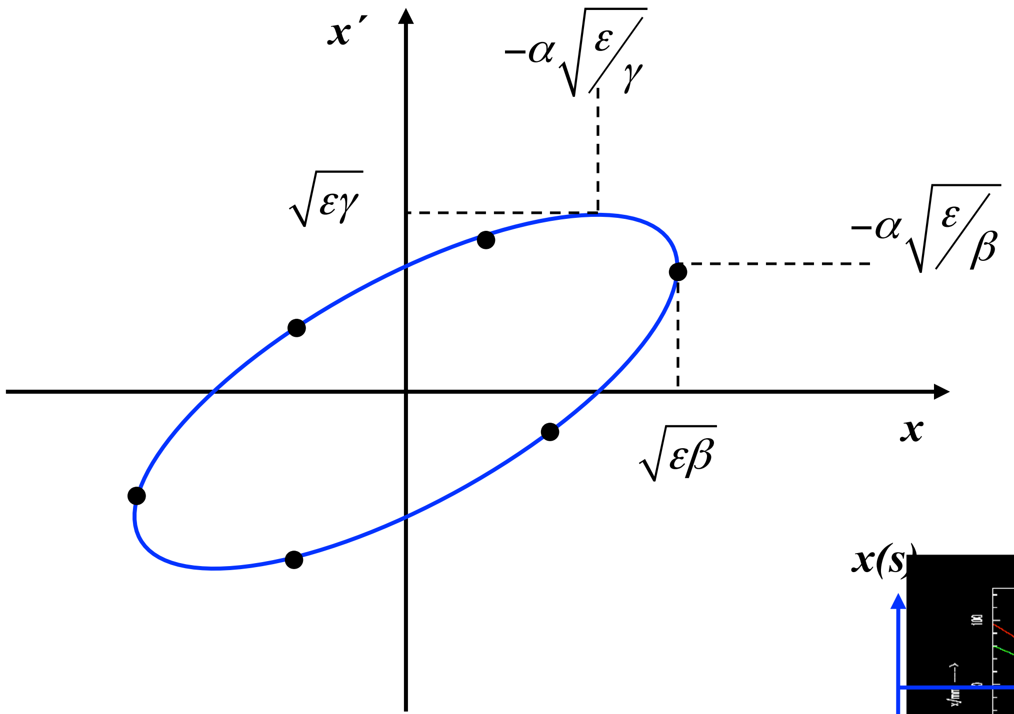
It reflects the periodicity of the magnet structure.



Beam Emittance and Phase Space Ellipse

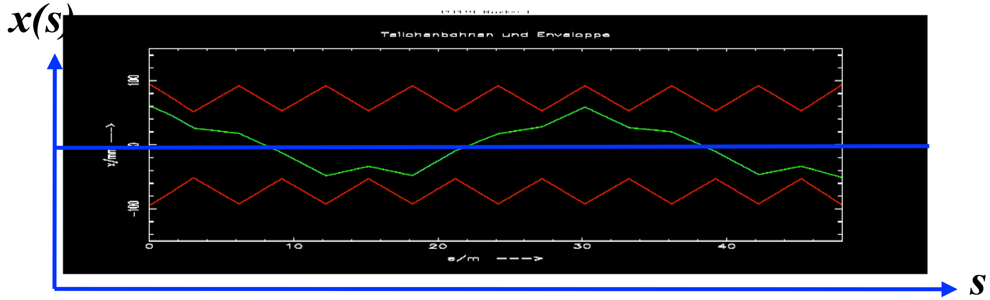
$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



Liouville: ***in reasonable storage rings***
area in phase space is constant.

A = π*ε=const

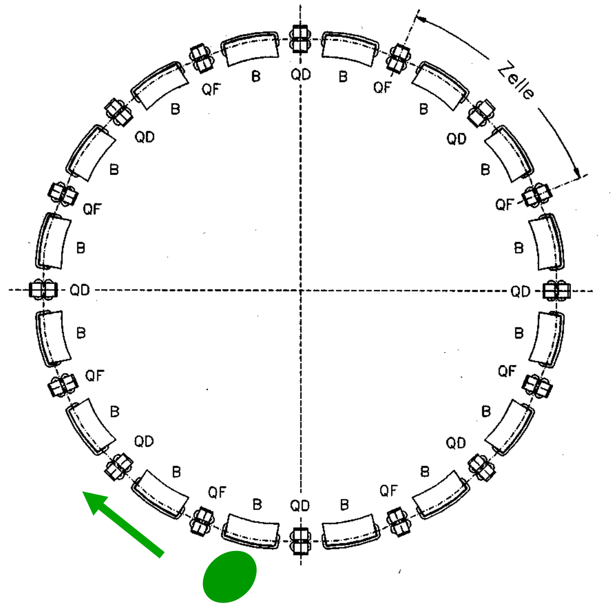


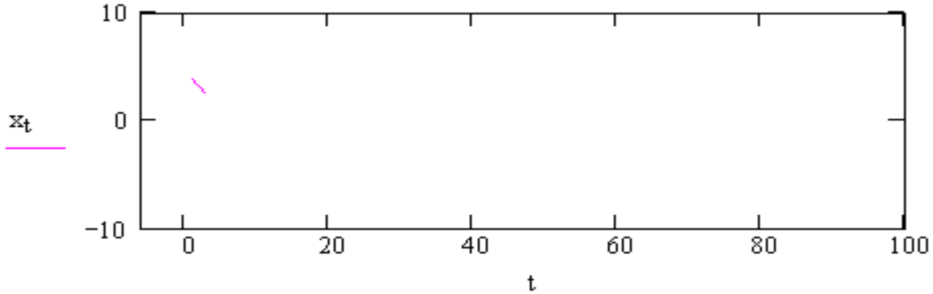
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely speaking: area covered in transverse x, x´ phase space … and it is constant !!!

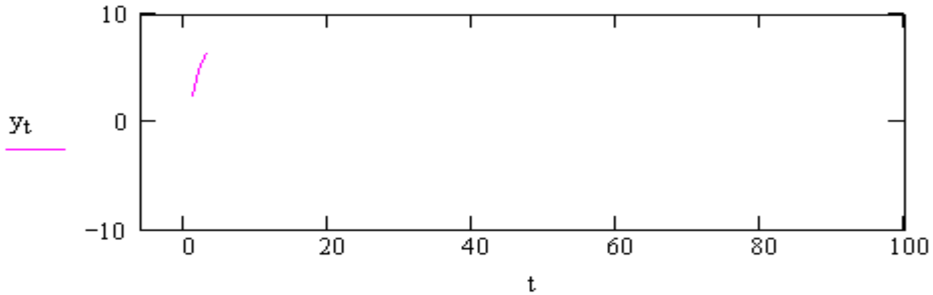
Particle Tracking in a Storage Ring

Calculate x, x´ for each linear accelerator element according to matrix formalism

