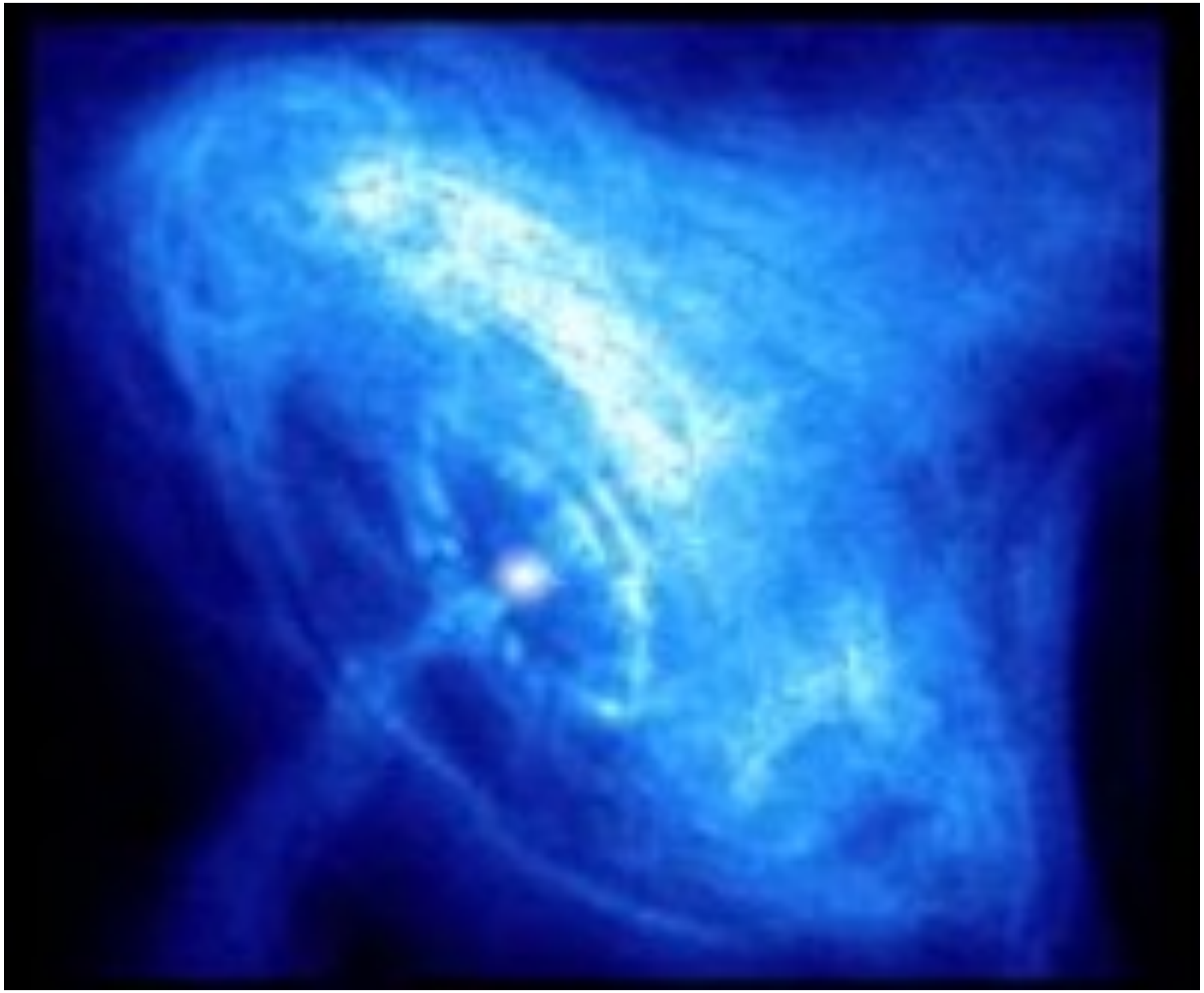
IV) … let´s talk about acceleration



crab nebula,

burst of charged particles E = 10 ^{20} eV

Energy Gain

... we have to start again from the basics

Lorentz force

$$\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$$

in long. direction the B-field creates no force

v || B

$$\vec{F} = \frac{d\vec{p}}{dt} = e\vec{E}$$

acc. force is given by the electr. Field

In relativistic dynamics, energy and momentum satisfy the relation:

$$E^2 = E_0^2 + p^2c^2$$

$$\left(E = E_0 + W\right)$$

Hence:

$$dE = \int F ds = v dp$$

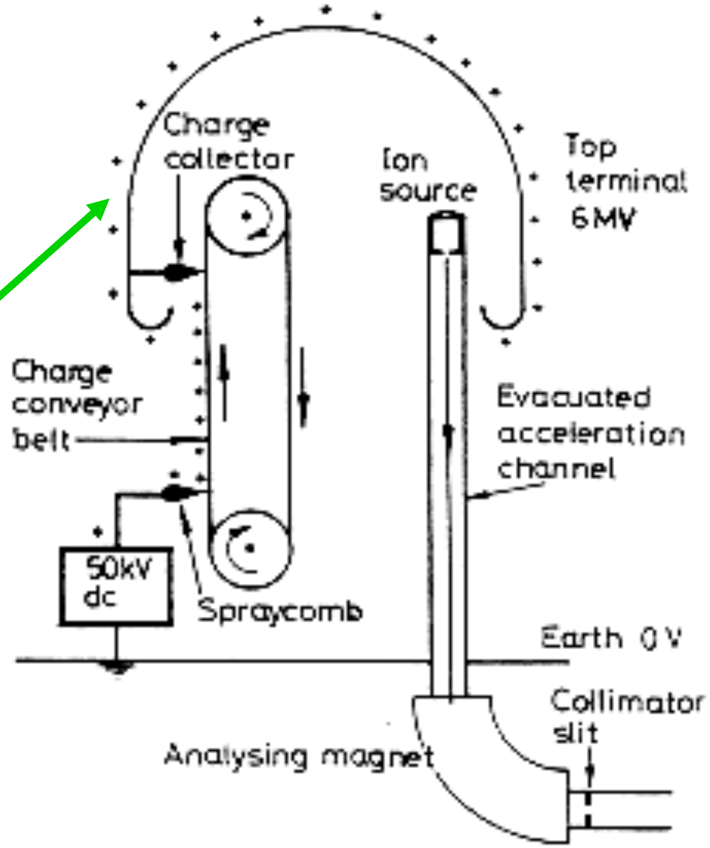
and the kinetic energy gained from the field along the z path is:

$$dW = dE = eE_z ds \qquad \Rightarrow \qquad W = e\int E_z ds = eV$$

11.) Electrostatic Machines

(Tandem –) van de Graaff Accelerator

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the
structure

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

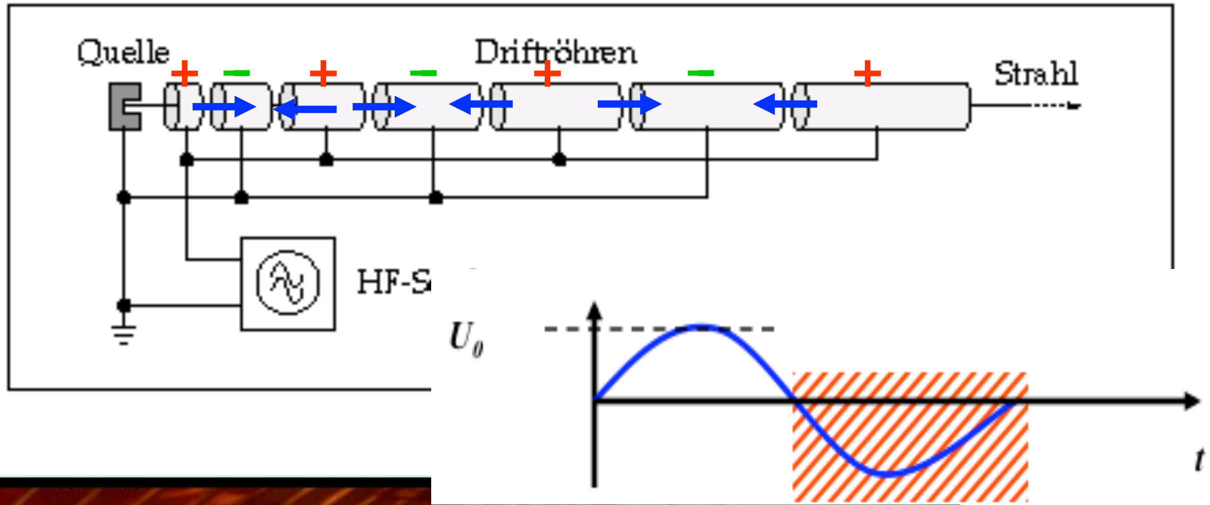
nota bene: all particles are "synchron" with the acceleration potential

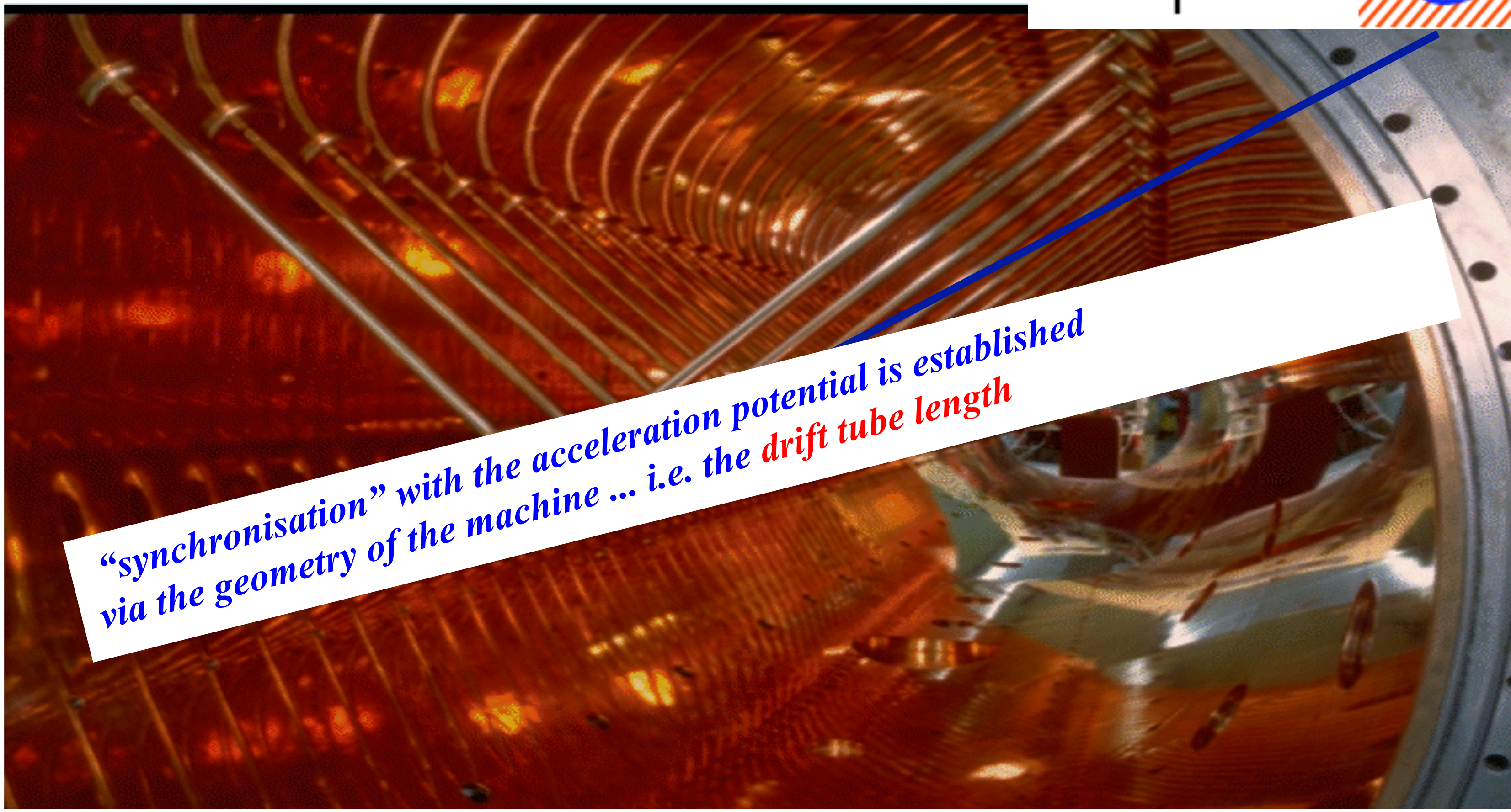
Electro Static Accelerator: 12 MV-Tandem van de Graaff
Accelerator at MPI Heidelberg

12.) Linear Accelerator 1928, Wideroe

Energy Gain per „Gap“:

$$W = q\,U_0 \sin \omega_{RF} t$$

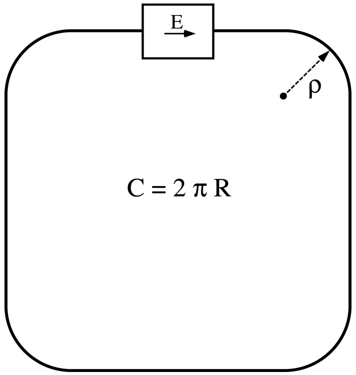




“synchronisation” with the acceleration potential is established via the geometry of the machine ... i.e. the drift tube length

drift tube structure at a proton linac (GSI Unilac)

The Synchrotron (Mac Millan, Veksler, 1945)



The synchrotron: Ring Accelerator of const. R where the increase in momentum (i.e. B-field) is automatically synchronised with the correct synchronous phase of the particle in the rf cavities

