Double parton scattering: evolution and matching

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In collaboration with Markus Diehl and Tomas Kasemets

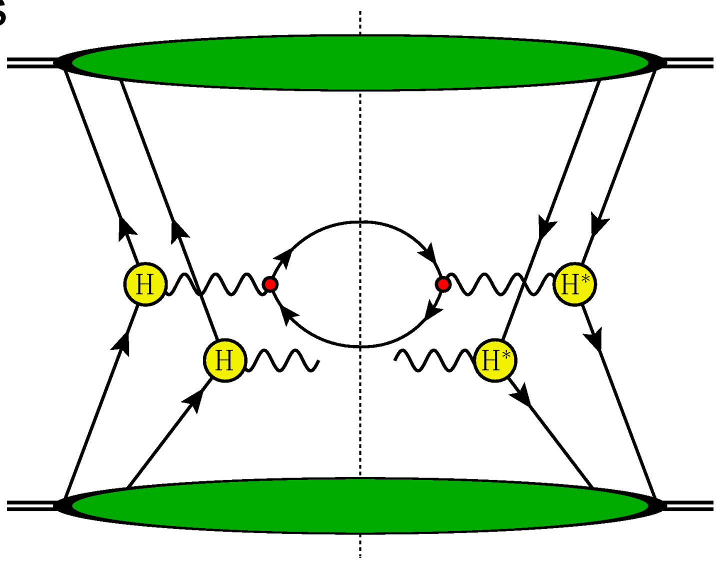
Workshop on Resummation, Evolution, Factorization
November 8, 2016

Content - outline

- Brief motivation
- Introduction
- Soft factor for DPDFs/DTMDs
- Evolution equations
  - Writing them down for DTMDs
  - Solving them
- Matching: cross section contributions for large **y**
- Conclusions

Our goals

- Factorization: stick to singlets in final states
  - Double Drell-Yan
  - Higgs + W/Z



- For perturbative $q_T \rightarrow$ significant predictive results
  - Short distance expansion

- Motivation and goals
  - Get a handle on soft factors
  - Write down evolution equations
  - Solve evolution equations
  - Matching equations for DPDFs/DTMDs

Diehl, Ostermeier, Schäfer, JHEP 1203 (2012) 089; MGAB, Diehl, Kasemets, a forthcoming paper

Introduction: DPS

- Hadronic interactions of the type

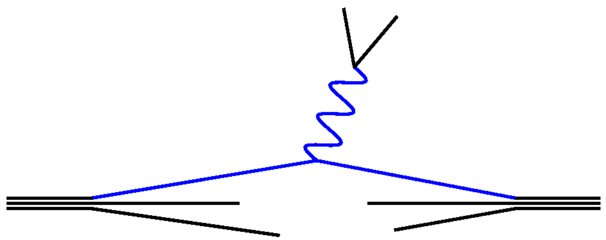
$$pp \to Y + X$$

- *Y* is produced in scattering and *X* has to be summed over

$$\sum_X \sigma\left(pp \to Y + X\right)$$

since factorization formulae are for inclusive cross sections.

- This *X* cannot be ignored if it is of high-energy itself.



Introduction: DPS

- Example: $pp \to H + Z \to b\bar{b} + Z$

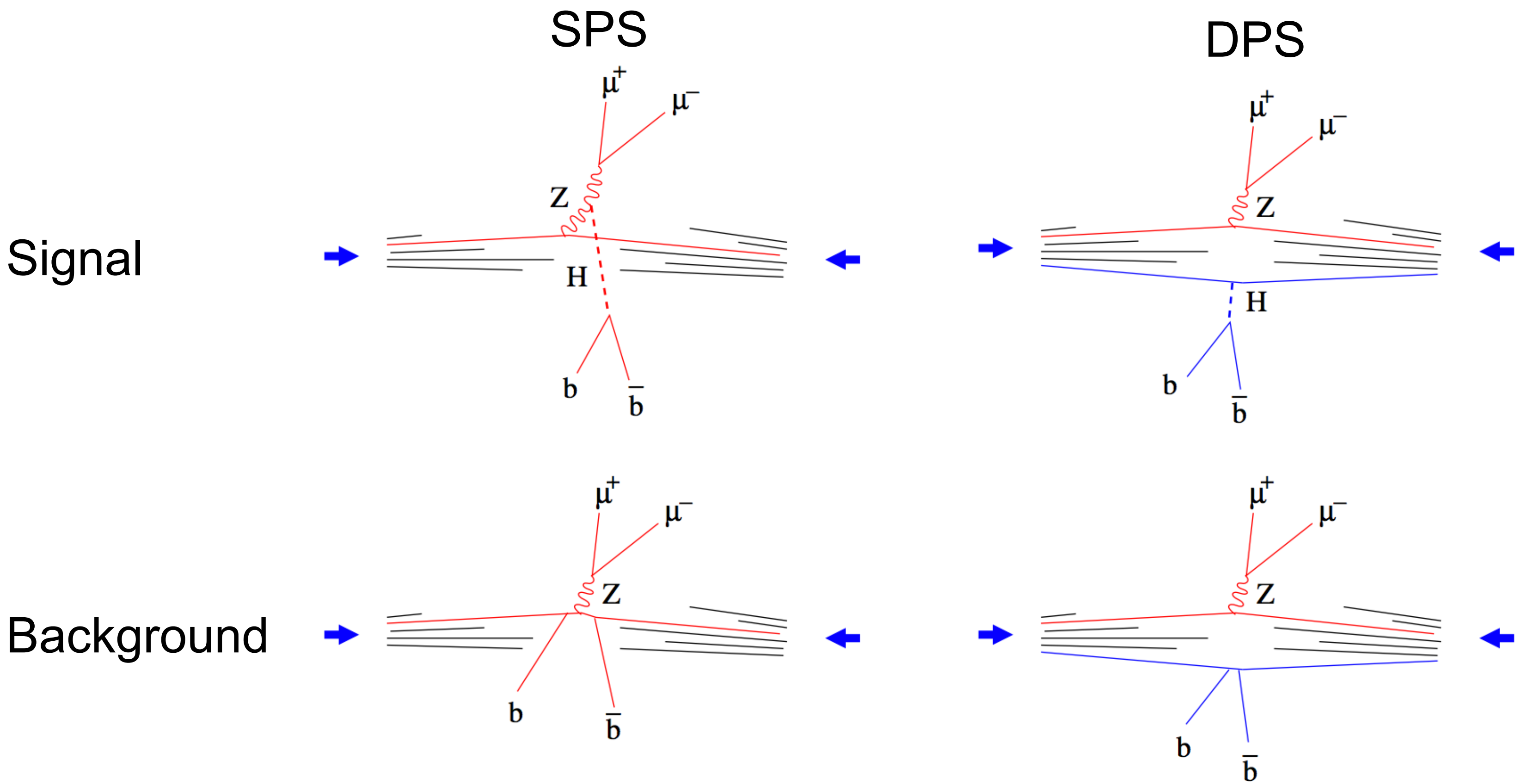


Figure: Diehl, talk QCD evolution workshop 2014

Del Fabbro, Treleani, PRD 61 (2000) 077502

Introduction: like sign W-pairs

- Consider like sign W-pair production
- Small cross section but clean
- Single scattering:
  $$qq \to qq + W^+W^+$$
  suppressed by α_s^2

