Workshop on Resummation, Evolution, Factorization (REF 2016)

7-10 November 2016
University of Antwerp

Reconstructing PDFs at large x

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in collaboration with
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based on the paper
arXiv:1608.07638

Field theoretical definition of PDF

nucleon with momentum $P^\mu = [P^-, P^+, \mathbf{0}_T]$

and long. polarization $P \cdot S = 0 \quad S^2 = -1$

$$f_1(x) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{-i\xi^- xP^+} \langle P|\bar{\psi}(\xi^-)\,\gamma^+\,U_{n_-}[\xi^-,0]\,\psi(0)|P\rangle$$

$$(\xi^-, 0, \mathbf{0}_T)$$

$$g_1(x) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{-i\xi^- xP^+} \langle PS|\bar{\psi}(\xi^-)\,\gamma^+\gamma_5\,U_{n_-}[\xi^-,0]\,\psi(0)|PS\rangle$$

gauge link operator $U_{n_-}[\xi^-,0] = \mathcal{P}\left[\exp\left(-ig\int_0^{\xi^-} dw^-\,A^+(w^-)\right)\right]$

hadronic matrix elements of nonlocal operators on light-cone

essentially nonperturbative objects

PDF nonperturbative object

PDF

factorization theorem (at some scale μ)
data = PDF ⊗ (something perturbatively calculable)

lattice

within $\mathcal{L}_{QCD}$

calculate them in QCD-inspired models

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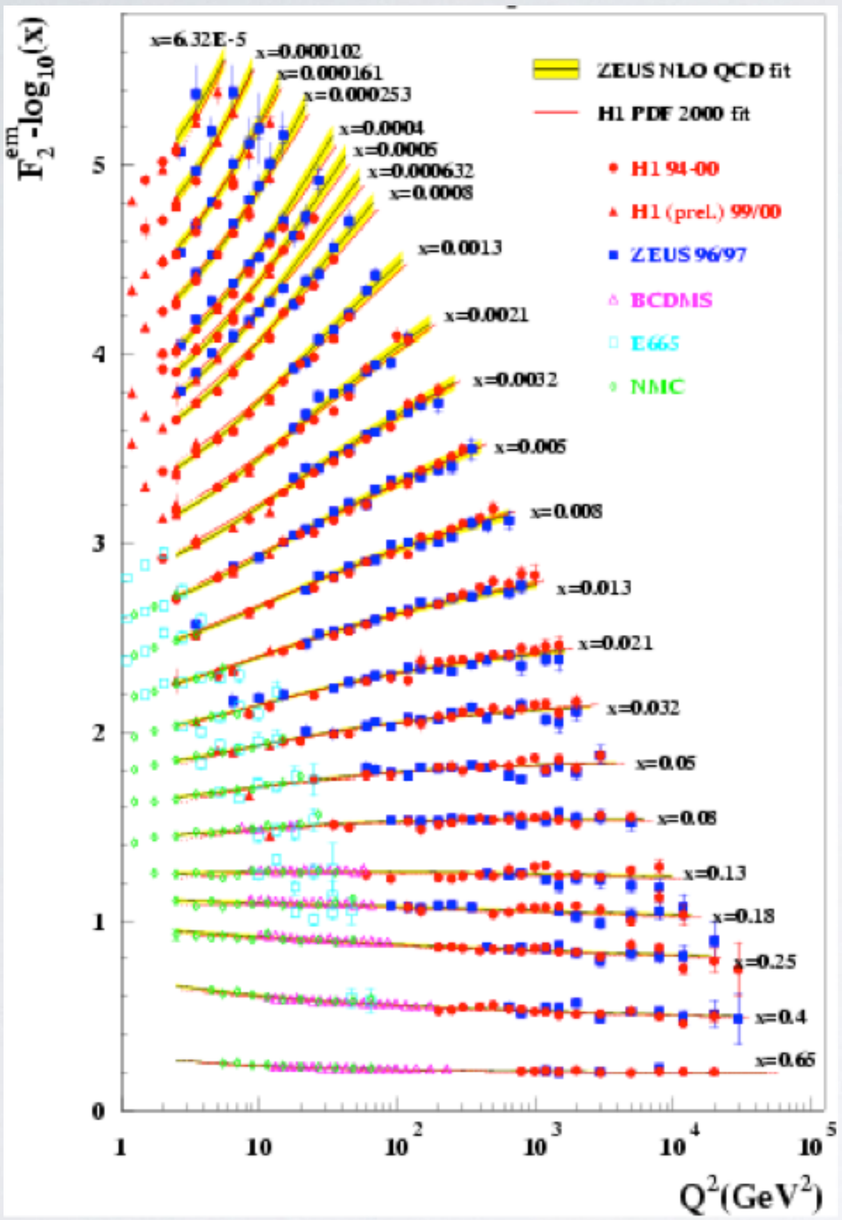
lattice

calculate them in QCD-inspired models

Nonperturbative object → 1) extract from data



slide from H.Montgomery, QCD Evolution 2016

World data for F_2^p in DIS

$f_1(x,Q^2)$ from fits of **thousands** data

~ 3000 **CT14**, *P.R.D***93** *(16) 033006*

~ 4000 **CJ12**, *P.R. D***84** *(11) 014008*

~ 4300 **NNPDF3.0**, *JHEP* **1504** *(15) 040*

….

\+ **many** future **constraints** from **LHC**

REACTION	OBSERVABLE	PDFS	x	Q
$pp \to W^\pm + X$	$d\sigma(W^\pm)/dy_l$	$q, \bar{q}$	$10^{-3} \lesssim x \lesssim 0.7$	$\sim M_W$
$pp \to \gamma^*/Z + X$	$d^2\sigma(\gamma^*/Z)/dy_{ll}dM_{ll}$	$q, \bar{q}$	$10^{-3} \lesssim x \lesssim 0.7$	5 GeV $\lesssim Q \lesssim 2$ TeV
$pp \to \gamma^*/Z + \text{jet} + X$	$d\sigma(\gamma^*/Z)/dp_T^{ll}$	q, g	$10^{-2} \lesssim x \lesssim 0.7$	200 GeV $\lesssim Q \lesssim 1$ TeV
$pp \to \text{jet} + X$	$d\sigma(\text{jet})/dp_T dy$	q, g	$10^{-2} \lesssim x \lesssim 0.8$	20 GeV $\lesssim Q \lesssim 3$ TeV
$pp \to \text{jet} + \text{jet} + X$	$d\sigma(\text{jet})/dM_{jj}dy_{jj}$	q, g	$10^{-2} \lesssim x \lesssim 0.8$	500 GeV $\lesssim Q \lesssim 5$ TeV
$pp \to t\bar{t} + X$	$\sigma(t\bar{t}), d\sigma(t\bar{t})/dM_{t\bar{t}}, \ldots$	g	$0.1 \lesssim x \lesssim 0.7$	350 GeV $\lesssim Q \lesssim 1$ TeV
$pp \to c\bar{c} + X$	$d\sigma(c\bar{c})/dp_{T,c}dy_c$	g	$10^{-5} \lesssim x \lesssim 10^{-3}$	1 GeV $\lesssim Q \lesssim 10$ GeV
$pp \to b\bar{b} + X$	$d\sigma(b\bar{b})/dp_{T,c}dy_c$	g	$10^{-4} \lesssim x \lesssim 10^{-2}$	5 GeV $\lesssim Q \lesssim 30$ GeV
$pp \to W + c$	$d\sigma(W + c)/d\eta_l$	$s, \bar{s}$	$0.01 \lesssim x \lesssim 0.5$	$\sim M_W$

*J. Rojo et al. (***PDF4LHC***), J.Phys.G***42** *(15) 103103*

Nonperturbative object → 1) extract from data

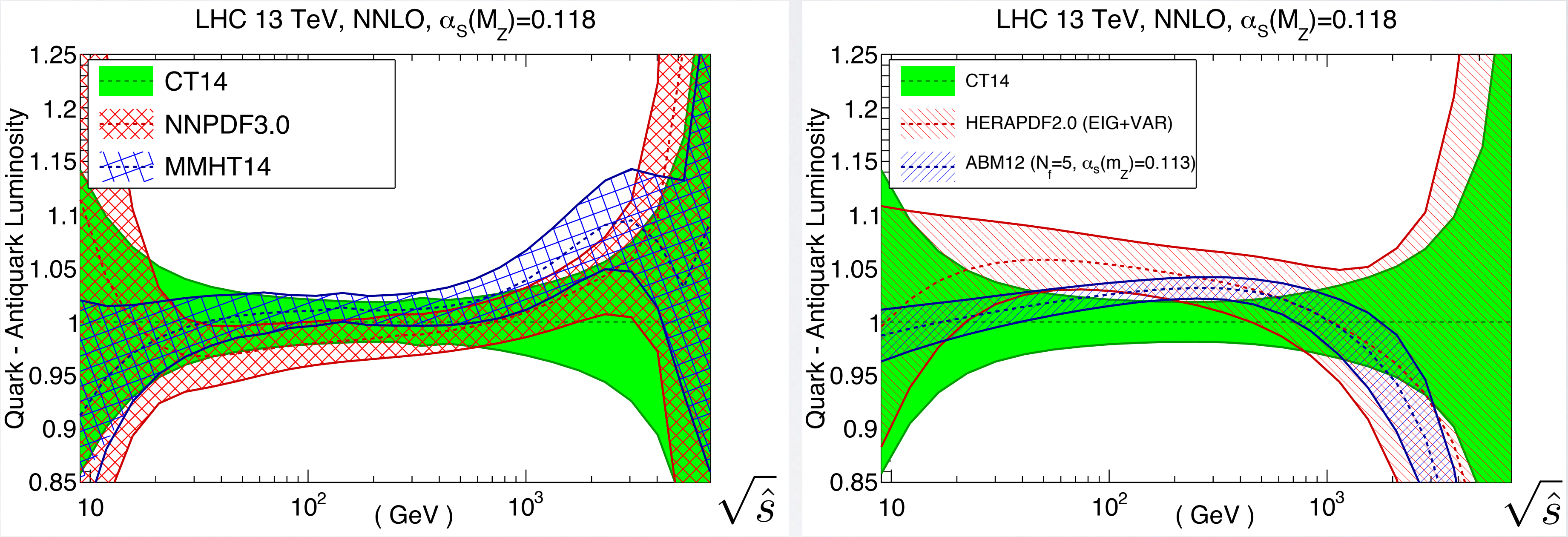
q - $\bar{q}$ luminosity

$$\mathcal{L}_{q\bar{q}}(\sqrt{\hat{s}}) = \frac{1}{s}\int_\tau^1 \frac{dx}{x}\sum_q \left[f_q(x,\hat{s})\, f_{\bar{q}}\left(\frac{\tau}{x},\hat{s}\right) + q \leftrightarrow \bar{q}\right]$$