"synchronisation" as basic principle of the particle dynamics

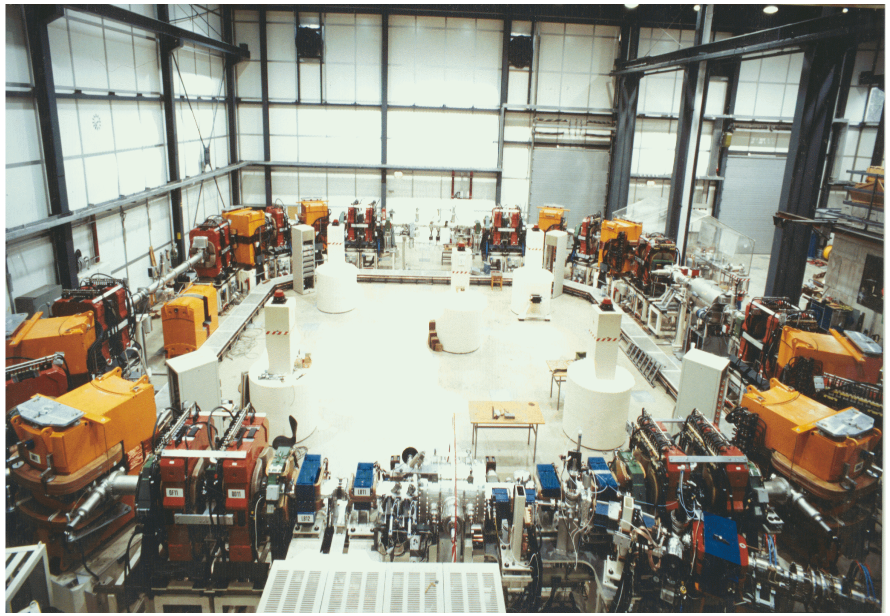
$e\hat{V}$ … → ***Energy gain per turn***

$\Phi = \Phi_s = cte$ → ***Synchronous particle***

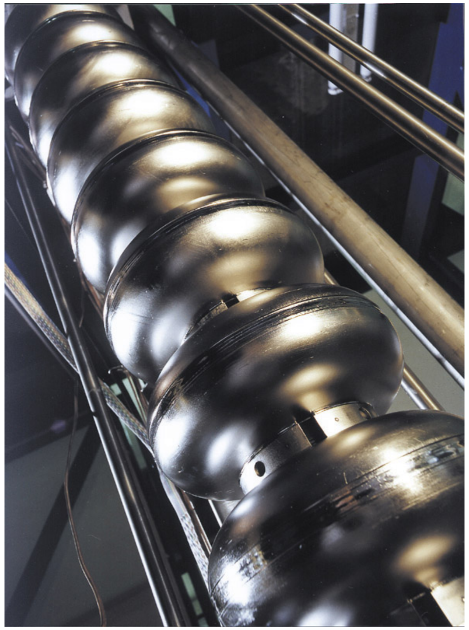
$\omega_{RF} = h\omega_r$ → ***RF synchronism***

$\rho = cte \quad R = cte$ → ***Constant orbit***

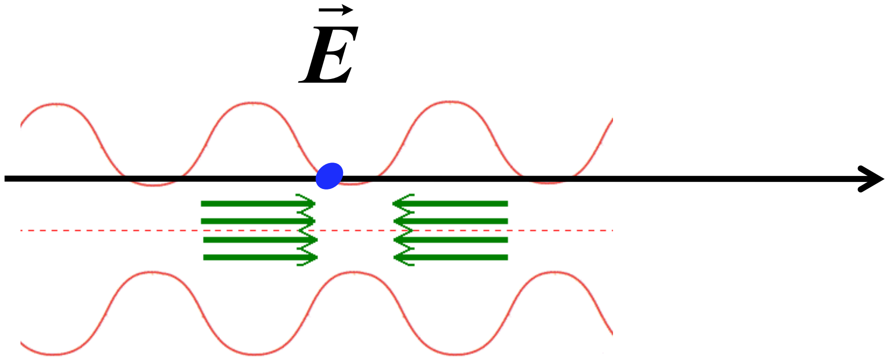
$B\rho = P/e \Rightarrow B$ → ***Variable magnetic field***



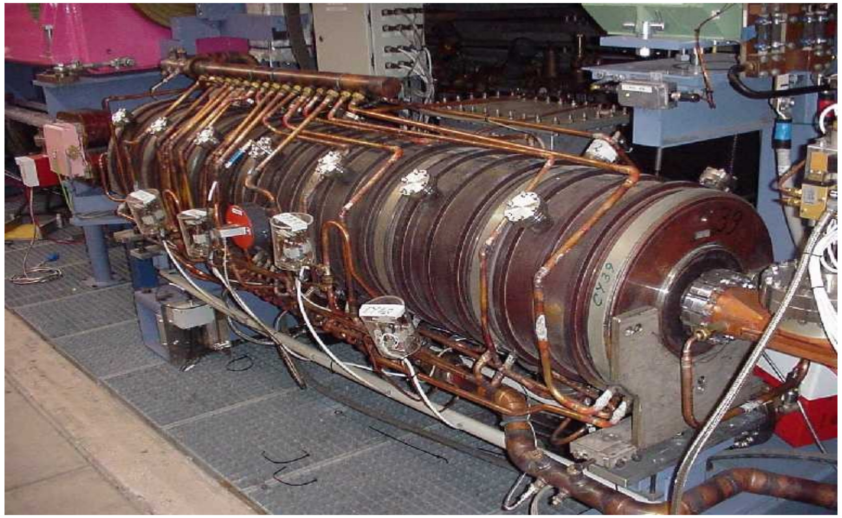
13.) The Acceleration

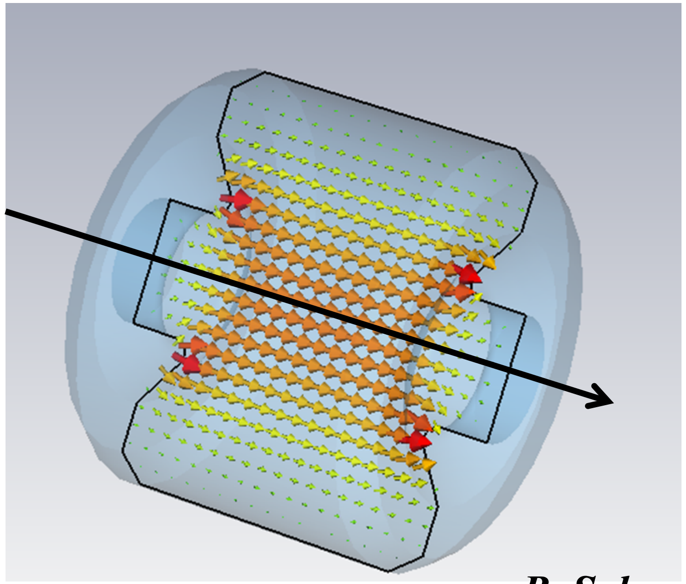


Where is the acceleration?
Install an RF accelerating structure in the ring and adjust the phase (the timing) between particle and RF-Voltage in the right way: "Synchronisation"



500 MHz cavities in an electron storage ring





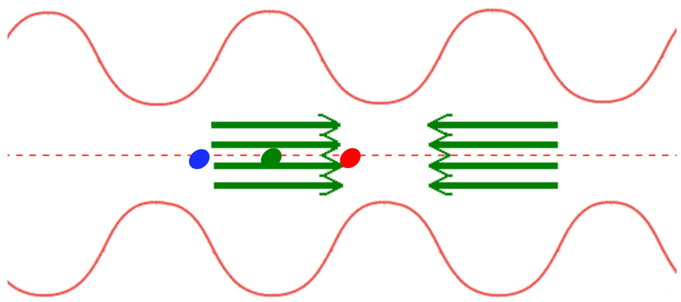
B. Salvant
N. Bianca

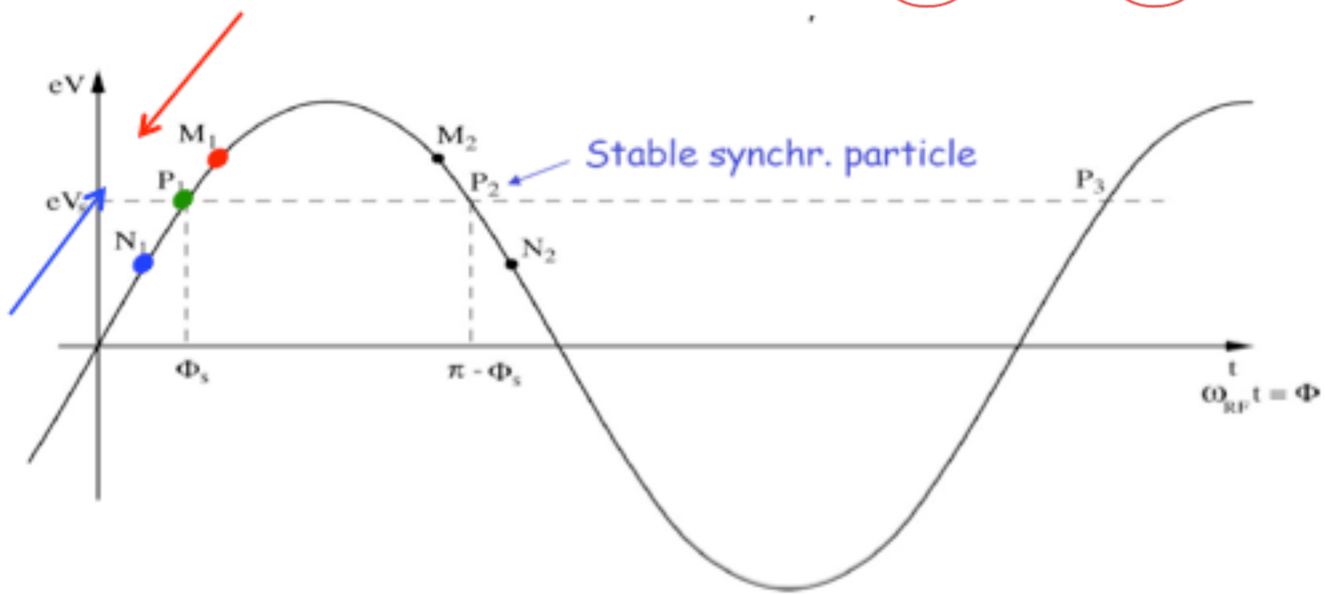
14.) The Acceleration for Δp/p≠0
"Phase Focusing" below transition

ideal particle •

particle with Δp/p > 0 • *faster*

particle with Δp/p < 0 • *slower*



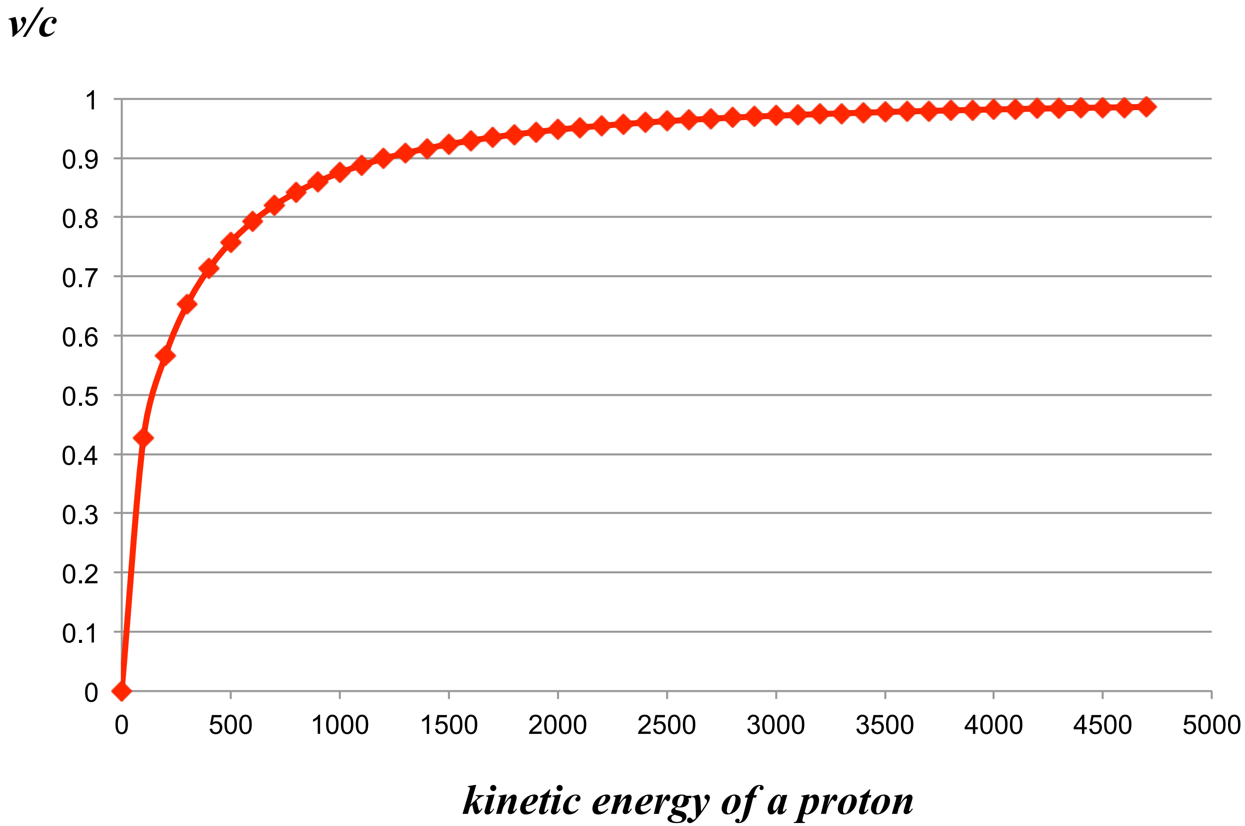


Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \longrightarrow \quad \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$



... some when the particles do not get faster anymore

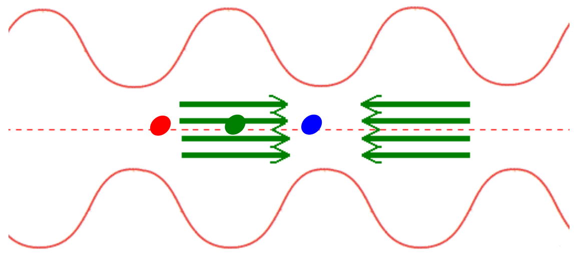
.... but heavier !