plot x, x´as a function of „s“

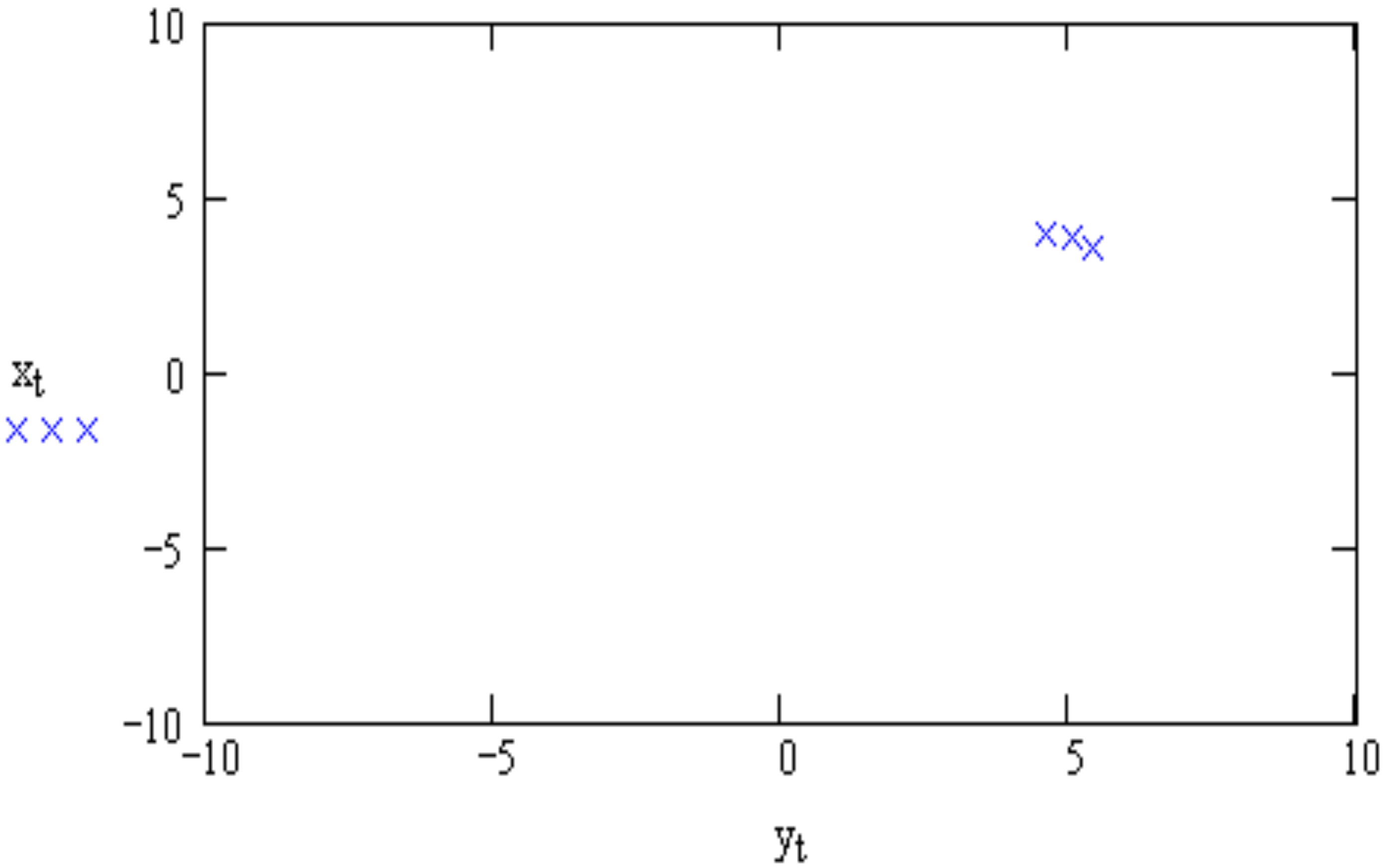






… and now the ellipse:

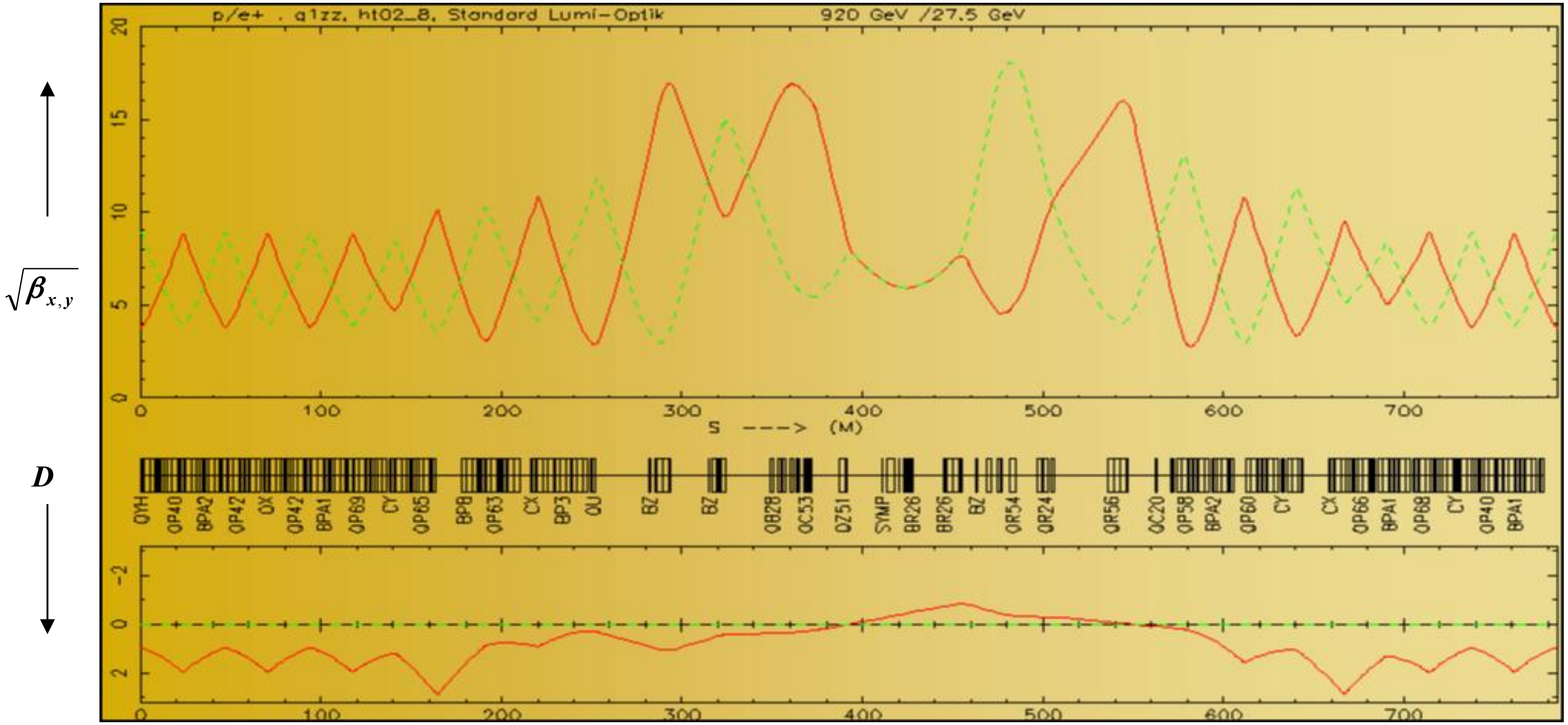
note for each turn x, x´at a given position „s_1“ and plot in the phase space diagram



… just as Big Ben

… and just as any harmonic pendulum

III.) The „not so ideal“ World
Storage Rings & Lattice Design



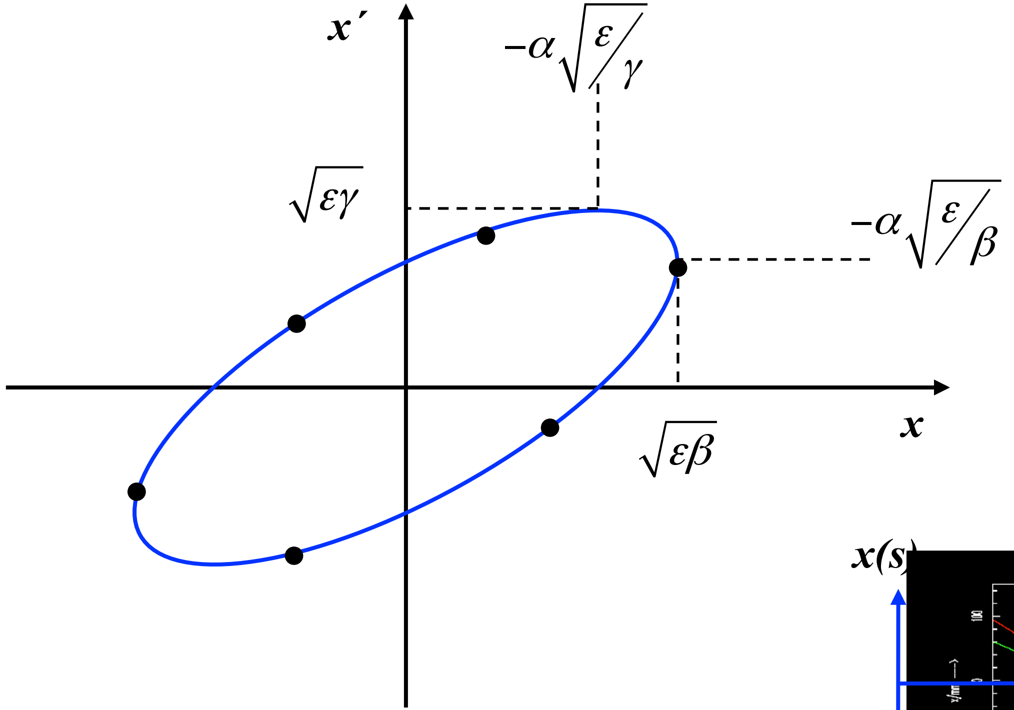
1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Beam Emittance and Phase Space Ellipse

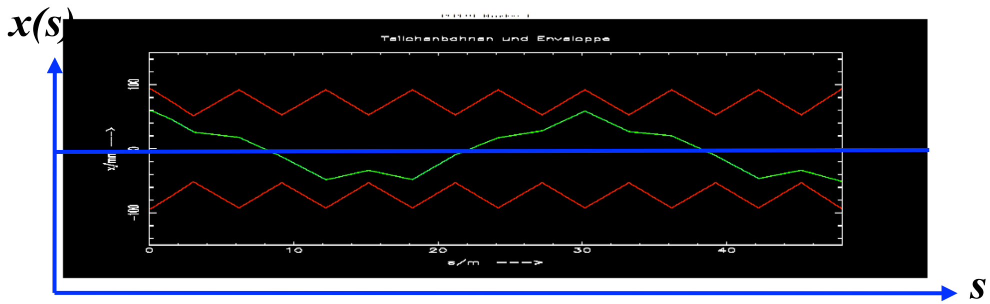
$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

$$\boldsymbol{\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\, x'^2(s)}$$