$\sqrt{\hat{s}}$ = partonic c.m. energy

$\sqrt{s}$ = collision c.m. energy = 13 TeV

$\tau = \frac{\hat{s}}{s}$



*J. Butterworth et al. (**PDF4LHC**), J.Phys.G**43** (16) 023001*

Nonperturbative object → 1) extract from data

still significant uncertainties among recent PDF extractions

q - $\bar{q}$ luminosity $$\mathcal{L}_{q\bar{q}}(\sqrt{\hat{s}}) = \frac{1}{s}\int_{\tau}^{1} \frac{dx}{x} \sum_{q} \left[f_q(x,\hat{s})\, f_{\bar{q}}\left(\frac{\tau}{x},\hat{s}\right) + q \leftrightarrow \bar{q} \right]$$

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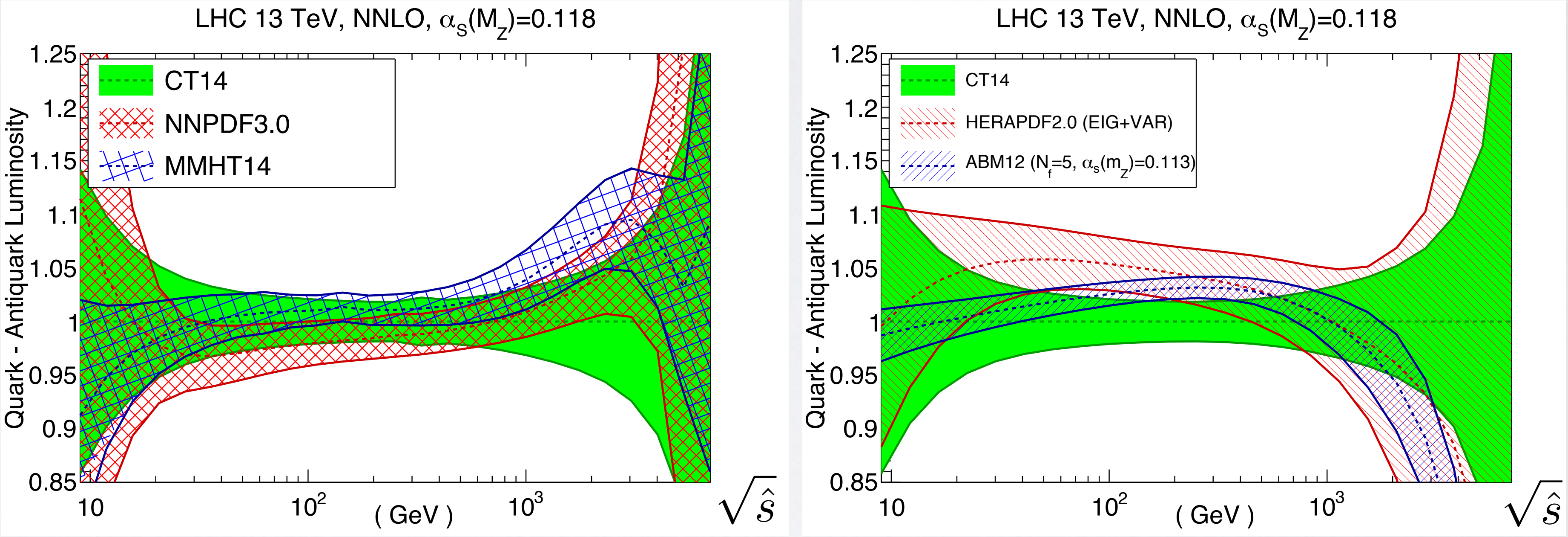


*J. Butterworth et al. (**PDF4LHC**), J.Phys.G**43** (16) 023001*

reflects in :
- less accurate extraction of SM quantities from LHC data (H coupling, M_W, $\sin\theta_{eff}$)
- limited sensitivity to BSM searches

PDF nonperturbative object

factorization theorem (at some scale μ)
data = PDF ⊗ (something perturbatively calculable)

PDF

lattice

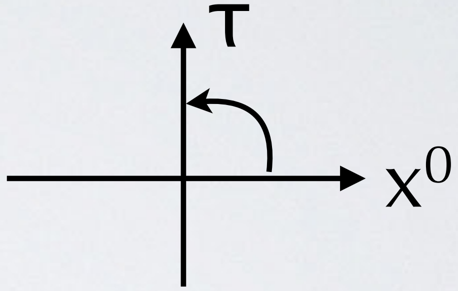
within $\mathcal{L}_{QCD}$

calculate them in
QCD-inspired models

Nonperturbative object → 2) compute on lattice

$$f_1(x,\mu^2) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi}\, e^{-i\xi^- x P^+}\, \langle P|\bar{\psi}(\xi^-)\,\gamma^+\, U_{n_-}[\xi^-,0]\,\psi(0)|P\rangle$$

Wick rotation: Euclidean time $\tau = i\, x^0$



light-cone distance ξ^- becomes complex! $0 \longrightarrow \xi^-$

PDFs cannot be computed on lattice

Nonperturbative object → 2) compute on lattice

Mellin moments of PDFs

$$\int_0^1 dx\, f_1(x,\mu^2) = \int_0^1 dx \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi}\, e^{-i\xi^- x P^+}\, \langle P|\bar\psi(\xi^-)\,\gamma^+\, U_{n_-}[\xi^-,0]\,\psi(0)|P\rangle$$

$$= \frac{1}{2}\,\langle P|\bar\psi(0)\,\gamma^+\,\psi(0)|P\rangle$$

hadronic matrix elements of local operators calculable on lattice

....

$$\int_0^1 dx\, x^{n-1}\, f_1(x,\mu^2) = \frac{1}{2}\, c^n(\mu^2/Q^2, g(\mu))\, O^n(\mu)$$ where

$$\langle P|\bar\psi(0)\gamma^{\{\mu_1}\,(i\overleftrightarrow{D})^{\mu_2}\cdots(i\overleftrightarrow{D})^{\mu_n\}}\psi(0) - \mathrm{Tr}'\mathrm{s}\,|P\rangle = 2\,O^n\,[P^{\mu_1}..P^{\mu_n} - M^2 P^{\mu_1}..P^{\mu_{n-2}}\ldots]$$

Nonperturbative object → 2) compute on lattice

Mellin moments of PDFs

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hadronic matrix elements of local operators calculable on lattice

but

- operator mixing (power divergences)
- discrete regulariz. ← matching ? → continuum renorm. scheme

limit calculations to n≤ 4

(for workarounds see, e.g., Z. Davoudi (MITPDF Coll.) talk at SPIN-2016)

The LaMET approach

*X. Ji, P.R.L. **110** (13) 262002*

Can we compute the x-dependence of PDFs on lattice ?

PDF

$$f_1(x,\mu^2) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi}\, e^{-i\xi^- x P^+} \langle P|\bar{\psi}(\xi^-)\,\gamma^+\, U_{n_-}[\xi^-,0]\,\psi(0)|P\rangle$$

light-cone correlation $0 \longrightarrow \xi^-$

eliminate time dependence

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light-cone correlation $0 \longrightarrow \xi^-$

eliminate time dependence

spatial correlation $0 \longrightarrow \xi^z$

quasi-PDF

$$\tilde{f}_1(x,\mu^2,P^z) = \int_{-\infty}^{\infty} \frac{d\xi^z}{4\pi}\, e^{i\xi^z x P^z} \langle P|\bar{\psi}(\xi^z)\,\gamma^z\, U_z[\xi^z,0]\,\psi(0)|P\rangle$$

$$U_z[\xi^z,0] = \mathcal{P}\left[\exp\left(-ig\int_0^{\xi^z} dw^z\, A^z(w^z)\right)\right]$$

The LaMET approach

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light-cone correlation $0 \longrightarrow \xi^-$

eliminate time dependence

$P^z \to \infty$

spatial correlation $0 \longrightarrow \xi^z$

quasi-PDF

$$\tilde{f}_1(x,\mu^2,P^z) = \int_{-\infty}^{\infty} \frac{d\xi^z}{4\pi}\, e^{i\xi^z x P^z} \langle P|\bar{\psi}(\xi^z)\,\gamma^z\, U_z[\xi^z,0]\,\psi(0)|P\rangle$$

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Large **M**omentum **E**ffective Field **T**heory

The LaMET approach

quasi-PDF and PDF have same IR behavior → match by perturb. coeff.

$$\tilde{f}_1(x,\mu^2,P^z) = \int_0^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\, \frac{\mu}{P^z},\, \frac{1}{a_L P^z}\right) f_1(y,\mu^2) \;+\; \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2},\, \frac{M^2}{(P^z)^2}\right)$$

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$$\rightarrow Z\,(\xi,..) = \delta(1-\xi) + \frac{\alpha_s}{2\pi}\, Z^{(1)}(\xi,..) + \ldots$$

checked at 1 loop for non-singlet PDF

*X. Xiong et al., P.R.D**90** (14) 014051*

- UV divergences renormalized at μ up to 2 loops
- power divergences (cutoff a_L) cancelled by δm at all orders

*Ji & Zhang, P.R.D**92** (15) 034006 ; Chen, Ji, Zhang, arXiv:1609.08102*

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Z is finite for finite P^z, at most terms ~ $\log(P^z/\mu)$
quasi-PDF calculable on lattice for finite P^z, then

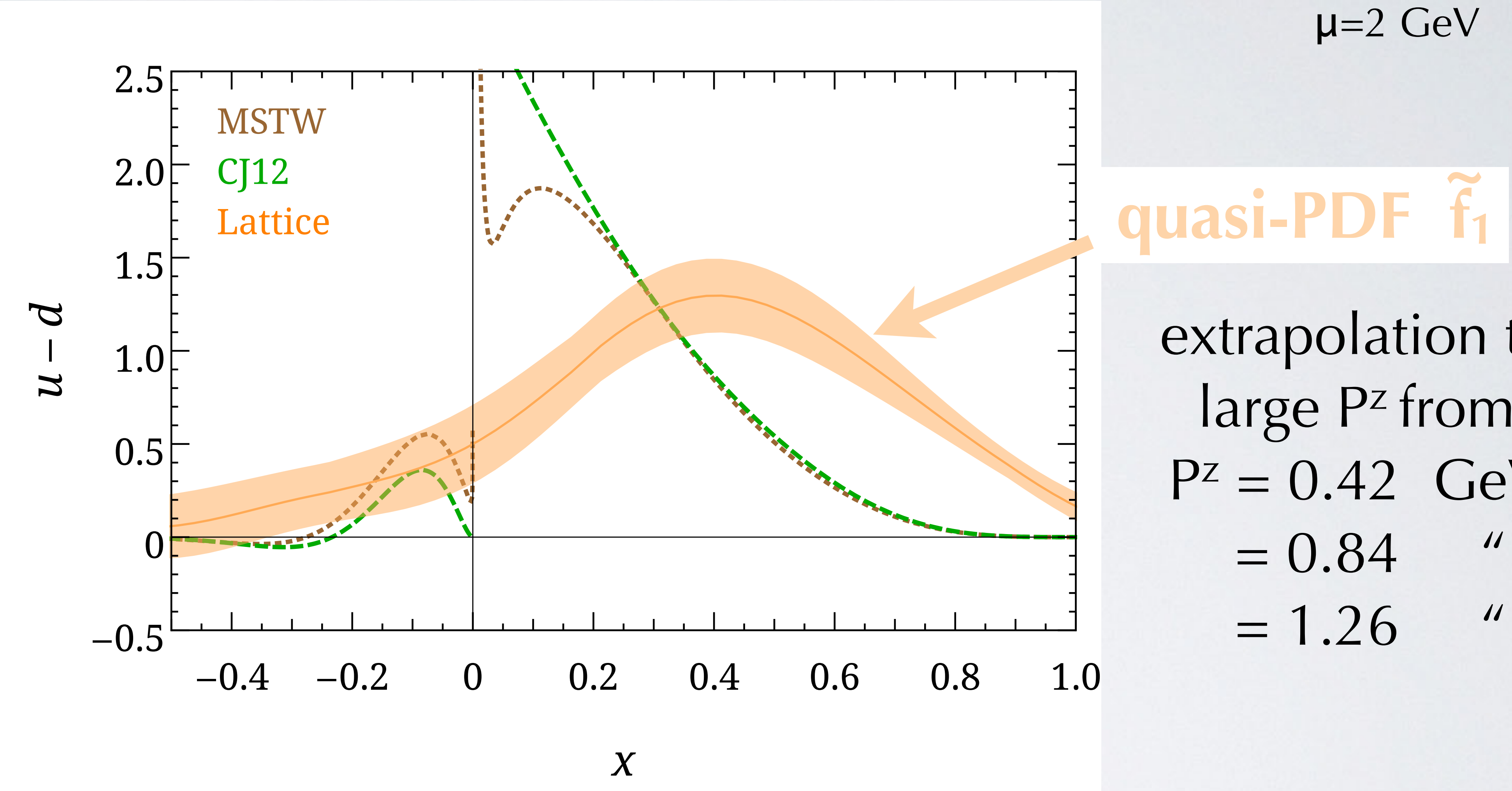
$$\lim_{P^z \to \infty} \text{quasi-PDF}\,(x, \mu^2, P^z) \;=\; \text{PDF}\,(x, \mu^2)$$