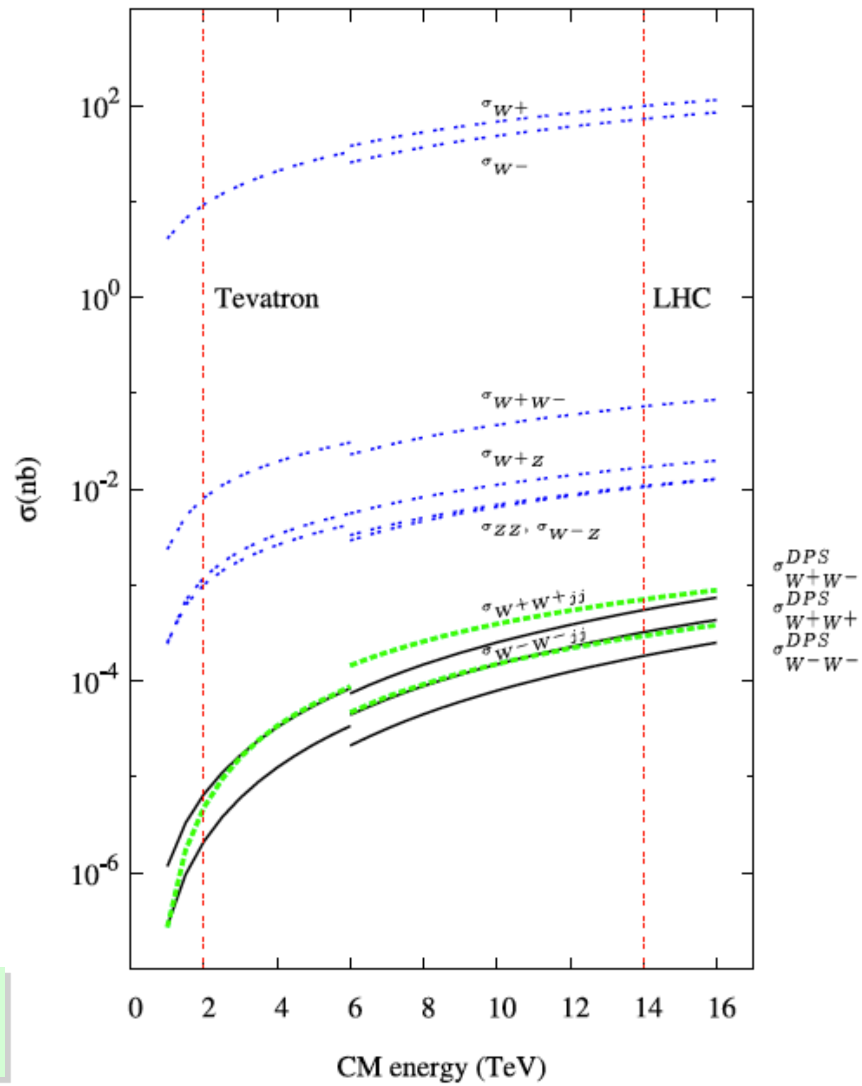


Figure: Gaunt, Kom, Kulesza, Stirling, Eur. Phys. J. C 69 (2010) 53

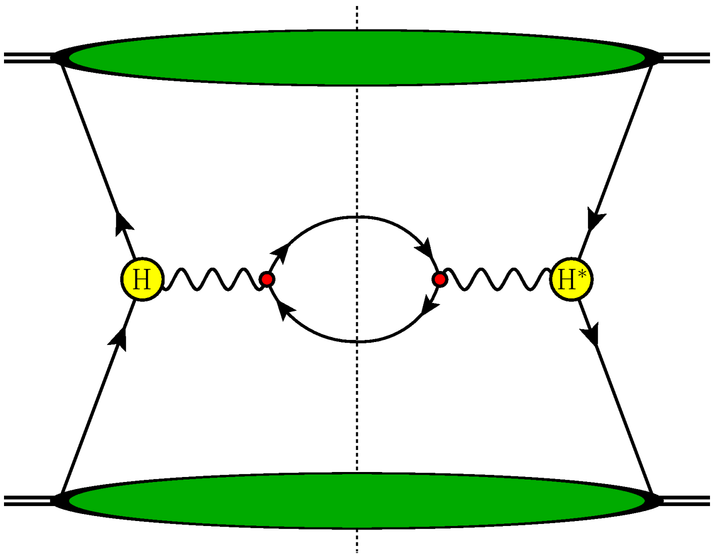
SPS: Drell-Yan

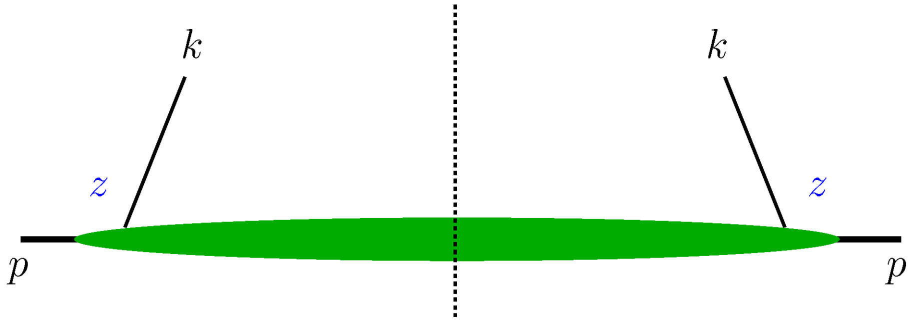
- Matching equation

$$F(x,\mathbf{z};\mu,\zeta) = C(x',\mathbf{z};\mu,\mu^2) \underset{x}{\otimes} F(x';\mu,\zeta)$$

with convolution defined as

$$C(x') \underset{x}{\otimes} F(x') = \int_x^1 \frac{dx'}{x'} C(x') F\left(\frac{x}{x'}\right)$$

