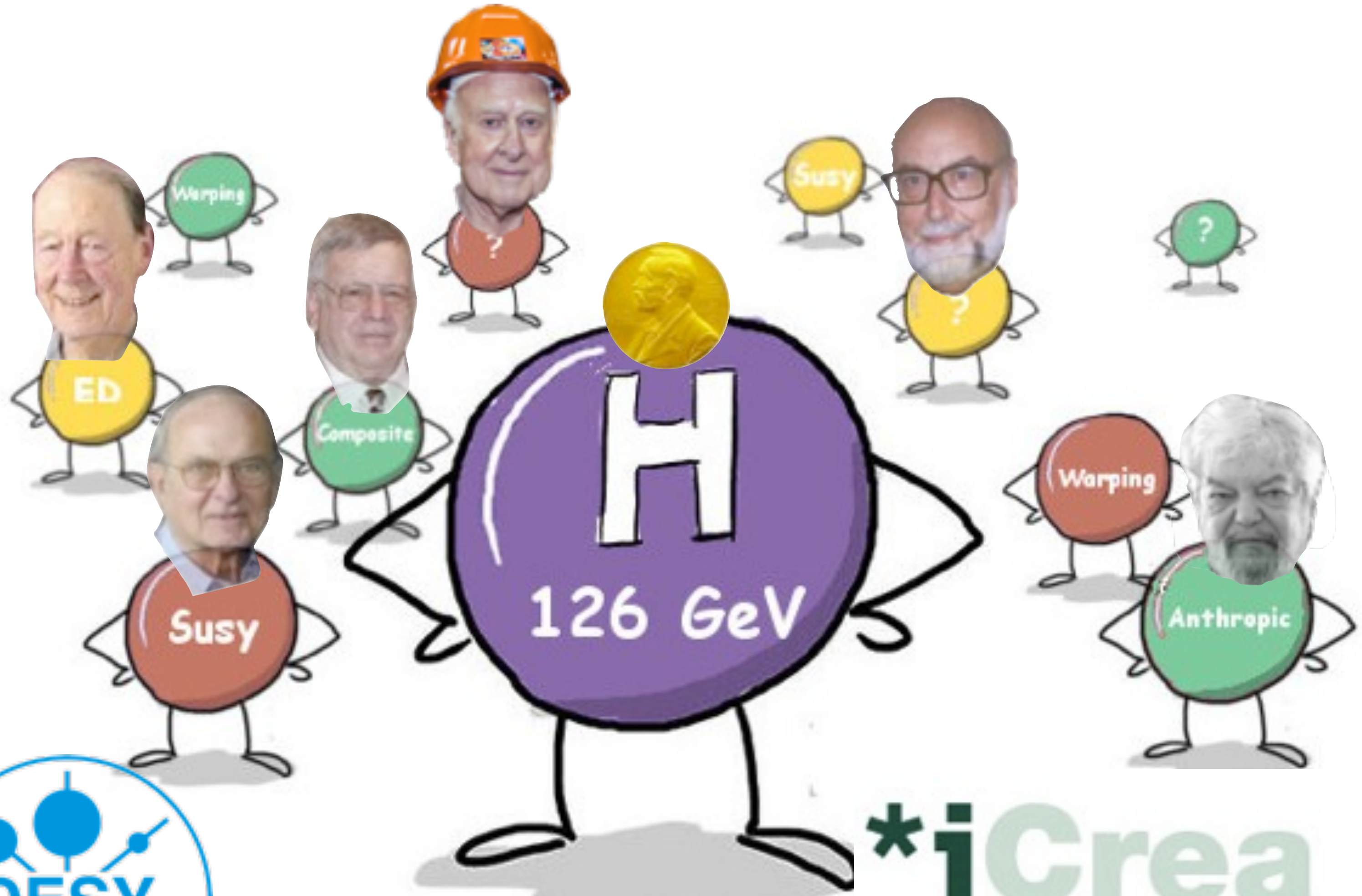
Beyond the Standard Model

CERN summer student lectures 2016



Lecture 2/4

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Outline

- Monday I
  - general introduction, units
  - Higgs physics as a door to BSM
- Monday II
  - Naturalness
  - Supersymmetry
  - (Grand unification, proton decay)
- Tuesday
  - Composite Higgs
  - Extra dimensions
  - Effective field theories
- Wednesday
  - Cosmological relaxation
  - Quantum gravity

HEP with a Higgs boson

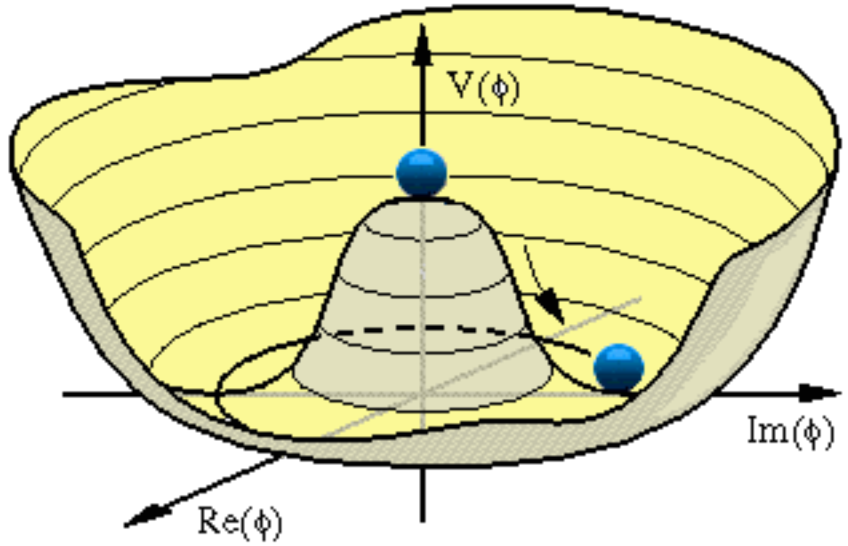
"If you don't have the ball, you cannot score"

Now with the Higgs boson in their feet,
particle physicists can... play as well as Barça players

Profound change in paradigm:

missing SM particle ⇨ tool to explore SM and venture into physics landscape beyond

Higgs and EW vacuum Stability

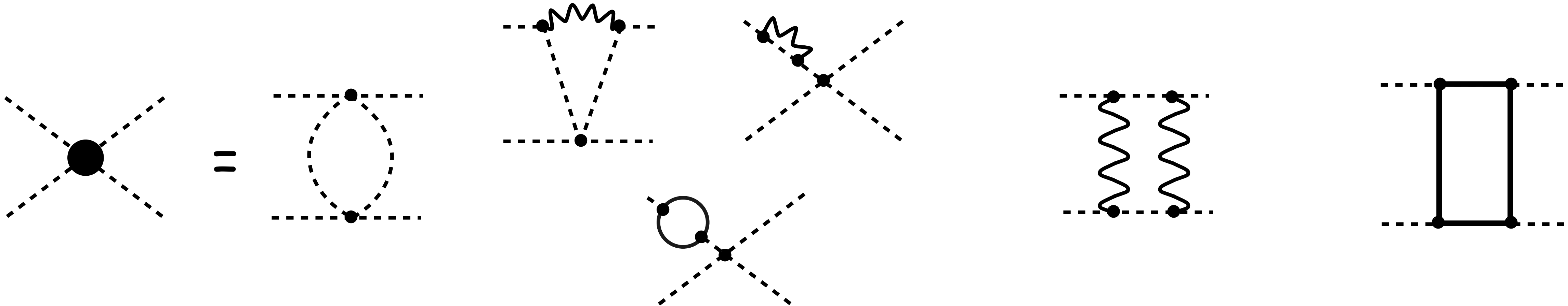


$$V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4$$

vev: $v^2 = \mu^2/\lambda$ mass: $m_H^2 = 2\lambda v^2$

the vacuum is not empty even classically $(\hbar \to 0)$

How is Quantum Mechanics changing the picture?



$$16\pi^2 \frac{d\lambda}{d\ln Q} = 24\lambda^2 - (3g'^2 + 9g^2 - 12y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 +$$