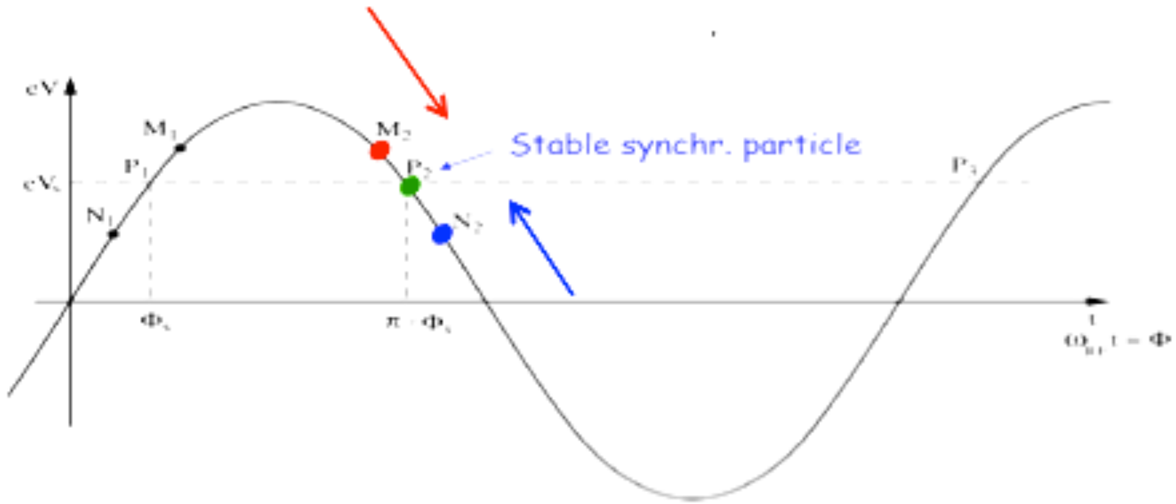
15.) The Acceleration for $\Delta p/p \neq 0$
"Phase Focusing" above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



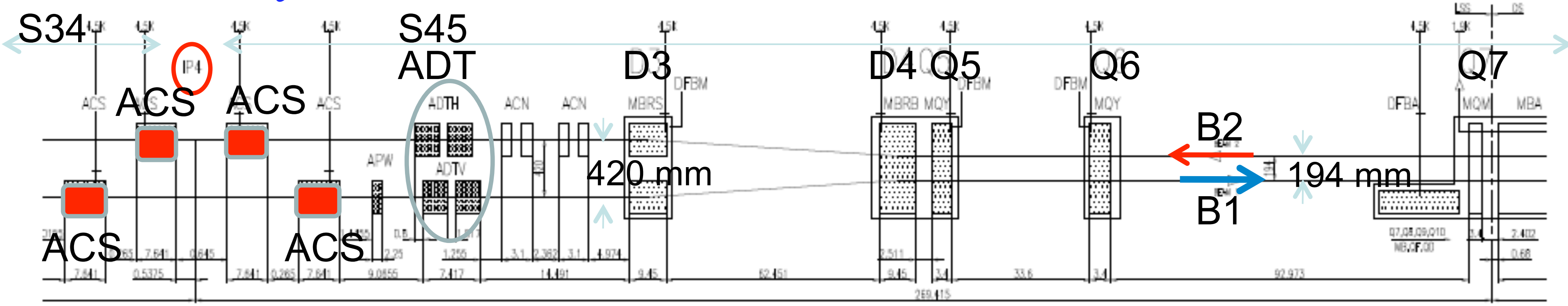


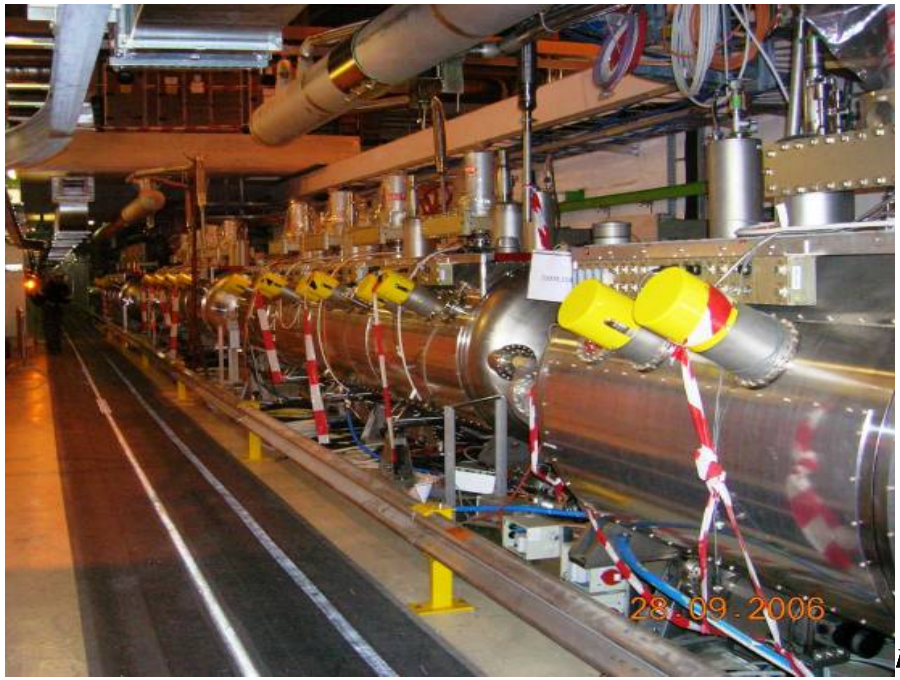
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





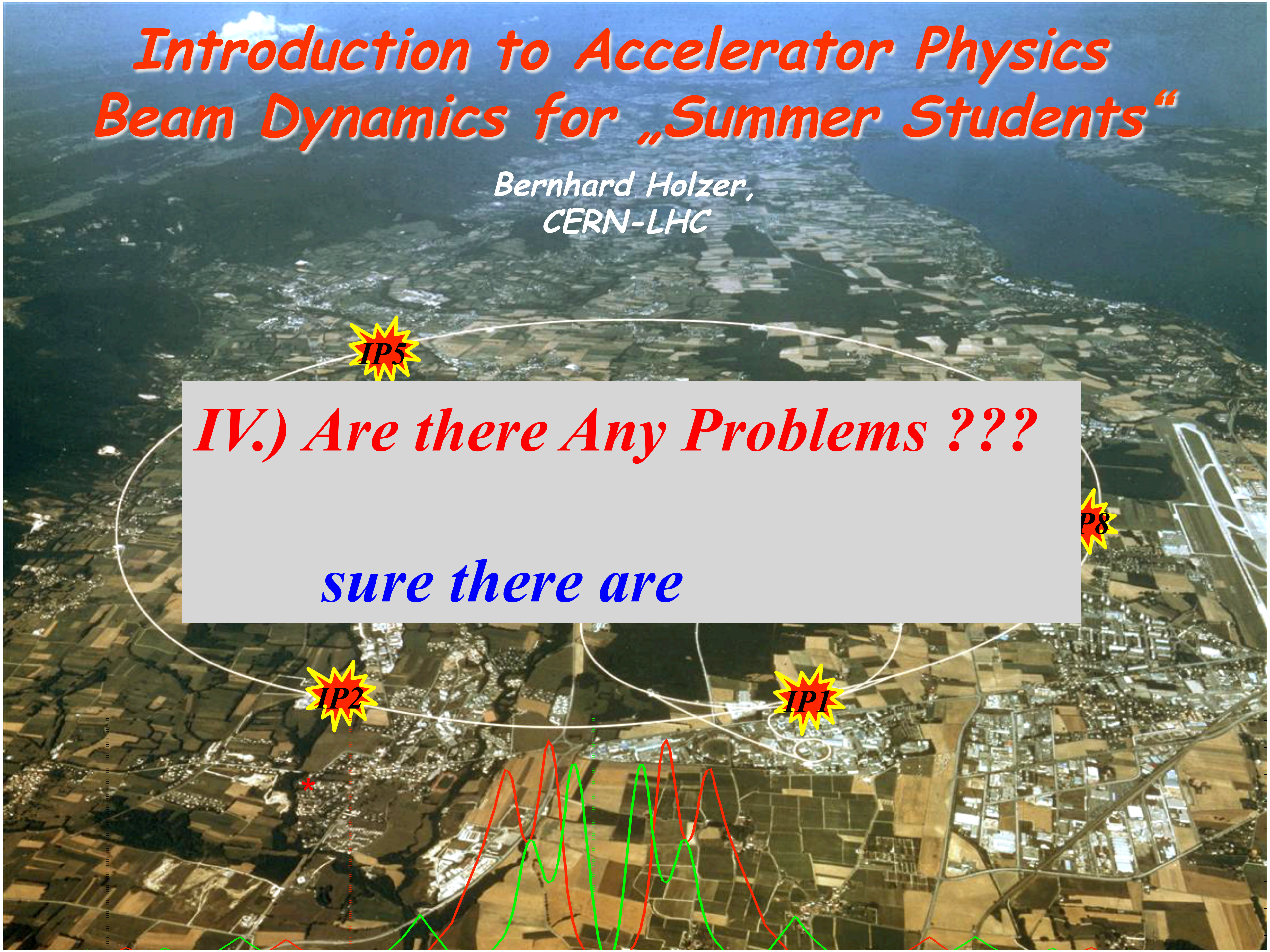
m

Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	*10^{-3}*	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

Introduction to Accelerator Physics
Beam Dynamics for „Summer Students“

Bernhard Holzer,
CERN-LHC

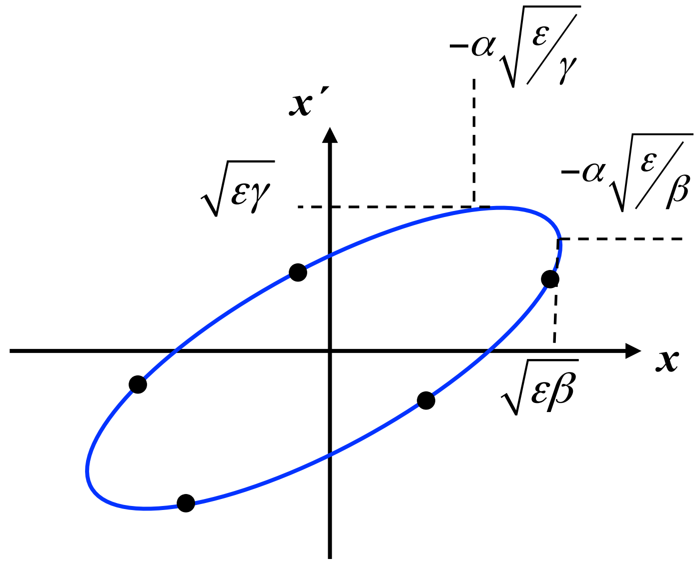


IV.) Are there Any Problems ???

sure there are

Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\, x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the x, x´ Phase Space Ellipse*

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq$ const !

Classical Mechanics:

phase space = diagram of the two canonical variables
position & momentum
x $\quad$ p_x

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad ; \quad L = T - V = kin.\,Energy - pot.\,Energy$$

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

$$\boldsymbol{q = position = x}$$
$$\boldsymbol{p = momentum = \gamma m v = m c \gamma \beta_x}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad \beta_x = \frac{\dot{x}}{c}$$

Liouvilles Theorem: $\int p\,dq = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta}$$

where $\beta_x = \frac{\dot{x}}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\int p dq = mc \int \gamma \beta_x dx$$

$$\int p dq = mc\gamma\beta \underbrace{\int x' dx}_{\varepsilon}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\boldsymbol{\varepsilon \sim 1/\gamma}$

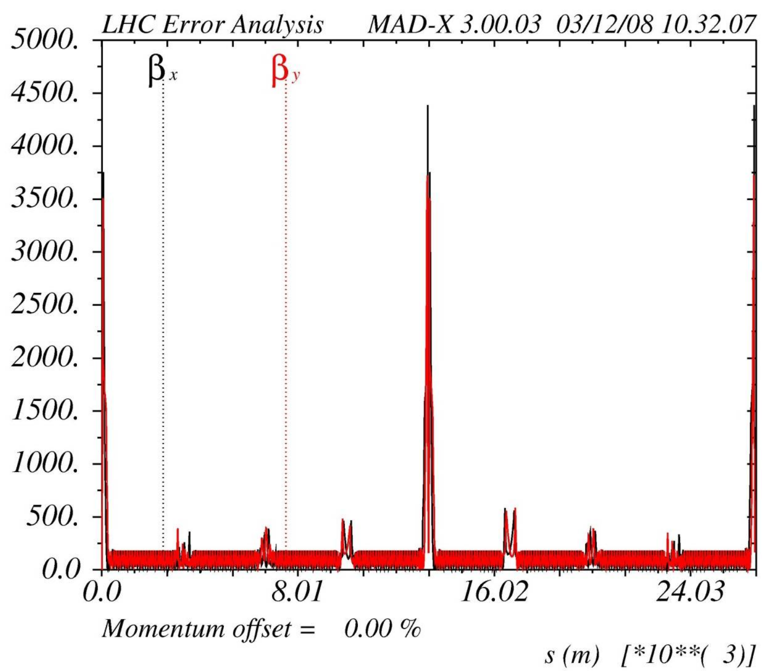
Nota bene:

1.) *A proton machine … or an electron linac … needs the highest aperture at injection energy !!!* *as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.*