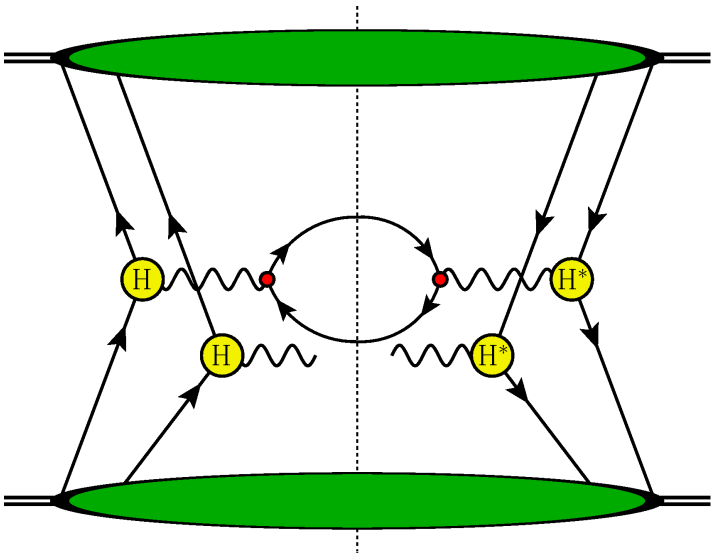




Collins, Soper, NPB 194 (1982) 445; Collins, Soper, NPB 193 (1981) 381 [B213 (1983) 545(E)]; Collins, Soper, Sterman, NPB 250 (1985) 199; Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042

Short-distance expansion: DPS

- Differences compared to TMDs
  - Two hard processes involved
  - Two coefficient functions per DTMD
  - Positions $\mathbf{z}_1$ and $\mathbf{z}_2$ (compare with $\mathbf{b}_T$ for the TMD case)
  - Additional distance $\mathbf{y}$



- Consider the limit
  - $|\mathbf{z}_1|, |\mathbf{z}_2|$ much smaller than $1/\Lambda$
  - $|\mathbf{z}_1|, |\mathbf{z}_2| \lll \mathbf{y}$, with $\mathbf{y}$ fixed

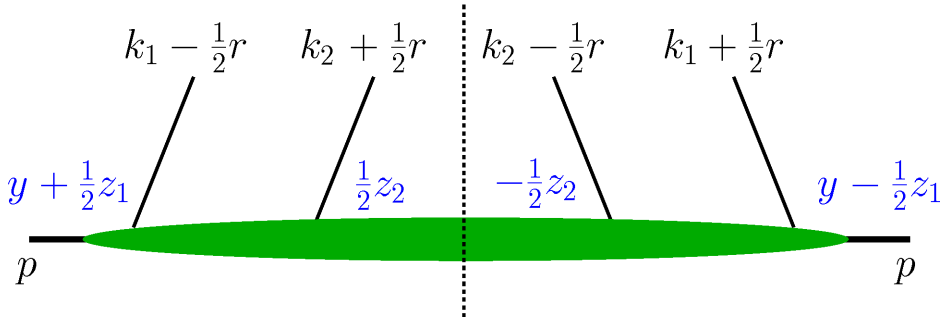


Figure: modified from Diehl, Ostermeier, Schäfer, JHEP03 (2012) 089

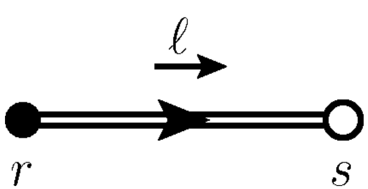
Wilson lines

- Wilson lines are path ordered exponentials

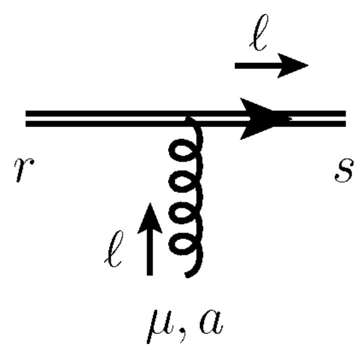
$$W(\mathbf{z}, v) = \mathcal{P} \exp \left[-igt^a \int_{-\infty}^{0} d\lambda v A^a(z + \lambda v) \right]^{z^+ = z^- = 0}$$

- *v* is a vector associated with the rapidity
  - Taken away from the light-cone to avoid rapidity divergences

- In Feynman diagram language Wilson lines are represented by Wilson line propagators and vertices, e.g. amplitude part (for quarks)



$$\frac{i\delta_{rs}}{\ell v + i0}$$



$$-igv^\mu (t_a)_{sr}$$

Wilson lines

- Consider double colorless-final state production

- Wilson line structure from factorization formula.

- Every double-line is a Wilson line

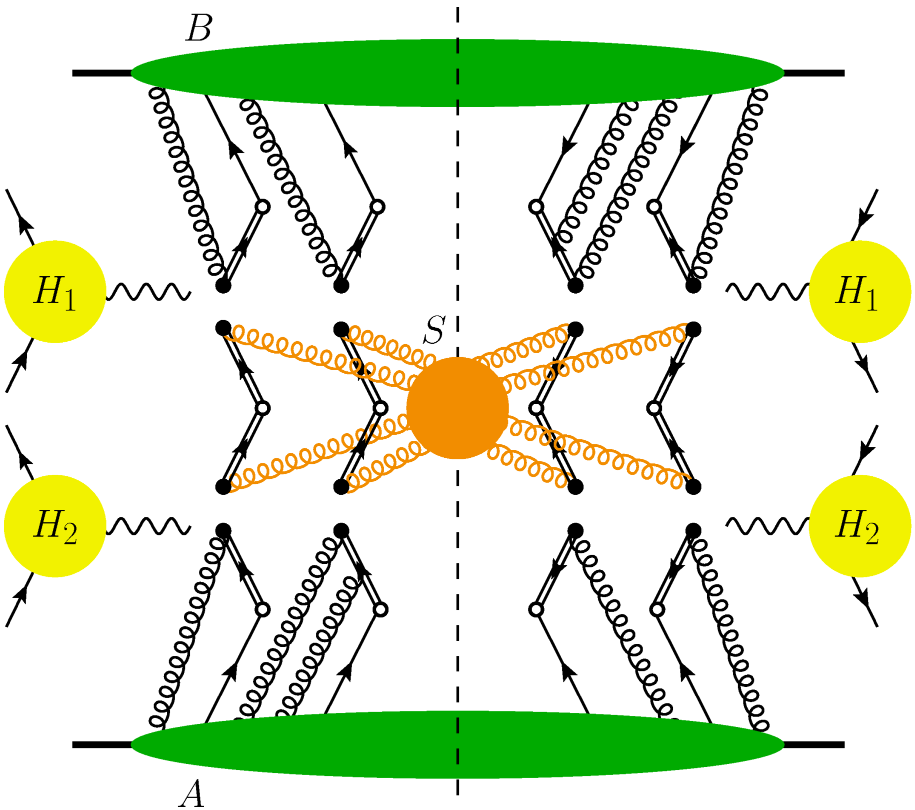


Figure: Diehl, Gaunt, Ostermeier, Plößl, Schäfer, JHEP01 (2016) 076

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042; Diehl, Gaunt, Ostermeier, Plößl, Schäfer, JHEP01 (2016) 076