But how large P^z to have quasi-PDF ≈ PDF ?

quasi-PDF on lattice

lattice $24^3 \times 64$
a_L~0.12 fm
m_π~310 MeV
μ=2 GeV

isovector $f_1^{\mathbf{u-d}}(x)$



extrapolation to large P^z from
P^z = 0.42 GeV
= 0.84 "
= 1.26 "

*Lin et al., P.R.D**91** (15) 054510*

confirmed also by
*Alexandrou et al. (ETMC), P.R.D**92** (15) 014502*
with P^z = 0.98 & 1.47 GeV

lattice $32^3 \times 64$
a_L~0.08 fm
m_π~370 MeV
μ=Λ=1/a_L~2.5 GeV

quasi-PDF on lattice

isovector $f_1^{\mathbf{u\text{-}d}}(x)$

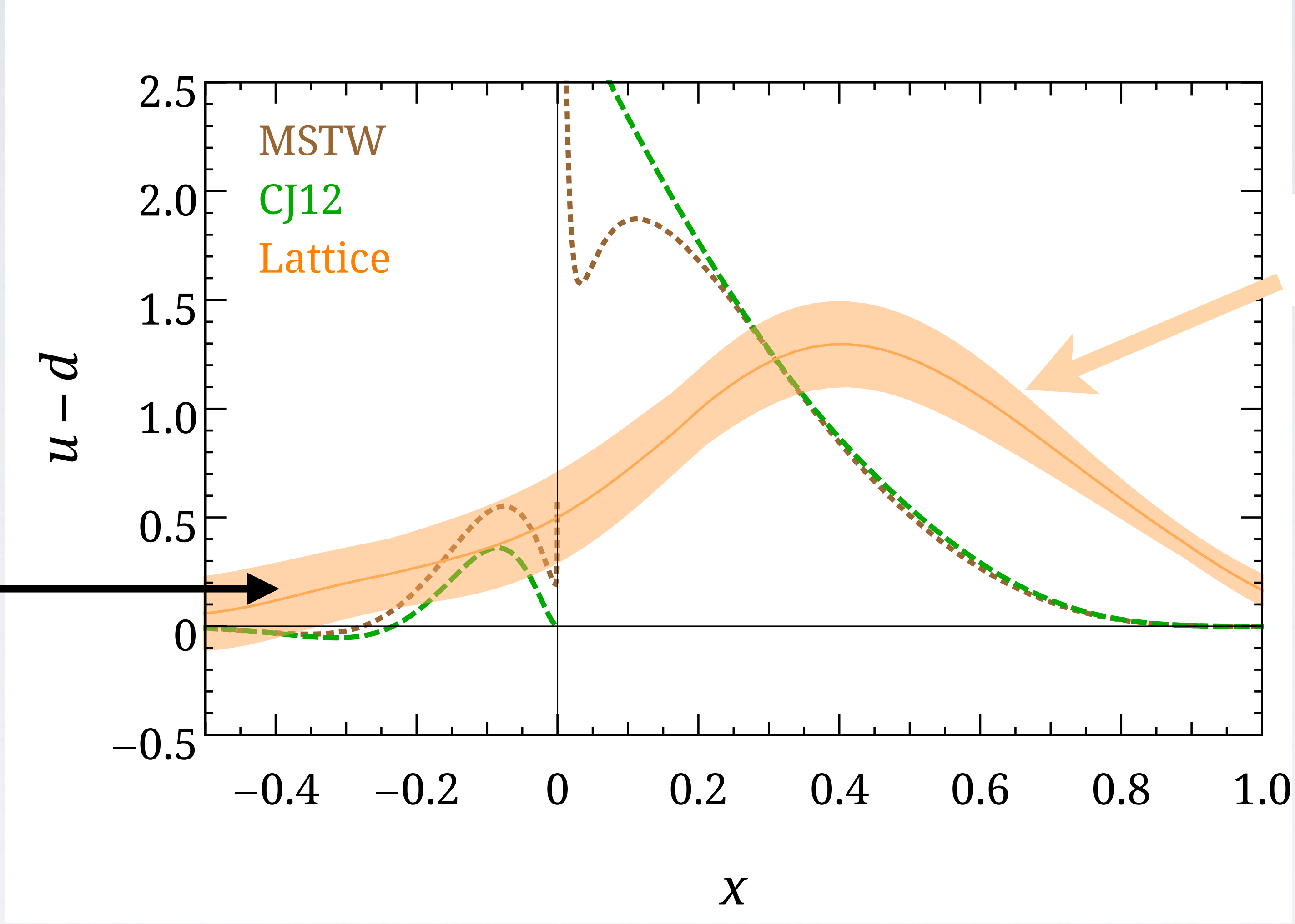
extending $\int_{-1}^{1} dy$

+

$\overline{q}(x) = - q(-x)$

antiquark

$\overline{d}(x) > \overline{u}(x)$



quasi-PDF $\tilde{f}_1$

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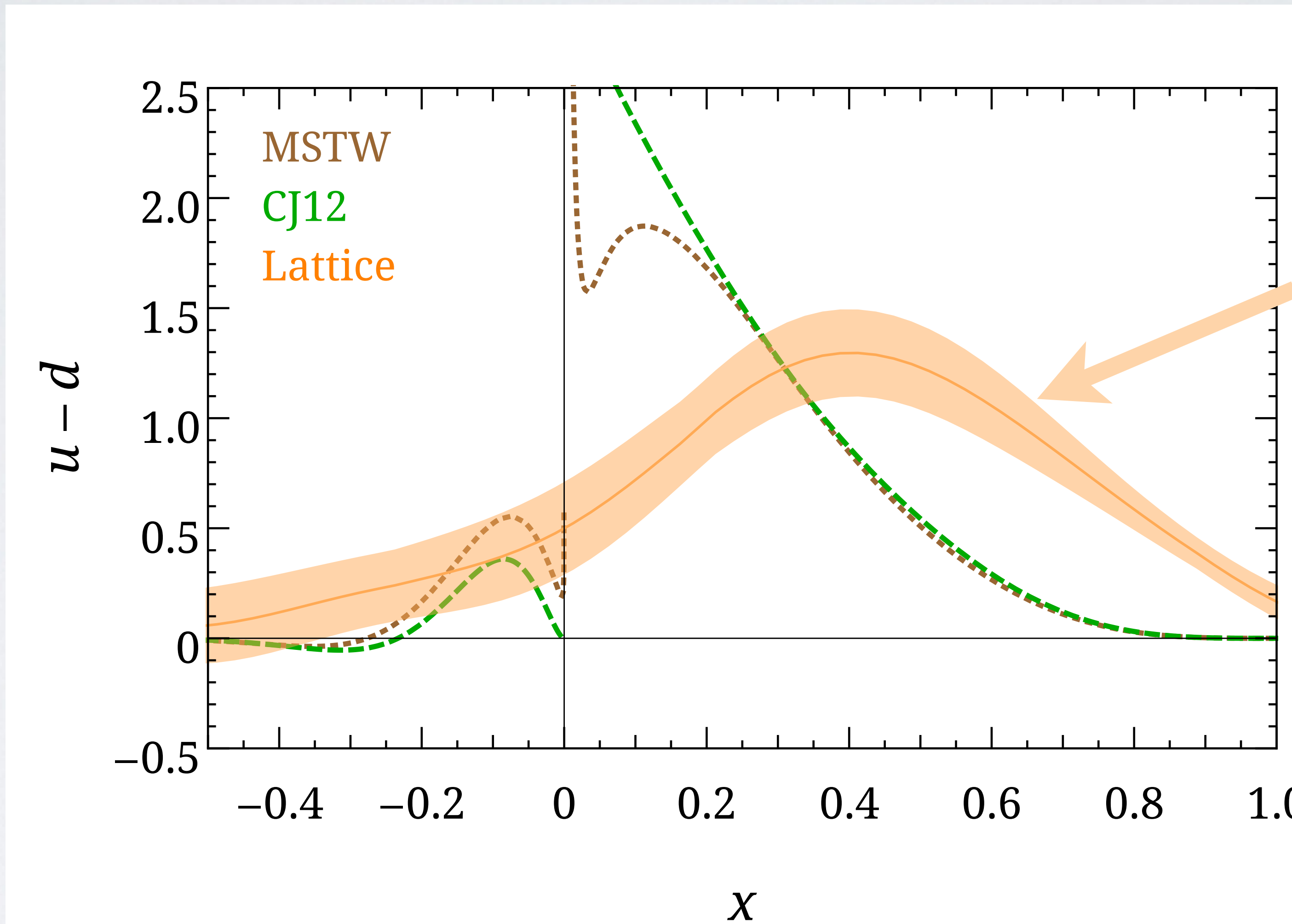