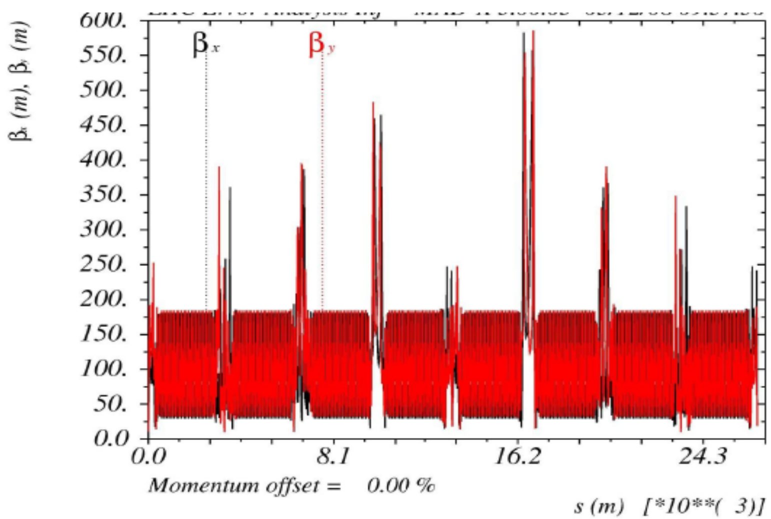
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) *At lowest energy the machine will have the major aperture problems,* → *here we have to minimise* $\hat{\beta}$

3.) *we need different beam optics adopted to the energy:* *A Mini Beta concept will only be adequate at flat top.*



LHC mini beta optics at 7000 GeV



LHC injection optics at 450 GeV

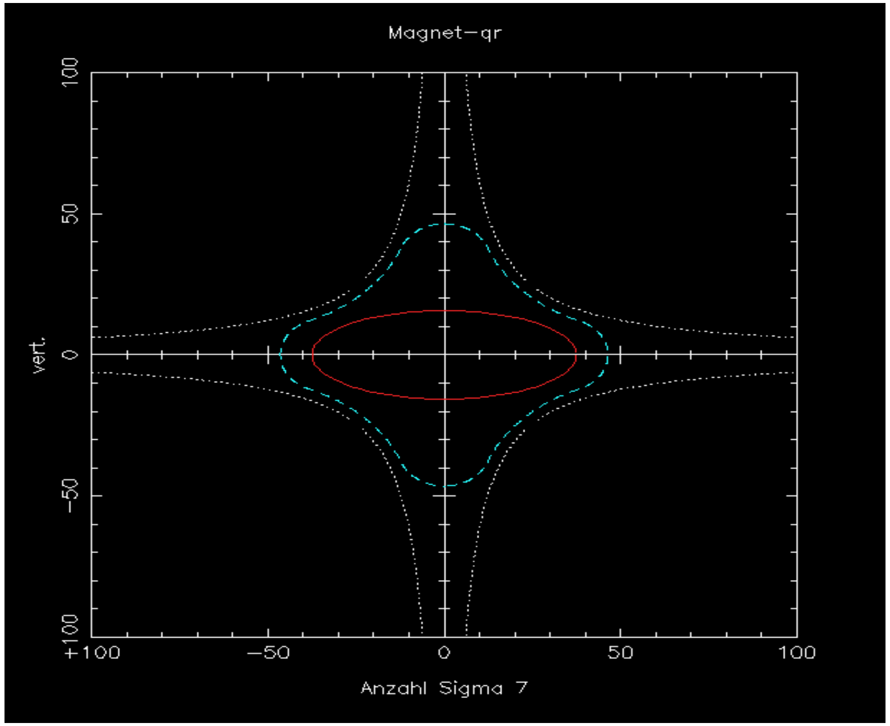
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$

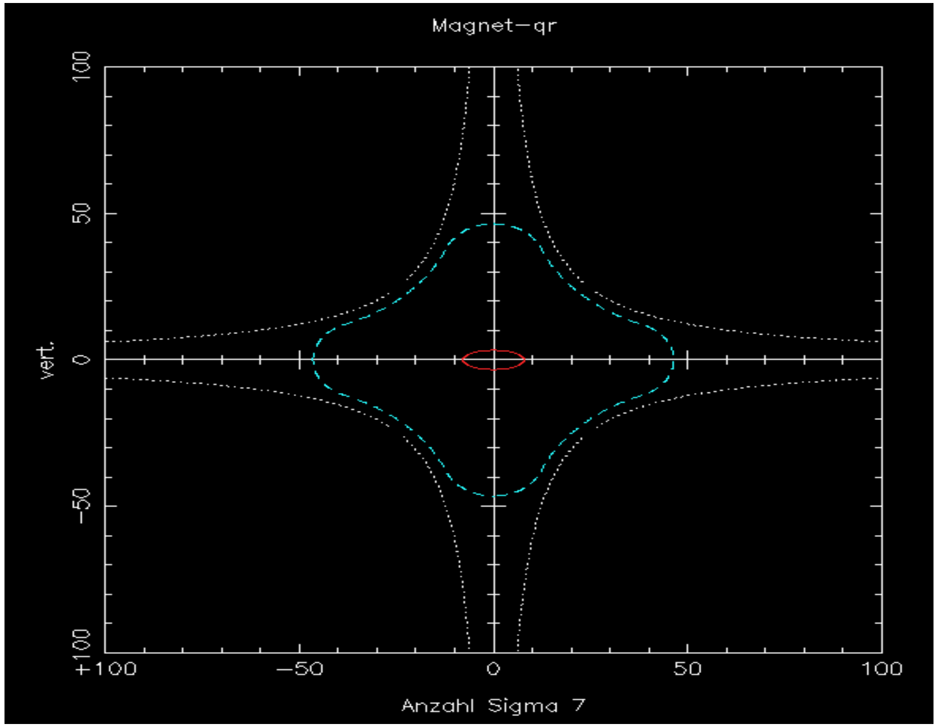
flat top energy: 920 GeV $\gamma = 980$

emittance ε *(40GeV)* $= 1.2 * 10^{-7}$

ε *(920GeV)* $= 5.1 * 10^{-9}$



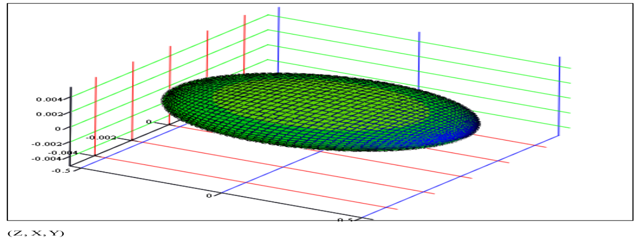
7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

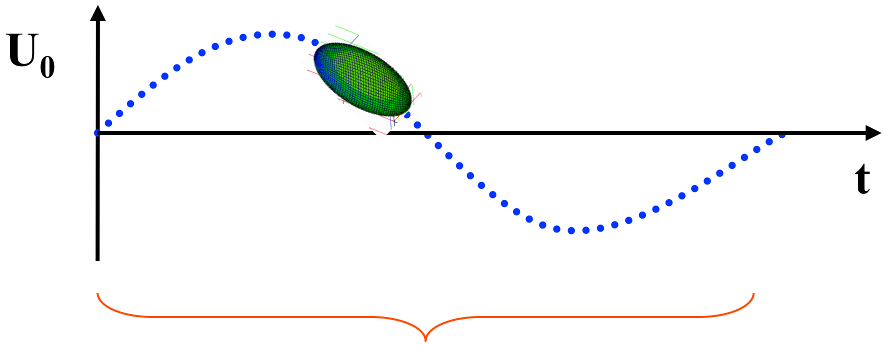
RF Acceleration-Problem: panta rhei !!!

(Heraklit: 540-480 v. Chr.)



Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$\lambda = 75\ cm$

$$\nu = 400\, MHz$$
$$c = \lambda\, \nu$$
$$\lambda = 75\ cm$$

$$\sin(90^o) = 1$$
$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

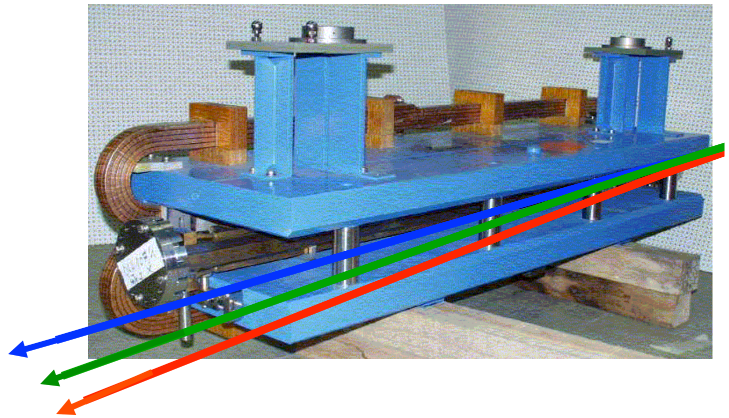
Sure there are !!!

font colors due to pedagogical reasons

17.) Dispersion and Chromaticity:
Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet $\alpha = \frac{\int B\,dl}{p/e}$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens $k = \frac{g}{p/e}$

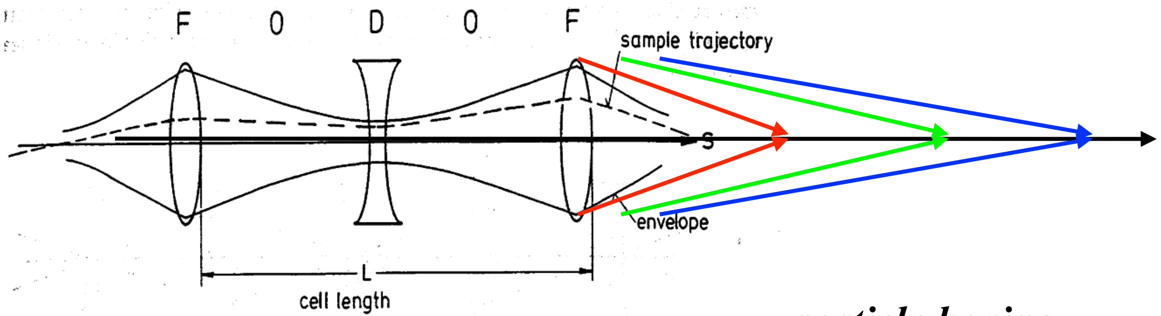
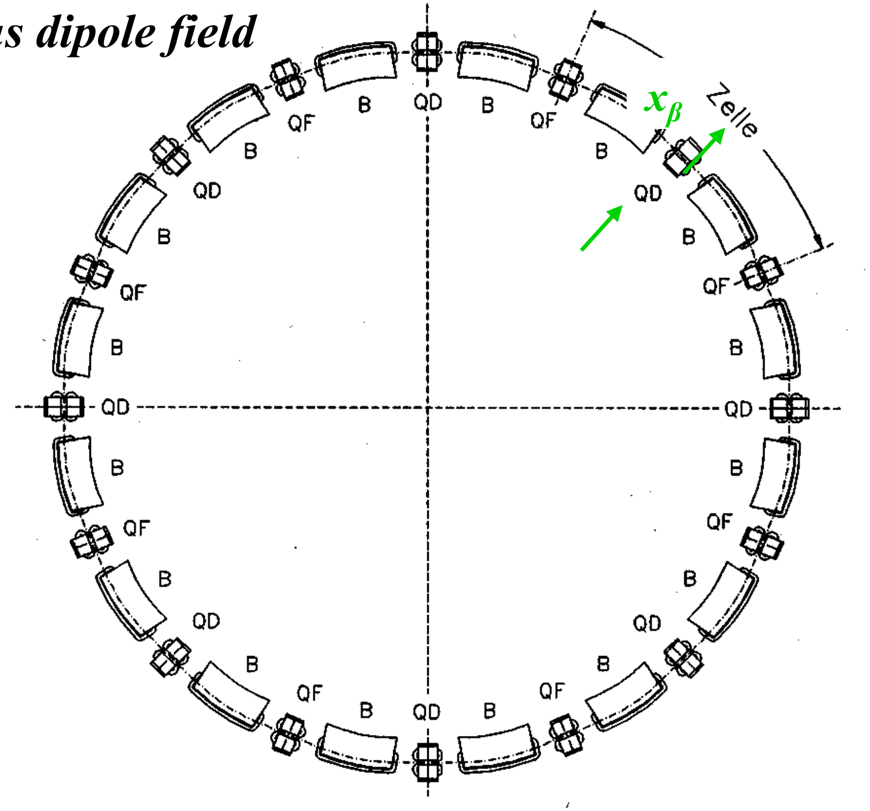


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

Dispersion

Example: homogeneous dipole field



ıit for Δp/p > 0

$$: D(s) \cdot \frac{\Delta p}{p}$$

Matrix formalism:

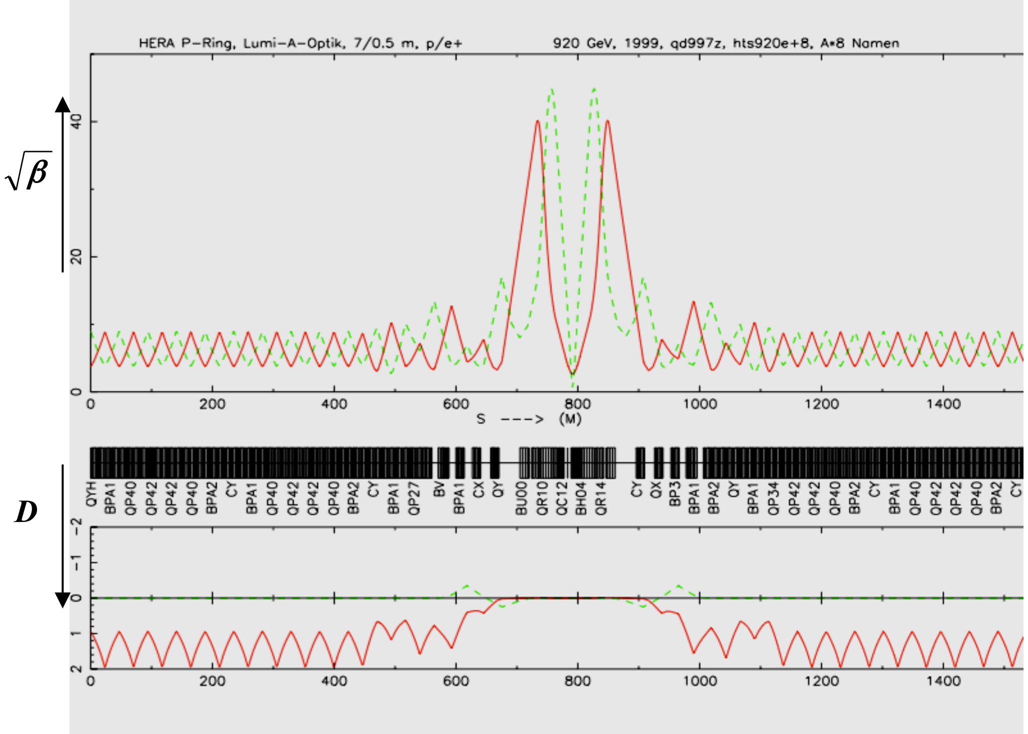
$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x_0' + D(s) \cdot \frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \dots 2\,mm$$
$$D(s) \approx 1 \dots 2\,m$$
$$\Delta p / p \approx 1 \cdot 10^{-3}$$