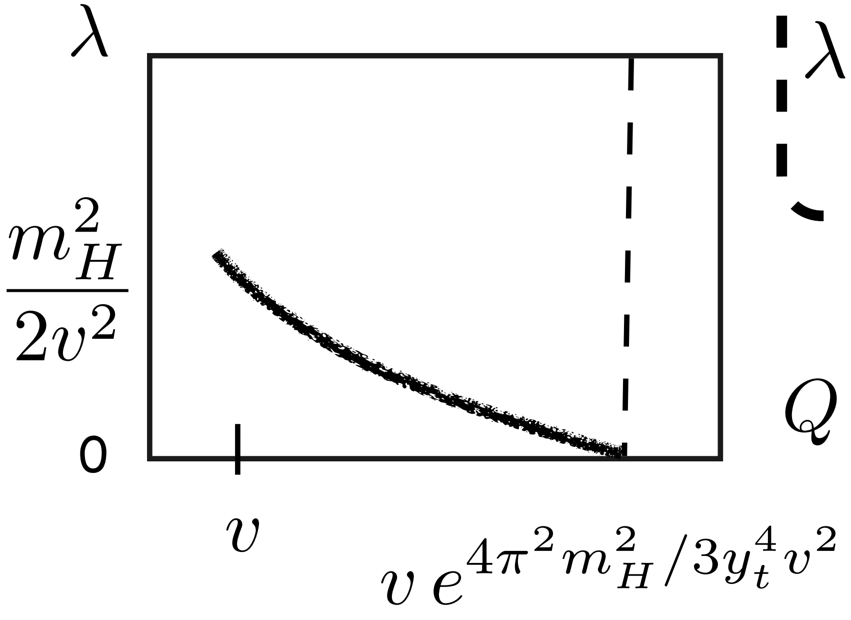
Higher loops
Small Yukawa

Higgs and EW vacuum Stability

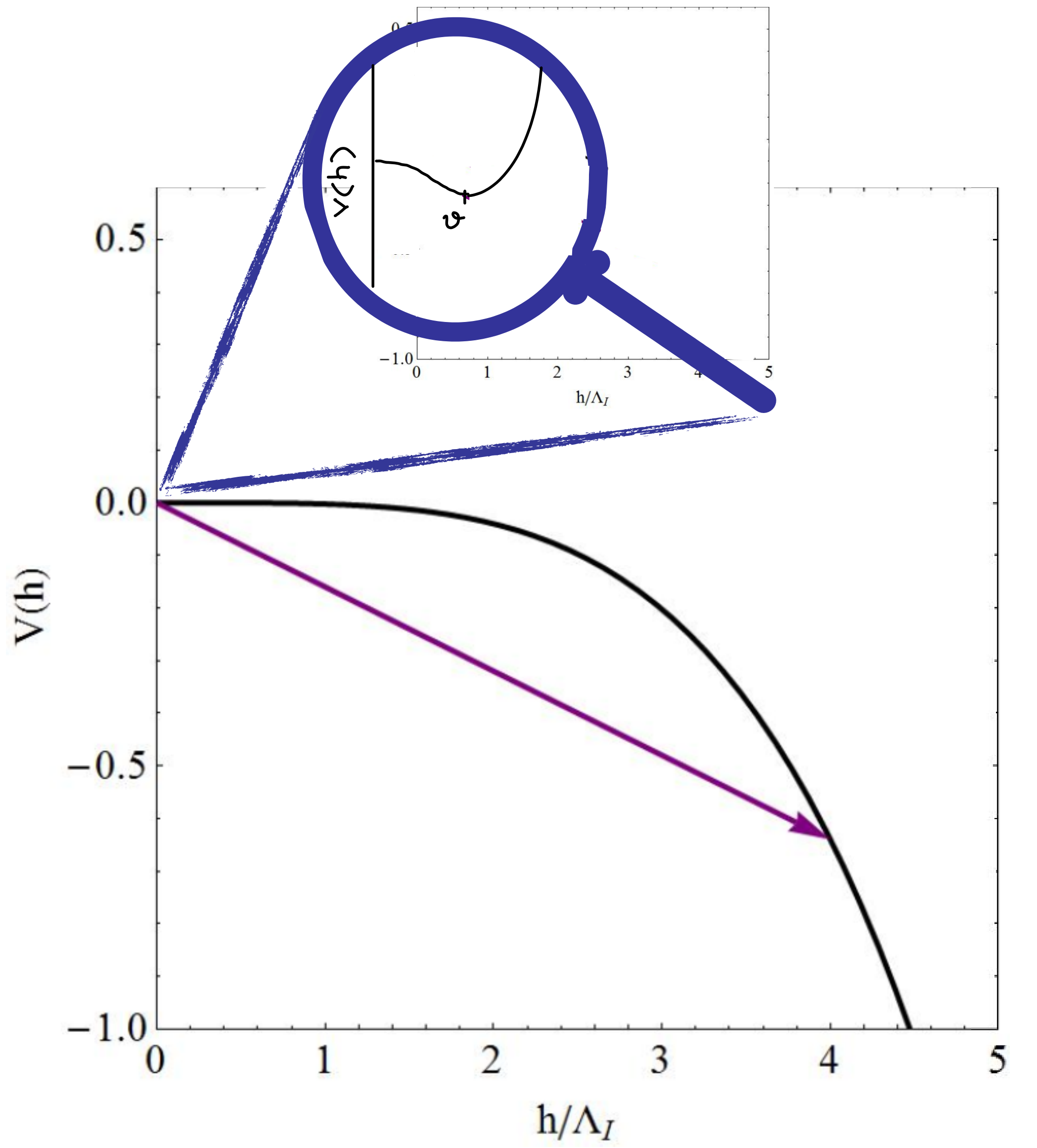
Small mass (y_t dominated RGE)

$$\boxed{\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}}$$

Linde ’76, ’80
Weinberg ’76
Maini et al ’78, ’79
Politzer, Wolfram ’79
Lindner ’86
+...



$\lambda < 0$ ⇛ potential unbounded from below



$$\Lambda \leq v\, e^{4\pi^2 m_H^2/3y_t^4 v^2}$$

New physics should appear before
that point to restore stability

Higgs and EW vacuum Stability

Small mass (y_t dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

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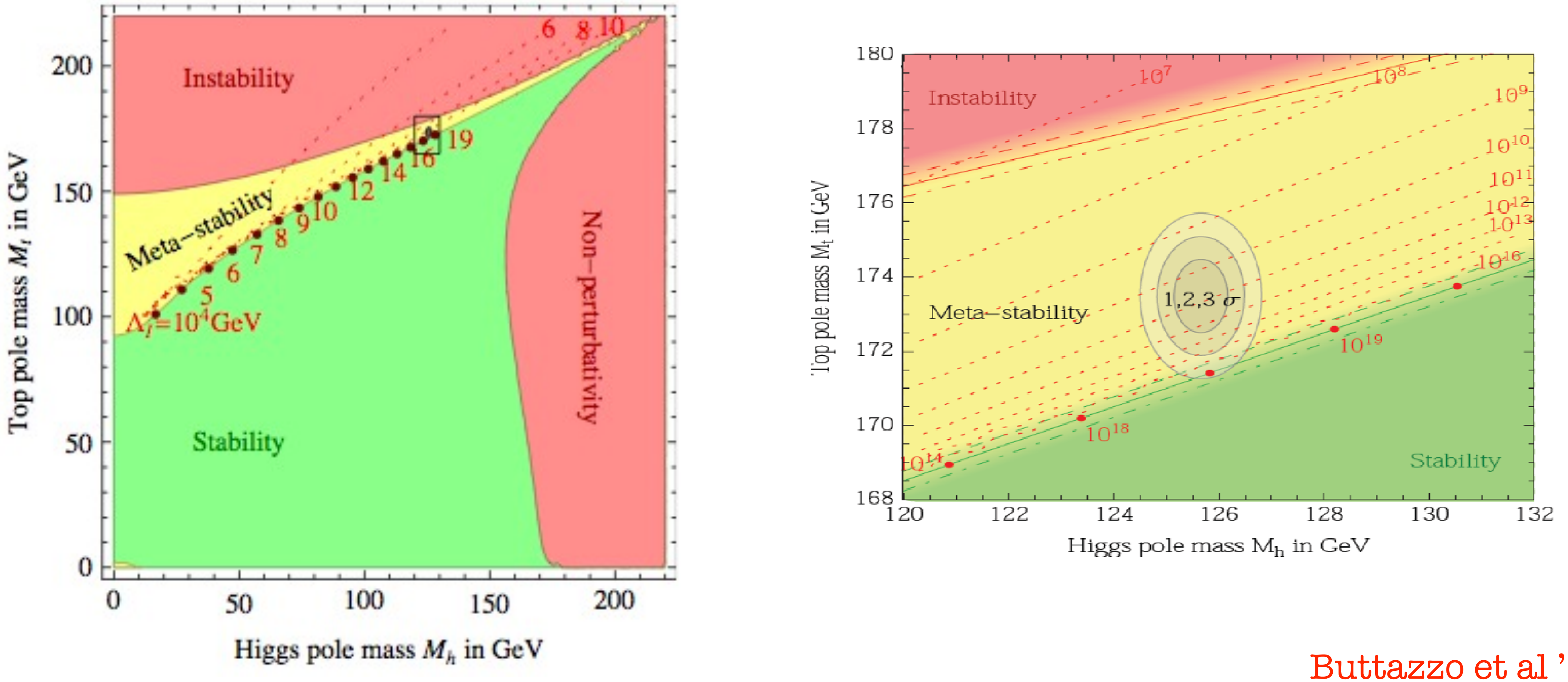
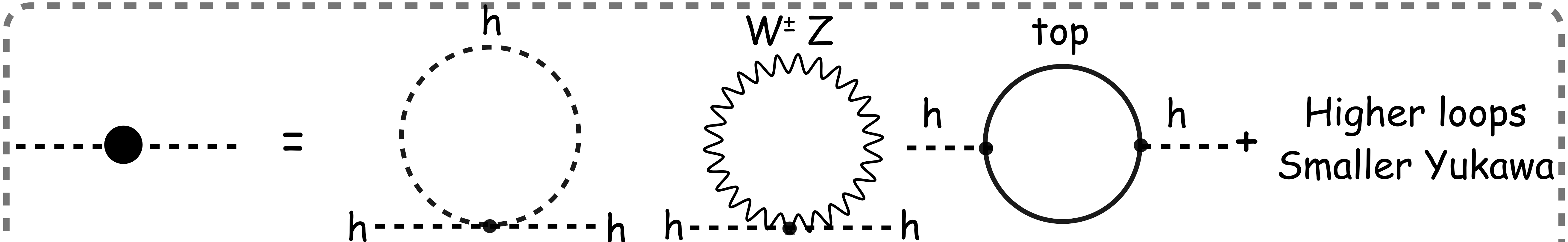


Buttazzo et al '13

Quantum Instability of the Higgs Mass

so far we looked only at the RG evolution of the Higgs quartic coupling (dimensionless parameter). The Higgs mass has a totally different behavior: it is highly dependent on the UV physics, which leads to the so called hierarchy problem



Higher loops
Smaller Yukawa

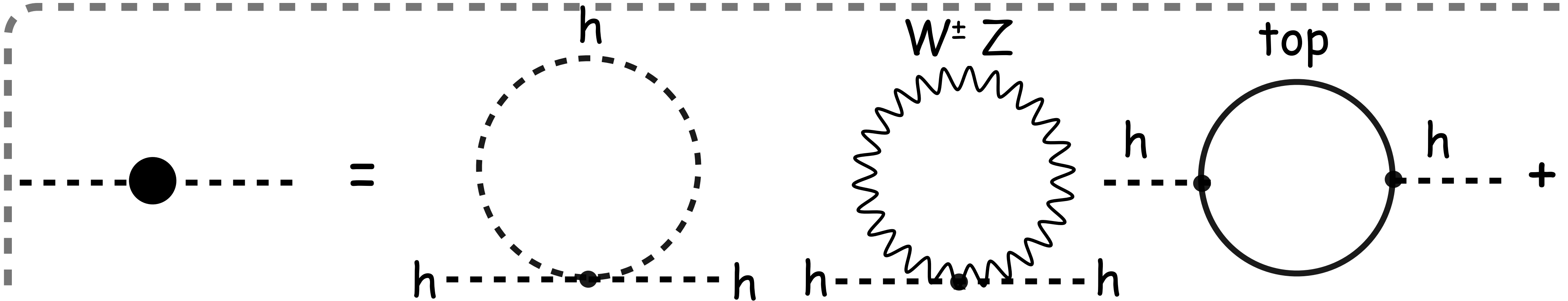
$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \propto \Lambda^2 \qquad \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2)^2} \propto \Lambda^2$$

Weisskopf '39
't hooft '79

$$\delta m_H^2 = \left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right) \frac{3G_F\Lambda^2}{8\sqrt{2}\pi^2}$$

$$m_H^2 \sim m_0^2 - (115\ \mathrm{GeV})^2 \left(\frac{\Lambda}{700\ \mathrm{GeV}}\right)^2$$