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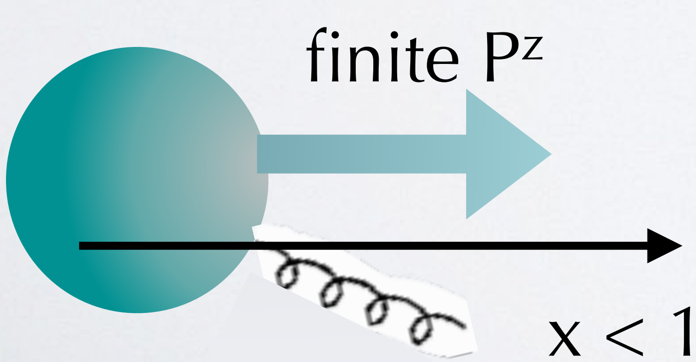
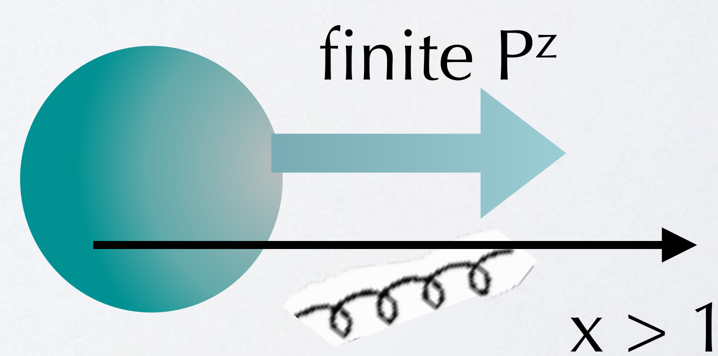
wrong support
$x > 1$



but also



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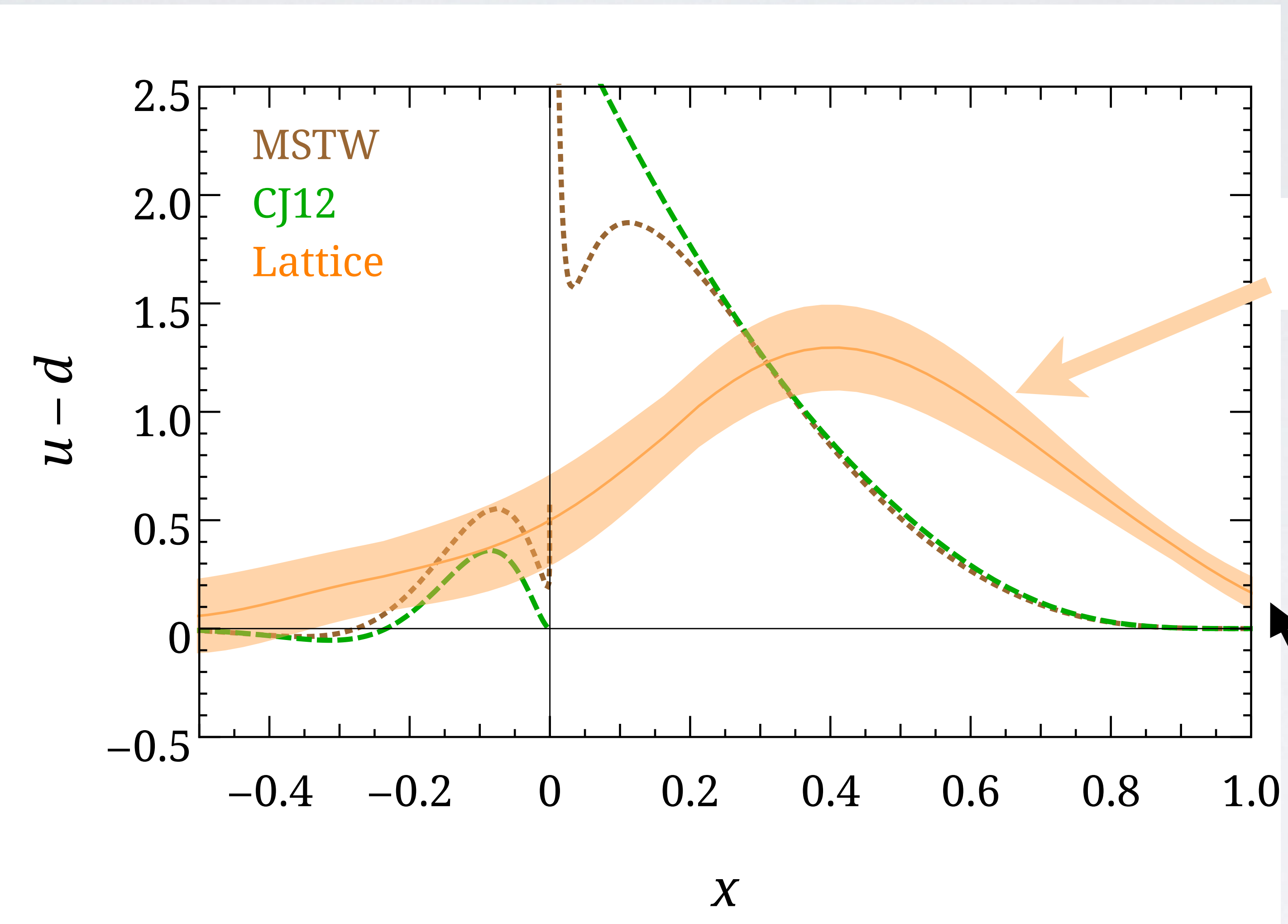
lattice 24^3 x 64
a_L~0.12 fm
m_π~310 MeV
μ=2 GeV

$P^z \sim M$
nucleon
mass



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quasi-PDF on lattice

present lattice calculations of quasi-PDF at **$P^z \sim M$**

$$\tilde{f}_1(x, \mu^2, P^z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}, \frac{1}{a_L P^z}\right) f_1(y, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{(P^z)^2}, \frac{M^2}{(P^z)^2}\right)$$

problem

quasi-PDF on lattice

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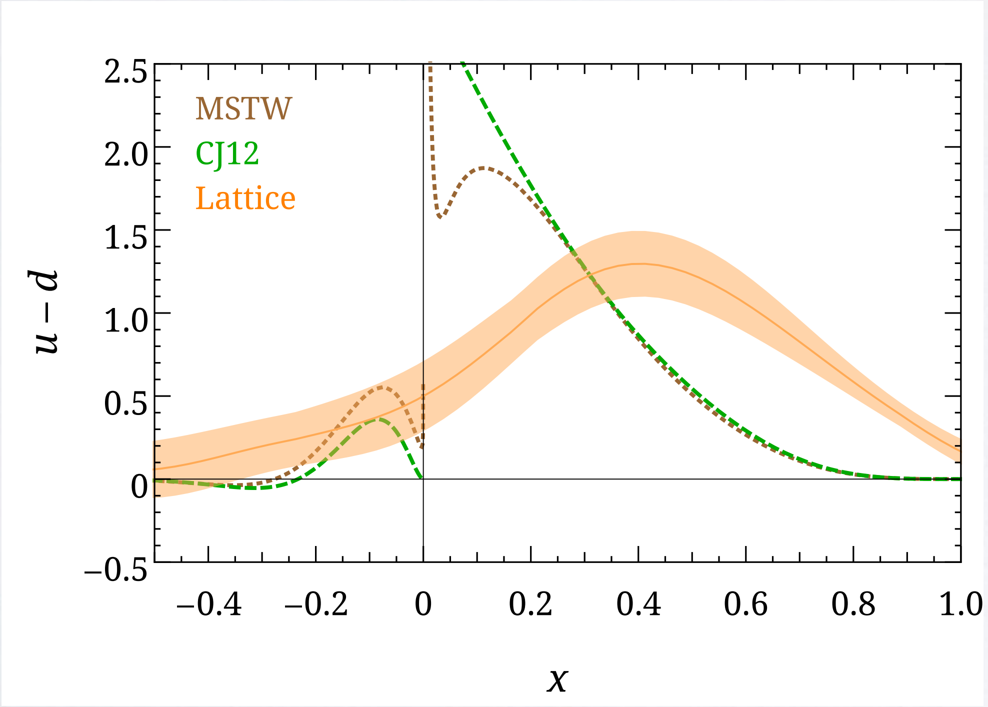
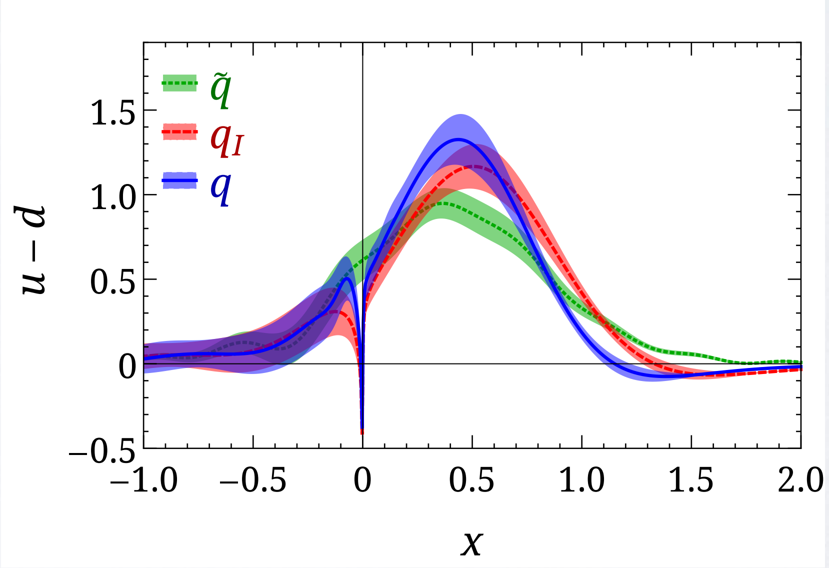
problem

recent attempts to compute power corrections at finite P^z

*Lin et al., P.R.D**91** (15) 054510*



*Chen et al., N.P.**B911** (16) 246*



final

P^z = 1.26 GeV

quasi-PDF on lattice

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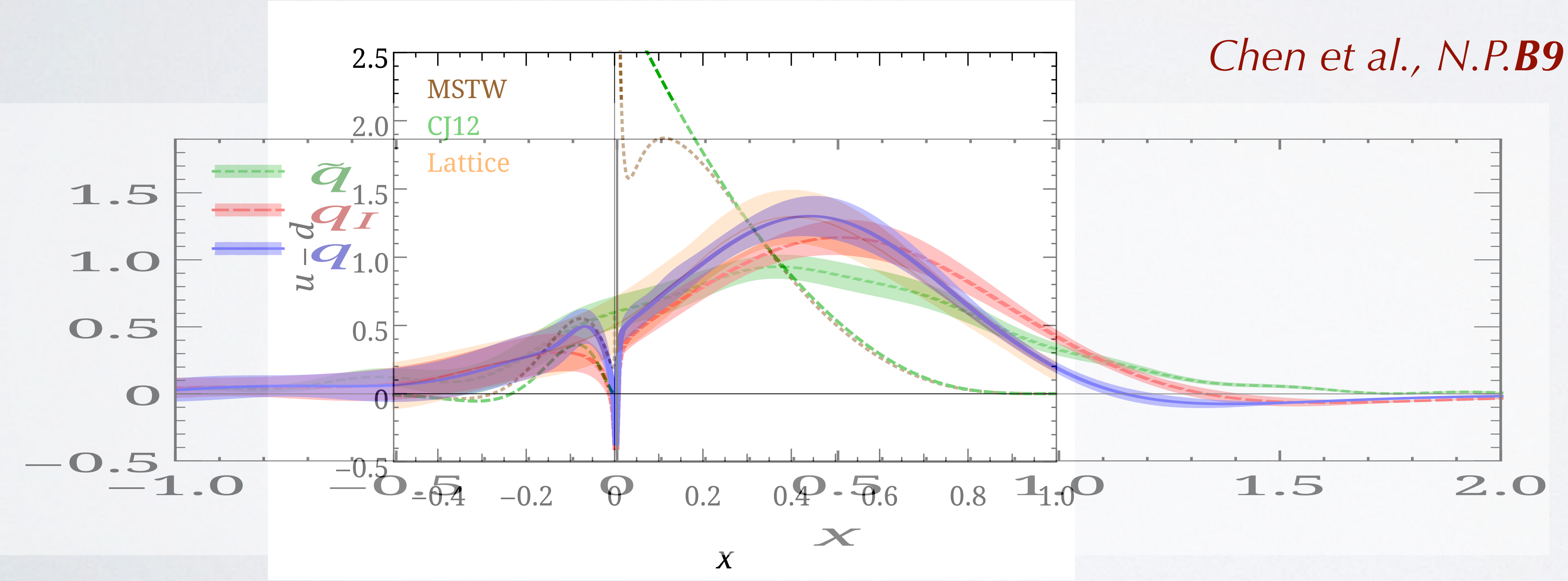