Soft function

- Soft function: orange part in figure
- Couples the two correlators *A* and *B* with each other through soft gluons
- Evolution kernel *K* related to soft function *S*
- Nontrivial color complications. Collinear and soft factors carry color indices.

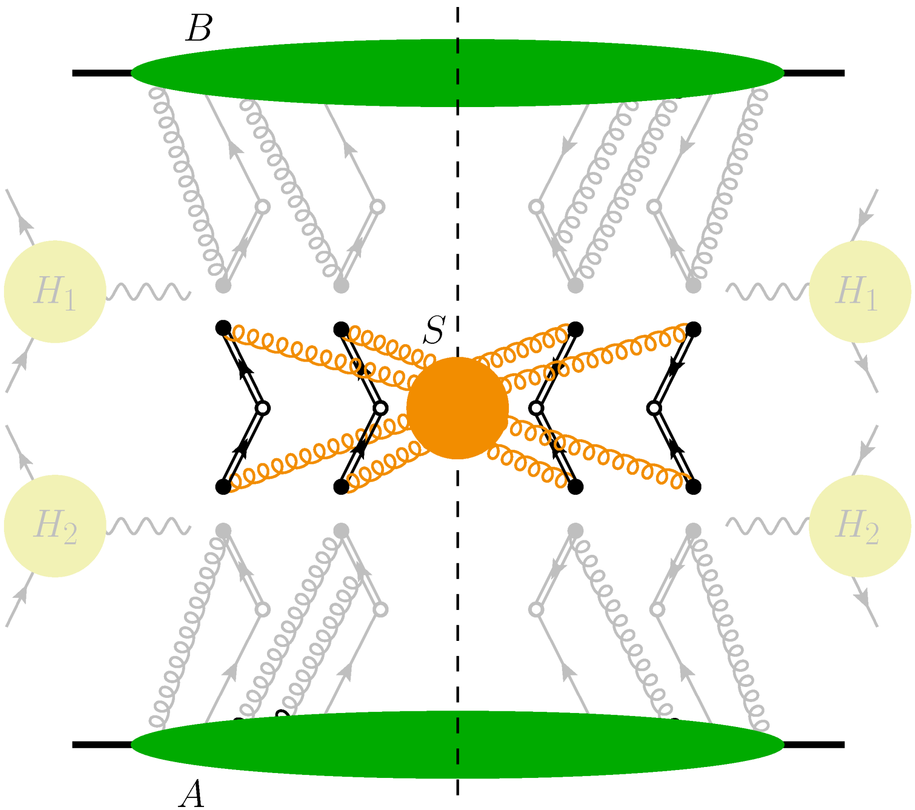


Figure: Diehl, Gaunt, Ostermeier, Plößl, Schäfer, JHEP01 (2016) 076

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042; Diehl, Gaunt, Ostermeier, Plößl, Schäfer, JHEP01 (2016) 076; Vladimirov, 1608.04920