Quantum Instability of the Higgs Mass



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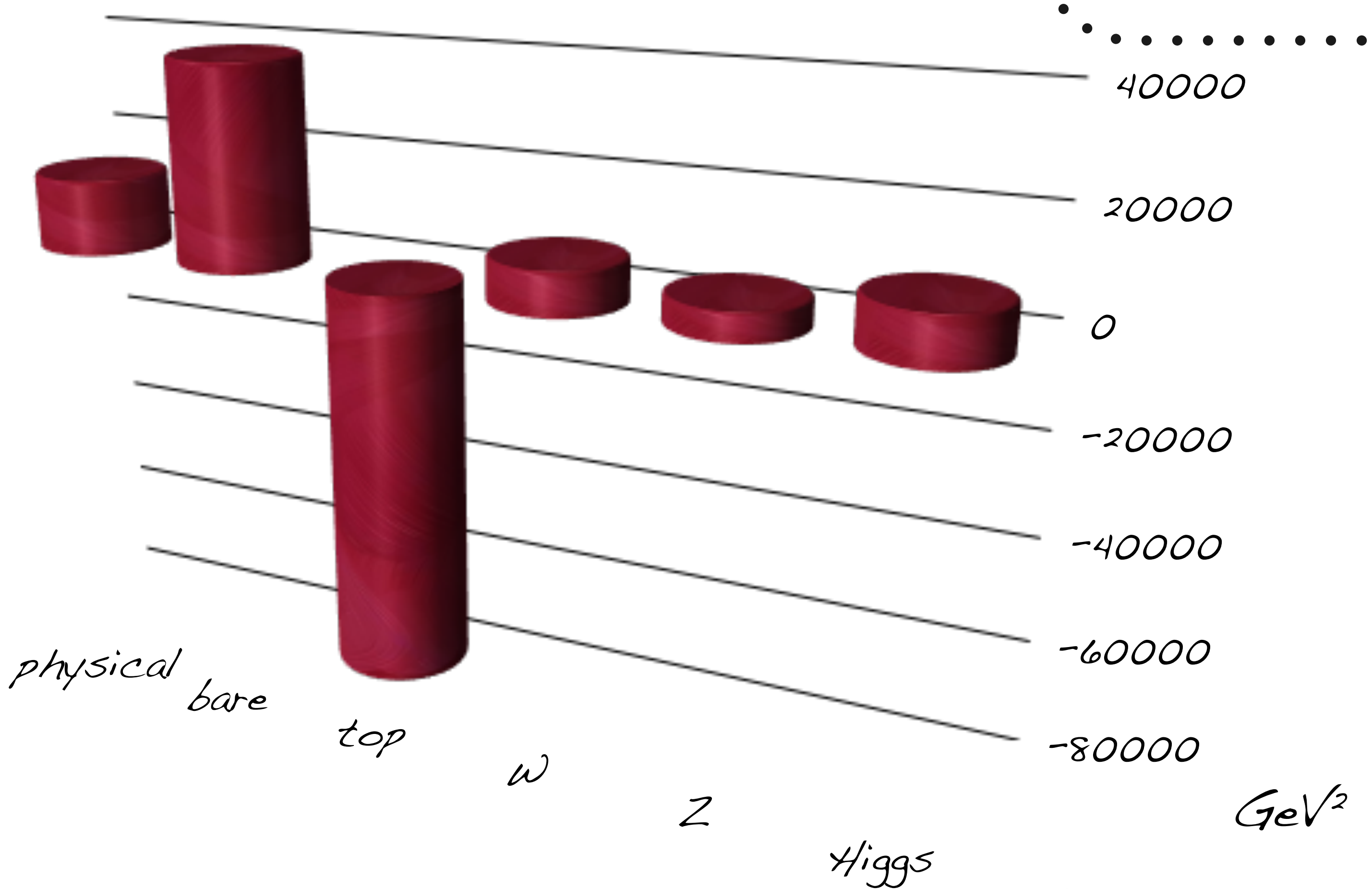
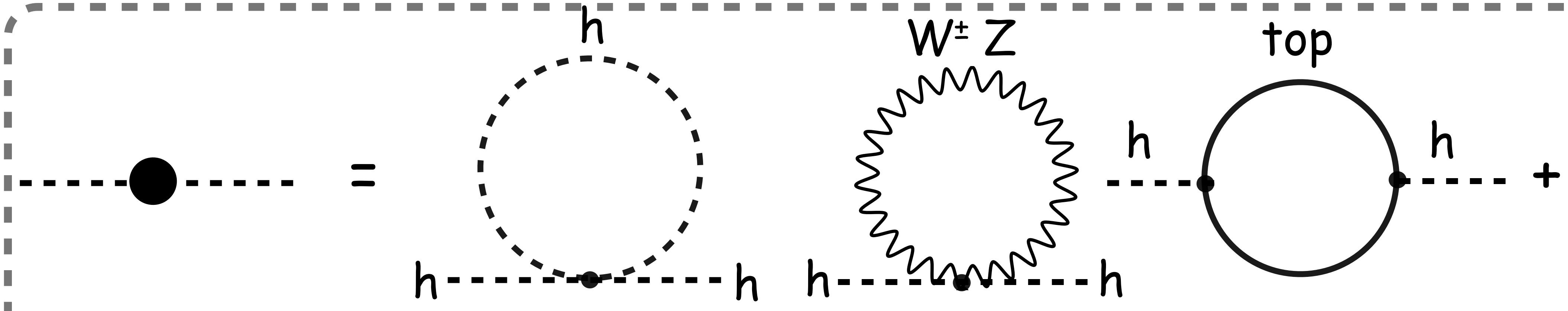
Λ = 1 TeV



Quantum Instability of the Higgs Mass



Higher loops
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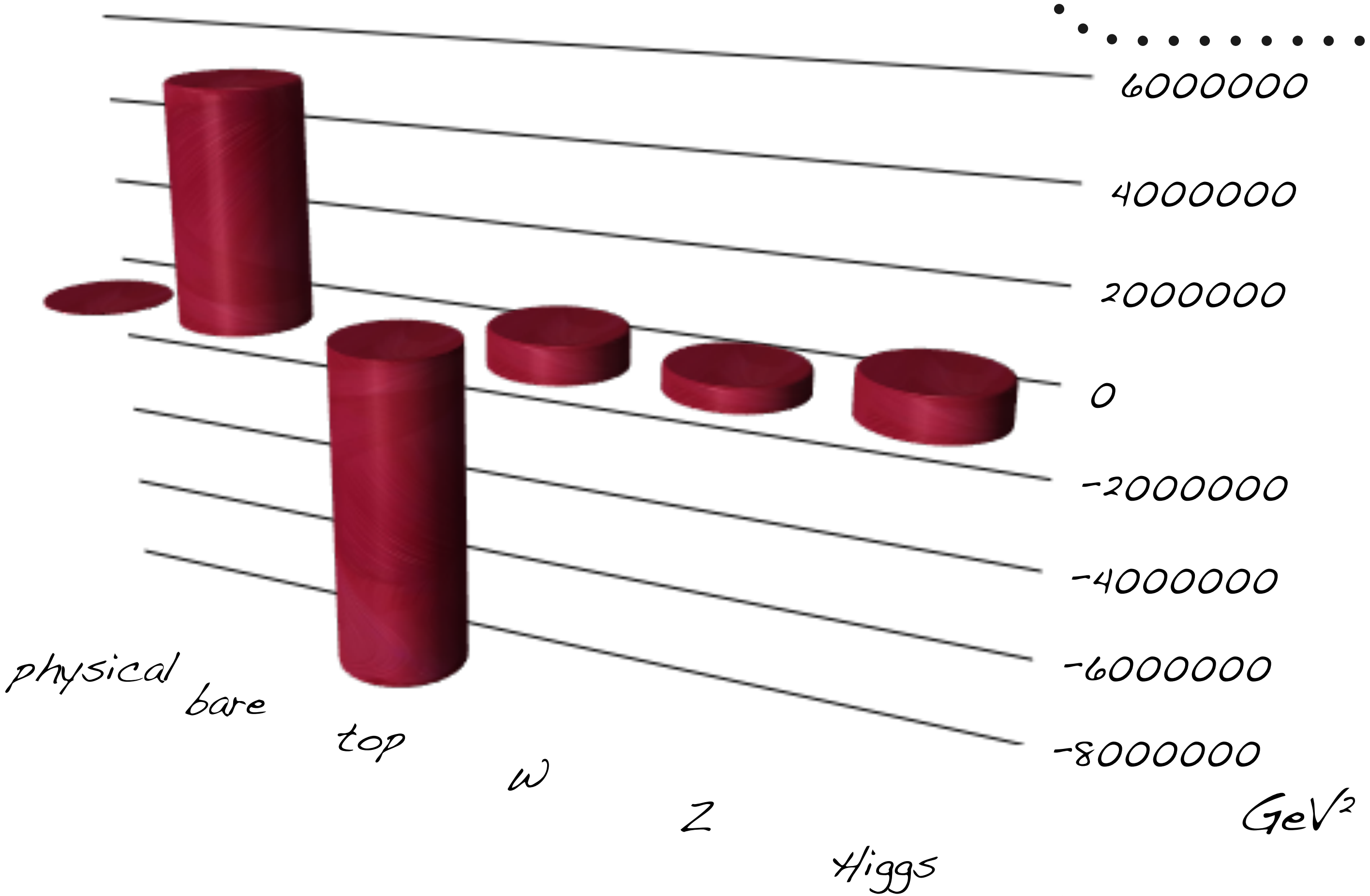
$$m_H^2 \sim m_0^2 - (115\ \mathrm{GeV})^2 \left(\frac{\Lambda}{700\ \mathrm{GeV}}\right)^2$$

Λ = 10 TeV



Naturalness principle @ work

Following the arguments of Wilson, 't Hooft (and others):
only small numbers associated to the breaking of a symmetry survive quantum corrections

Beautiful examples of naturalness to understand the need of "new" physics

see for instance Giudice '13 (and refs. therein) for an account

- the need of the positron to screen the electron self-energy: $\Lambda < m_e/\alpha_{\rm em}$
- the rho meson to cutoff the EM contribution to the charged pion mass: $\Lambda < \delta m_\pi^2/\alpha_{\rm em}$
- the kaon mass difference regulated by the charm quark: $\Lambda^2 < \frac{\delta m_K}{m_K}\frac{6\pi^2}{G_F^2 f_K^2 \sin^2\theta_C}$
- the light Higgs boson to screen the EW corrections to gauge bosons self-energies
- ...
- New physics at the weak scale to cancel the UV sensitivity of the Higgs mass?

Playing with cracks: The way forward

Small numbers are not necessarily theoretically inconsistent
but they require some conspiracy at different scales

Better to find an explanation with new degrees of freedom that cancel the sensitivity to the details of the physics at high-energy

Theoretical inconsistencies

- ✳ 4 Fermi interactions to describe muon decay
  $\mathcal{A} \sim G_{\mathcal{F}} E^2$ ➢ W boson
- ✳ $W_L W_L$ scattering
  $\mathcal{A} \sim E^2/v^2$ ➢ H boson

Naturalness arguments

- ✳ positron
- ✳ rho
- ✳ charm quark
- ✳ susy?

How to Stabilize the Higgs Potential

Goldstone's Theorem

spontaneously broken global symmetry ⇨ massless scalar

... but the Higgs has sizable non-derivative couplings

The spin trick

a particle of spin s: 2s+1 polarization states

...with the only exception of a particle moving at the speed of light

... fewer polarization states

Spin 1 Gauge invariance ⟶ no longitudinal polarization ↘ m=0

Spin 1/2 Chiral symmetry ⟶ only one helicity ↗ m=0

... but the Higgs is a spin 0 particle

Symmetries to Stabilize a Scalar Potential

Supersymmetry

fermion ~ boson

Higher Dimensional Lorentz invariance

$\Lleftarrow$ gauge-Higgs unification models

[Manton '79, Fairlie 79, Hosotani '83 +...]

$$A_\mu \sim A_5$$

4D spin 1

4D spin 0

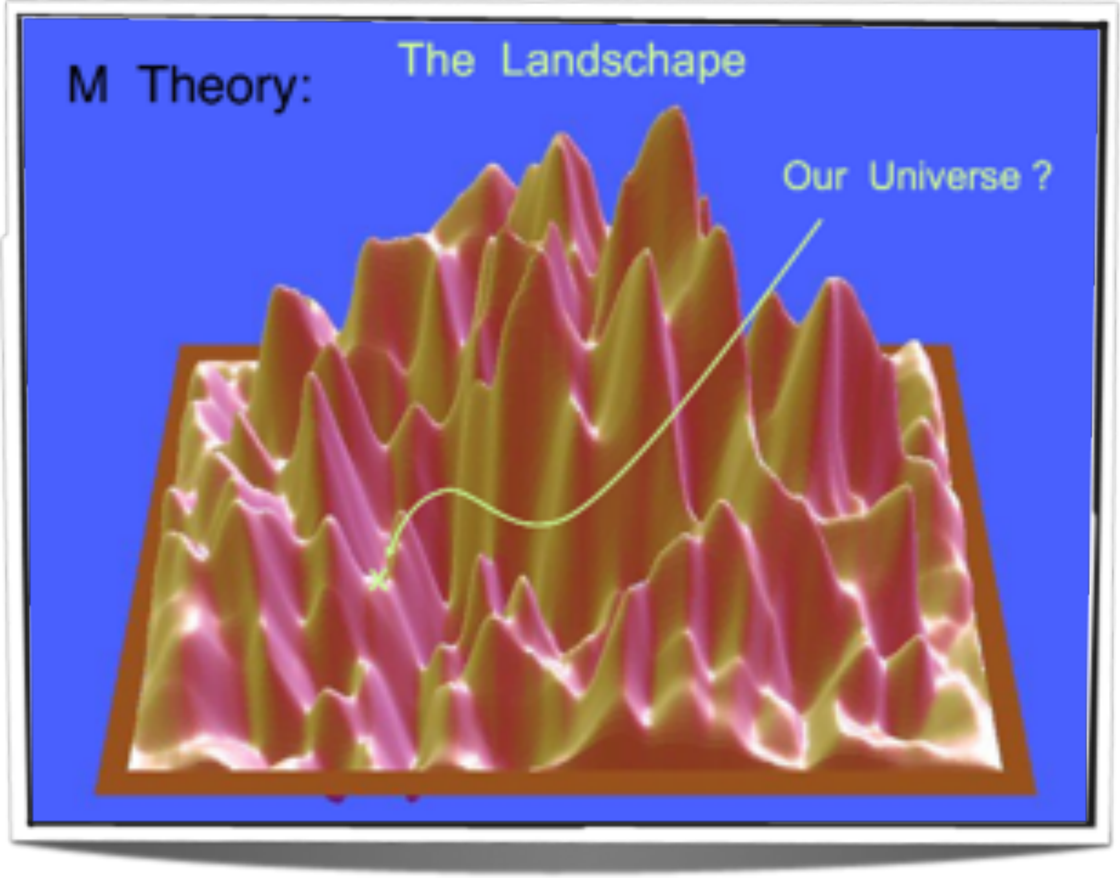
These symmetries cannot be exact symmetry of the Nature. They have to be broken. We want to look for a soft breaking in order to preserve the stabilization of the weak scale.