final

P^z = 1.26 GeV

quasi-PDF in models

factorization theorem (at some scale μ)

data = PDF ⊗ (something perturbatively calculable)

within $\mathcal{L}_{QCD}$

PDF

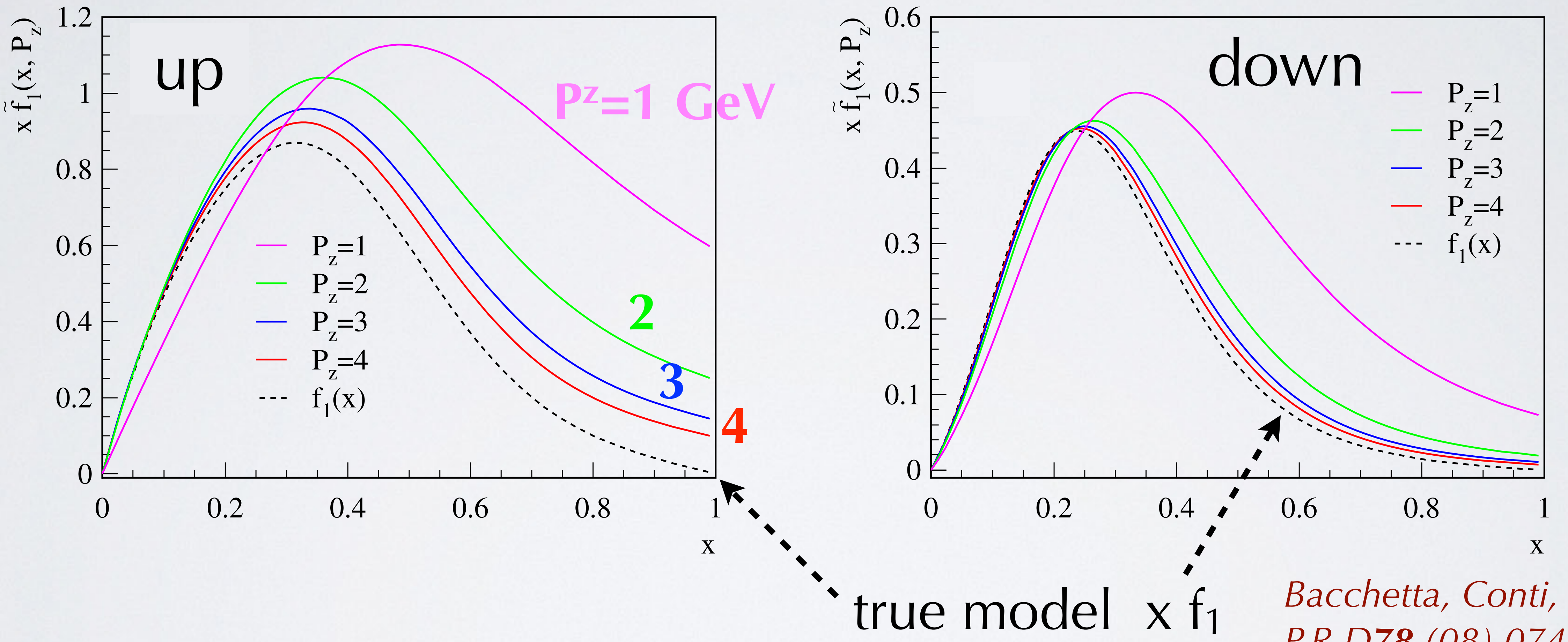
lattice

calculate them in QCD-inspired models

quasi-PDF ≈ PDF ?

(spectator-diquark) model calculations of quasi-PDF $x\,\tilde{f}_1$

*Gamberg et al., P.L.**B743** (15) 112*

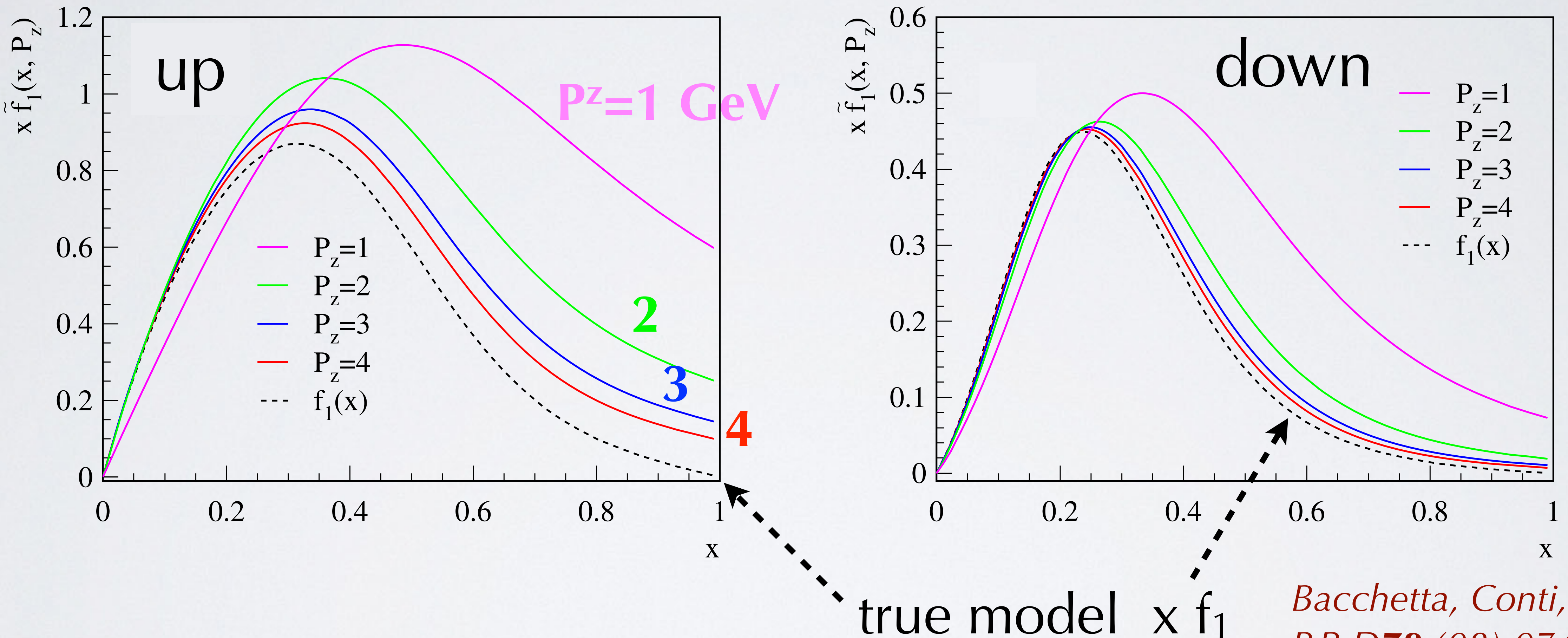


*Bacchetta, Conti, Radici, P.R.D**78** (08) 074010*

quasi-PDF $\approx$ PDF ?

(spectator-diquark) model calculations of $x\,\tilde{f}_1$

*Gamberg et al., P.L.**B743** (15) 112*



*Bacchetta, Conti, Radici, P.R.D**78** (08) 074010*

quasiPDF ≈ PDF for x ≲ 0.2 only if $P^z \sim (4\div5)$ M

quasi-PDF in spectator-diquark model

analytic calculation of quasi-TMD

*Gamberg et al., P.L.**B743** (15) 112*

$$\begin{cases} \tilde{f}_1^{D=s,a,a'}(x,\mathbf{p}_T,P^z) \\ \tilde{g}_1^{D=s,a,a'}(x,\mathbf{p}_T,P^z) \end{cases} \Rightarrow \begin{cases} \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) \\ \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) \end{cases}$$

verify that TMD are recovered

$$\begin{cases} \lim\limits_{P^z\to\infty} \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) = f_1^{u,d}(x,\mathbf{p}_T) \\ \lim\limits_{P^z\to\infty} \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) = g_1^{u,d}(x,\mathbf{p}_T) \end{cases}$$

from *Bacchetta, Conti, Radici, P.R.D**78** (08) 074010*

same for quasi-PDF

$$\begin{cases} \int d\mathbf{p}_T\, \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) = \tilde{f}_1^{u,d}(x,P^z) \\ \int d\mathbf{p}_T\, \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) = \tilde{g}_1^{u,d}(x,P^z) \end{cases} \xrightarrow{P^z\to\infty} \begin{matrix} f_1^{u,d}(x) \\ g_1^{u,d}(x) \end{matrix}$$

analytic expressions in Appendix of

Bacchetta et al., arXiv:1608.07638

quasi-PDF in spectator-diquark model

analytic calculation of quasi-TMD

*Gamberg et al., P.L.**B743** (15) 112*

$$\begin{cases} \tilde{f}_1^{D=s,a,a'}(x,\mathbf{p}_T,P^z) \\ \tilde{g}_1^{D=s,a,a'}(x,\mathbf{p}_T,P^z) \end{cases} \Rightarrow \begin{cases} \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) \\ \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) \end{cases}$$

contain terms $\sim (1-x)^2\,(P^z)^2$

verify that TMD are recovered

$$\begin{cases} \lim\limits_{P^z\to\infty} \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) = f_1^{u,d}(x,\mathbf{p}_T) \\ \lim\limits_{P^z\to\infty} \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) = g_1^{u,d}(x,\mathbf{p}_T) \end{cases}$$

large P^z expansion
critical if $x\to 1$

from *Bacchetta, Conti, Radici, P.R.D**78** (08) 074010*

same for quasi-PDF

$$\begin{cases} \int d\mathbf{p}_T\, \tilde{f}_1^{u,d}(x,\mathbf{p}_T,P^z) = \tilde{f}_1^{u,d}(x,P^z) \\ \int d\mathbf{p}_T\, \tilde{g}_1^{u,d}(x,\mathbf{p}_T,P^z) = \tilde{g}_1^{u,d}(x,P^z) \end{cases} \xrightarrow{P^z\to\infty} \begin{matrix} f_1^{u,d}(x) \\ g_1^{u,d}(x) \end{matrix}$$