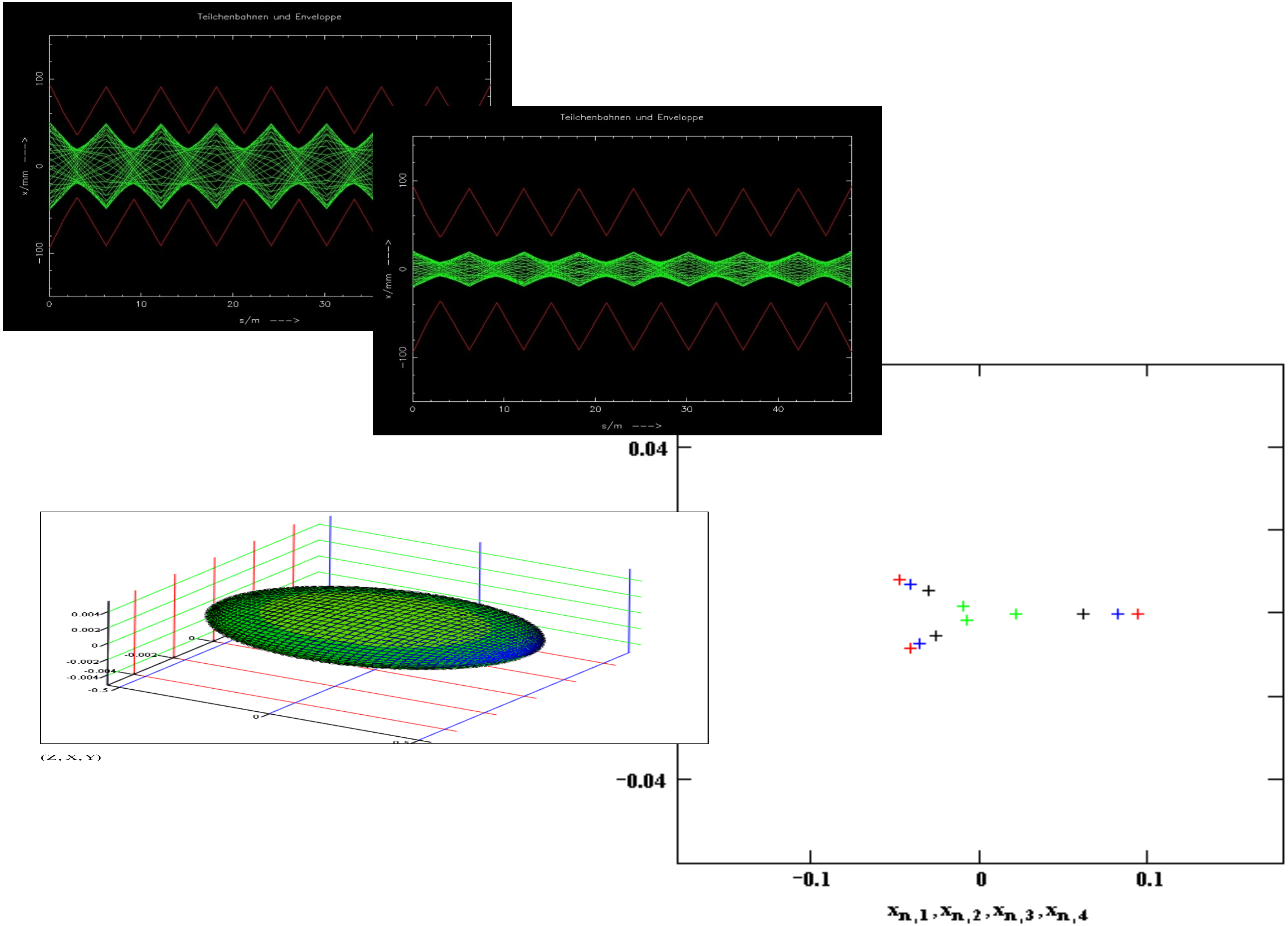
***Liouville:* in reasonable storage rings**
area in phase space is constant.

A = π*ε=const



ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely speaking: area covered in transverse x, x´ phase space … and it is constant !!!

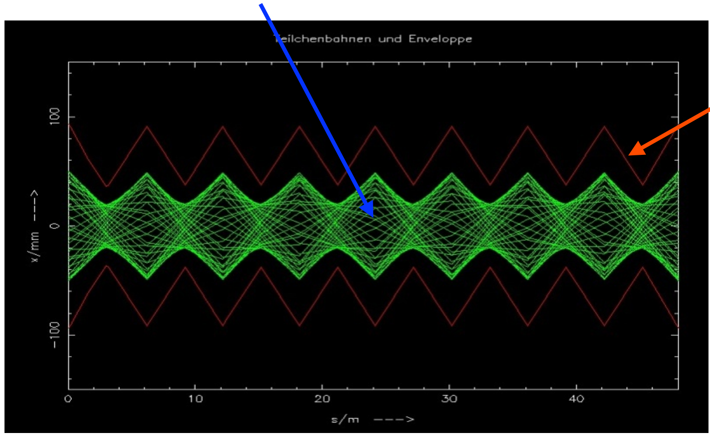
Emittance of the Particle Ensemble:



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi)$$

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

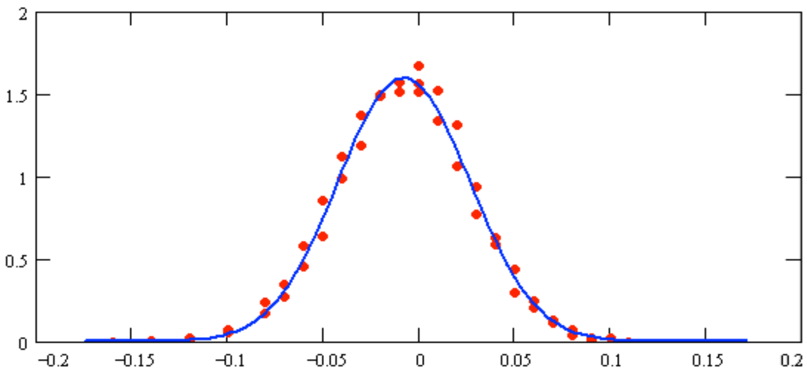
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

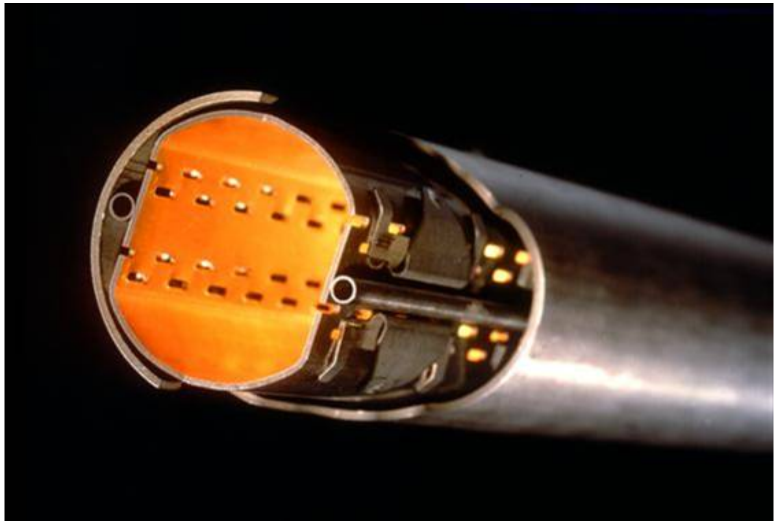
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5 * 10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\,m * 180\,m} = 0.3\,mm$$





aperture requirements: $r_0 = 12 * \sigma$

8.) Lattice Design: „… how to build a storage ring“

Geometry of the ring: $B * \rho = p / e$

p = momentum of the particle,
ρ = curvature radius

Bρ= beam rigidity

Circular Orbit: bending angle of one dipole

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$

The angle run out in one revolution must be 2π, so for a full circle

$$\alpha = \frac{\int Bdl}{B\rho} = 2\pi$$

$$\int Bdl = 2\pi \frac{p}{q}$$

… defines the integrated dipole field around the machine.