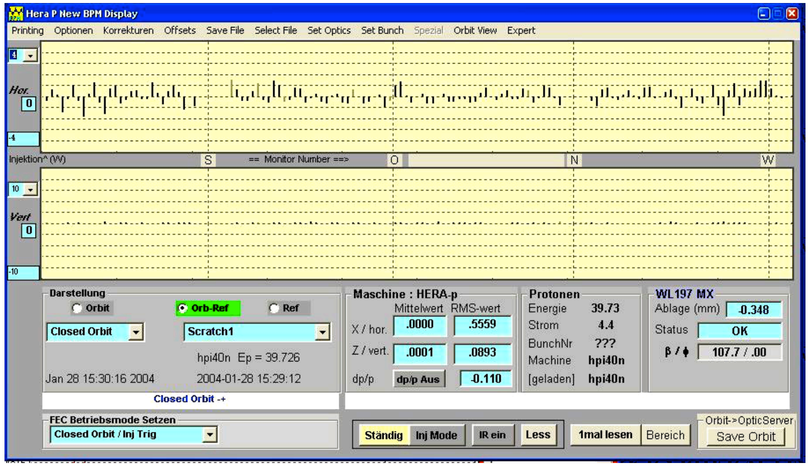
Amplitude of Orbit oscillation

contribution due to Dispersion ≈ beam size

→ Dispersion must vanish at the collision point

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

Dispersion is visible



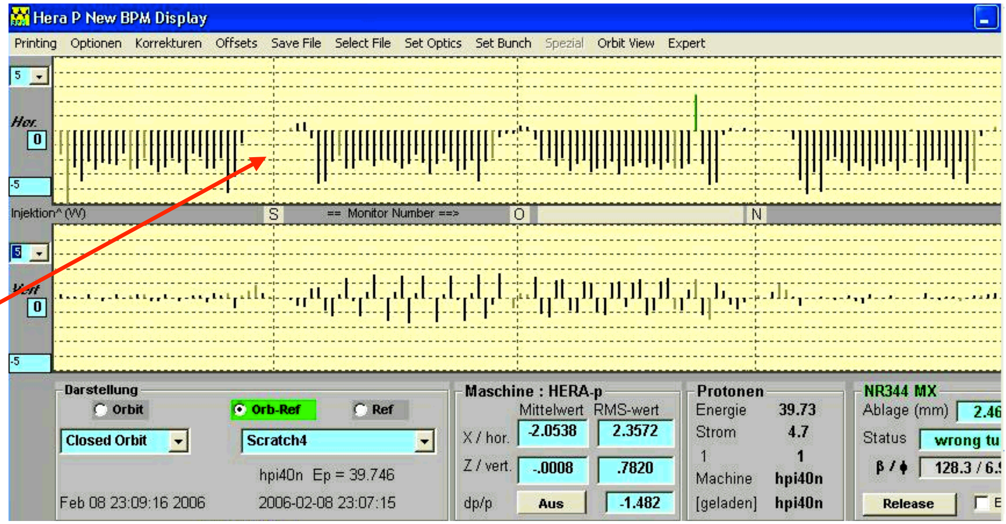
HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam

→ closed orbit is moved to a dispersions trajectory

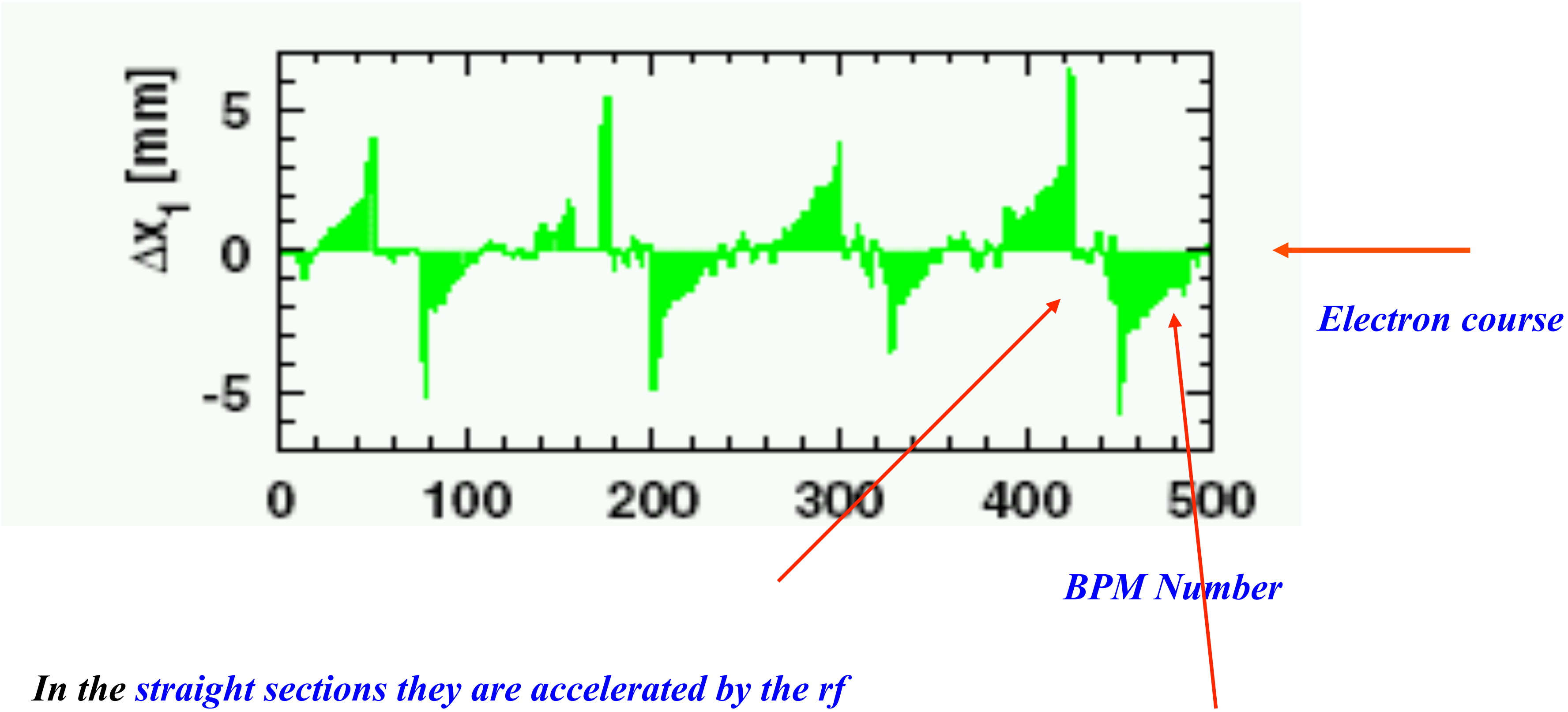
$$x_D = D(s) * \frac{\Delta p}{p}$$



Attention: at the Interaction Points we require D=D′= 0

Periodic Dispersion:

„Sawtooth Effect“ at LEP (CERN)



In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.

In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.

26.) Chromaticity:
A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

focusing lens $$k = \frac{g}{p/e}$$

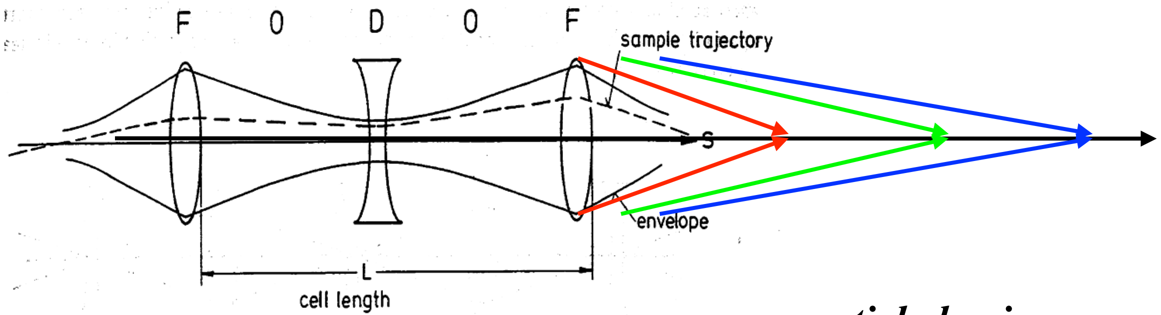


Figure 29: FODO cell

particle having ...

to high energy

to low energy

ideal energy

... which acts like a quadrupole error in the machine and leads to a tune spread:

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds$$

definition of chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p} \quad ; \quad Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)\,ds$$

… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ *it is determined by the focusing strength k of all quadrupoles*

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)\,ds$$

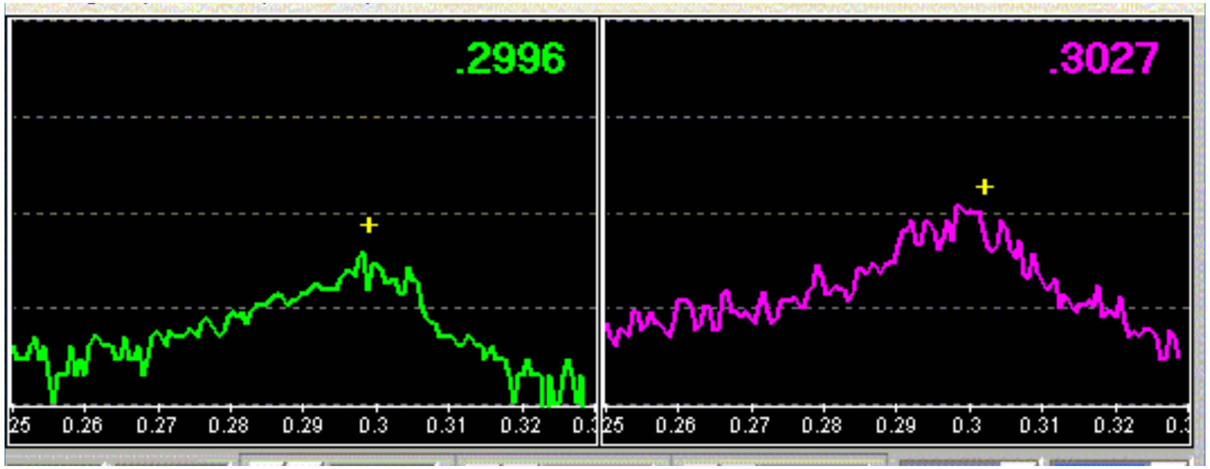
k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields

Example: LHC

$Q' = 250$
$\Delta p/p = +/- 0.2 * 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

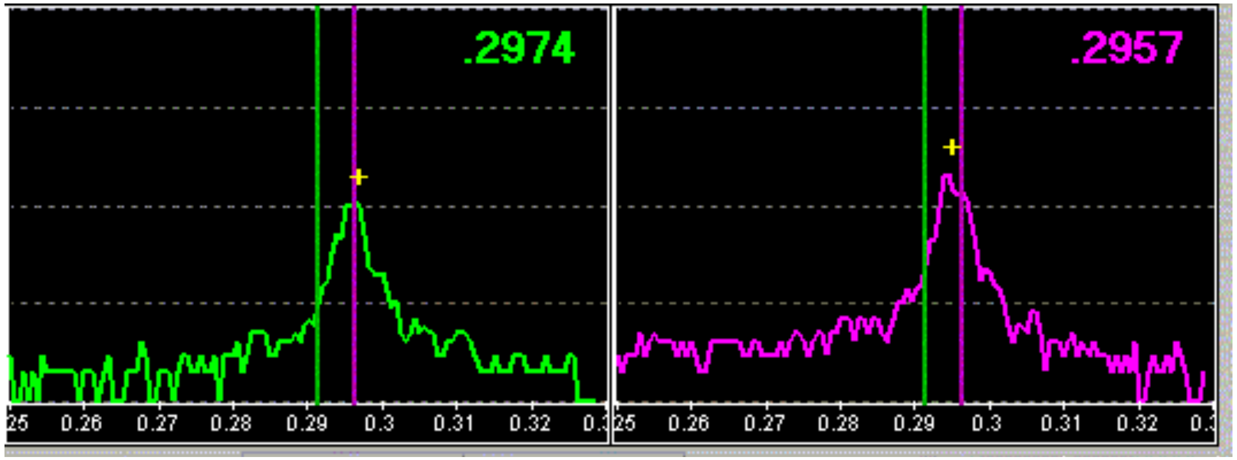
→ *Some particles get very close to resonances and are lost*

in other words: the tune is not a point
it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)

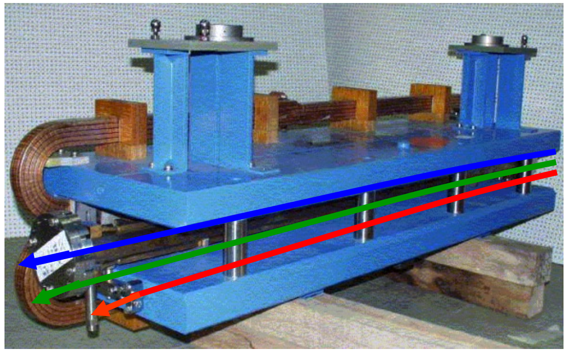


Correction of Q':