analytic expressions in Appendix of

Bacchetta et al., arXiv:1608.07638

our matching procedure

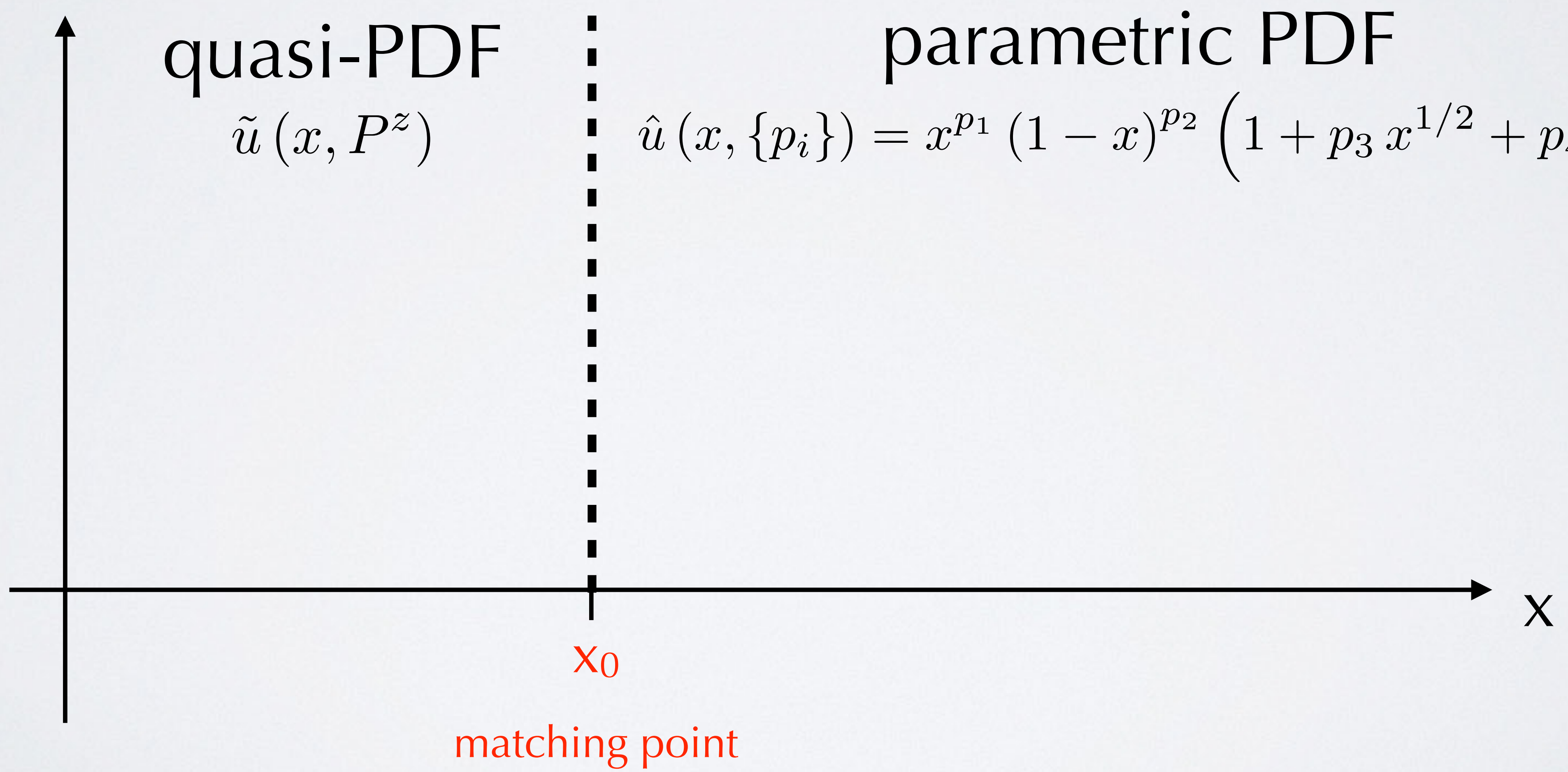
Bacchetta et al., arXiv:1608.07638

<u>Definition</u> for $u \equiv f_1^u$ (similarly for f_1^d and $g_1^{u,d}$)

reconstructed PDF
$$\mathring{u}(x, P^z) = \begin{cases} \tilde{u}(x, P^z) & 0 \le x \le x_0 \\ \hat{u}\,(x; \{p_i\}) & x_0 < x \le 1 \end{cases}$$

quasi-PDF
$$\tilde{u}\,(x, P^z)$$

parametric PDF
$$\hat{u}\,(x, \{p_i\}) = x^{p_1}\,(1-x)^{p_2}\left(1 + p_3\,x^{1/2} + p_4\,x + p_5\,x^{3/2}\right)$$



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<u>constraint #1</u>

quasi-PDF

$$\tilde{u}\left(x, P^z\right)$$

parametric PDF

$$\hat{u}\left(x, \{p_i\}\right) = x^{p_1}\left(1-x\right)^{p_2}\left(1 + p_3\, x^{1/2} + p_4\, x + p_5\, x^{3/2}\right)$$

matching

$$\tilde{u}\left(x_0, P^z\right) = \hat{u}\left(x_0; \{p_i\}\right)$$

$$\frac{d}{dx}\tilde{u}\left(x, P^z\right)\Big|_{x=x_0} = \frac{d}{dx}\hat{u}\left(x; \{p_i\}\right)\Big|_{x=x_0}$$

only 3 parameters
$\{p_1, p_2, p_5\}$

x

x_0

matching point

our matching procedure

Bacchetta et al., arXiv:1608.07638

<u>Definitions</u> for $u \equiv f_1^u$ (similarly for f_1^d and $g_1^{u,d}$)

(truncated) Mellin n-moment

$$u^n = \int_0^1 dx\, x^{n-1}\, u(x)$$

$$\tilde{u}^n(P^z) = \int_0^{x_0} dx\, x^{n-1}\, \tilde{u}(x, P^z)$$

$$\hat{u}^n\left(\{p_1, p_2, p_5\}\right) = \int_{x_0}^1 dx\, x^{n-1}\, \hat{u}\left(\{p_1, p_2, p_5\}\right)$$

our matching procedure

Bacchetta et al.,
arXiv:1608.07638

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constraint #2

fix parameters $\{p_1, p_2, p_5\}$ by minimizing squared distance χ^2 for n=2,3,4

$$\chi^2\left(\{p_1, p_2, p_5\}\right) = \sum_{n=2}^{4} \frac{\left[\hat{u}^n\left(\{p_1, p_2, p_5\}\right) + \tilde{u}^n(P^z) - u^n\right]^2}{\left[\tilde{u}^n(P^z) - u^n\right]^2}$$

our matching procedure

Bacchetta et al., arXiv:1608.07638

constraints #1 (matching at x_0) and #2 (χ^2_{min})
are valid at any scale μ^2

In principle, each step is possible on lattice.
At present, it's not.

Proof of concept: use spectator-diquark model for PDF and quasi-PDF to test the method

explore arbitrary choices of P^z, x_0

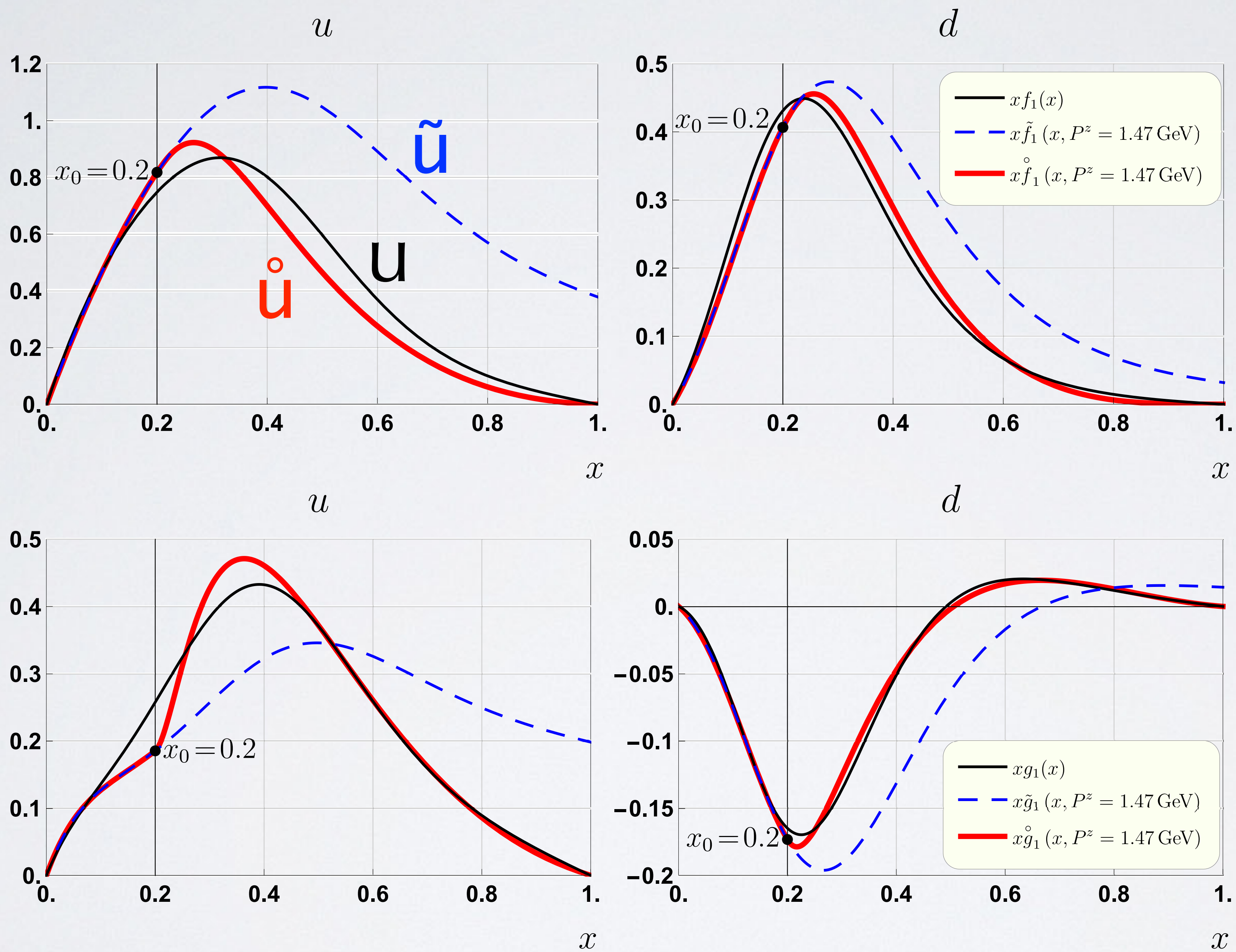
results (at model scale Q_0^2=0.3 GeV^2)

Bacchetta et al., arXiv:1608.07638

matching $x_0 = 0.2$; $P^z = 1.47$ GeV

f_1

g_1



true u

quasi ũ

reconstructed ů

results (at model scale Q_0^2=0.3 GeV^2)