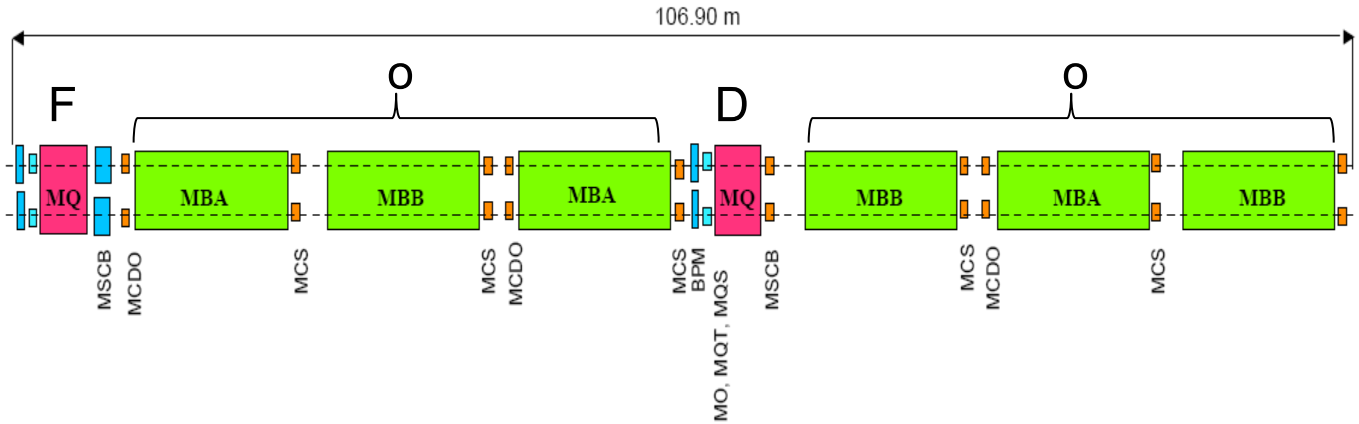
Example LHC:

7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

$$\int B\, dl \approx N\, l\, B = 2\pi\; p / e$$

$$B \approx \frac{2\pi \;\; 7000 \;\; 10^{9}\, eV}{1232 \;\; 15\, m \;\; 3 \;\; 10^{8} \, \frac{m}{s} \;\;\; e} = 8.3\; Tesla$$

LHC: Lattice Design
the ARC 90° FoDo in both planes



equipped with additional corrector coils

MB: main dipole
MQ: main quadrupole
MQT: Trim quadrupole
MQS: Skew trim quadrupole
MO: Lattice octupole (Landau damping)
MSCB: Skew sextupole
Orbit corrector dipoles
MCS: Spool piece sextupole
MCDO: Spool piece 8 / 10 pole
BPM: Beam position monitor + diagnostics

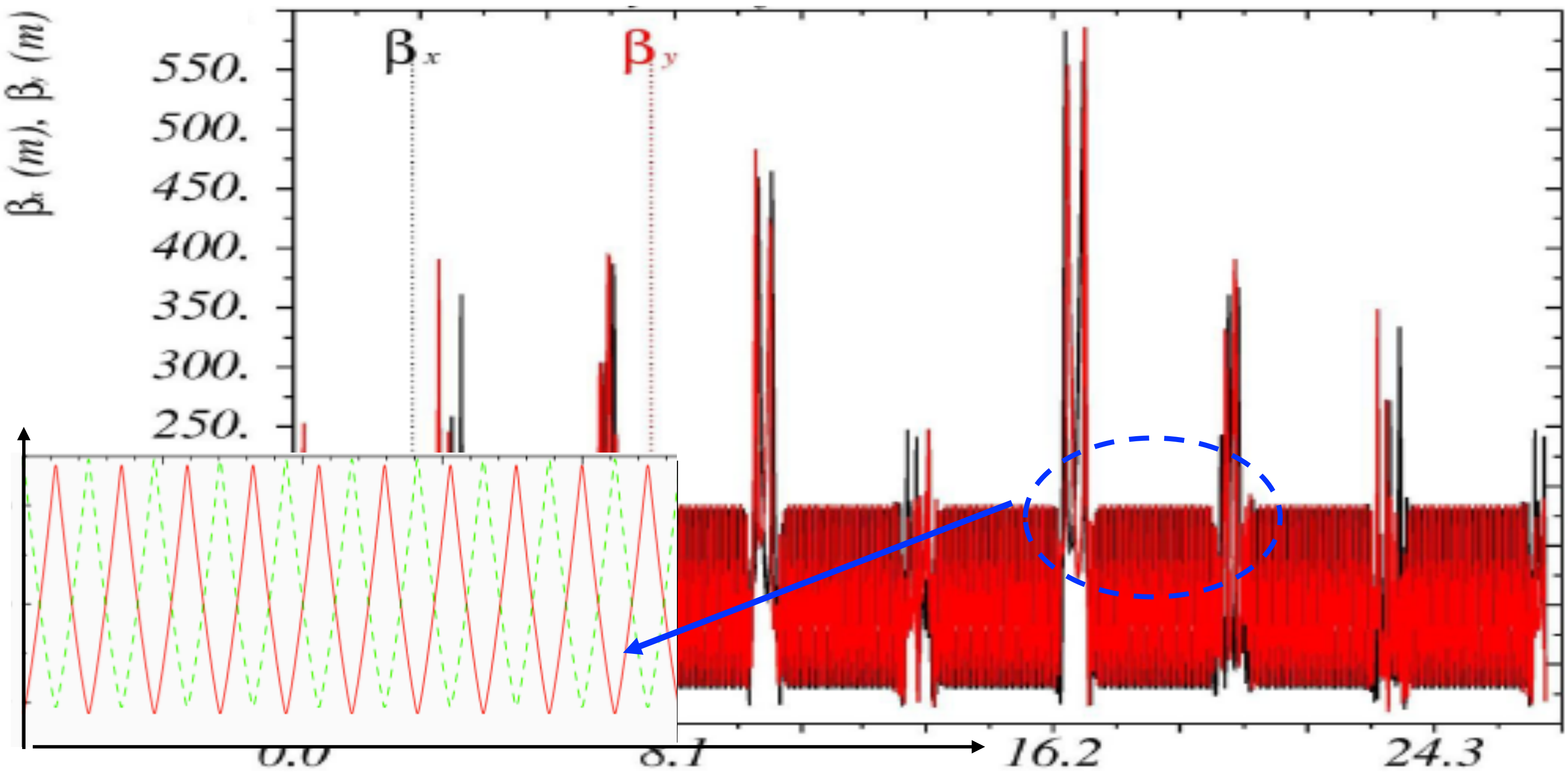
Magnets for the LHC, total budget, every magnet has a role in the optics design

Name	Quantity	Purpose
MB	1232	Main dipoles
MQ	400	Main lattice quadrupoles
MSCB	376	Combined chromaticity/ closed orbit correctors
MCS	2464	Dipole spool sextupole for persistent currents at injection
MCDO	1232	Dipole spool octupole/decapole for persistent currents
MO	336	Landau octupole for instability control
MQT	256	Trim quad for lattice correction
MCB	266	Orbit correction dipoles
MQM	100	Dispersion suppressor quadrupoles
MQY	20	Enlarged aperture quadrupoles

In total 6628 cold magnets ...

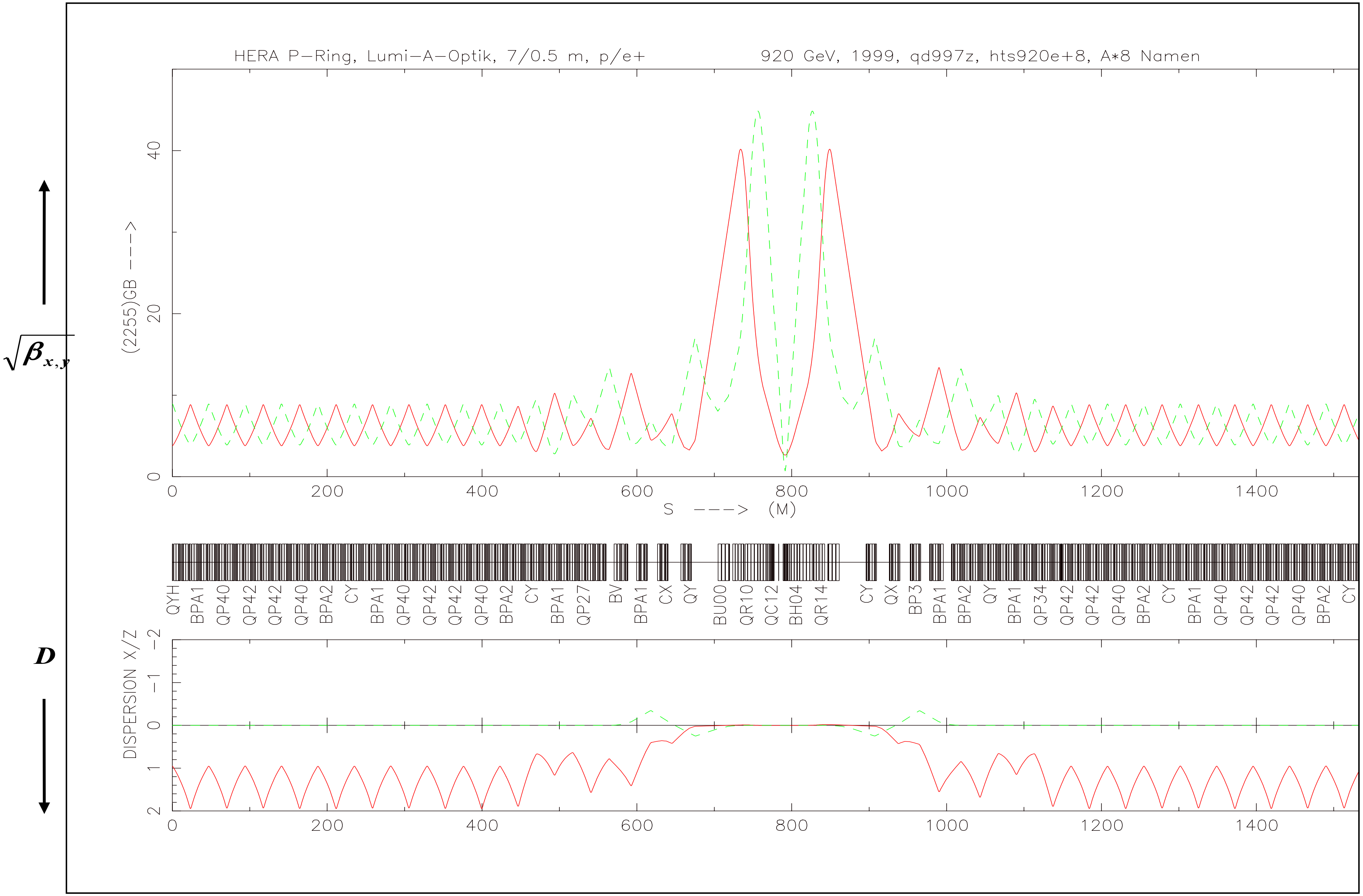
FoDo-Lattice

A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in   .
(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)