EWSB might be unnatural

nothing to say but the usual words:

- cosmological constant problem...
- multiverse...
- landscape of vacua...
- laws of physics are environmental...
- anthropic solution...
- end of reductionism...

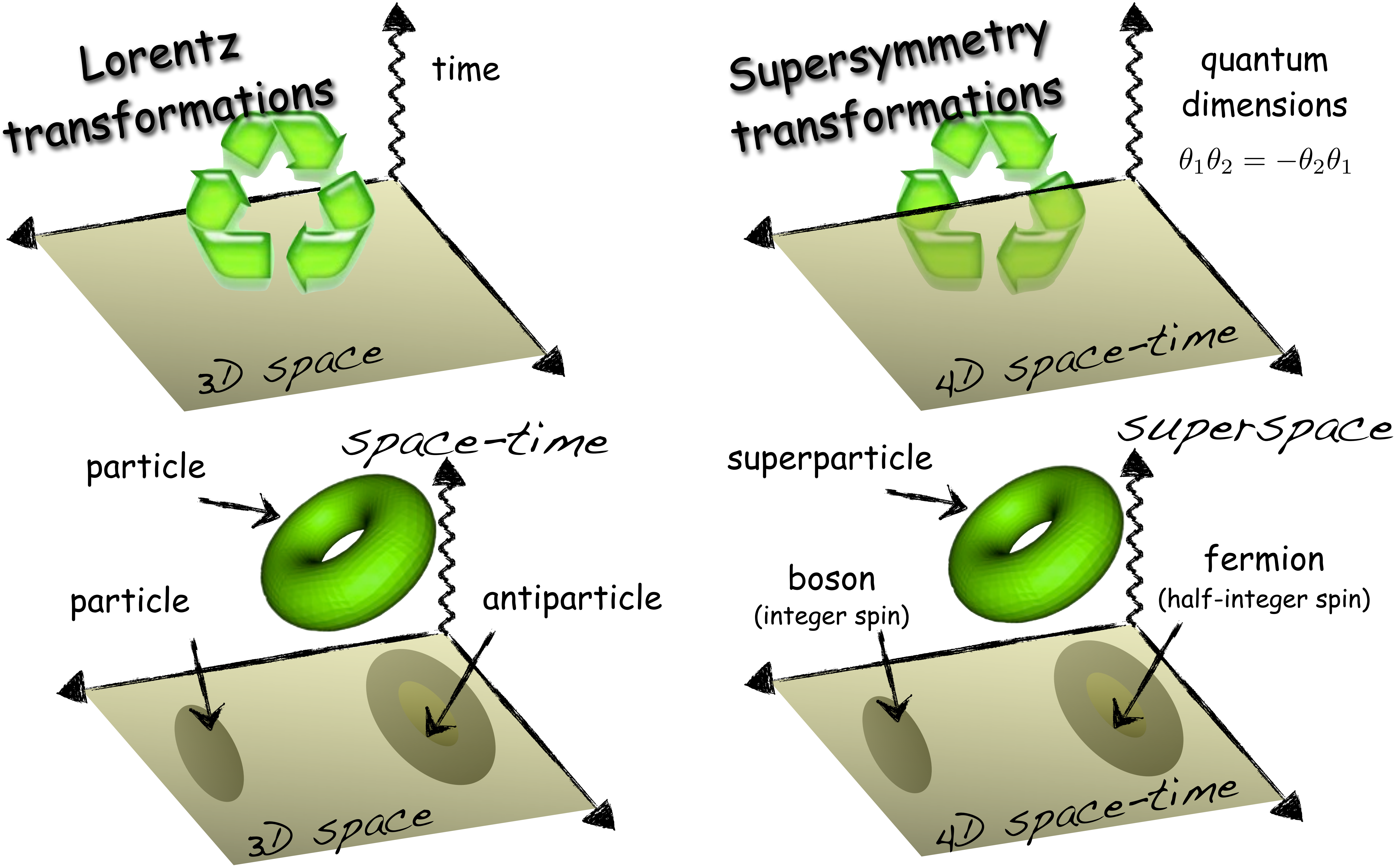
will be tested to an unprecedented level (10^{-4})



Supersymmetry

SUSY: a quantum space-time

(G. Giudice HCPSS'09)



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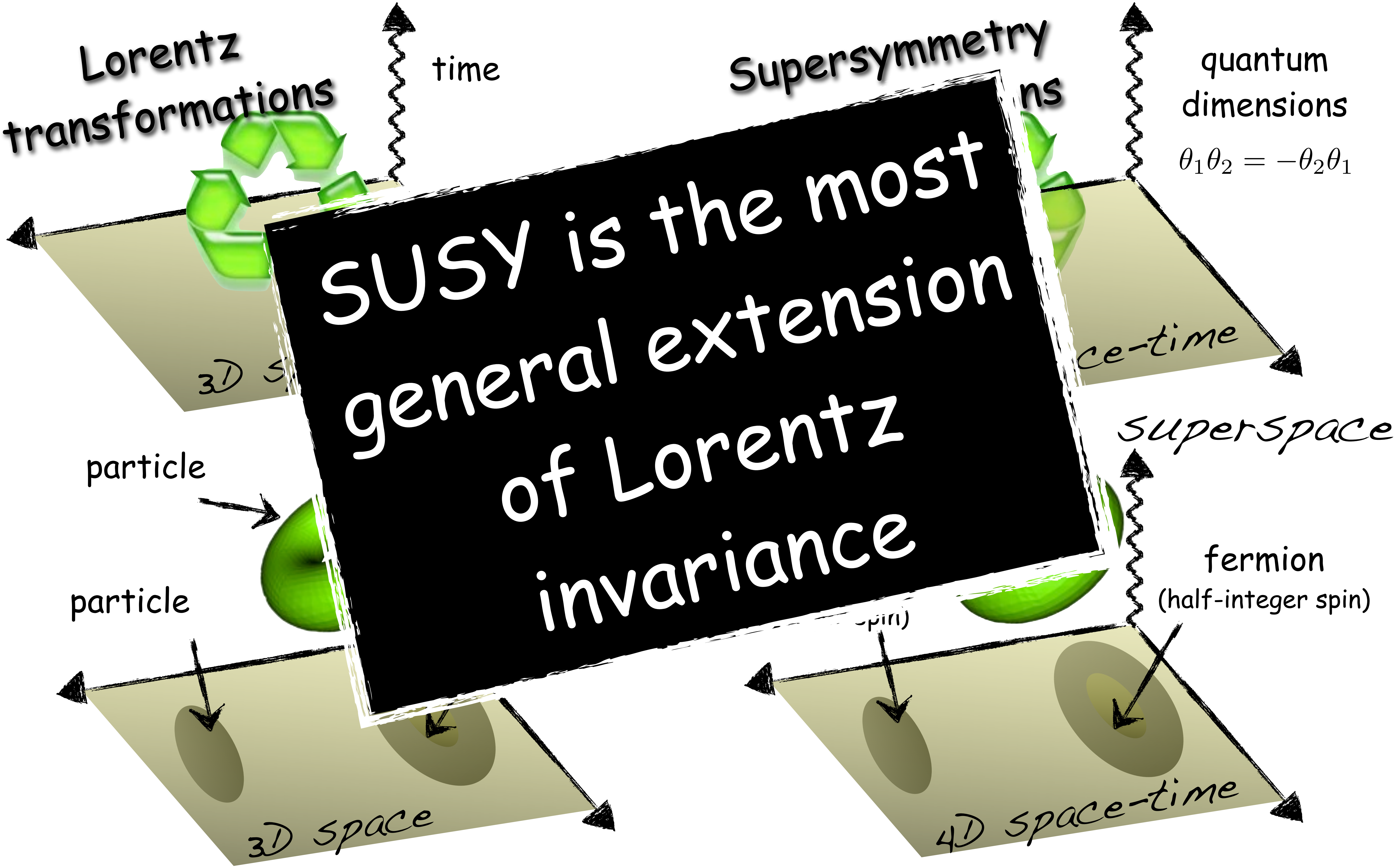


SUSY is the most general extension of Lorentz invariance

SUSY 1.0.1

Wess, Zumino '74

fermion ⇔ boson

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$

- susy transformations:

$$\delta\phi = \bar{\epsilon}\psi$$
$$\delta\psi = -i\left(\gamma^\mu \partial_\mu \phi\right)\epsilon$$

$$\delta\mathcal{L} = \text{total derivative}$$

- susy algebra:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = -i\left(\bar{\epsilon_2}\gamma^\mu \epsilon_1\right)\partial_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

susy^2 = 4D translation

How to introduce interactions?

Superspace

$$\left(x^{\mu}, \theta, \bar{\theta}\right)$$

usual 4D
space-time coordinates

new fermionic/Grassmanian
coordinates

A general superfield can be Taylor-expanded in the superspace

$$F(x,\theta,\bar{\theta}) = f(x) + \theta\chi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}\bar{m}(x) + \theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}d(x)$$

complex spin-0 fields:	$f(x), m(x), \bar{m}(x), d(x)$	4x2=8 real off-shell degrees of freedom
complex spin-1 fields:	$v_{\mu}(x)$	1x8=8 real off-shell degrees of freedom
Weyl spin-1/2 fields:	$\chi(x), \bar{\chi}, \lambda(x), \bar{\lambda}(x)$	4x4=16 real off-shell degrees of freedom

- **Chiral superfield** $\bar{D}_{\dot{\alpha}}F = 0$

covariant derivative
ie commute with supersymmetry

$\Rightarrow$

$$F = \phi(x) + \theta\psi(x) + \theta\theta f(x)$$

off-shell dof	2	4	2
on-shell dof	2	2	0

chiral fermion!