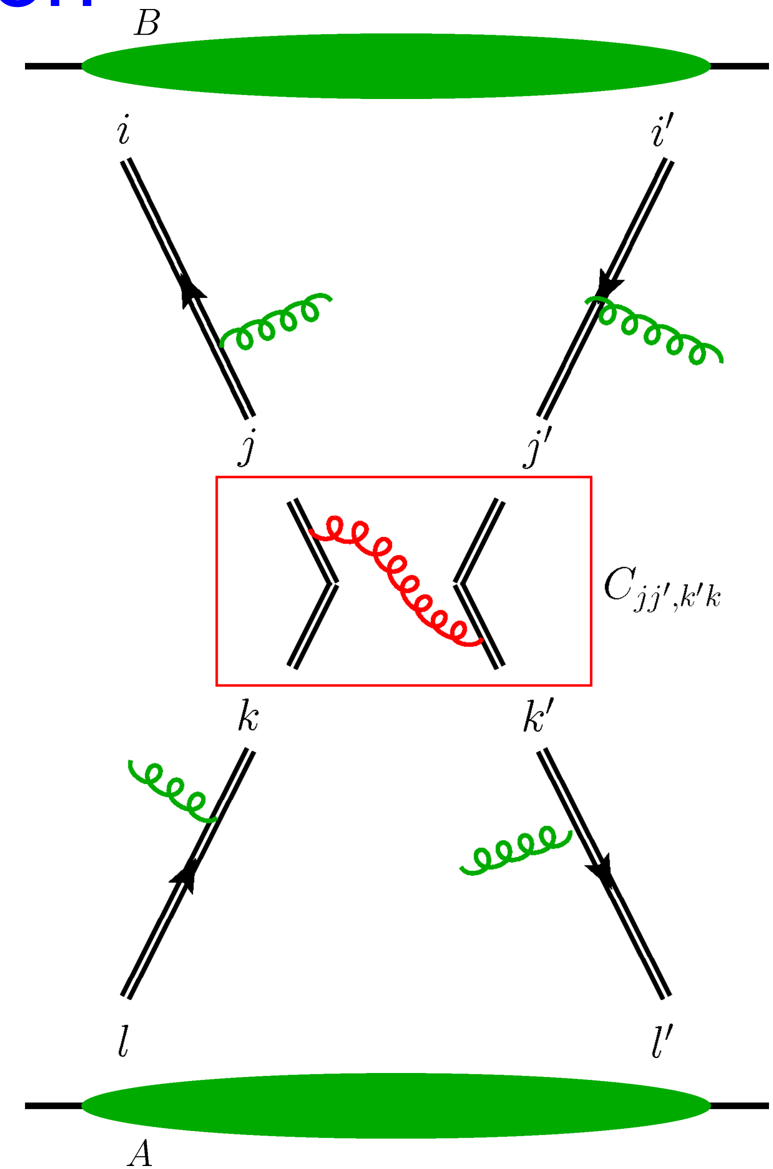
Soft function

- Uncontracted indices in the middle

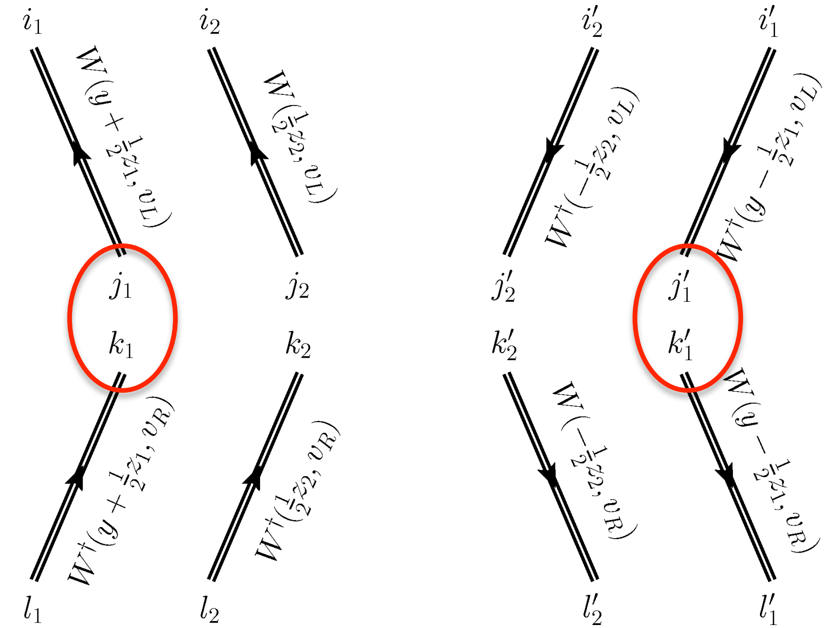
- Soft factor for DTMDs factorizes in the limit $|\mathbf{z}_1|, |\mathbf{z}_2| \ll \mathbf{y}$, as
  $$S(\mathbf{z_1}, \mathbf{z_2}, \mathbf{y}) = C_s(\mathbf{z_1})\, C_s(\mathbf{z_2})\, S(\mathbf{y})$$

- Boxed object $C_{jj',kk'}$ will appear in matching equations

- We require a simplification of the color indices.



For the expert: soft function



- Recall full Wilson line structure
- Hard scattering couples four parton lines, insert color projectors
- Examples of color projectors
  - Quarks:

$$p_1^{j_1 j'_1\, k_1 k'_1} = \frac{1}{N_c}\delta_{j_1 j'_1}\delta_{k_1 k'_1}$$

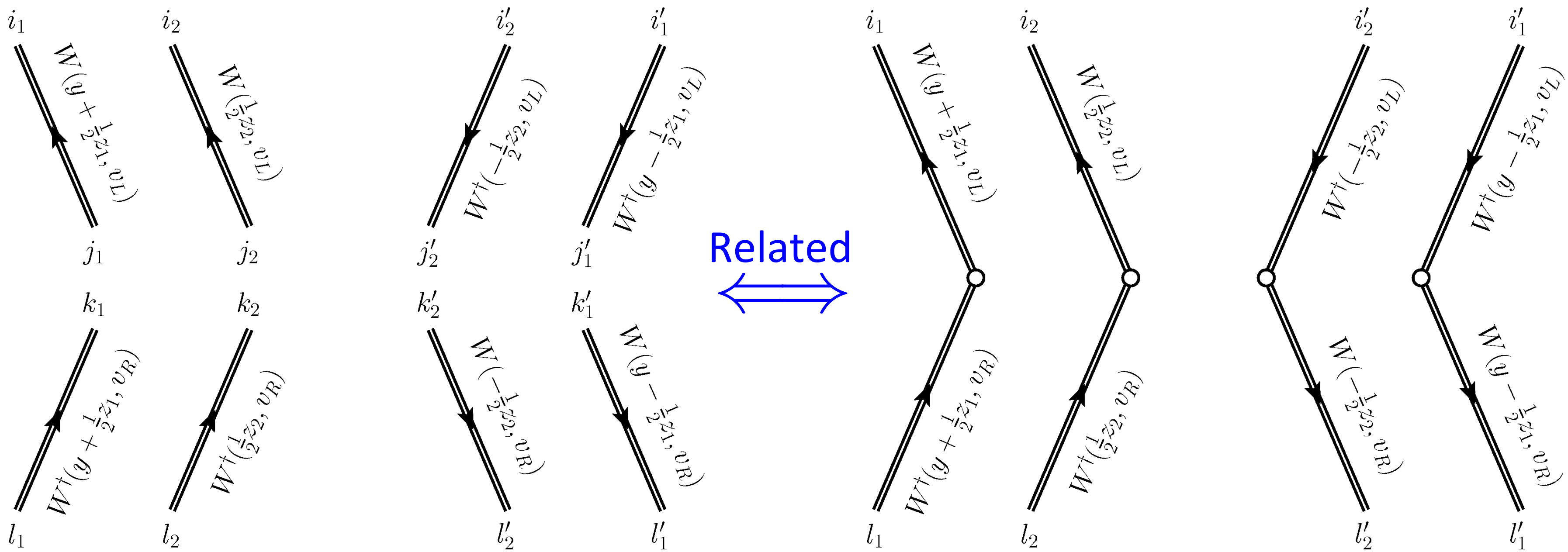
$$p_8^{j_1 j'_1\, k_1 k'_1} = 2t^a_{j_1 j'_1} t^a_{k_1 k'_1}$$

  - For gluons: more possibilities, we label them p_R
  - Mixed quark-gluon projectors also exist

- Highly nontrivial whether color structure can be factorized.