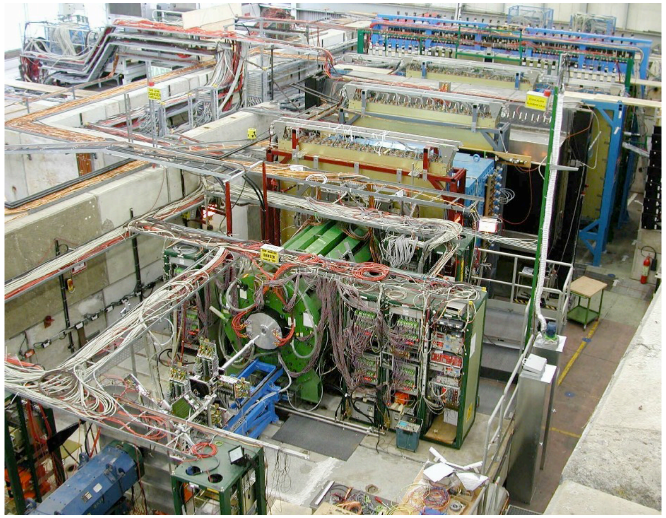


Starting point for the calculation: in the middle of a focusing quadrupole
Phase advance per cell $\mu = 45°$,
→ calculate the twiss parameters for a periodic solution

9.) Insertions

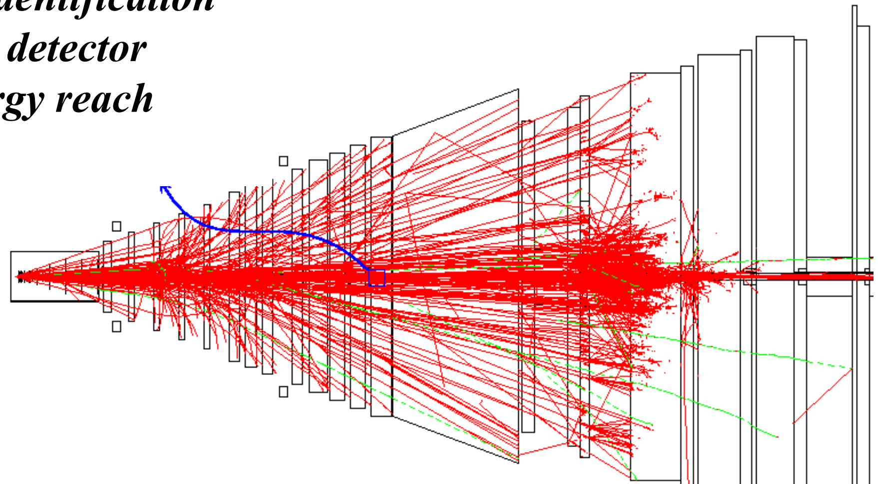


Fixed target experiments:



HARP Detector, CERN

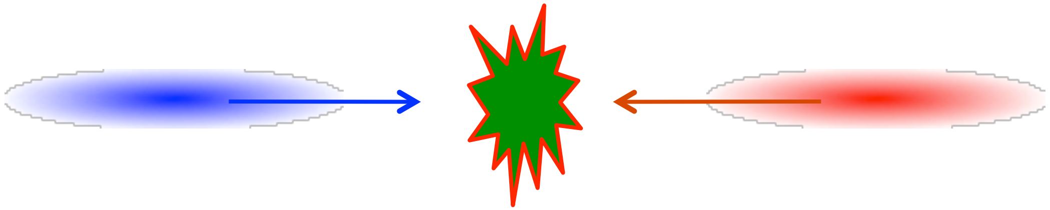
high event rate
easy track identification
asymmetric detector
limited energy reach

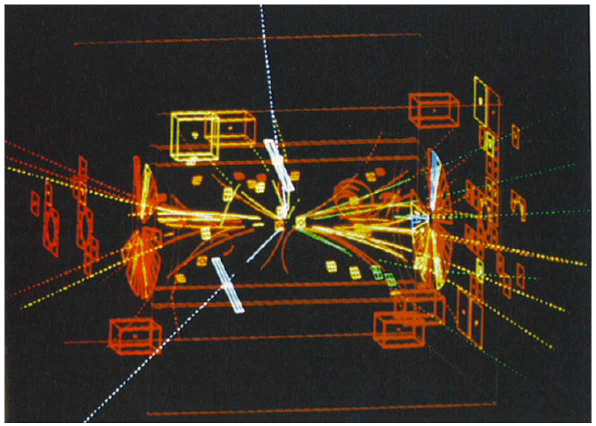


fixed target event p + W -> xxxxx

Collider experiments:

$E=mc^2$



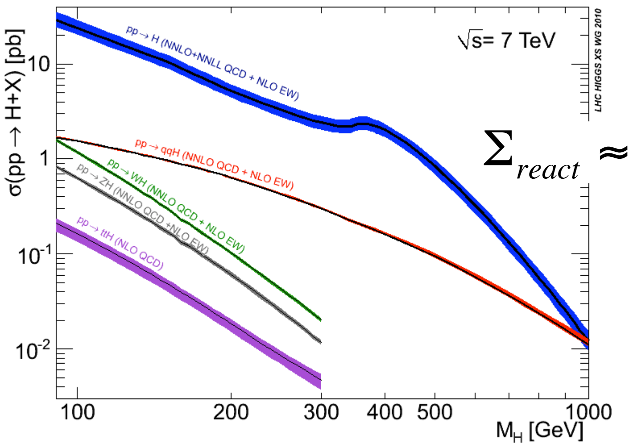


low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$

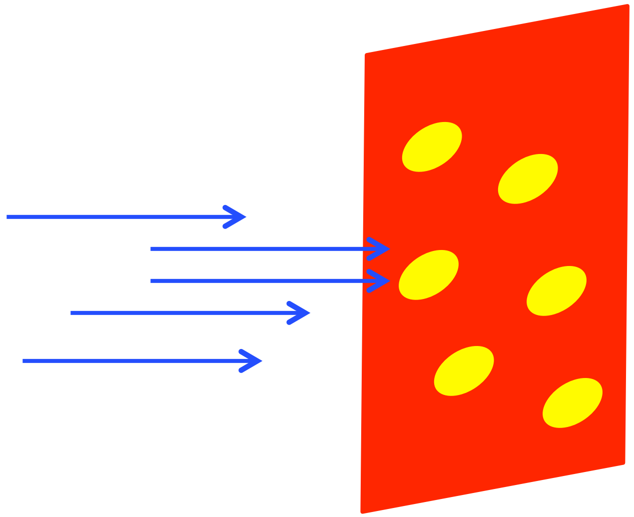
Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e+e− pair, shown as white dashed lines.

Problem: Our particles are VERY small !!

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\,pb$$

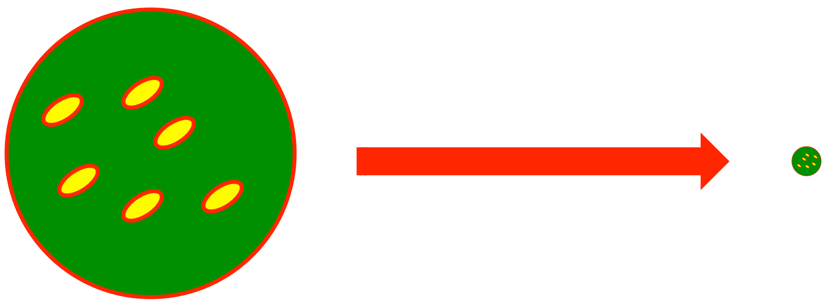


$$1b \;= 10^{-24}\,cm^2$$

$$1pb = \;\; 10^{-12} * 10^{-24}\,cm^2 = 1/mio * 1/mio * 1/mio * 1/mio * 1/mio * 1/10000\;mm^2$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP

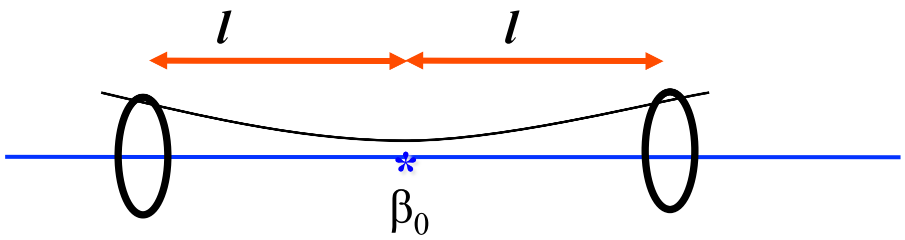


LHC typical:

σ = 0.1 mm → 16 μm

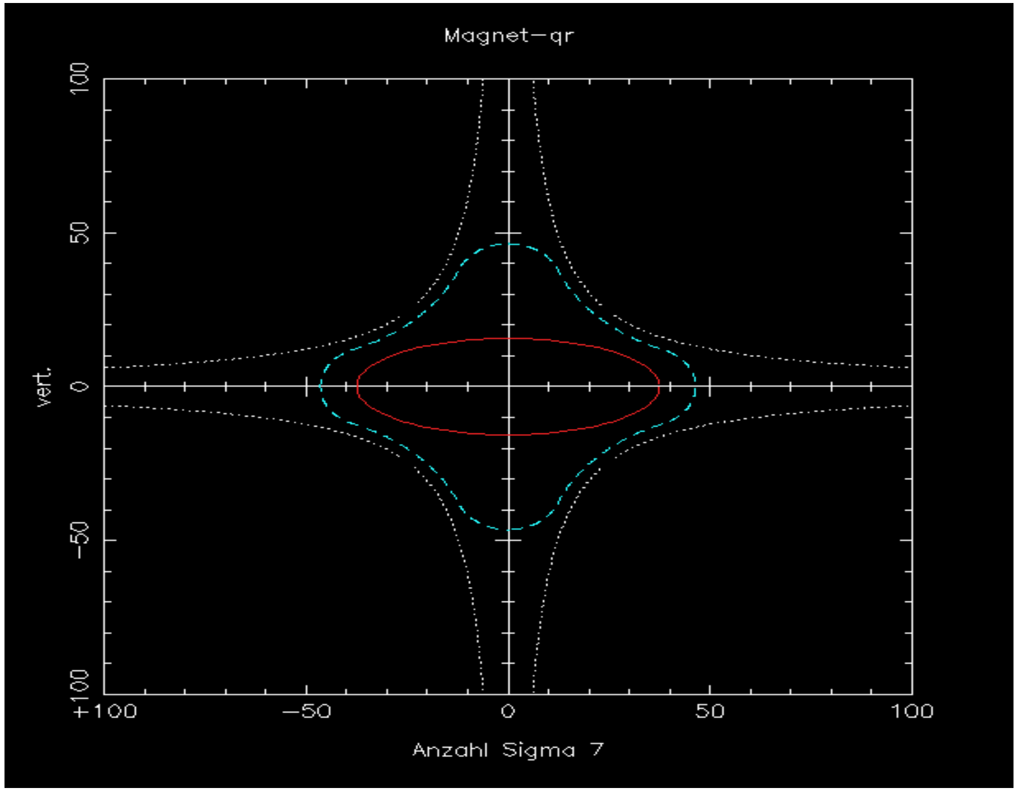
β- function in a drift

$$\beta(l) = \beta_0 + \frac{l^2}{\beta_0}$$



At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

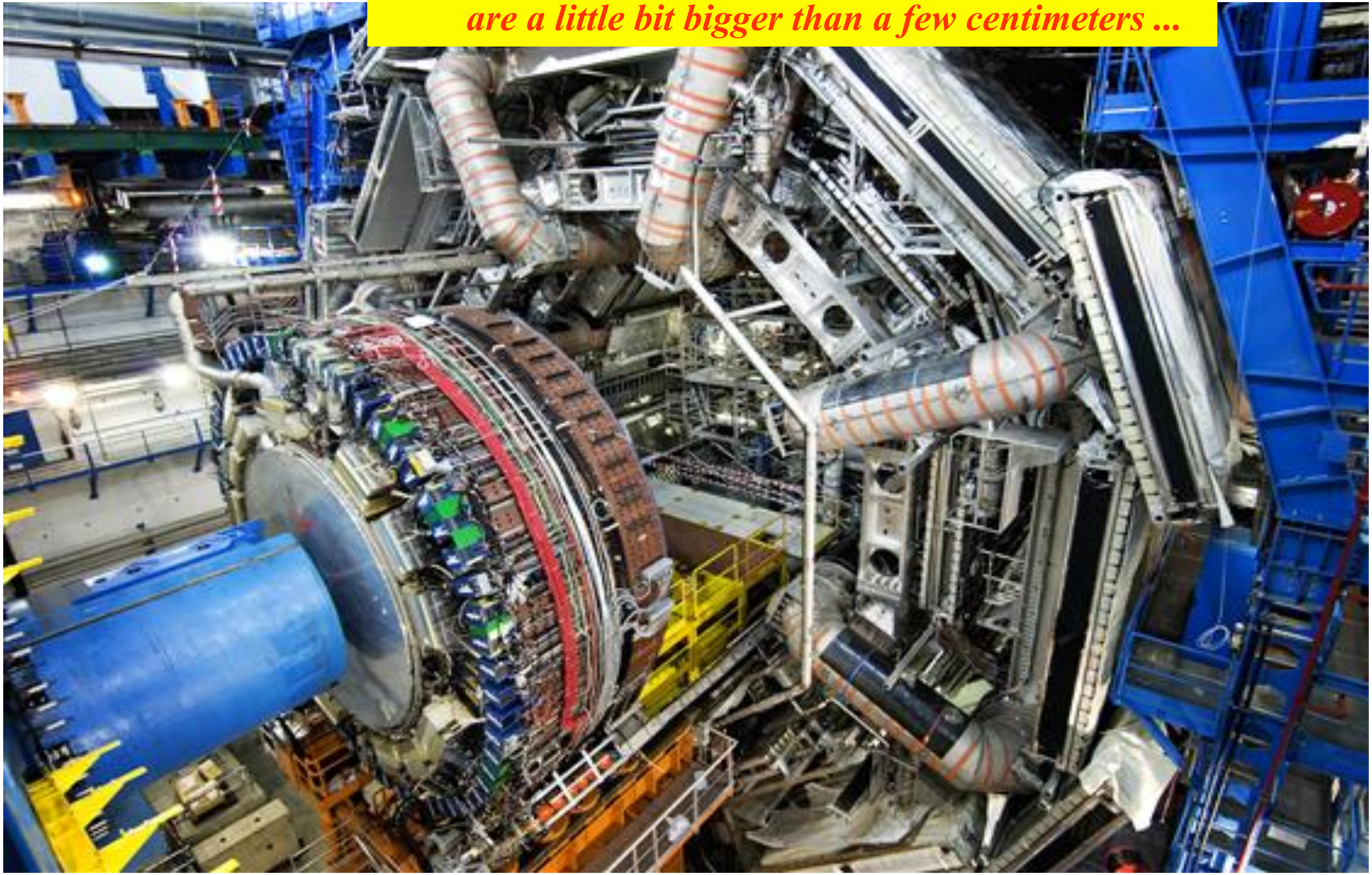
-> keep l as small as possible



7 sigma beam size inside a mini beta quadrupole

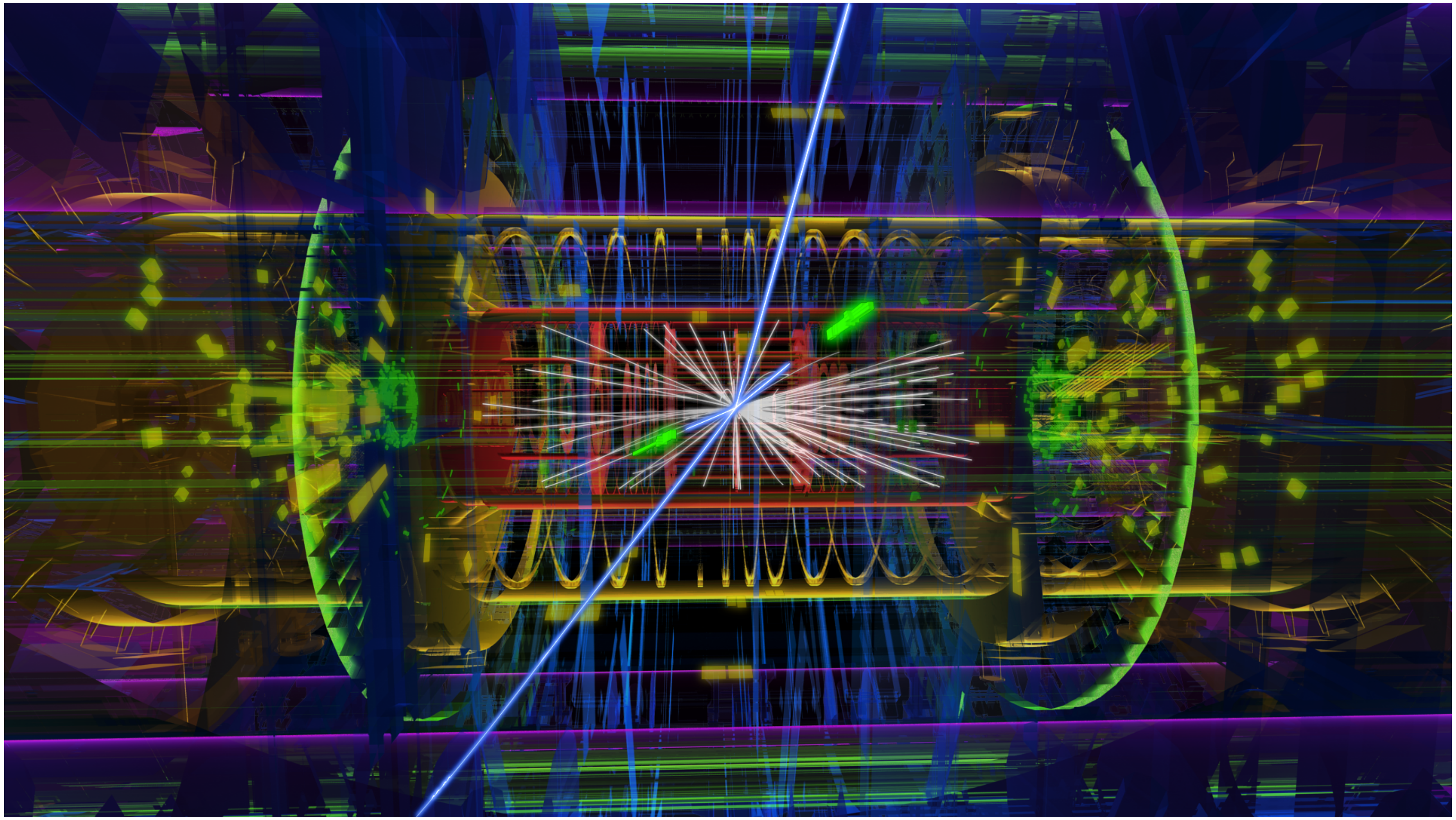
... clearly there is an

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



... and why all that ??

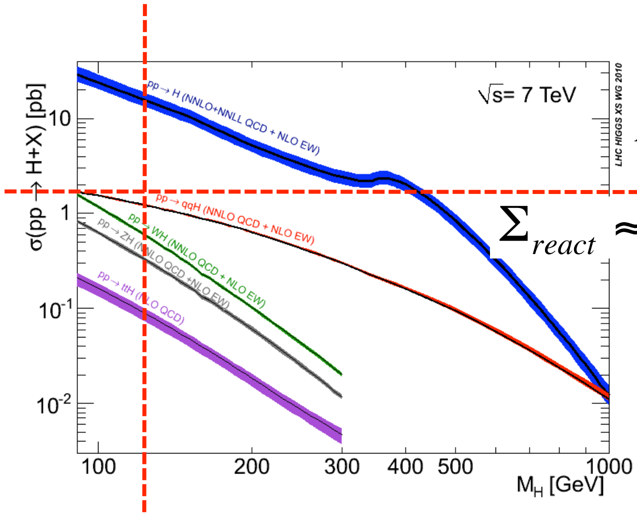
High Light of the HEP-Year 2012 / 13 naturally the HIGGS



ATLAS event display: Higgs => two electrons & two muons

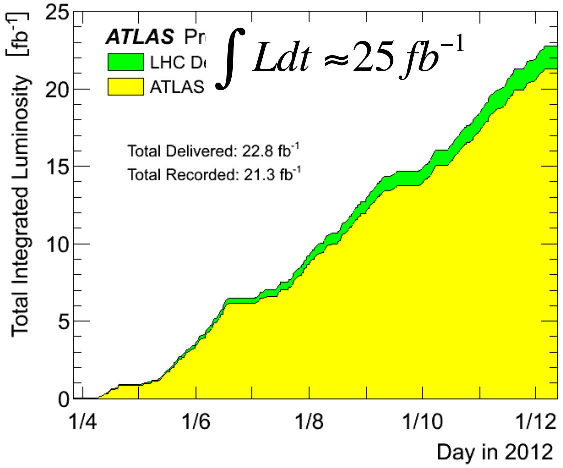
The High light of the year

production rate of events **is** ***determined by*** **the** ***cross section*** Σ_{react}
and a parameter L that is given by the design of the accelerator:
… the luminosity



$$\Sigma_{react} \approx 1\, pb$$

"typical particle size"
i.e. cross section for