- **Vector superfield**

$$F = F^{\dagger}$$

$\Rightarrow$

$$F = \theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}d(x)$$

off-shell dof	3	4	1
on-shell dof	2	2	0

massless gauge field

MSSM - Matter Content

		particles		Sparticles
Chiral superfields	quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R d_R	squarks	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$ $\tilde{u}_R$ $\tilde{d}_R$
	leptons	$\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$ e_R	sleptons	$\begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}$ $\tilde{e}_R$
	Higgs doublets	H_1 (hypercharge $= -1$) H_2 (hypercharge $= +1$)	Higgsinos	$\tilde{H}_1$ $\tilde{H}_2$
vector superfields		$W_\mu^\pm, W_\mu^3$	winos	$\tilde{\omega}^\pm, \tilde{\omega}^3$
		B_μ	bino	$\tilde{b}$
		G_μ^A $A = 1, \ldots, 8$	gluinos	$\tilde{g}^A$

(G. Giudice HCPSS'09)

SUSY Interactions - Superpotential

superpotential W = holomorphic fct of chiral superfields

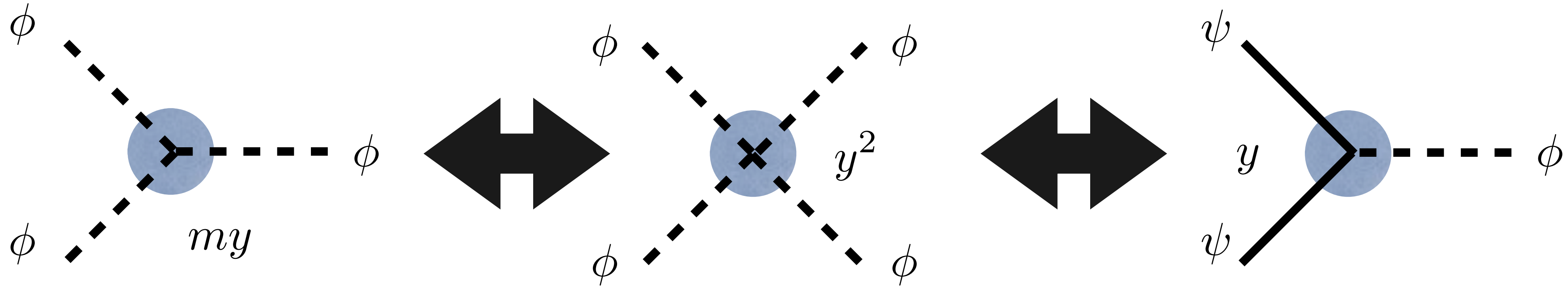
$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \left|\frac{\partial W}{\partial \phi}\right|^2_{|\theta=0} - \frac{1}{2}\frac{\partial^2 W}{\partial \phi^2}_{|\theta=0} \psi\psi + h.c.$$

is invariant under susy

example: susy Yukawa interaction

$$W = \frac{1}{2}m\phi^2 + \frac{1}{3!}y\phi^3 \qquad \partial_\phi W = m\phi + \frac{1}{2}y\phi^2 \qquad \partial^2_\phi W = m + y\phi$$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \left|m\phi + \frac{1}{2}y\phi^2\right|^2 - \frac{1}{2}\left(m + y\phi\right)\psi\psi + h.c.$$



will be modified by soft susy breaking

will survive soft susy breaking

MSSM Superpotential

the most general ("renormalizable") superpotential of the MSSM

$$W = H_u Q D + H_u Q U + H_d L E + \mu H_u H_d + LQD + UDD + LLE + \mu_L L H_u$$

$\cancel{B}, \cancel{L}$

lead to fast p decay

R parity forbids all the dangerous terms

superfields

$Q, D, U, L : -1$

$H_u, H_d : +1$

R-parity

doesn't commute with susy

$\theta : -1$

fields

$\phi_{\text{SM}} : +1$

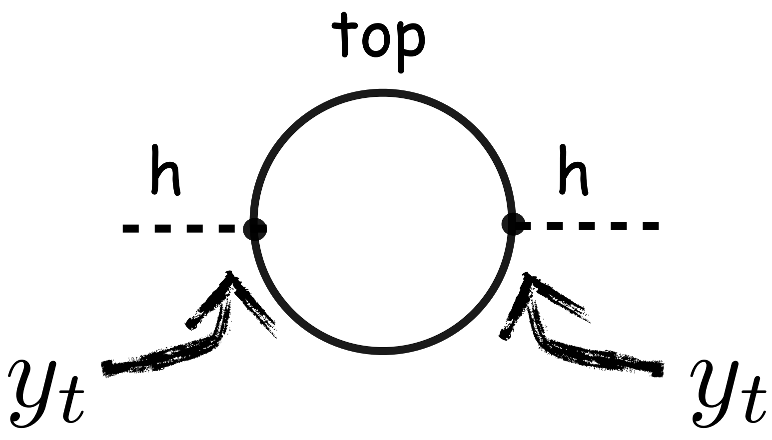
$\phi_{\text{superpartner}} : -1$

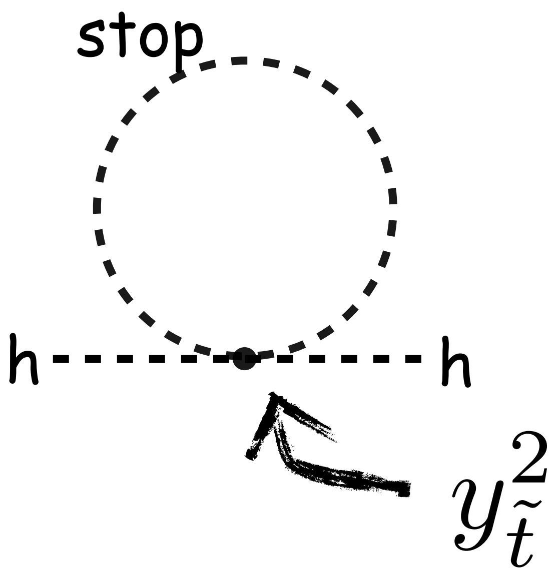
nice consequences:

- superpartners are pair-produced
- Lightest Supersymmetric Particle is stable → DM?

SUSY and the (big) hierarchy problem

(DE Kaplan HCPSS'07)





$$\delta m_H^2 \propto \left(y_t^2 - y_{\tilde{t}}^2\right)\Lambda^2 + \left(m_t^2 - m_{\tilde{t}}^2\right)\log\Lambda$$

$y_t \neq y_{\tilde{t}}$

⇩

Λ^2 dv

hard susy breaking

$m_t \neq m_{\tilde{t}}$

⇩

$\log\Lambda$ dv

soft susy breaking

SUSY biggest pb: how to dynamically generate soft breaking terms compatible with exp constraints?

SUSY little hierarchy problem

SUSY needs new (super)particles that haven't been seen (yet?)
SUSY (at least MSSM) predicts a (very) light Higgs

$$V = (|\mu|^2 + m_{H_u}^2) \left|H_u^0\right|^2 + (|\mu|^2 + m_{H_d}^2) \left|H_d^0\right|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8} \left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2 \right)^2$$

$$m_h^2 = m_Z^2 \cos^2 2\beta$$

tree-level

excluded

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$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

one-loop level

$m_H > 115$ GeV ⇒ $\tilde{m}_t > 1$ TeV

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$$V = \left(|\mu|^2 + m_{H_u}^2\right)\left|H_u^0\right|^2 + \left(|\mu|^2 + m_{H_d}^2\right)\left|H_d^0\right|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2+g'^2}{8}\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2$$

one-loop level

$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

$$m_Z^2/2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

$m_H > 115$ GeV ⇒ $\tilde{m}_t > 1$ TeV

$$\delta m_{H_u}^2 = -\frac{3\sqrt{2} G_F m_t^2 m_{\tilde{t}}^2}{4\pi^2} \log \frac{\Lambda}{m_{\tilde{t}}}$$

requires some fine-tuning O(1%) in m_Z

fine-tuned

susy
little hierarchy
problem

The MSSM Higgs mass and stop searches

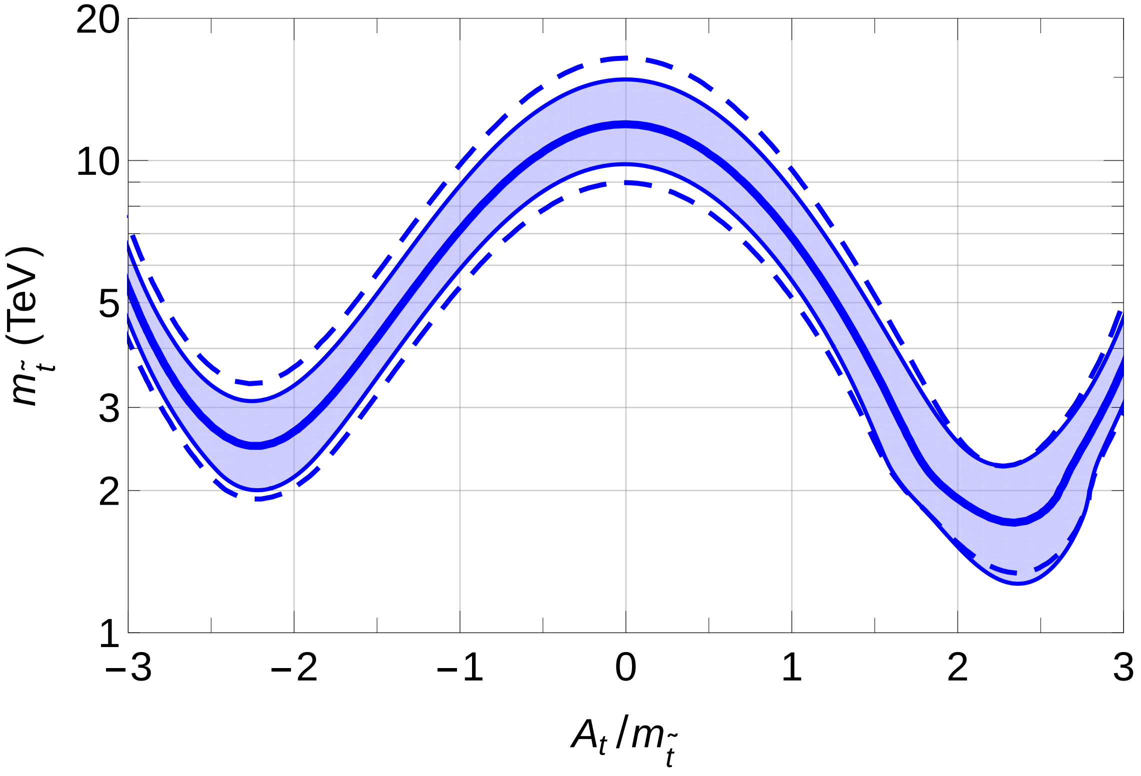
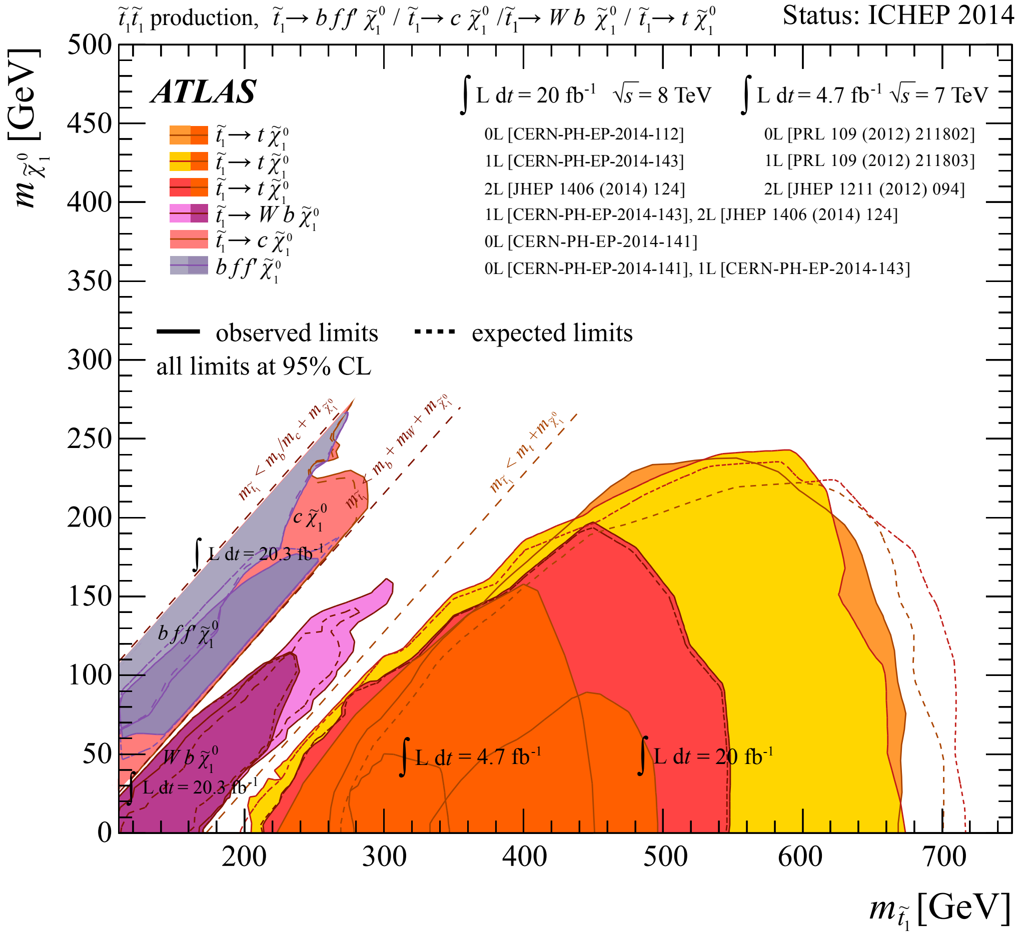