For the expert: soft function

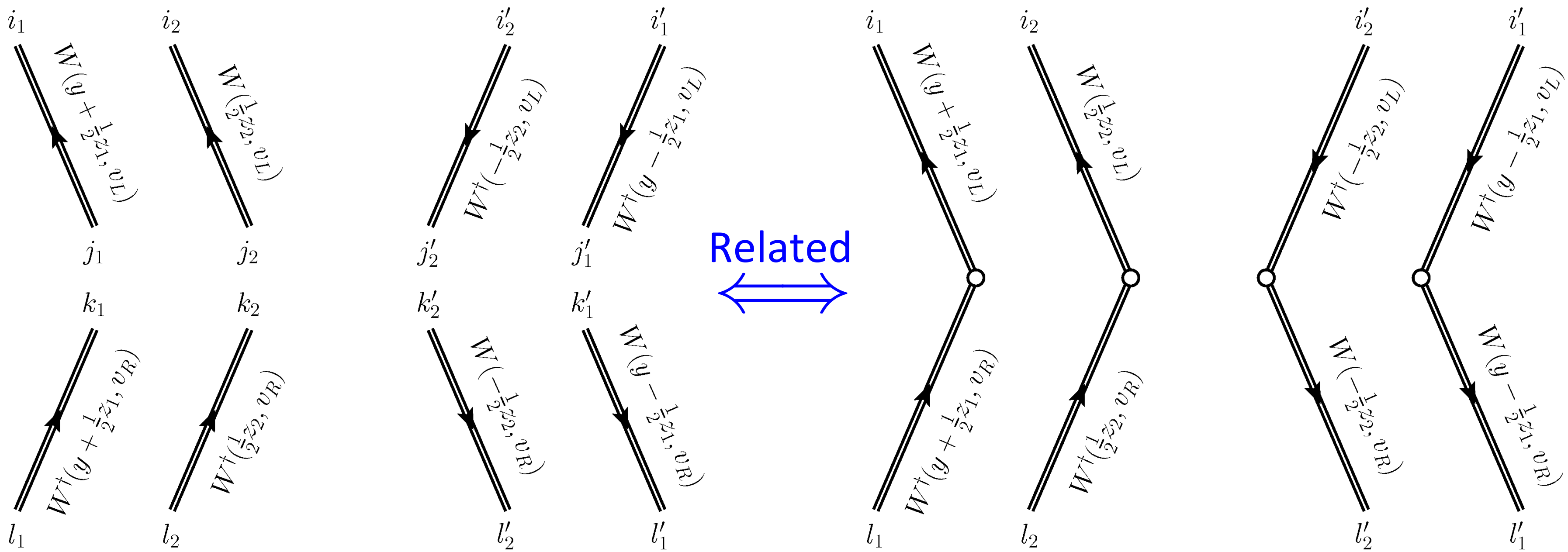
- Color projection of fields at infinity rather than $\xi^+ = \xi^- = 0$.



- Allows for relating most general soft function with open indices in the middle with soft function with contracted indices in the middle.
- Fully nonperturbative statement, since it involves color algebra
- For collinear factorization case only!

For the expert: soft function

- Color projection of fields at infinity rather than $\xi^+ = \xi^- = 0$.



- Then, in the short-distance expansion

$$^{RR'}S(\mathbf{z_1}, \mathbf{z_2}, \mathbf{y}) = {}^{R}C_s(\mathbf{z_1})\,{}^{R}C_s(\mathbf{z_2})\,{}^{RR}S(\mathbf{y})\delta_{RR'}$$

where *R* is a color representation index (singlet, octet, etc.)

Evolution: TMDs vs DTMDs

PDF/TMDs

- Soft function not matrix valued
- Just the position of one parton
- Renormalization scale μ
- Rapidity evolution scale ζ
- One coefficient function per TMD

DPDF/DTMDs

- Soft function matrix valued
- Positions of two partons and the distance y
- Renormalization scales μ_1, μ_2
- Rapidity evolution scale ζ
  - ζ dependence also for collinear distribution if $R \neq 1$.
- Two coefficient functions per DTMD

Renormalization and rapidity evolution

- TMDs

$$\frac{\partial}{\partial \log \mu} F(x, \mathbf{z}; \mu, \zeta) = \gamma_F(\mu, \zeta) F(x, \mathbf{z}; \mu, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} F(x, \mathbf{z}, \mu, \zeta) = \frac{1}{2} K(\mathbf{z}; \mu) F(x, \mathbf{z}, \mu, \zeta)$$

- DTMDs

$$\frac{\partial}{\partial \log \mu_1} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_1, x_1\zeta/x_2) {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \mu_2} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_2, x_2\zeta/x_1) {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}, \mu_i, \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'}K(\mathbf{z}_i, \mathbf{y}; \mu_i) {}^{R'}F(x_i, \mathbf{y}, \mu_i, \zeta)$$

- DTMD renormalizations are independent, since they are separated.

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042

Renormalization and rapidity evolution

- TMDs

$$\frac{\partial}{\partial \log \mu} F(x, \mathbf{z}; \mu, \zeta) = \gamma_F(\mu, \zeta) F(x, \mathbf{z}; \mu, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} F(x, \mathbf{z}, \mu, \zeta) = \frac{1}{2} K(\mathbf{z}; \mu) F(x, \mathbf{z}, \mu, \zeta)$$

$F =$ DTMDs

- DTMDs

$$\frac{\partial}{\partial \log \mu_1} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_1, x_1\zeta/x_2) {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \mu_2} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_2, x_2\zeta/x_1) {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}, \mu_i, \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'} K(\mathbf{z}_i, \mathbf{y}; \mu_i) {}^{R'} F(x_i, \mathbf{y}, \mu_i, \zeta)$$

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Renormalization and rapidity evolution

- TMDs

$$\frac{\partial}{\partial \log \mu} F(x, \mathbf{z}; \mu, \zeta) = \gamma_F(\mu, \zeta) F(x, \mathbf{z}; \mu, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} F(x, \mathbf{z}, \mu, \zeta) = \frac{1}{2} K(\mathbf{z}; \mu) F(x, \mathbf{z}, \mu, \zeta)$$

$F =$ DTMDs

$K =$ Collins Soper evolution kernel

- DTMDs

$$\frac{\partial}{\partial \log \mu_1} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_1, x_1\zeta/x_2)\, {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \mu_2} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_2, x_2\zeta/x_1)\, {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}, \mu_i, \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'}K(\mathbf{z}_i, \mathbf{y}; \mu_i)\, {}^{R'}F(x_i, \mathbf{y}, \mu_i, \zeta)$$

- DTMD renormalizations are independent, since they are separated.