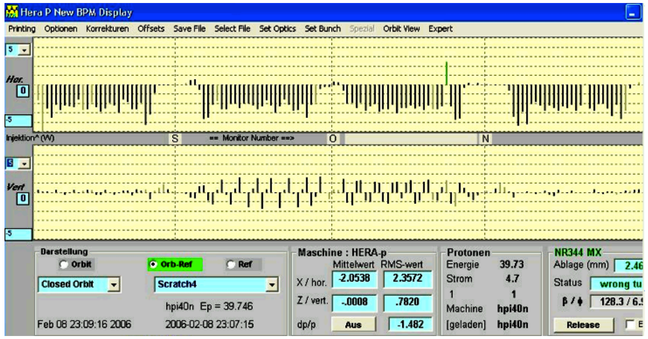
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



***2.) apply a** magnetic field that rises quadratically **with x** (sextupole field)*

$$\left.\begin{aligned} B_x &= \tilde{g}xz \\ B_z &= \frac{1}{2}\tilde{g}(x^2 - z^2) \end{aligned}\right\} \qquad \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \tilde{g}x$$

linear rising „gradient“:

Correction of Q':

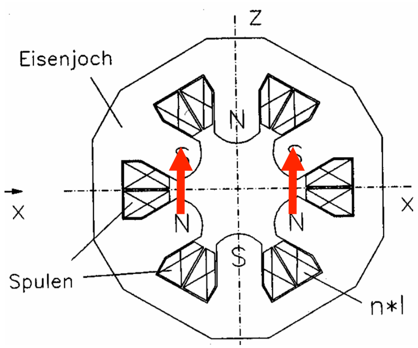
k_1 ***normalised quadrupole strength***

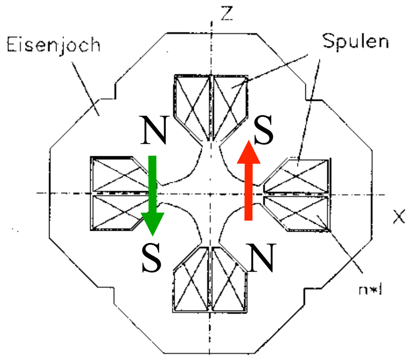
k_2 ***normalised sextupole strength***

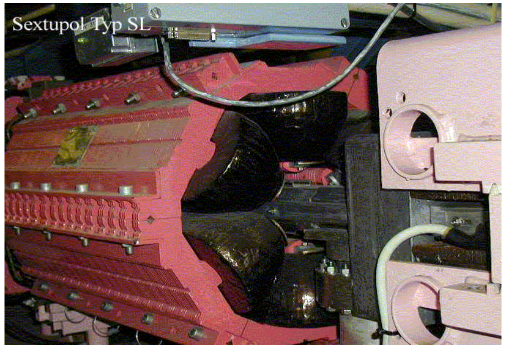
Sextupole Magnets:

$$k_{sext} = \frac{\tilde{g}x}{p/e} = m_{sext}.x$$

$$k_{sext} = m_{sext}.D\frac{\Delta p}{p}$$







corrected chromaticity

considering a single cell:

$$Q'_{cell\_x} = \frac{-1}{4\pi}\left\{k_{qf}\hat{\beta}_x l_{qf} - k_{qd}\breve{\beta}_x l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

$$Q'_{cell\_y} = \frac{-1}{4\pi}\left\{-k_{qf}\breve{\beta}_y l_{qf} + k_{qd}\hat{\beta}_y l_{qd}\right\} - \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_y^F + \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_y^D$$

Some Golden Rules to Avoid Trouble

I.) Golden Rule number one: do not focus the beam !

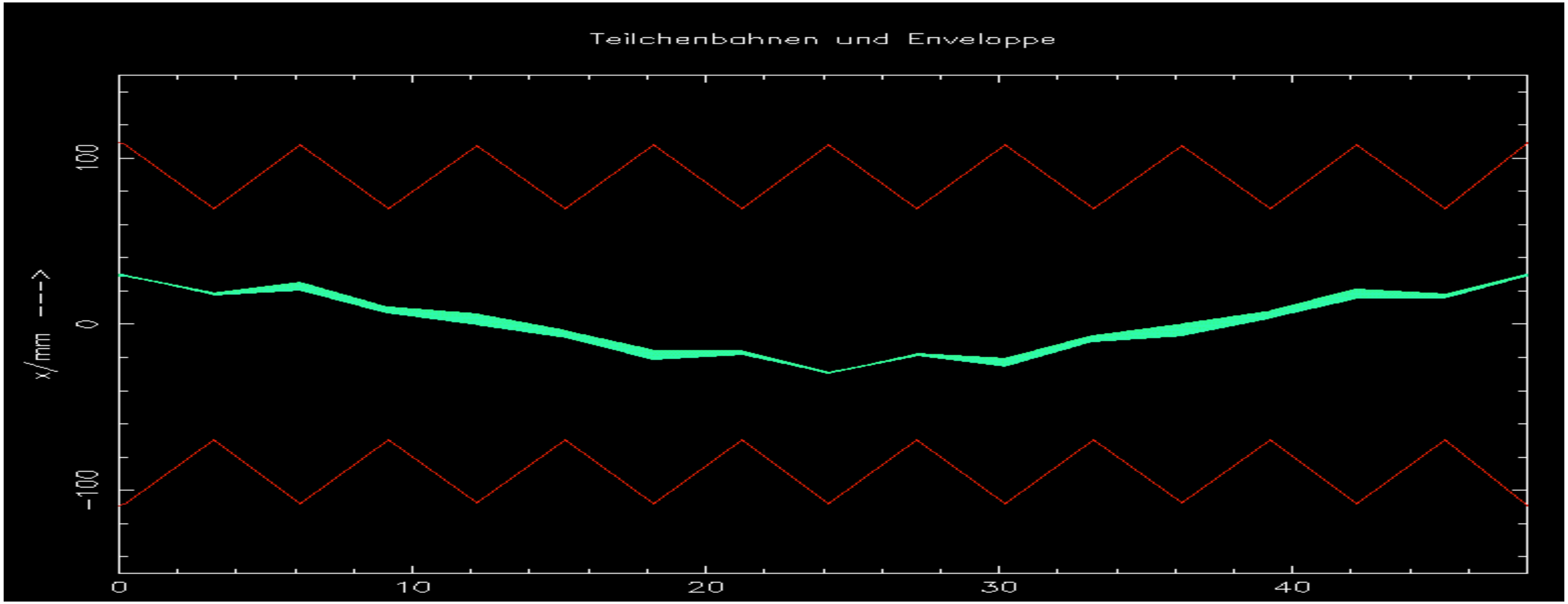
Problem: Resonances

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{s1}} \sqrt{\beta_{s1}} * \cos(\psi_{s1} - \psi_s - \pi Q)\, ds}{2 \sin \pi Q}$$

Assume: Tune = integer $Q = 1 \rightarrow 0$

Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.

Qualitatively spoken:

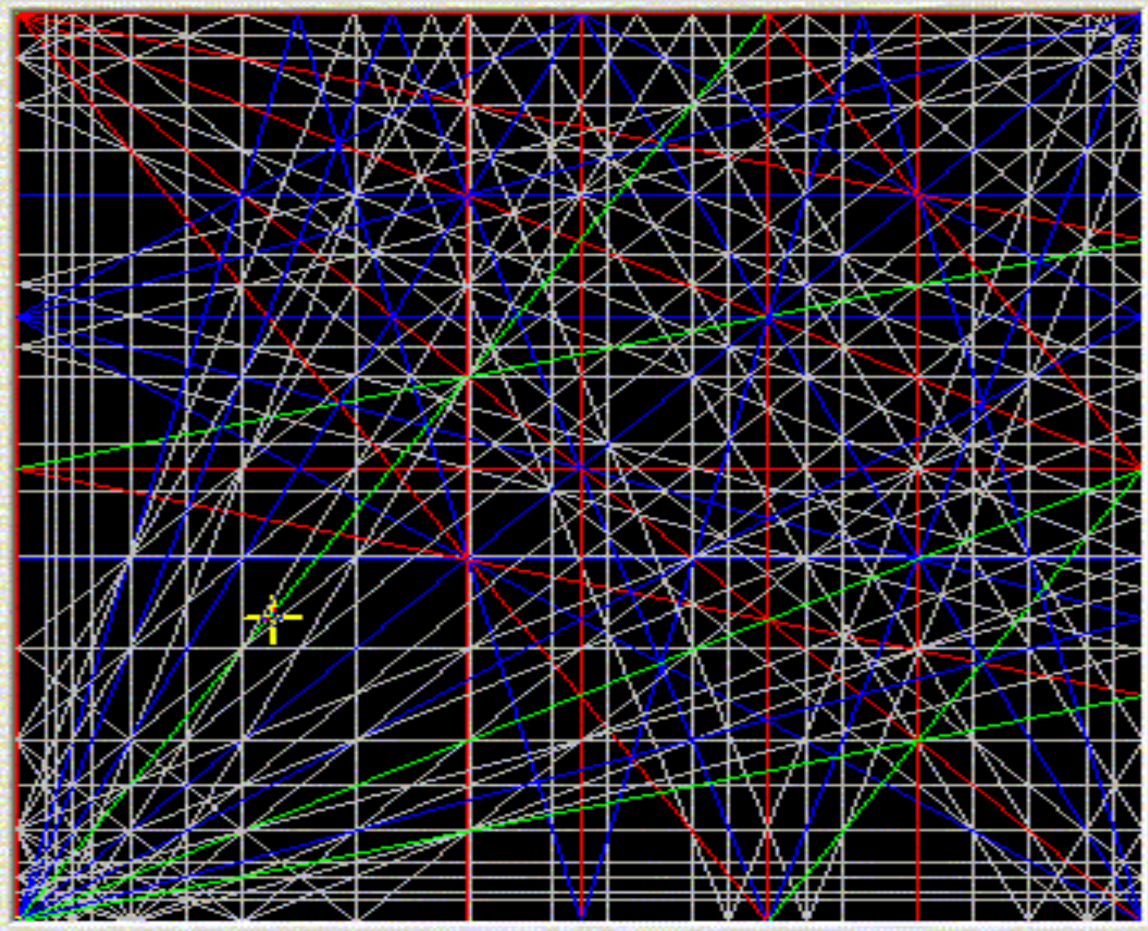


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

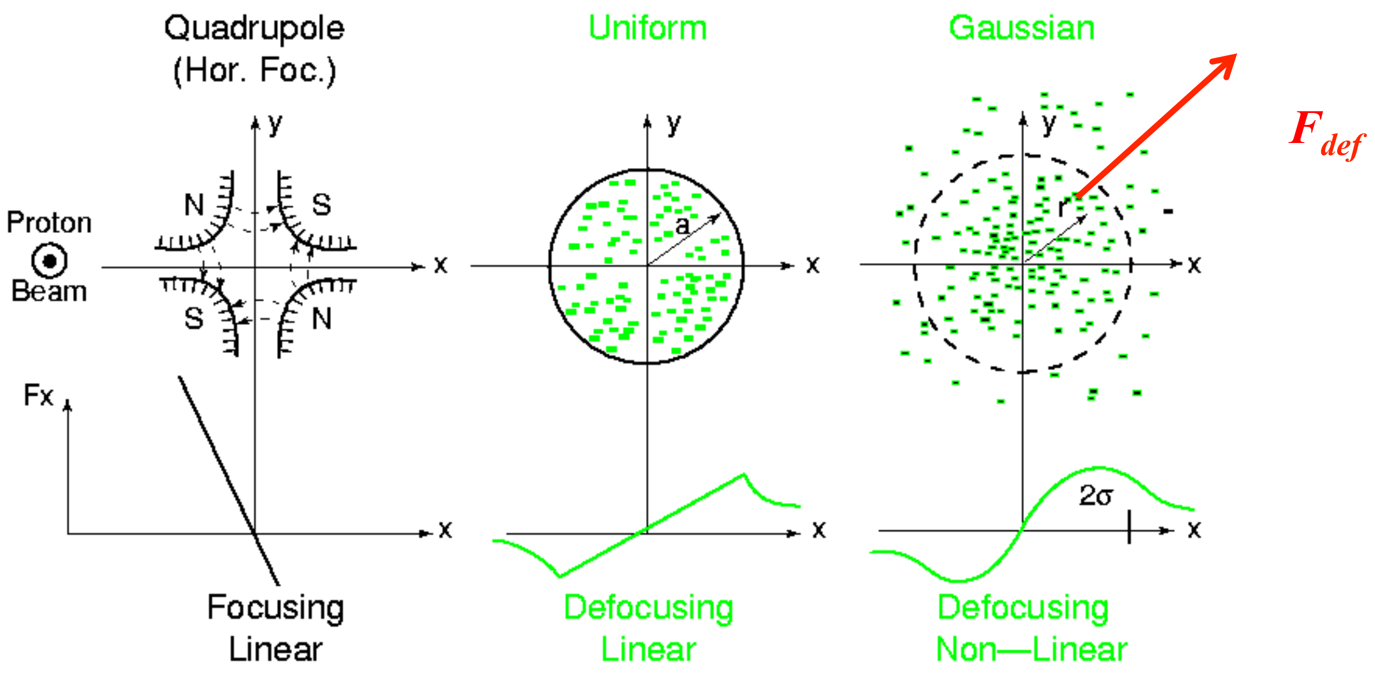
Tune diagram up to 3rd order

… and up to 7th order



Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive

II.) Golden Rule number two: Never accelerate charged particles !