Figure 5: *Allowed values of the OS stop mass reproducing $m_h = 125$ GeV as a function of the stop mixing, with $\tan\beta = 20$, $\mu = 300$ GeV and all the other sparticles at 2 TeV. The band reproduce the theoretical uncertainties while the dashed line the 2σ experimental uncertainty from the top mass. The wiggle around the positive maximal mixing point is due to the physical threshold when $m_{\tilde{t}}$ crosses $M_3 + m_t$.*

Pardo Vega, Villadoro '15 + many others

One needs heavy stop(s)
to obtain a 125GeV Higgs
(within the MSSM)

Current and future
bounds on stop mass



The MSSM Higgs mass and stop searches

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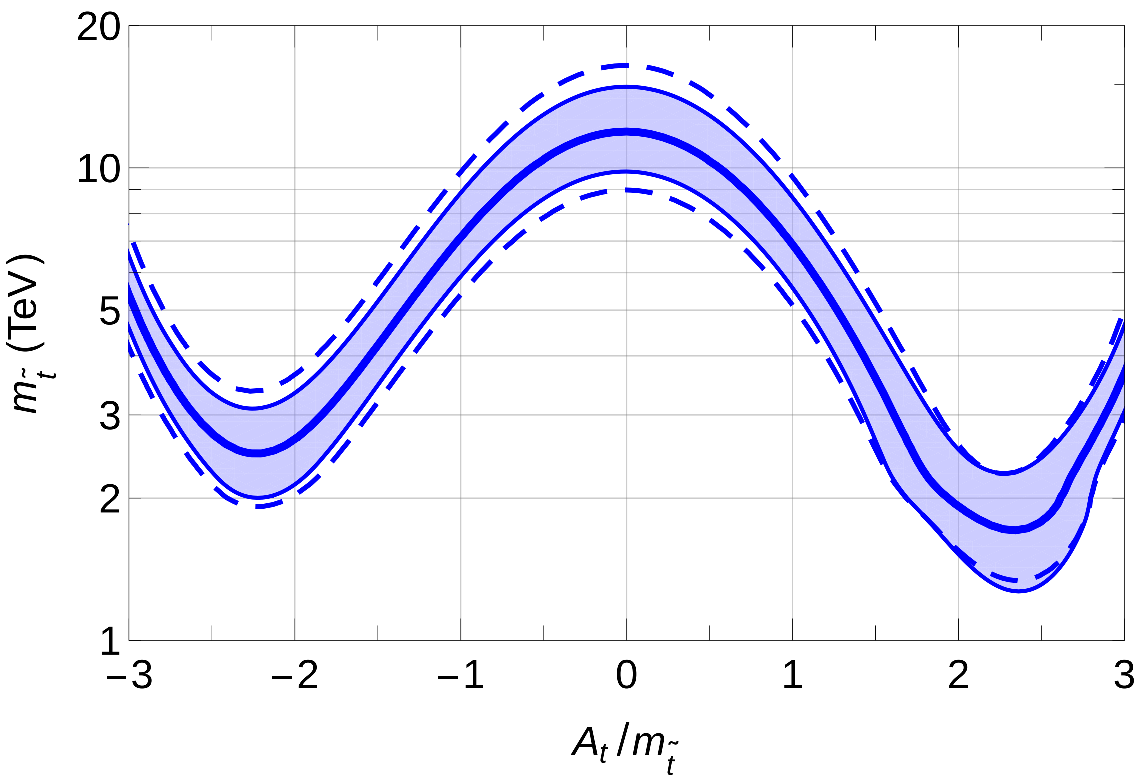

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One needs heavy stop(s)
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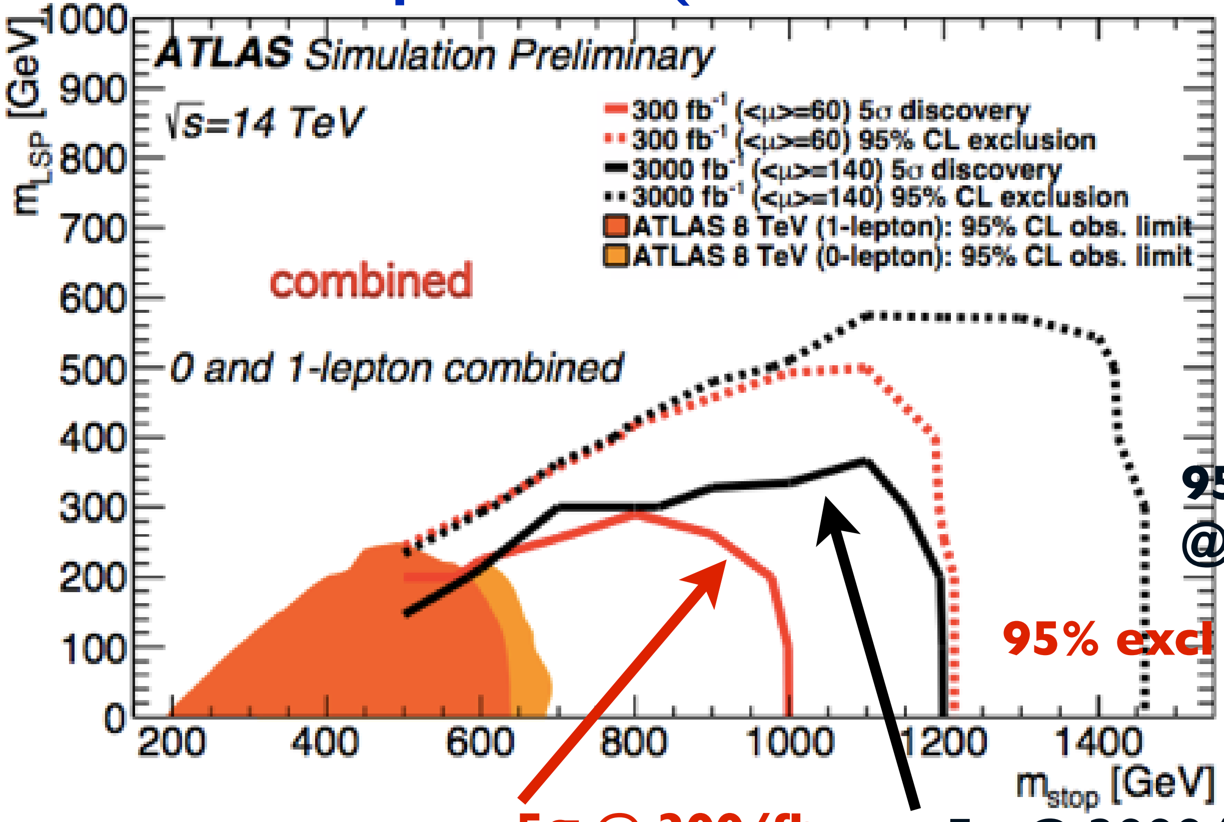
(within the MSSM)

Current and future
bounds on stop mass

HL-LHC (2030)

Direct stop searches (ATLAS Snowmass doc)