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042

Renormalization and rapidity evolution

- TMDs

$$\frac{\partial}{\partial \log \mu} F(x, \mathbf{z}; \mu, \zeta) = \gamma_F(\mu, \zeta) F(x, \mathbf{z}; \mu, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} F(x, \mathbf{z}, \mu, \zeta) = \frac{1}{2} K(\mathbf{z}; \mu) F(x, \mathbf{z}, \mu, \zeta)$$

$F =$ DTMDs

$K =$ Collins Soper evolution kernel

$\gamma_F =$ anomalous dimension of F

- DTMDs

$$\frac{\partial}{\partial \log \mu_1} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_1, x_1\zeta/x_2) {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \mu_2} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_2, x_2\zeta/x_1) {}^R F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

$$\frac{\partial}{\partial \log \zeta} {}^R F(x_i, \mathbf{z}_i, \mathbf{y}, \mu_i, \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'} K(\mathbf{z}_i, \mathbf{y}; \mu_i) {}^{R'} F(x_i, \mathbf{y}, \mu_i, \zeta)$$

- DTMD renormalizations are independent, since they are separated.

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042

Solving evolution equations for DTMDs

DTMDs: μ_1 and μ_2 scale evolution

- μ_1 scale evolution governed by an equation of the form

$$\frac{\partial}{\partial \log \mu_1} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = \gamma_F(\mu_1, x_1\zeta/x_2)\, {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta)$$

  and similarly for μ_2.

- For the starting values:
  - Starting scales μ_{10} and μ_{20} for μ_1 and μ_2.
  - We define the ζ value as the geometric mean

- We get the result

$$ {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{0i}, \zeta) \times \exp\left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \gamma_F(\mu, x_1\zeta/x_2) + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \gamma_F(\mu, x_2\zeta/x_1) \right\}$$

- Note the additive structure

Solving evolution equations for DTMDs

DTMDs: μ_1 and μ_2 scale evolution *and* ζ evolution

- ζ evolution governed by

$$\frac{\partial}{\partial \log\zeta} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}, \mu_i, \zeta) = \frac{1}{2}\sum_{R'} {}^{RR'}K(\mathbf{z}_i, \mathbf{y}; \mu_i)\, {}^{R'}F(x_i, \mathbf{y}, \mu_i, \zeta)$$

- Solving for rapidity dependence, we then get the result

$$\begin{aligned} {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) = {}^{R}F(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{0i}, \zeta) & \\ \times \exp\Bigg\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu}\left[\gamma_F(\mu, x_1\zeta/x_2) - \gamma_K(\mu)\log\frac{\sqrt{x_1\zeta/x_2}}{\mu}\right] & \\ + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu}\left[\gamma_F(\mu, x_2\zeta/x_1) - \gamma_K(\mu)\log\frac{\sqrt{x_2\zeta/x_1}}{\mu}\right] & \\ + \left[{}^{R}K(\mathbf{z}_1, \mu_{01}) + {}^{R}K(\mathbf{z}_2, \mu_{02}) + {}^{R}J(\mathbf{y}, \mu_{0i})\right]\log\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \Bigg\} & \end{aligned}$$

- The *K*-kernel splitting in three separate contributions is crucial, but only true in limit where we can do the DTMD → DPDF matching.

DTMD/DPDF matching

- The matching equation is given by

$$^{R}F(x_i, \mathbf{z_i}, \mathbf{y}; \mu_i, \zeta) = {}^{R}C(x_1', \mathbf{z_1}; \mu_1, \mu_1^2) \underset{x_1}{\otimes} {}^{R}C(x_2', \mathbf{z_2}; \mu_2, \mu_2^2) \underset{x_2}{\otimes} {}^{R}F(x_i', \mathbf{y}; \mu_i, \zeta)$$

- Convolution defined as

$$C(x') \underset{x}{\otimes} F(x') = \int_x^1 \frac{dx'}{x'} C(x') F\left(\frac{x}{x'}\right)$$

- The coefficient functions *C* are the same as for TMD/PDF matching

- Splitting kernels from PDF/TMDs can largely be recycled

$C_{q/g}(x', \mathbf{z}; \mu_0, \mu_0^2)$ $C_{g/g}(x', \mathbf{z}; \mu_0, \mu_0^2)$ $C_{g/\delta g}(x', \mathbf{z}; \mu_0, \mu_0^2)$ etc.

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042; Bacchetta, Prokudin, NPB 875 (2013) 536; Echevarría, Kasemets, Mulders, Pisano, JHEP 1507 (2015) 158; MGAB, Diehl, Kasemets, *work in progress*.

Combining matching and evolution

- The evolution of DTMDs is in the short-distance matching given by

$$
\begin{aligned}
&{}^{R}F(x_i,\mathbf{z}_i,\mathbf{y};\mu_1,\mu_2,\zeta) \\
&= \exp\Bigg\{\int_{\mu_{01}}^{\mu_1}\frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2)-\gamma_K(\mu)\log\frac{\sqrt{x_1\zeta/x_2}}{\mu}\right]+{}^{R}K(\mathbf{z}_1,\mu_{01})\log\frac{\sqrt{x_1\zeta/x_2}}{\mu_{01}} \\
&\quad+\int_{\mu_{02}}^{\mu_2}\frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2)-\gamma_K(\mu)\log\frac{\sqrt{x_2\zeta/x_1}}{\mu}\right]+{}^{R}K(\mathbf{z}_2,\mu_{02})\log\frac{\sqrt{x_2\zeta/x_1}}{\mu_{02}} \\
&\quad+{}^{R}J(\mathbf{y},\mu_{01},\mu_{02})\log\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Bigg\} \\
&\times\,{}^{R}C(x_1',\mathbf{z}_1;\mu_{01},\mu_{01}^2)\underset{x_1}{\otimes}{}^{R}C(x_2',\mathbf{z}_2;\mu_{02},\mu_{02}^2)\underset{x_2}{\otimes}{}^{R}F(x_i',\mathbf{y};\mu_{01},\mu_{02},\zeta_0)
\end{aligned}
$$

- From additive structure of the Collins-Soper evolution kernel we have the sum for the two contributions for the μ_1 and μ_2 dependences.

- $K(\mathbf{z}_1,\mathbf{z}_2,\mathbf{y})$-kernel splits in three separate contributions: $K(\mathbf{z}_1,\mu_{01})$, $K(\mathbf{z}_2,\mu_{02})$ and $J(\mathbf{y},\mu_{01},\mu_{02})$ when collinear soft function becomes diagonal.