Bacchetta et al., arXiv:1608.07638

matching $x_0 = 0.2$; $P^z = 1.47$ GeV

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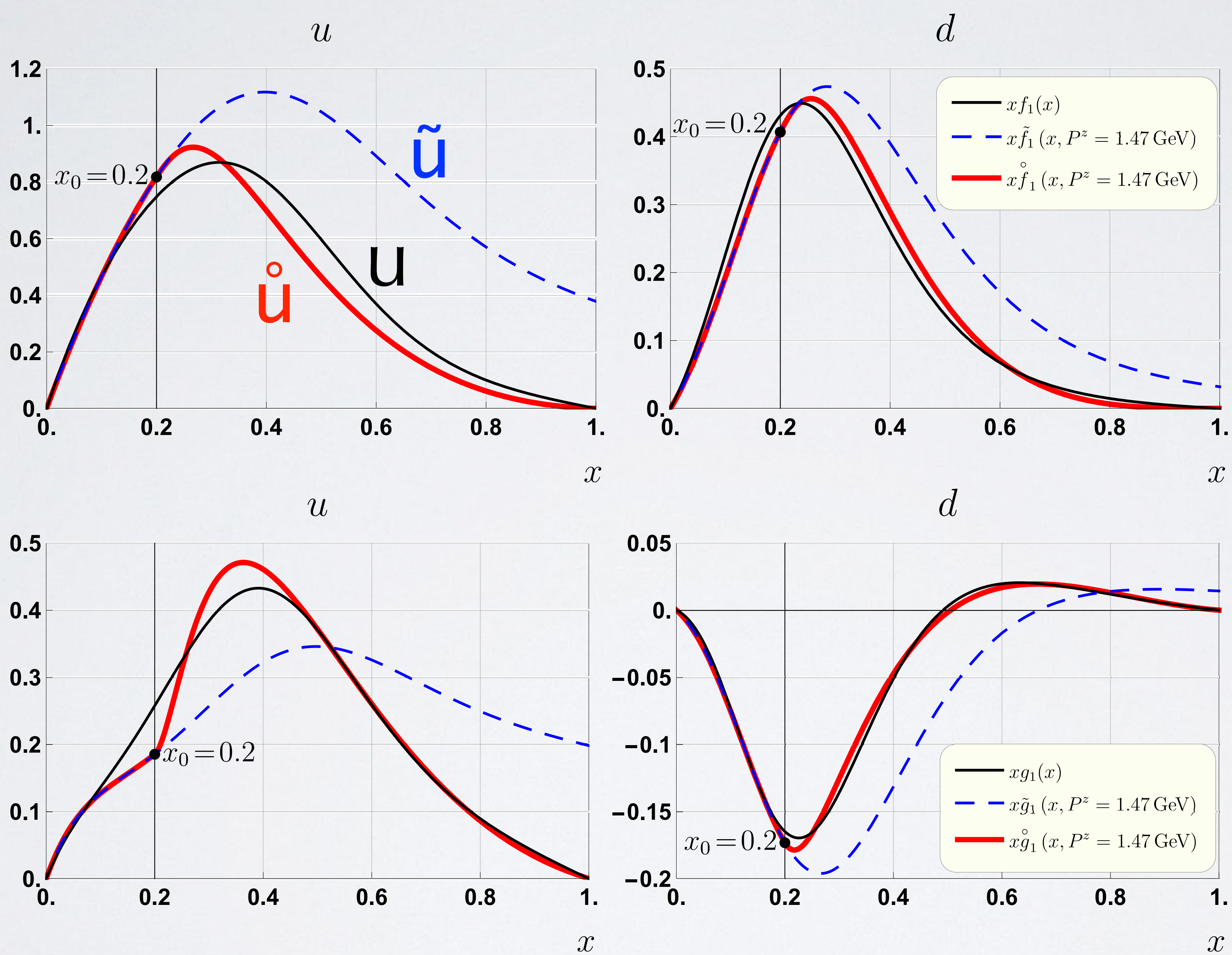


true u

quasi ũ

reconstructed ů

ů much better than ũ

but still problems for f_1^u & g_1^u

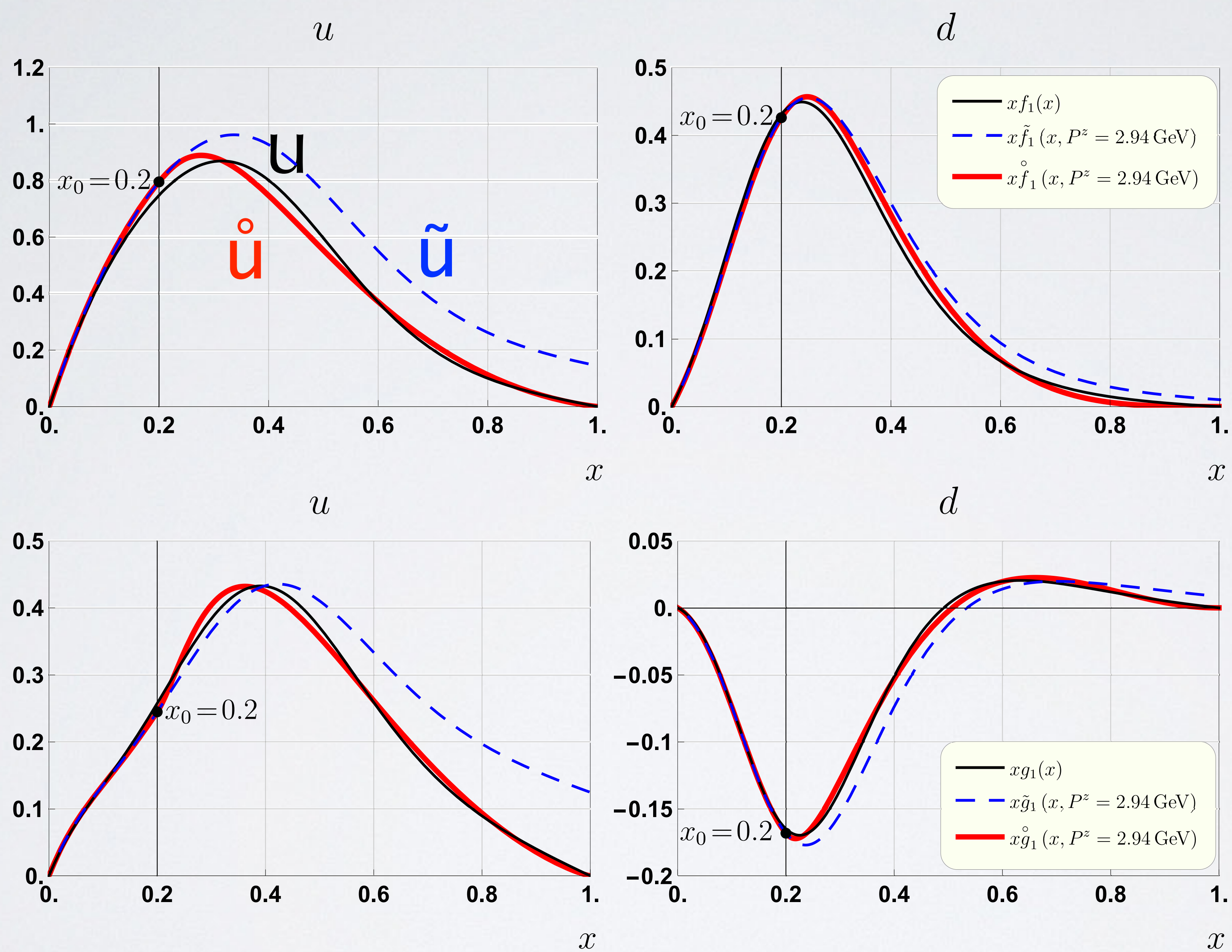
results (at model scale $Q_0^2=0.3\ \mathrm{GeV}^2$)

Bacchetta et al., arXiv:1608.07638

matching $x_0 = 0.2$; $P^z = 2.94$ GeV

f_1

g_1



true u

quasi ũ

reconstructed ů

increasing P^z beneficial for ũ

still ů is better

quantitative comparison

matching $x_0 = 0.2$

<u>Definitions</u> for $u \equiv f_1^u$ (similarly for f_1^d and $g_1^{u,d}$)

relative distance with respect to true u

reconstructed ů $\mathring{r}[\mathring{u}] = \frac{\int_0^1 dx\, [\mathring{u}(x,P^z) - u(x)]^2}{\int_0^1 dx\, u(x)^2}$

quasi ũ $\tilde{r}[\tilde{u}] = \frac{\int_0^1 dx\, [\tilde{u}(x,P^z) - u(x)]^2}{\int_0^1 dx\, u(x)^2}$

quantitative comparison

matching $x_0 = 0.2$

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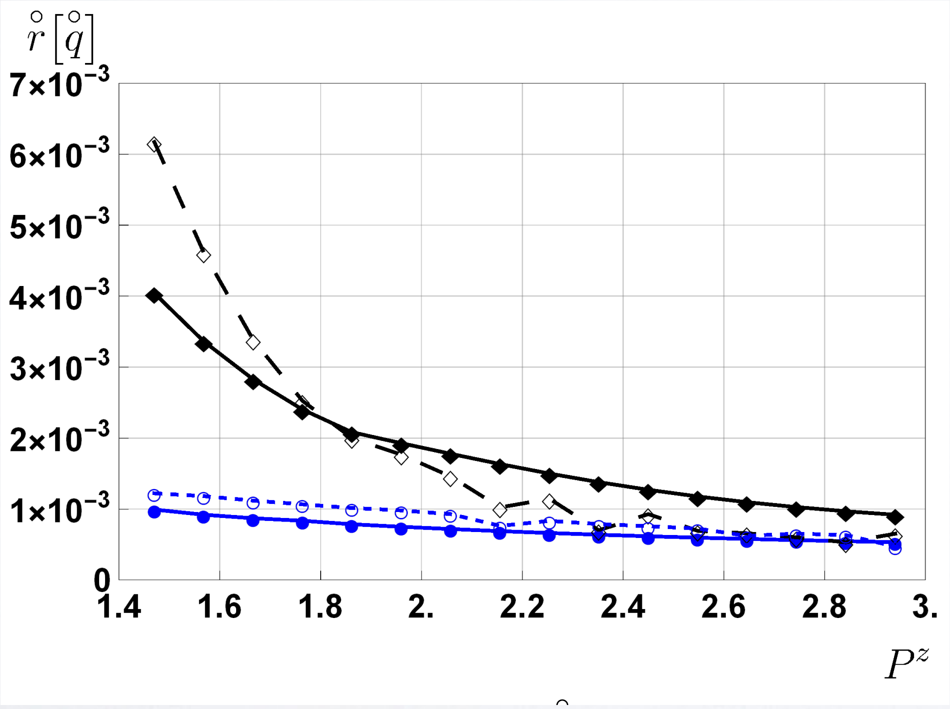
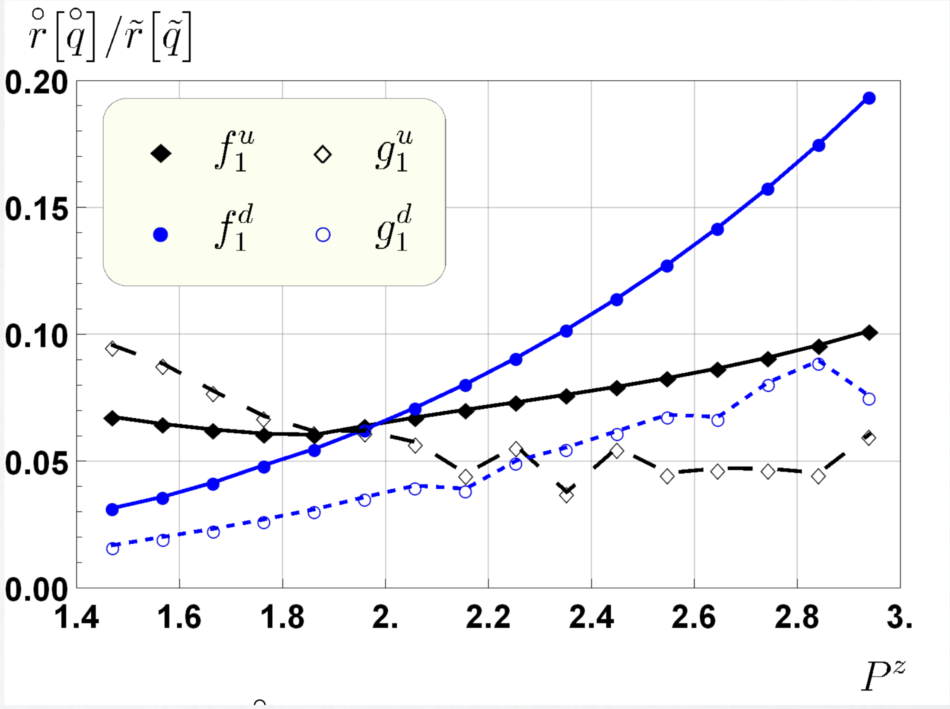
quasi $\tilde{u}$ $$\tilde{r}[\tilde{u}] = \frac{\int_0^1 dx\, [\tilde{u}(x,P^z) - u(x)]^2}{\int_0^1 dx\, u(x)^2}$$