95% excl @ 3000/fb

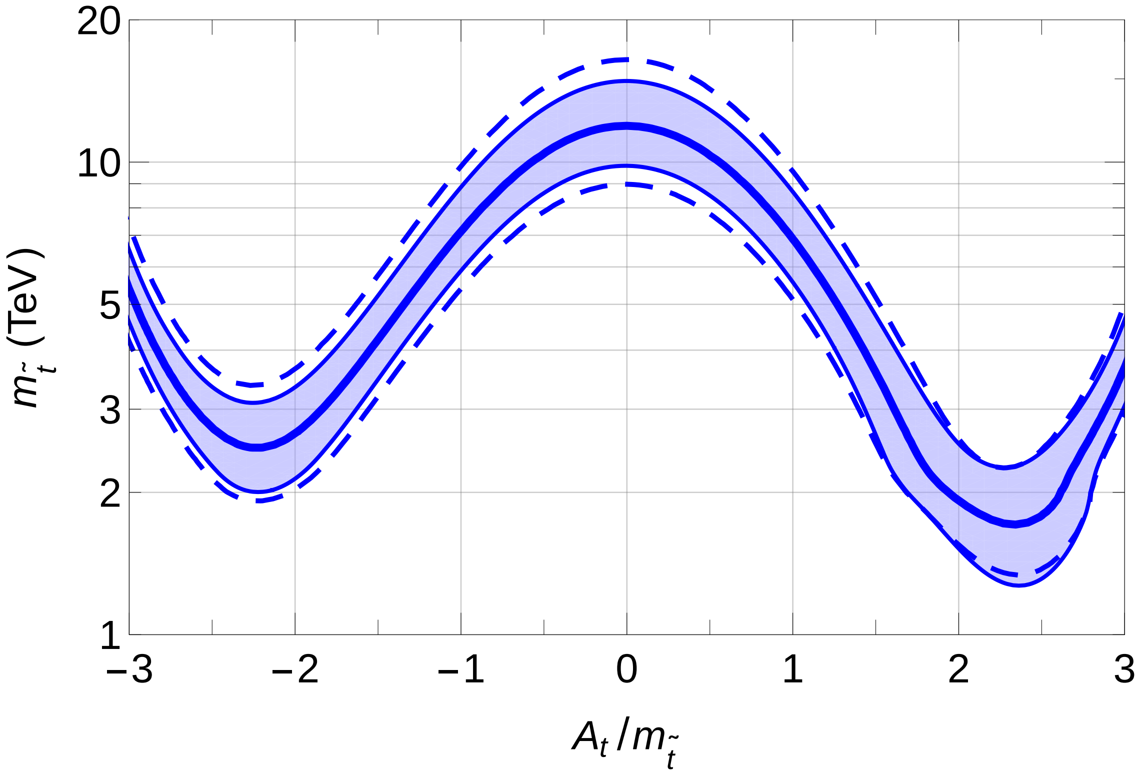
95% excl @ 300/fb

5σ @ 300/fb

5σ @ 3000/fb

The MSSM Higgs mass and stop searches

Pardo Vega, Villadoro '15 + many others



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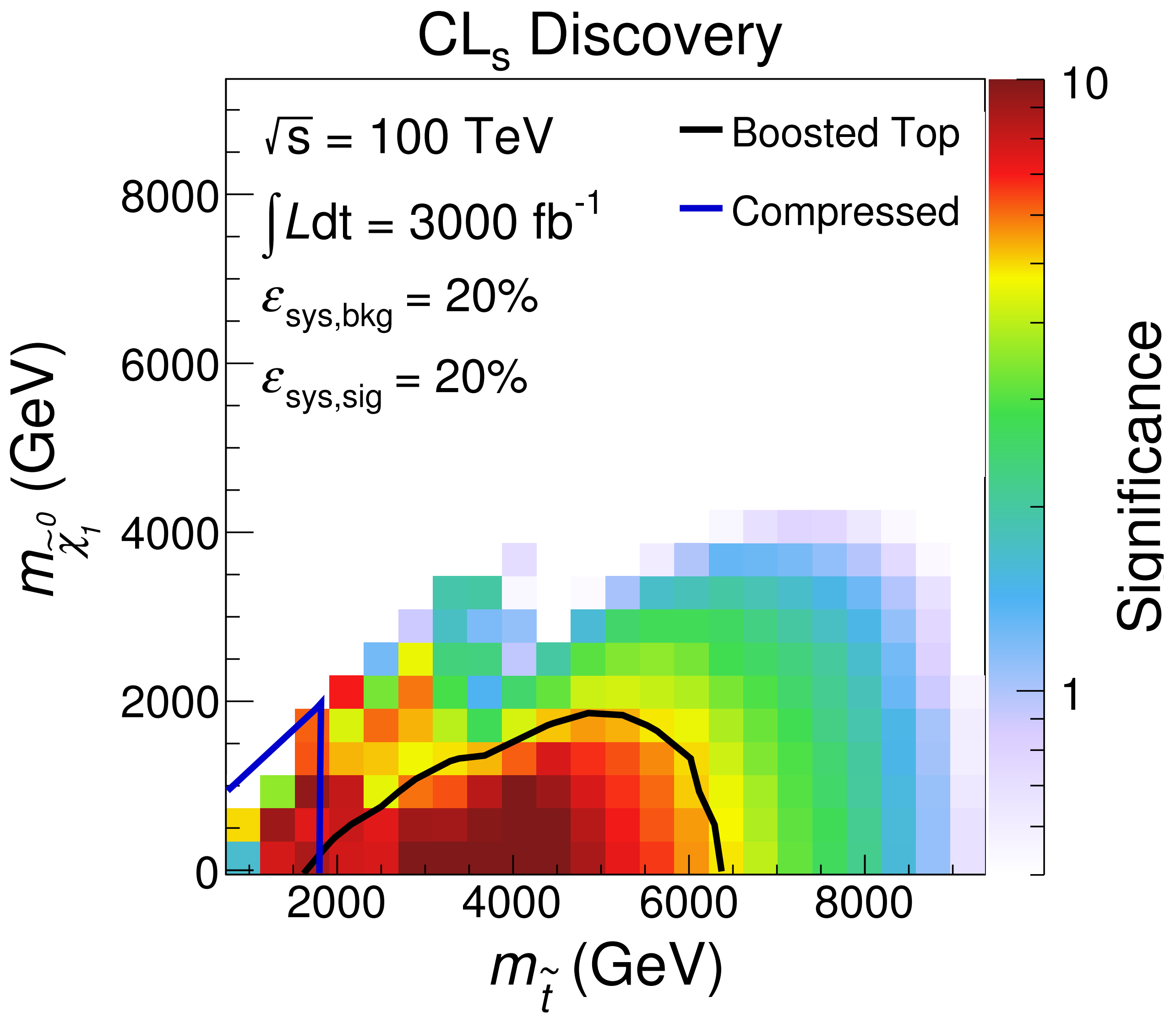
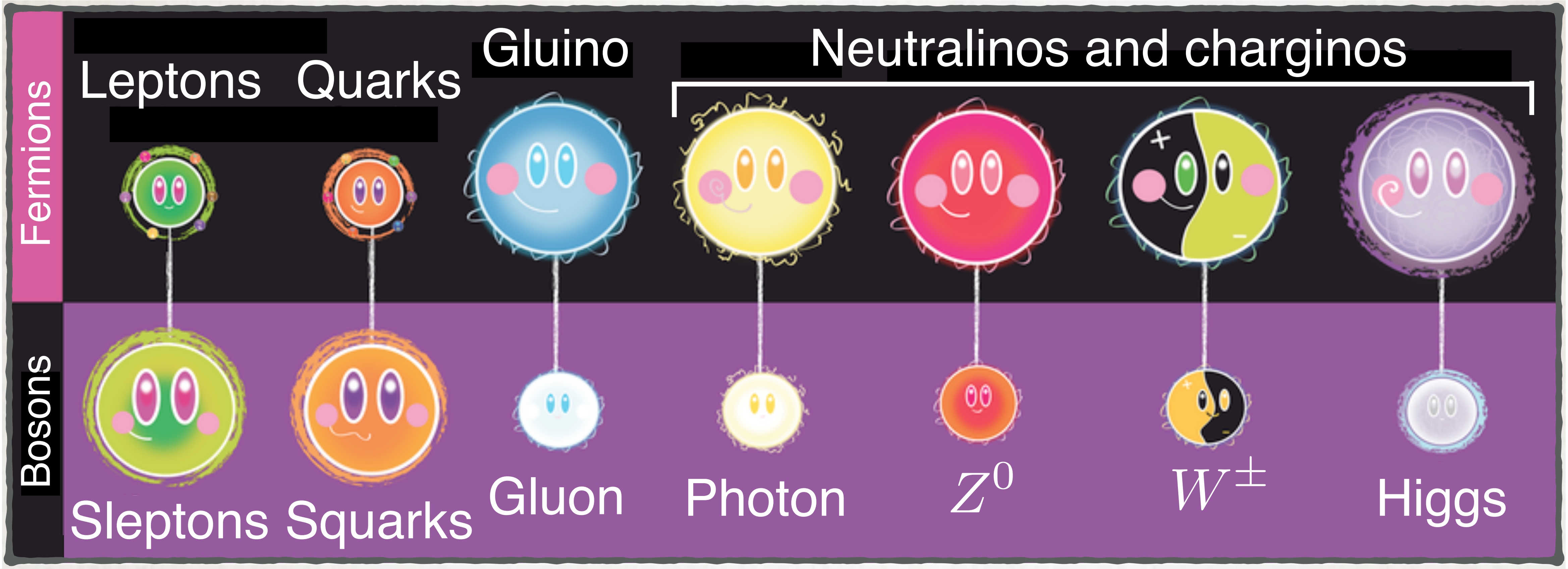
One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Current and future bounds on stop mass

FCC-hh @ 100TeV (2050)



Natural SUSY: where is everybody



Two-loops: gluinos $\mathcal{O}\left(\frac{16\,\pi^2}{\text{Log}}m_h\right)$

One-loop: stops $\mathcal{O}\left(\sqrt{\frac{16\,\pi^2}{\text{Log}}}m_h\right)$

Tree-level: Higgsinos $\mathcal{O}(m_h)$

Tim Cohen [University of Oregon]

HIGGSINO

$$\mu \lesssim 200\,\text{GeV}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

STOP

$$m_{\text{stop}} \lesssim 500\,\text{GeV}\,\frac{\sin\beta}{\sqrt{1+(A_t/m_{\text{stop}})^2}}\sqrt{\frac{3}{\log(\Lambda/\text{TeV})}}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

GLUINO

$$m_{\text{gluino}} \lesssim 1000\,\text{GeV}\,\sin\beta\,\frac{3}{\log(\Lambda/\text{TeV})}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

Tuning: $\Delta \equiv \dfrac{2\,\delta m_H^2}{m_h^2}$

Saving SUSY

SUSY is Natural
but not plain vanilla

- ~~CMSSM~~
- pMSSM
- NMSSM
- colorless stops ("folded susy")
- Hide SUSY, e.g. smaller phase space
  - reduce production (eg. split families) Mahbubani et al
  - reduce MET (e.g. R-parity, Csaki et al compressed spectrum)
  - dilute MET (decay to invisible particles with more invisible particles)
  - soften MET (stealth susy, stop -top Fan et al degeneracy)

LHC$_{100fb^{-1}}$ will tell!

Good coverage of

hidden natural susy

- mono-top searches (DM, flavored naturalness - mixing among different squark flavors-, stop-higgsino mixings)
- mono-jet searches with ISR recoil (compressed spectra)
- precise tt inclusive measurement+ spin correlations (stop → top + very soft neutralino)
- multi-hard-jets (RPV, hidden valleys, long decay chains)

Grand Unified Theory: SM vs MSSM

Evolution of coupling constants

Classical physics: the forces depend on distances

Quantum physics : the charges depend on distances

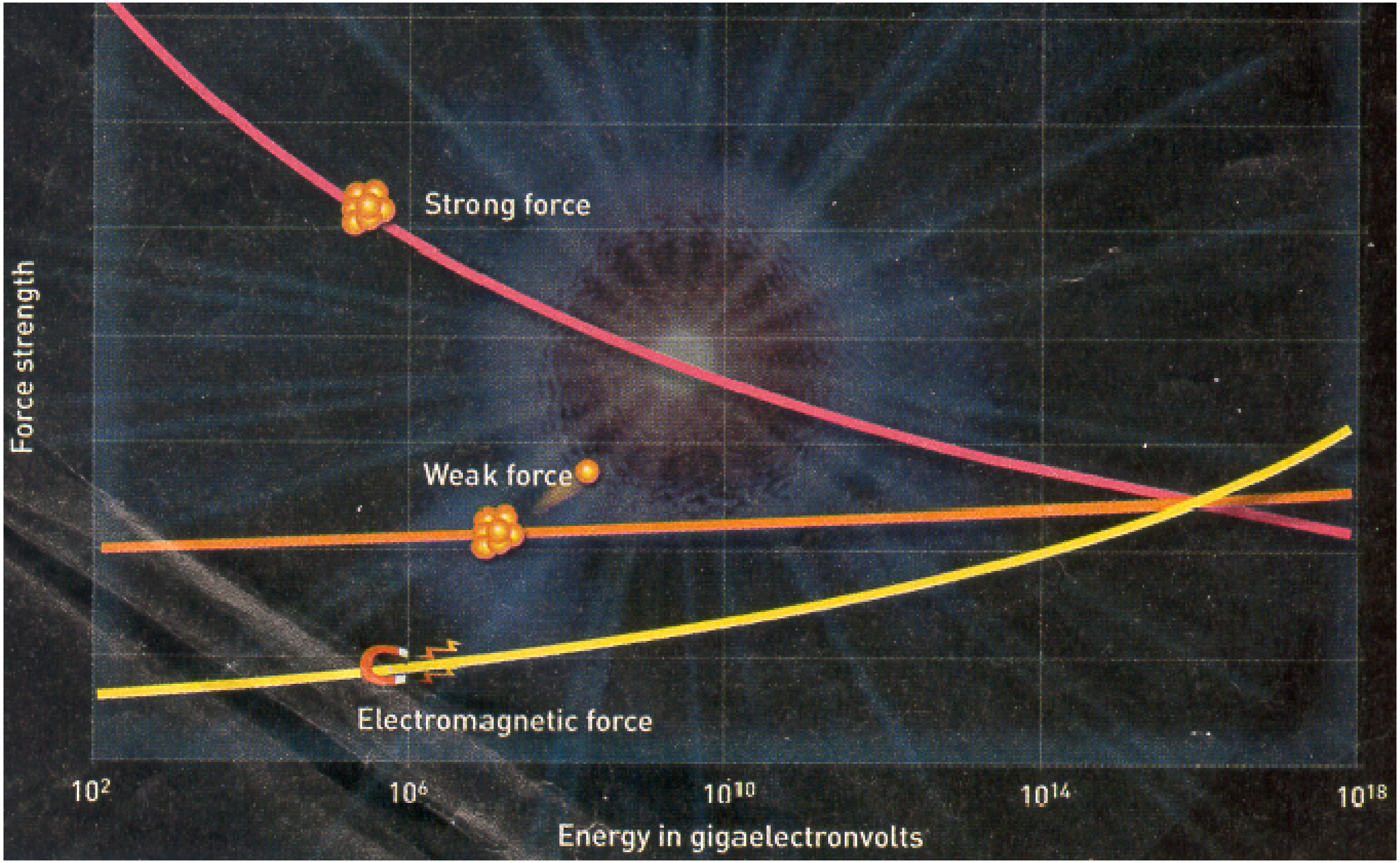
QED: virtual particles screen the electric charge: α ↘ when d ↗

QCD: virtual particles (quarks and *gluons*) screen the strong charge: α_s ↗ when d ↗

'asymptotic freedom'

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11N_c}{6}+\frac{N_f}{3}\right)$$

Grand Unified Theories



A single form of matter
A single fundamental interaction

SU(5) GUT: Gauge Group Structure

$SU(3)_c$x$SU(2)_L$x$U(1)_Y$: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

How can you ever remember all these numbers?