Combining matching and evolution

- Cross section contribution given by

$$
\begin{aligned}
W_{\text{large } \mathbf{y}} = \sum_R \exp\Bigg\{ & \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2) - \gamma_K(\mu)\log\frac{Q_1^2}{\mu^2}\right] + {}^R K(\mathbf{z}_1,\mu_{01})\log\frac{Q_1^2}{\mu_{01}^2} \\
& + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2) - \gamma_K(\mu)\log\frac{Q_2^2}{\mu^2}\right] + {}^R K(\mathbf{z}_2,\mu_{02})\log\frac{Q_2^2}{\mu_{02}^2} \Bigg\} \\
& \times {}^R C(\overline{x}'_1,\mathbf{z}_1;\mu_{01},\mu_{01}^2) \underset{\overline{x}_1}{\otimes} {}^R C(\overline{x}'_2,\mathbf{z}_2;\mu_{02},\mu_{02}^2) \underset{\overline{x}_2}{\otimes} \\
& \times {}^R C(x'_1,\mathbf{z}_1;\mu_{01},\mu_{01}^2) \underset{x_1}{\otimes} {}^R C(x'_2,\mathbf{z}_2;\mu_{02},\mu_{02}^2) \underset{x_2}{\otimes} \\
& \times \left[\Phi(\nu\mathbf{y})\right]^2 \exp\left[{}^R J(\mathbf{y},\mu_{0i})\log\frac{\sqrt{Q_1^2 Q_2^2}}{\zeta_0}\right] {}^R F(\overline{x}_i,\mathbf{y};\mu_{0i},\zeta_0)\, {}^R F(x_i,\mathbf{y};\mu_{0i},\zeta_0)
\end{aligned}
$$

- The $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{y}$ contributions nicely factorize.

- There is ζ – dependence for color non-singlet DPDFs, even if there is no transverse momenta anymore

Combining matching and evolution

- Cross section contribution given by

$$
\begin{aligned}
W_{\text{large }\mathbf{y}} = \sum_R \exp\Bigg\{ &\int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2) - \gamma_K(\mu)\log\frac{Q_1^2}{\mu^2}\right] + {}^R K(\mathbf{z}_1,\mu_{01})\log\frac{Q_1^2}{\mu_{01}^2} \\
&+ \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu}\left[\gamma_F(\mu,\mu^2) - \gamma_K(\mu)\log\frac{Q_2^2}{\mu^2}\right] + {}^R K(\mathbf{z}_2,\mu_{02})\log\frac{Q_2^2}{\mu_{02}^2}\Bigg\} \\
\times\, &{}^R C(\overline{x}'_1,\mathbf{z}_1;\mu_{01},\mu_{01}^2) \underset{\overline{x}_1}{\otimes} {}^R C(\overline{x}'_2,\mathbf{z}_2;\mu_{02},\mu_{02}^2) \underset{\overline{x}_2}{\otimes} \\
\times\, &{}^R C(x'_1,\mathbf{z}_1;\mu_{01},\mu_{01}^2) \underset{x_1}{\otimes} {}^R C(x'_2,\mathbf{z}_2;\mu_{02},\mu_{02}^2) \underset{x_2}{\otimes} \\
\times\, &\left[\Phi(\nu\mathbf{y})\right]^2 \exp\left[{}^R J(\mathbf{y},\mu_{0i})\log\frac{\sqrt{Q_1^2 Q_2^2}}{\zeta_0}\right] {}^R F(\overline{x}_i,\mathbf{y};\mu_{0i},\zeta_0)\, {}^R F(x_i,\mathbf{y};\mu_{0i},\zeta_0)
\end{aligned}
$$

- Sudakov suppression for everything other than color singlet configur.

- Similar Sudakov suppressions for collinear DPDFs

Manohar, Waalewijn, PRD 85 (2012) 114009; Mekhfi, Artru, PRD 37 (1988) 2618.

Polarizations

- Including parton labels in equations for DTMDs and cross section. E.g.

$$
\begin{aligned}
&{}^{R}F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_1, \mu_2, \zeta) \\
&\quad = \sum_{b_1 b_2} \exp\Bigg\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \Bigg[\gamma_{F,a_1}(\mu, \mu^2) - \gamma_{K,a_1}(\mu) \log \frac{\sqrt{x_1 \zeta / x_2}}{\mu} \Bigg] + {}^{R}K_{a_1}(\mathbf{z}_1, \mu_{01}) \log \frac{\sqrt{x_1 \zeta / x_2}}{\mu_{01}} \\
&\qquad + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \Bigg[\gamma_{F,a_2}(\mu, \mu^2) - \gamma_{K,a_2}(\mu) \log \frac{\sqrt{x_2 \zeta / x_1}}{\mu} \Bigg] + {}^{R}K_{a_2}(\mathbf{z}_2, \mu_{02}) \log \frac{\sqrt{x_2 \zeta / x_1}}{\mu_{02}} \\
&\qquad + {}^{R}J(\mathbf{y}, \mu_{01}, \mu_{02}) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \Bigg\} \\
&\quad \times {}^{R}C_{a_1 b_1}(x_1', \mathbf{z}_1; \mu_{01}, \mu_{01}^2) \underset{x_1}{\otimes} {}^{R}C_{a_2 b_2}(x_2', \mathbf{z}_2; \mu_{02}, \mu_{02}^2) \underset{x_2}{\otimes} {}^{R}F_{b_1 b_2}(x_i', \mathbf{y}; \mu_{01}, \mu_{02}, \zeta_0)
\end{aligned}
$$

- Parton labels like a_1 not only q, $\bar{q}$ and g, but also δq, Δq, δg, Δg, etc.
- Splitting kernels from PDF/TMDs can largely be recycled

$$C_{q/g}(x', \mathbf{z}; \mu_0, \mu_0^2) \qquad C_{g/g}(x', \mathbf{z}; \mu_0, \mu_0^2) \qquad C_{g/\delta g}(x', \mathbf{z}; \mu_0, \mu_0^2)$$ etc.

Collins, *Foundations of perturbative QCD*, (2011); Aybat, Rogers, PRD 83 (2011) 114042; Bacchetta, Prokudin, NPB 875 (2013) 536; Echevarría, Kasemets, Mulders, Pisano, JHEP 1507 (2015) 158; MGAB, Diehl, Kasemets, *work in progress*.

Conclusions

- We use short-distance expansion
  - $|\mathbf{z}_1|$, $|\mathbf{z}_2|$ much smaller than $1/\Lambda$
  - $|\mathbf{z}_1|, |\mathbf{z}_2| \ll \mathbf{y}$

  although part of our results are also valid outside this region.

- Description for soft function
  - Separation in a **y**-dependent contribution and two pieces depending on either $\mathbf{z}_1$ or $\mathbf{z}_2$.

- Matching equations for DPDs
  - Evolution equations for DTMDs.
  - Expression for matching at level of individual DTMDs/DPDFs and cross section.

- Explicit expressions for matching of all polarization-modes: results from TMD/PDF matching can in part be recycled.