Transport line with quadrupoles

$$x'' + K(s)x = 0$$

Transport line with quadrupoles and space charge

$$x'' + (K(s) + K_{SC}(s))x = 0$$

$$x'' + \left(K(s) - \frac{2r_0 I}{ea^2\beta^3\gamma^3 c} \right) x = 0$$

K_{SC}

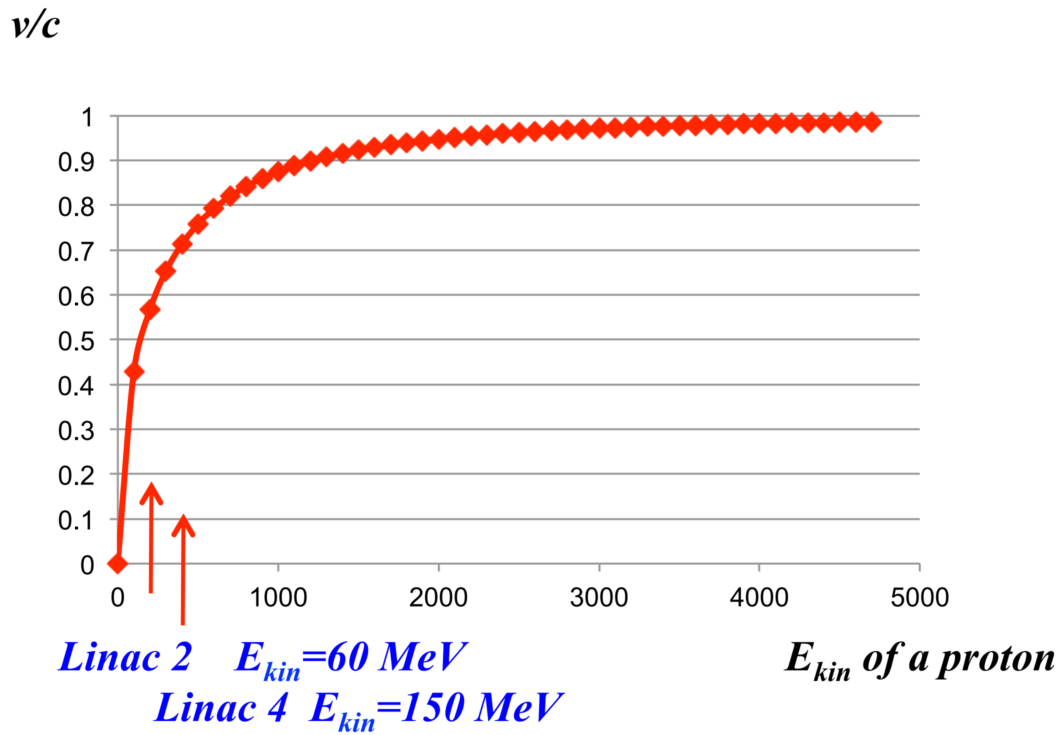
Golden Rule number two: *Never accelerate charged particles !*

Tune Shift due to Space Charge Effect

Problem at low energies

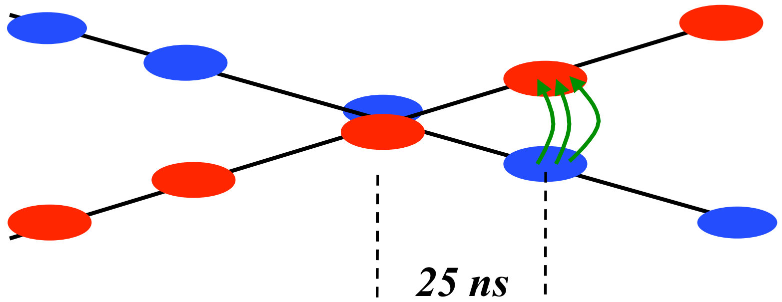
$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi\varepsilon_{x,y}\beta\gamma^2}$$

... at low speed the particles repel each other

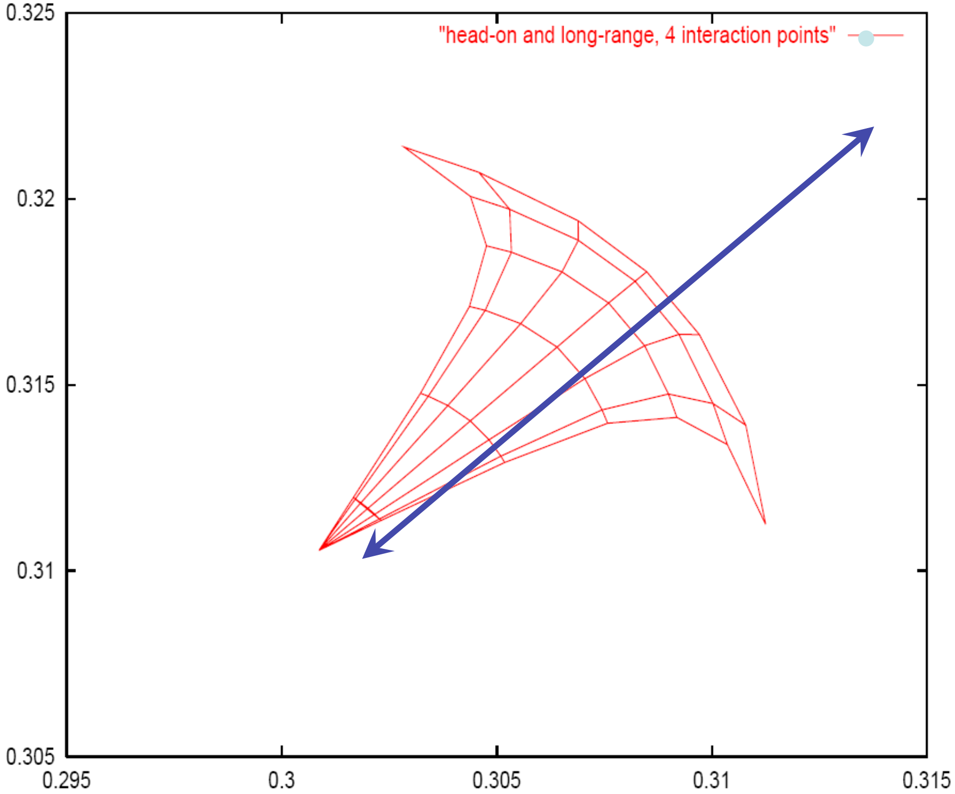


III.) Golden Rule number three:

Never Collide the Beams !



the colliding bunches influence each other
→ *change the focusing properties of the ring !!*



Q_x

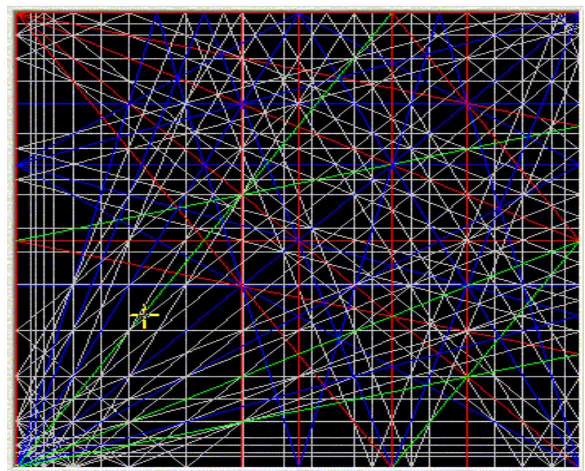
Courtesy W. Herr

Q_x

most simple case:
linear beam beam tune shift

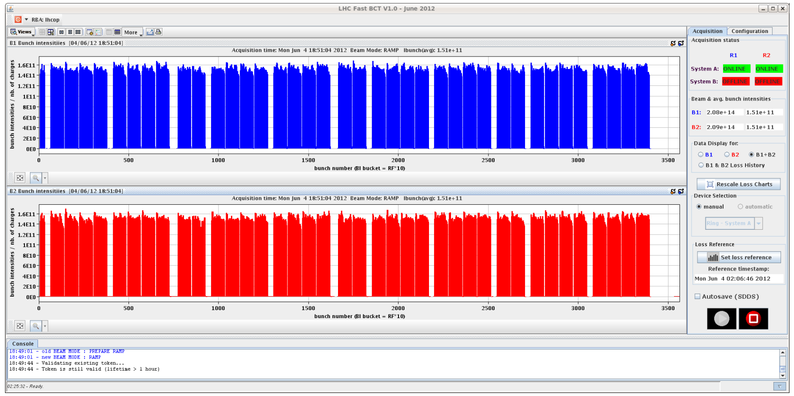
$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi \gamma_p (\sigma_x + \sigma_y) * \sigma_x}$$

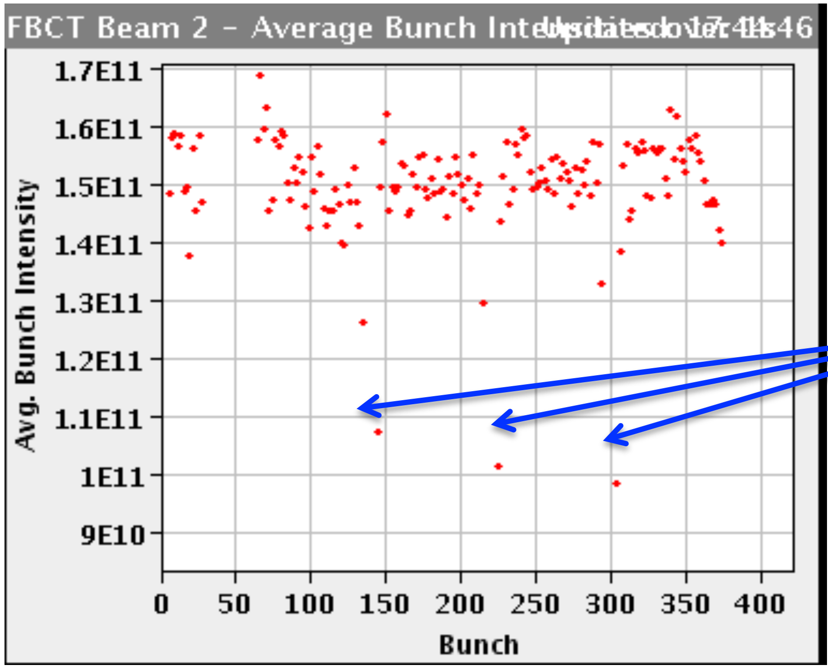
and again the resonances !!!



LHC logbook: Sat 9-June "Late-Shift"

18:18h injection for physics
clean injection !

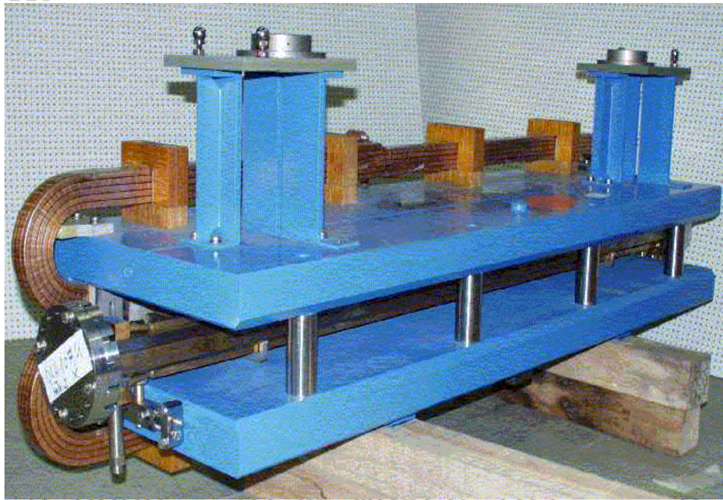




but particle losses when beams are brought into collision

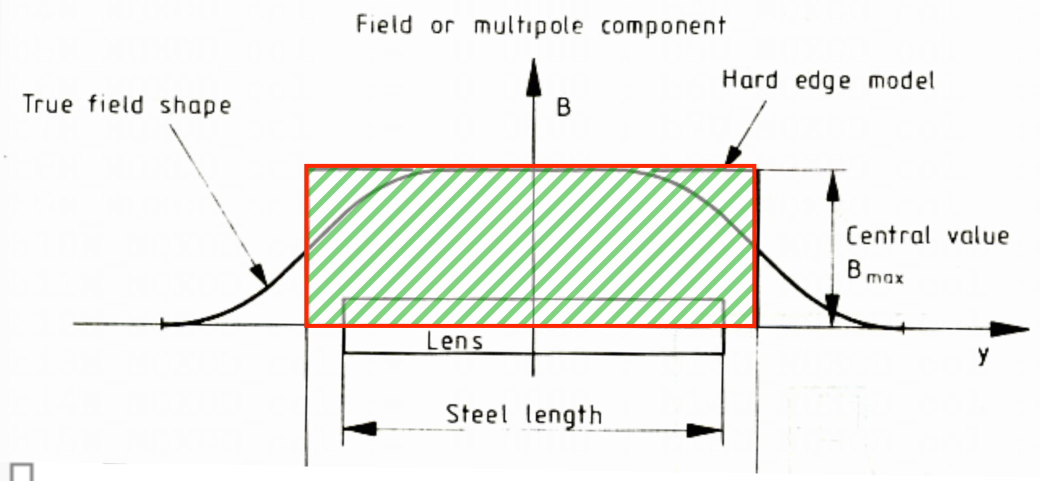
IV.) Golden Rule Number 4: Never use Magnets

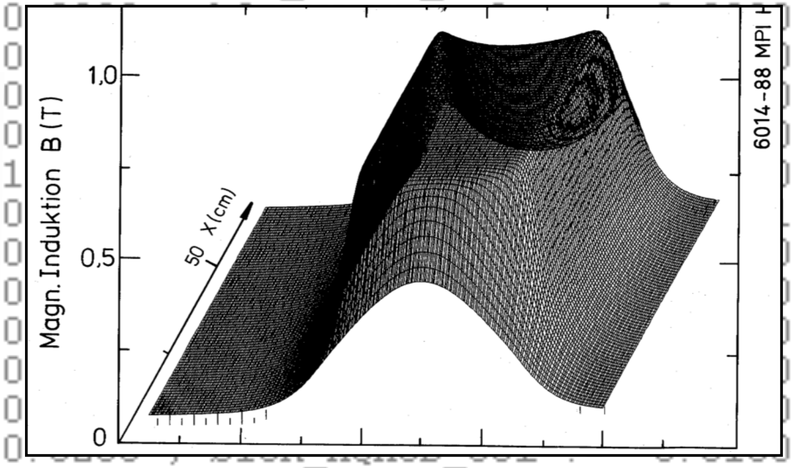
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{r_0} \right)^{n-1}$$