$SU(3)_c$x$SU(2)_L$x$U(1)_Y \subset SU(5)$

SU(5)

Adjoint rep.

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

additional U(1) factor that commutes with SU(3)xSU(2)

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right)$$

$$\boxed{\begin{gathered} \bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}} \\ \bar{5} = L + d_R^c \end{gathered}}$$

$$\boxed{\begin{gathered} 10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}} \\ 10 = u_R^c + Q_L + e_R^c \end{gathered}}$$

$$T^{12} = \sqrt{\frac{3}{5}} Y \qquad g_5\sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$$\sin^2\theta_W = \frac{3}{8}$$ @ M_{GUT}

SU(5) GUT: Gauge Group Structure

SU(3)cxSU(2)LxU(1)Y: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ \end{pmatrix} \quad = (1,1)_1$$

How can you even r

the SM matter fits nicely into representations of SU(5), even more nicely into SO(10) unification baryon-lepton

†

2)

Ac

Tr(

$$\begin{pmatrix} & \\ \hline -1/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

$$= (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$$

$$\bar{5} = L + d_R^c$$

$$T^{12} = \sqrt{\frac{3}{5}} Y \qquad g_5 \sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$10 = (5 \times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$$

$$10 = u_R^c + Q_L + e_R^c$$

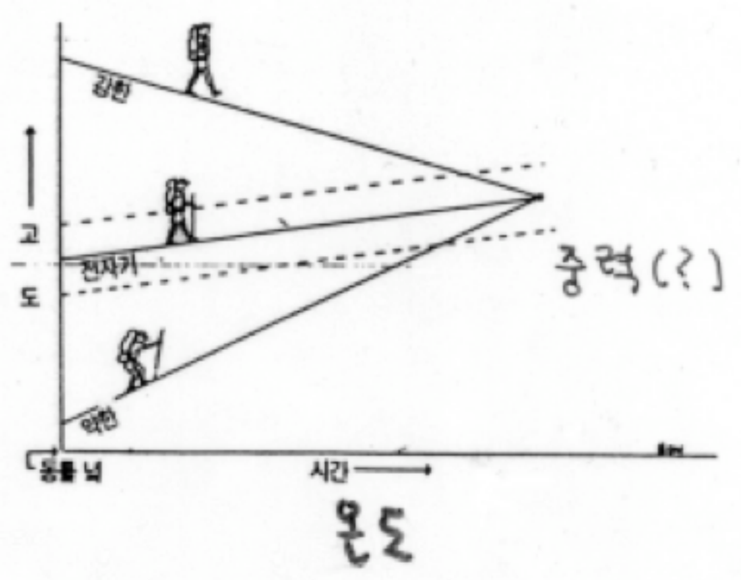
$$g_5 T^{12} = g' Y$$

$\sin^2 \theta_W = \frac{3}{8}$ @ MGUT

SU(5) GUT: SM β fcts

g, g′ and g$_s$ are different but it is a low energy artifact!

$$\boxed{\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2}bg^3 + \ldots}$$



$$\boxed{\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2}\ln\frac{Q^2}{Q_0^2}}$$

$$b = \frac{11}{3}T_2(\texttt{spin-1}) - \frac{2}{3}T_2(\texttt{chiral spin-1/2}) - \frac{1}{3}T_2(\texttt{complex spin-0})$$

$$\mathrm{Tr}\left(T^a(R)T^b(R)\right) = T_2(R)\delta^{ab} \qquad T_2(\texttt{fund}) = \frac{1}{2} \qquad T_2(\texttt{adj}) = N$$

$$\boxed{b_{SU(3)}} = \overset{g}{\frac{11}{3}\times 3} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2}\times 2\times 3} + \overset{u_R^c}{\frac{1}{2}\times 1\times 3} + \overset{d_R^c}{\frac{1}{2}\times 1\times 3}\right) = \boxed{7}$$

$$\boxed{b_{SU(2)}} = \overset{W^\pm, Z}{\frac{11}{3}\times 2} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2}\times 3\times 3} + \overset{L}{\frac{1}{2}\times 1\times 3}\right) - \frac{1}{3}\times\frac{1}{2} = \boxed{\frac{19}{6}}$$

$$b_Y = -\frac{2}{3}\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3\times 2\times 3} + \overset{u_R^c}{\left(-\frac{2}{3}\right)^2 3\times 3} + \overset{d_R^c}{\left(\frac{1}{3}\right)^2 3\times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2\times 3} + \overset{e_R^c}{(1)^2\times 3}\right) - \frac{1}{3}\left(\frac{1}{2}\right)^2\times 2 = -\frac{41}{6} \Rightarrow \boxed{b_{T^{12}} = -\frac{41}{10}}$$

SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ experimental inputs

b_3, b_2, b_1 predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation for unification

$$\epsilon_{ijk} \frac{b_j - b_k}{\alpha_i(M_Z)} = 0 \quad \Rightarrow \quad \boxed{\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1} \frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}$$

$$\alpha_{em}(M_Z) \approx \frac{1}{128} \qquad \alpha_s(M_Z) \approx 0.1184 \pm 0.0007$$

$$\Downarrow$$

$$\sin^2\theta_W \approx 0.207$$

not so bad...

SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ ⟵ experimental inputs

b_3, b_2, b_1 ⟵ predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation for unification

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 7 \times 10^{14}\ \text{GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

self-consistent computation:

- ○ $M_{GUT} < M_{Pl}$ safe to neglect quantum gravity effects
- ○ $\alpha_{GUT} \ll 1$ perturbative computation

SU(5) GUT: SM vs MSSM β fcts

chiral superfield

complex spin-0
Weyl spin-1/2
in same representation R of gauge group

vector superfield

Weyl spin-1/2
real spin-1
in same representation V of gauge group

$$b = \frac{11}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{chiral}) - \frac{1}{3} T_2(\texttt{chiral}) = 3\, T_2(\texttt{vector}) - T_2(\texttt{chiral})$$

MSSM Chiral Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad U = (\bar{3},1)_{-2/3}, \quad D = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad E = (1,1)_1, \quad H_u = (1,2)_{1/2}, \quad H_d = (1,2)_{-1/2}$$

$$b_{SU(3)} = \overset{g}{3 \times 3} - \left(\overset{Q_L}{\frac{1}{2} \times 2 \times 3} + \overset{U}{\frac{1}{2} \times 1 \times 3} + \overset{D}{\frac{1}{2} \times 1 \times 3} \right) = \boxed{3}$$

$$b_{SU(2)} = \overset{W^\pm, Z}{3 \times 2} - \left(\overset{Q_L}{\frac{1}{2} \times 3 \times 3} + \overset{L}{\frac{1}{2} \times 1 \times 3} \right) - \overset{H_u}{\frac{1}{2}} - \overset{H_d}{\frac{1}{2}} = \boxed{-1}$$

$$b_Y = - \left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3 \times 2 \times 3} + \overset{U}{\left(-\frac{2}{3}\right)^2 3 \times 3} + \overset{D}{\left(\frac{1}{3}\right)^2 3 \times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2 \times 3} + \overset{E}{(1)^2 \times 3} \right) - \overset{H_u}{\left(\frac{1}{2}\right)^2 \times 2} - \overset{H_d}{\left(\frac{1}{2}\right)^2 \times 2} = -11 \quad \Rightarrow \quad \boxed{b_{T^{12}} = -\frac{33}{5}}$$

SU(5) GUT: MSSM GUT

$$b_3 = 3, \quad b_2 = -1, \quad b_1 = -33/5$$

low-energy consistency relation for unification

$$\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1}\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)} \approx 0.23$$

squarks and sleptons form complete SU(5) reps ➙ they don't improve unification!
gauginos and higgsinos are improving the unification of gauge couplings

GUT scale predictions

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 2 \times 10^{16}\ \text{GeV}$$

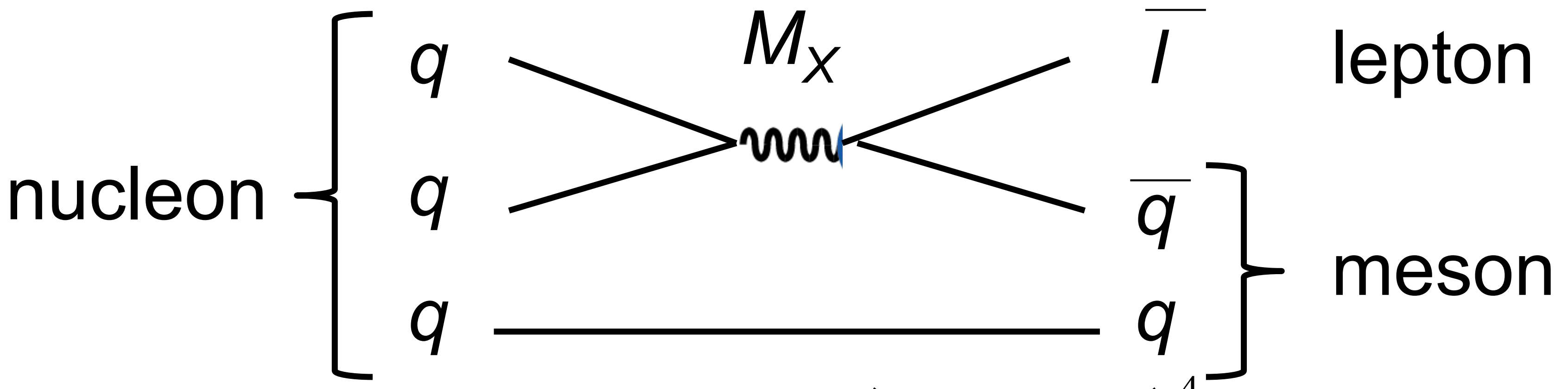
$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 24.3$$

Proton Decay

(G. Giudice SSLP'15)

in GUT, matter is unstable

decay of proton mediated by new SU(5)/SO(10) gauge bosons



$$\text{GUT:}\quad \tau_p(p \to e^+\pi^0) = \left(\frac{M_X}{10^{15}\ \text{GeV}}\right)^4 10^{31-32}\ \text{yr}$$

$$\text{Exp:}\quad \tau_p(p \to e^+\pi^0) > 8.2\times 10^{33}\ \text{yr}$$