particle production



$$\int L dt \approx 25\, fb^{-1}$$

accumulated
collision rate
in LHC run 1

$$1b = 10^{-24} cm^2 \quad = \quad 1/mio * 1/mio * 1/mio * \frac{1}{100} mm^2$$

The particles are "very small"

$$R = L * \Sigma_{react} \quad \approx 10^{-12} b \cdot 25 \frac{1}{10^{-15} b} = some\, 1000\, H$$

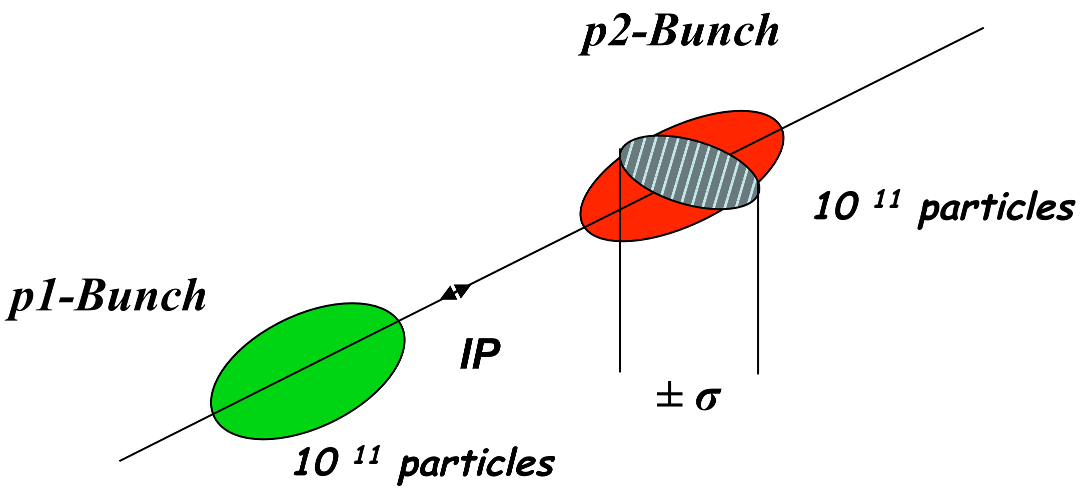
During collider run we had in Run 1 …
1400 bunches circulating,
with 800 Mio proton collisions per second in the experiments
and collected only 450 Higgs particles in three years.

10.) Luminosity

$$R = L * \Sigma_{react}$$

production rate of events is determined by the cross section Σ_{react} and a parameter L that is given by the design of the accelerator:

… the luminosity



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$I_p = 584\ mA$

$f_0 = 11.245\ kHz$

$n_b = 2808$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

beam sizes in the order of my cat's hair !!