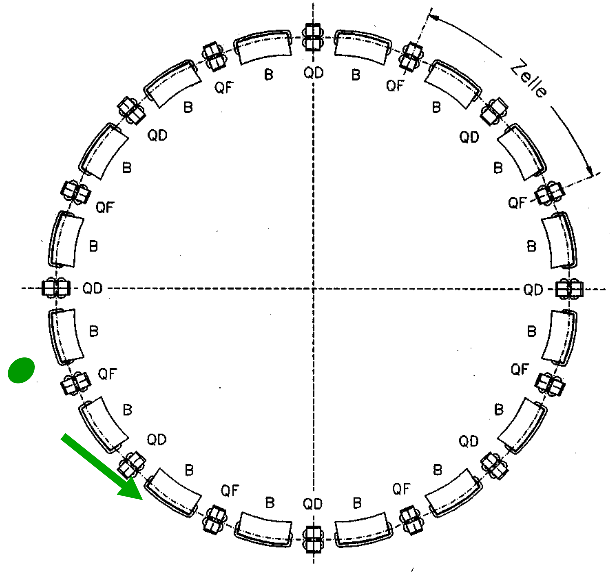
"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B\,ds$$





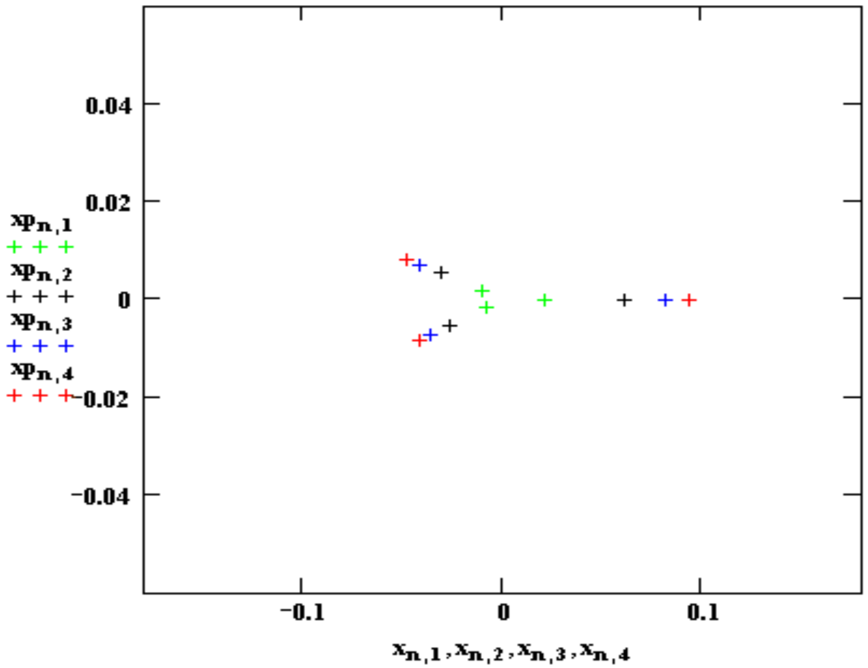
Clearly there is another problem ...
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

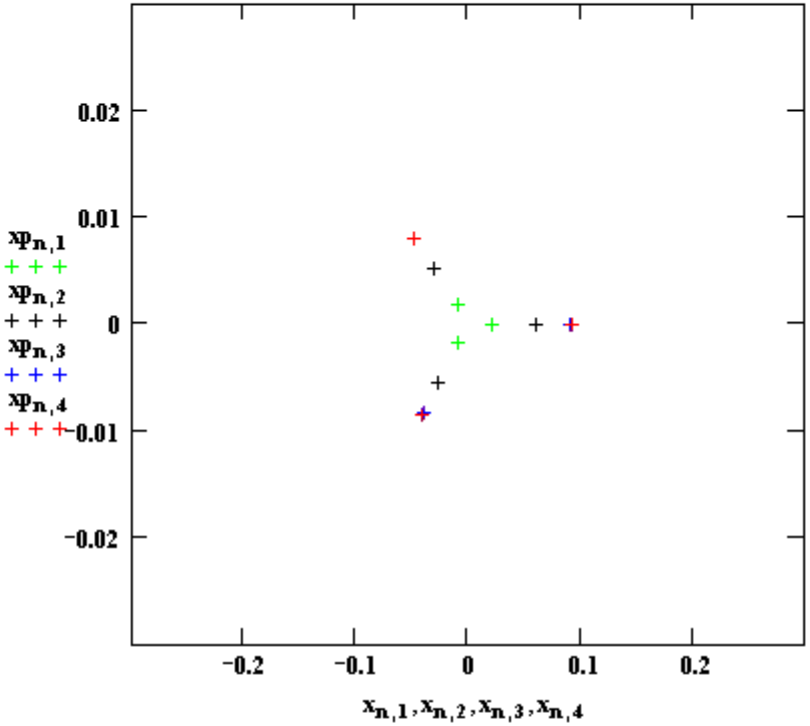


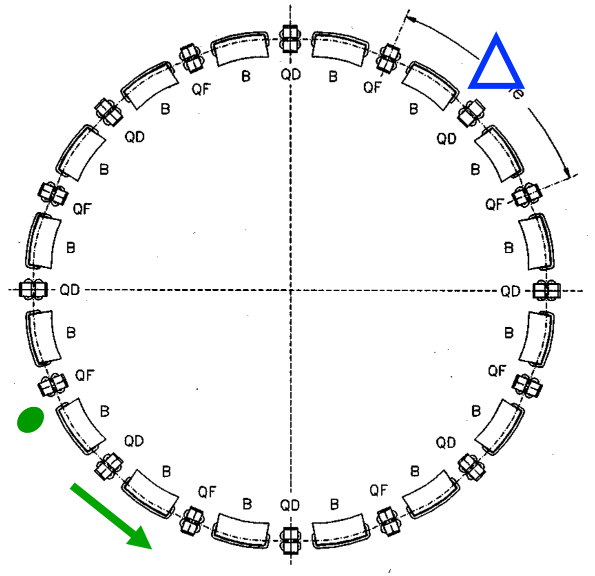
A beam of 4 particles

– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

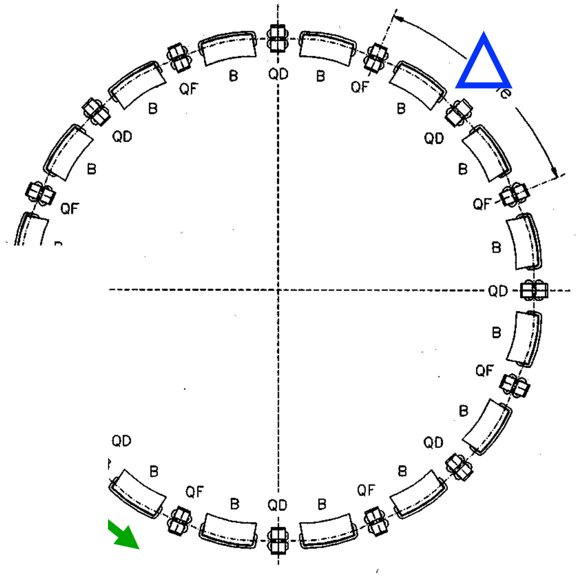
The good news: sextupole fields in accelerators
cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation
„ particle tracking "

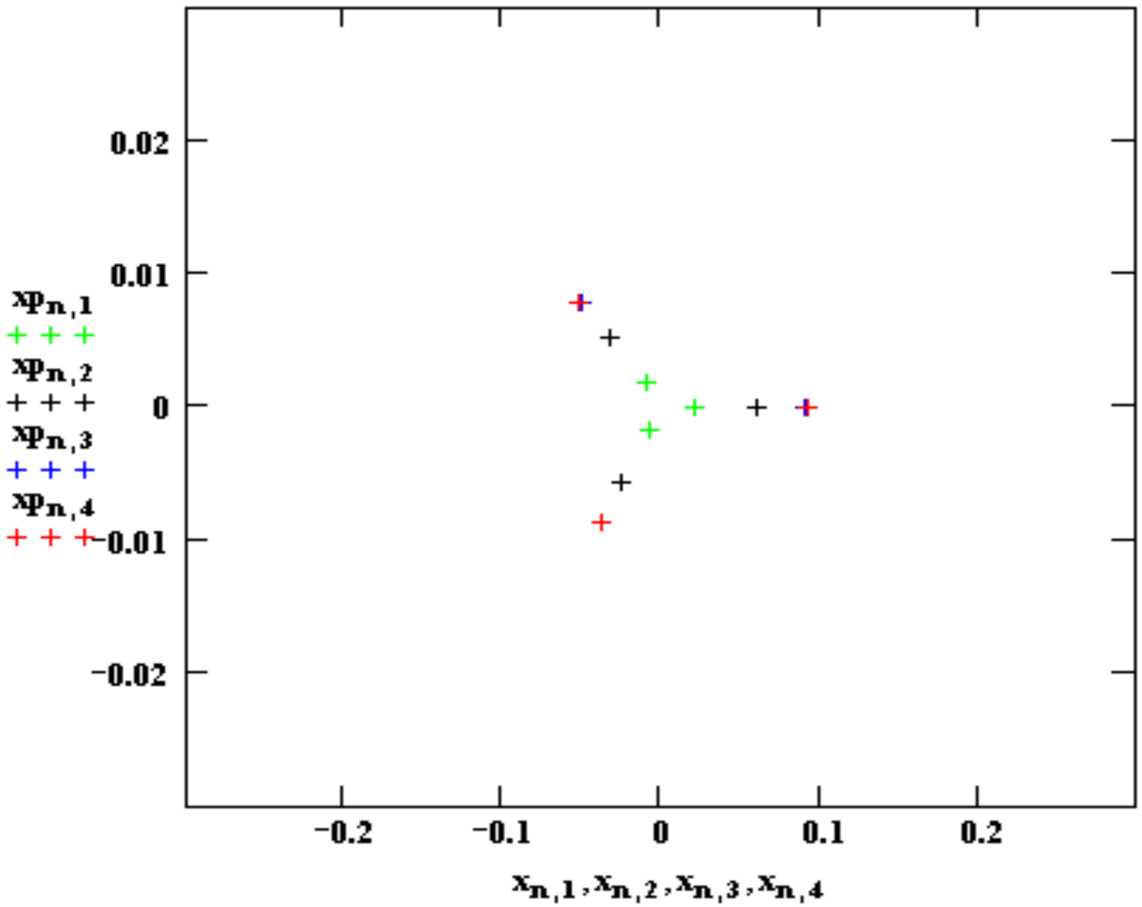




Effect of a strong (!!!) Sextupole …

→ Catastrophy !

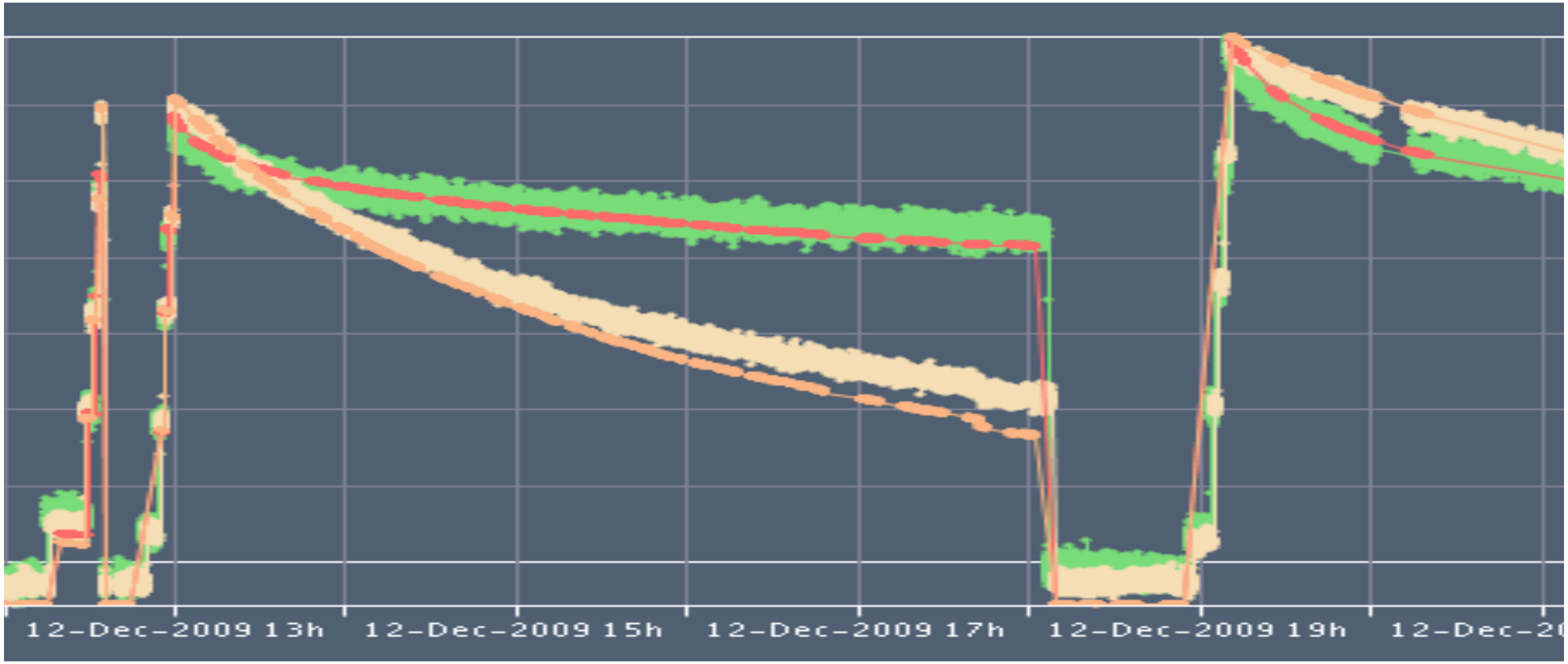




„dynamic aperture"

Golden Rule XXL: COURAGE

and with a lot of effort from Bachelor / Master / Diploma / PhD and Summer-Students the machine is running !!!



thank'x for your help and have a lot of fun