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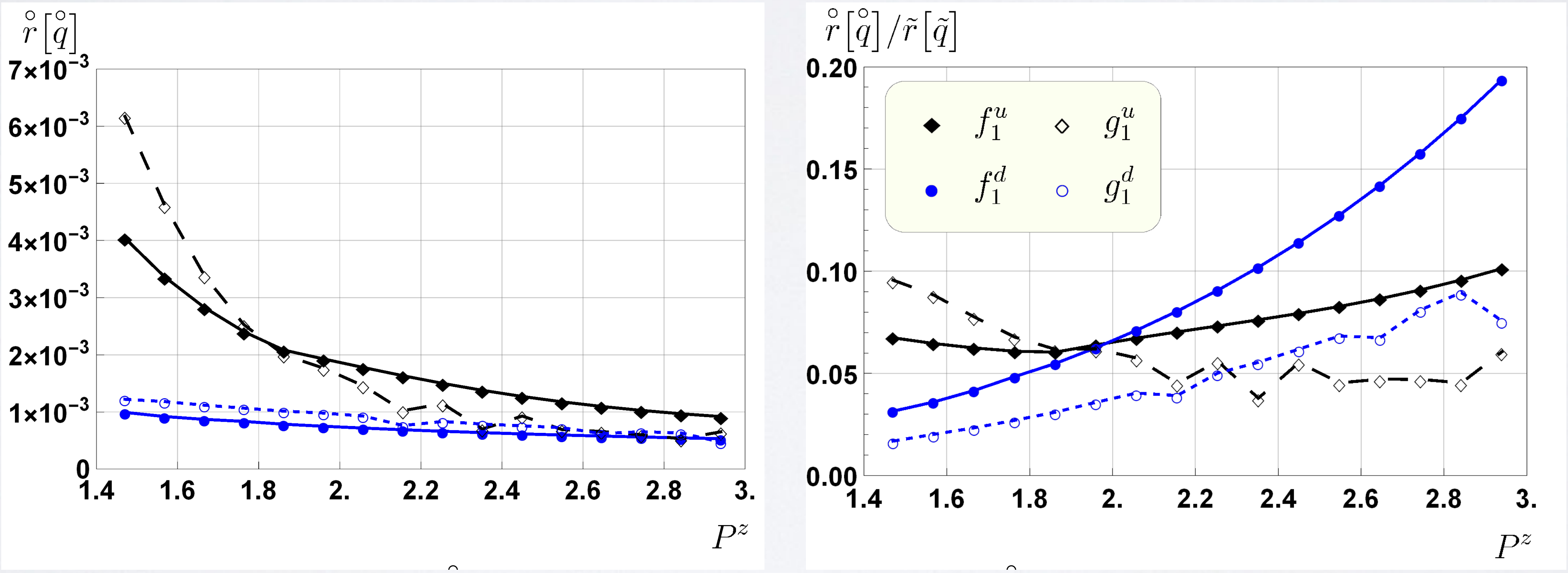

Bacchetta et al., arXiv:1608.07638

on average, ů↔u closer than ũ↔u
by a factor 10 already at P^z ~ 1.5 M

stability check

check for $u \equiv f_1^u$ (similarly for f_1^d and $g_1^{u,d}$)

1. shift by δ the matching conditions of quasi-PDF

$$\tilde{u}(x_0, P^z) \to (1+\delta)\tilde{u}(x_0, P^z)$$

$$\frac{d}{dx}\tilde{u}(x, P^z)\Big|_{x=x_0} \to (1+\delta)\frac{d}{dx}\tilde{u}(x, P^z)\Big|_{x=x_0}$$

2. shift by δ the distance of its Mellin moments

$$\tilde{u}^n(P^z) - u^n \to (1+\delta)\left(\tilde{u}^n(P^z) - u^n\right)$$

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$$\tilde{u}^n(P^z) - u^n \to (1+\delta)\left(\tilde{u}^n(P^z) - u^n\right)$$

3. minimize χ^2 → get new {p′₁, p′₂, p′₅} → new $\hat{u}(x, \delta) = \hat{u}(x, \{p_1', p_2', p_5'\})$

unperturbed $\hat{u}(x, 0) = \hat{u}(x, \{p_1, p_2, p_5\})$

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unperturbed $\hat{u}(x, 0) = \hat{u}(x, \{p_1, p_2, p_5\})$

4. define relative distance

$$\mathrm{r}(\delta) = \frac{\int_{x_0}^1 dx\, [\hat{u}(x,\delta) - \hat{u}(x,0)]^2}{\int_{x_0}^1 dx\, [\hat{u}(x,0)]^2}$$

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4. define relative distance

$|\delta| \leq 0.1 \;\to\; r(\delta) \lesssim 0.01$

$$\mathrm{r}(\delta) = \frac{\int_{x_0}^1 dx\, \left[\hat{u}(x,\delta) - \hat{u}(x,0)\right]^2}{\int_{x_0}^1 dx\, \left[\hat{u}(x,0)\right]^2}$$

change of input by 10% → change of reconstructed û ≲1%

Conclusions

- reconstruct PDF over $x \in [0,1]$ by matching at point x_0
  quasi-PDF for $0 \leq x \leq x_0$ with
  parametric form fitted to n=2,3,4 Mellin moments of PDF for $x_0 < x \leq 1$

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when lattice will get (good) quasi-PDF at $P^z \sim 2$ GeV,
our method can reconstruct PDF 10 times better than quasi-PDF

backup slides

caveat

why neglect the n = 1 (truncated) Mellin moment ?

because lattice calculation of quasi-PDF
not reliable at small x

lattice can simulate the minimum x as

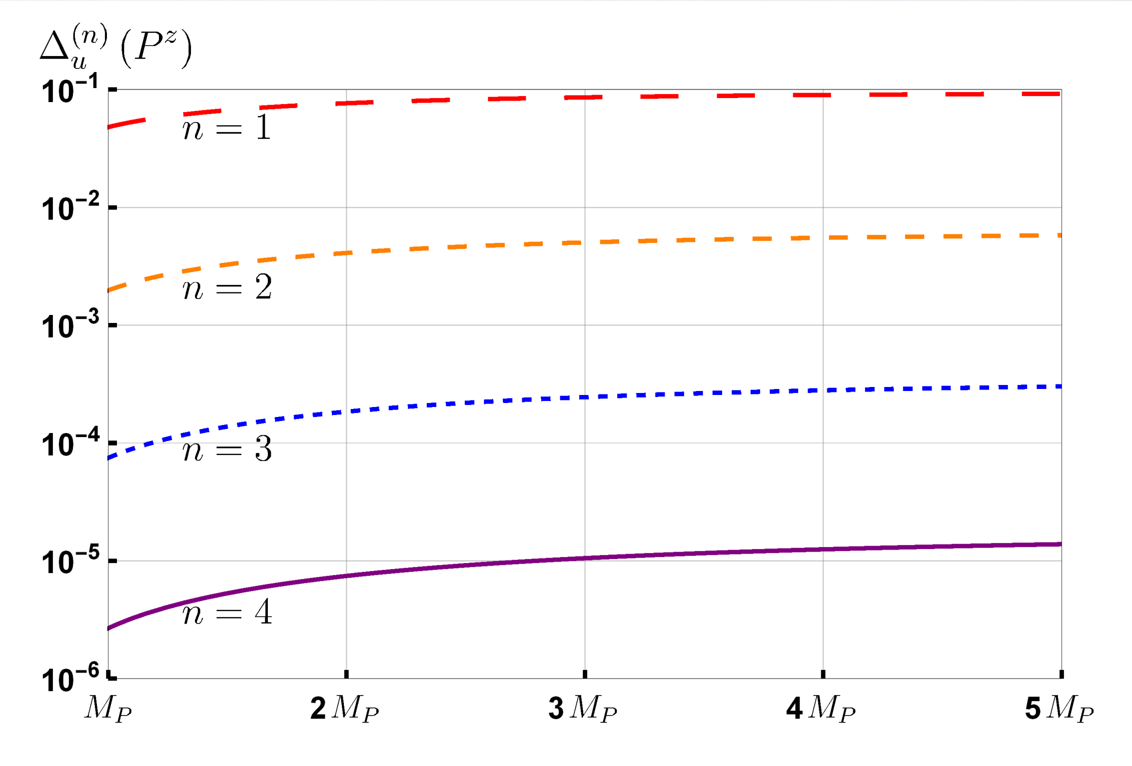
$$x_{\min} = \frac{P^z_{\min}}{P^z_{\max}} = \frac{[a_L L_z]^{-1}}{a_L^{-1}} = \frac{1}{L_z} = \frac{1}{32} \sim 0.03$$

32^3 x 64

*Alexandrou et al. (ETMC), P.R.D**92** (15) 014502*

$$\Delta_u^{(n)}(P^z) = \frac{\int_0^{x_{\min}} dx\, x^{n-1} \tilde{u}(x, P^z)}{\int_0^1 dx\, x^{n-1} \tilde{u}(x, P^z)}$$

% importance of "neglected" moment



n = 1 as large as 10%
n ≥ 2 irrelevant

Bacchetta et al., arXiv:1608.07638