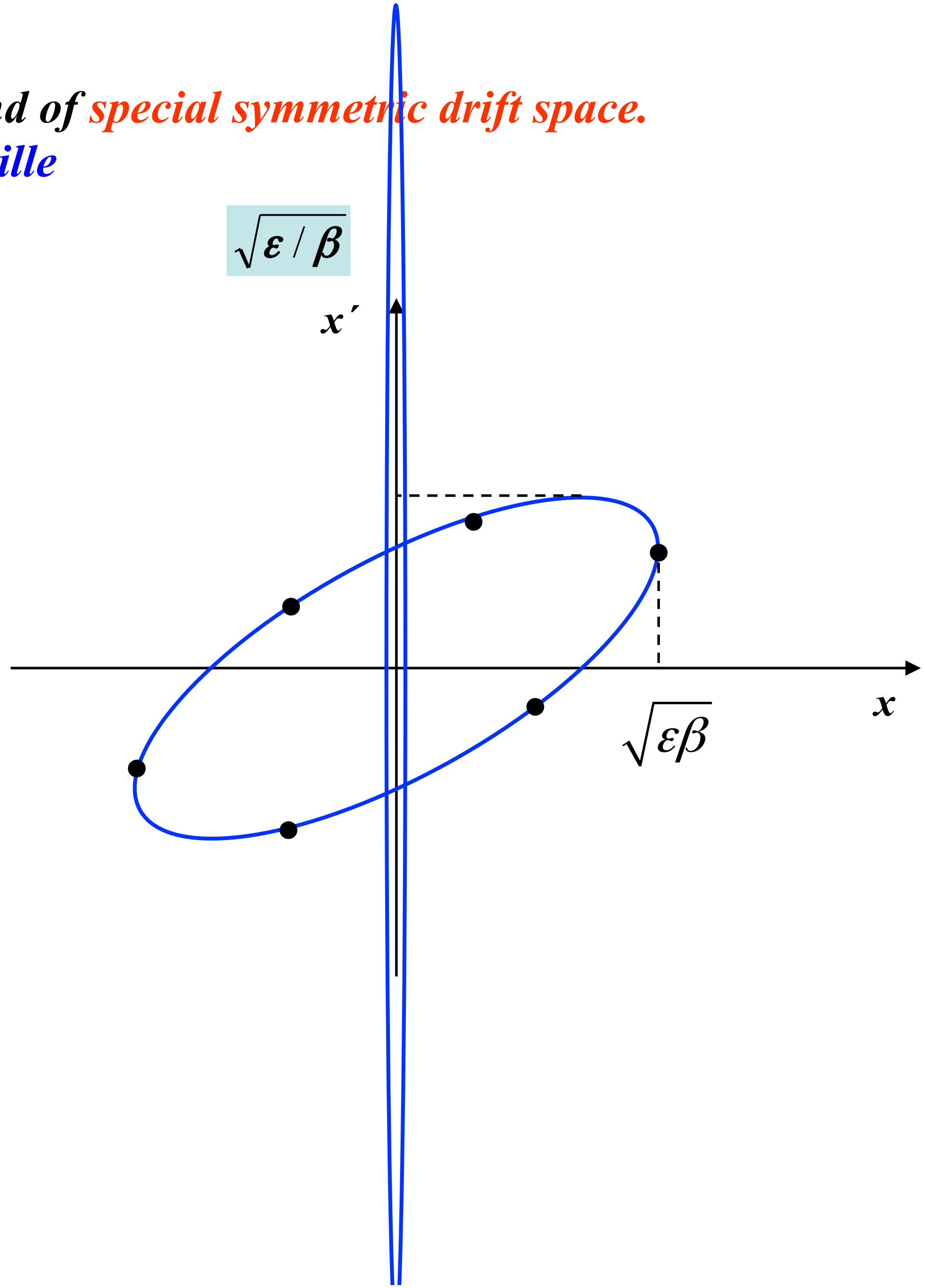
Mini-β insertions

A mini-β insertion is always a kind of special symmetric drift space.

→greetings from Liouville

the smaller the beam size

the larger the bam divergence



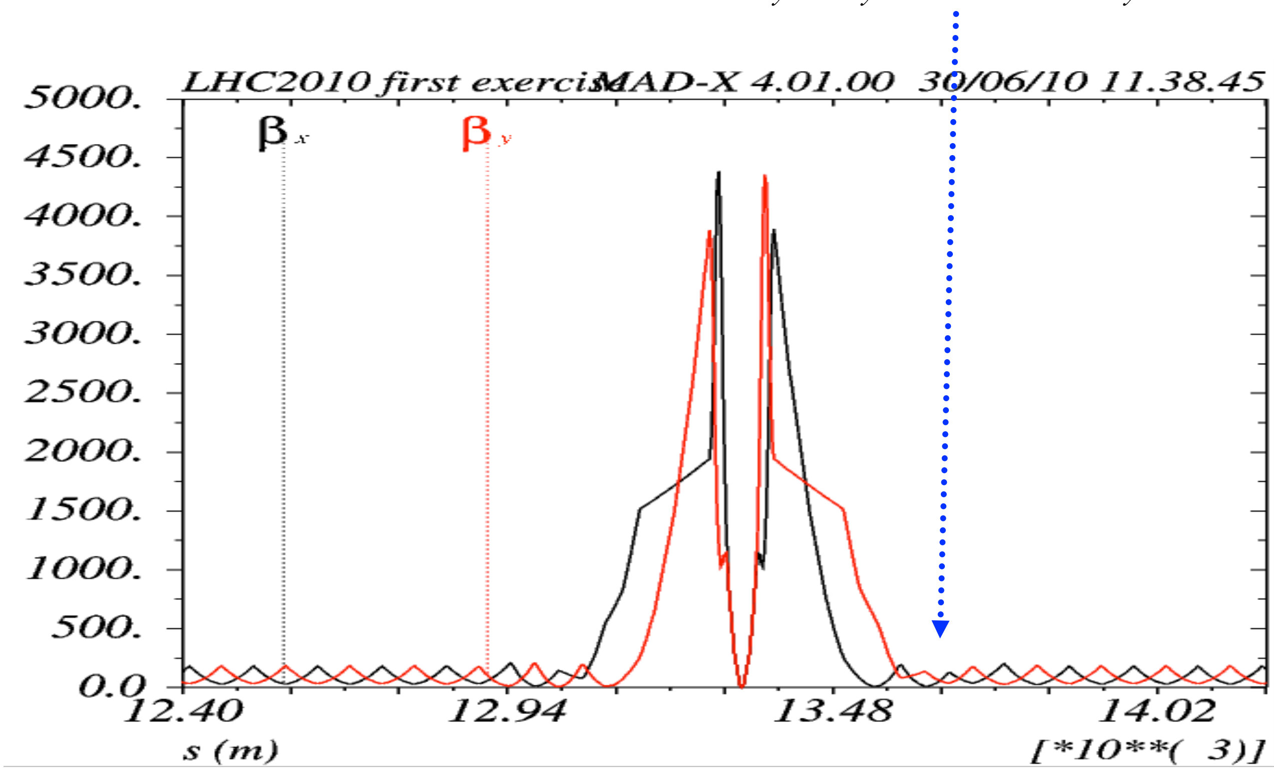
Mini-β Insertions: some guide lines

* *calculate the periodic solution in the arc*
* *introduce the drift space needed for the insertion device (detector ...)*
* *put a quadrupole doublet (triplet ?) as close as possible*
* *introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure*

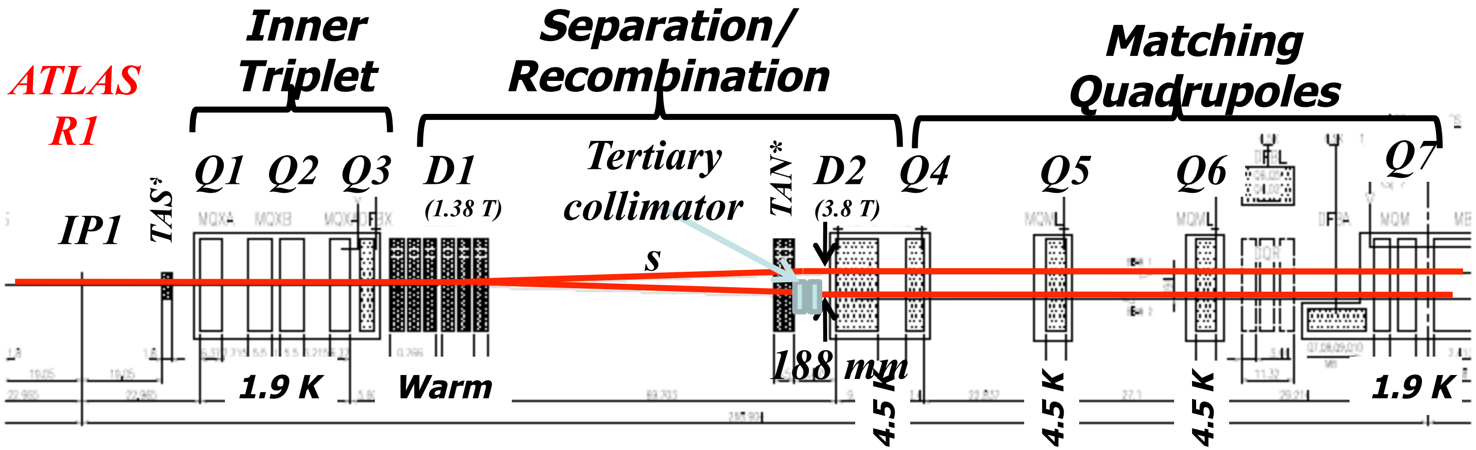
parameters to be optimised & matched to the periodic solution:

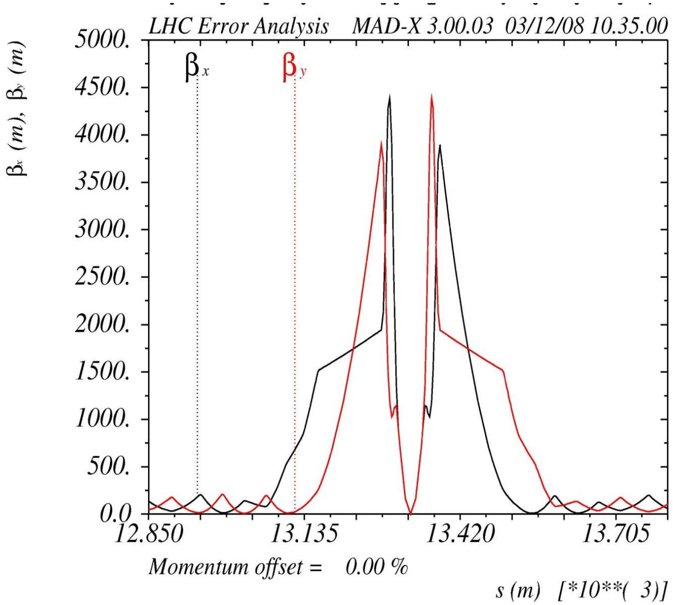
$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$
$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y$$

8 individually powered quad magnets are needed to match the insertion
(... at least)



The LHC Insertions





mini β optics