q_T resummation for Higgs boson pair production with top-quark mass effects

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Based on:
G. F. & J. Pires, arXiv:1609.01691 [hep-ph]

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Outline

1. **Higgs pair production in hadronic collisions**

2. **Transverse-momentum resummation**

3. **HH resummed q_T distribution with M_t effects at the LHC**

Motivations

The observation of the Higgs boson opened the window for testing the EWSB mechanism of the Standard Model.

- So far all the measured properties of the Higgs boson and its couplings with vector bosons and heavy fermions are consistent with the SM predictions.
- Particular important is the determination of the (trilinear and quartic) couplings of the Higgs boson to itself, which allow to reconstruct the Higgs scalar potential:

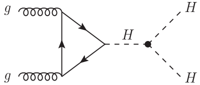
$$V(H) = \frac{1}{2} M_H^2 H^2 + v\lambda H^3 + \frac{1}{4}\lambda' H^4 \,,$$

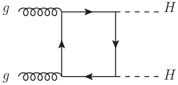
  where $M_H \simeq 125\,\mathrm{GeV}$, $v \simeq 246\,\mathrm{GeV}$ and (in the SM) $\lambda = \lambda' = M_H^2/(2v^2)$.
- Quartic coupling direct measurement not possible at the LHC and extremely difficult even at future colliders.
- Trilinear coupling measurement very challenging.
  It could be directly constrained at the high-luminosity LHC through Higgs boson pair production: $pp \to HH$.

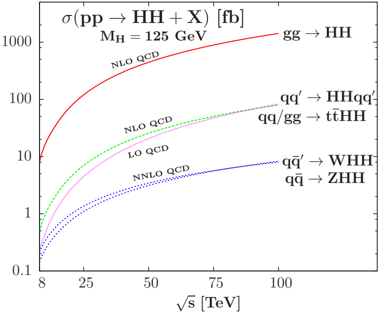
Higgs pair production in hadronic collisions

Higgs boson pair hadroproduction

- Gluon fusion main production mechanism: $\sigma \sim 35$ fb at LHC $\sqrt{s} = 14$ TeV (10 times larger than other HH production channels but 10^3 times smaller than single H).
- At LO, triangle (via trilinear Higgs self-coupling) and box heavy-quark loops contributions. Top-quark contribution dominates, bottom-quark effects negligible ($\sim 0.3\%$):







Total cross sections for HH prod. from [Baglio et al.('13)].

- Loop induced process: calculations in full theory (FT) beyond LO extremely difficult. Great simplification within the HEFT ($M_t \to \infty$ limit with top-quark loops shrinked to a tree-level gluons-Higgs interaction).

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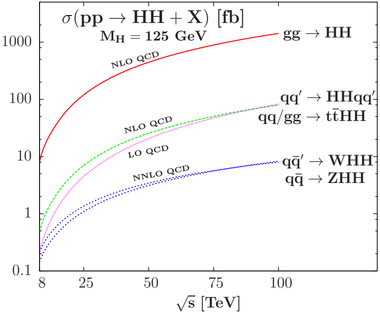

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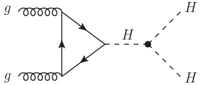

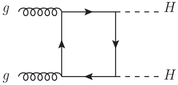

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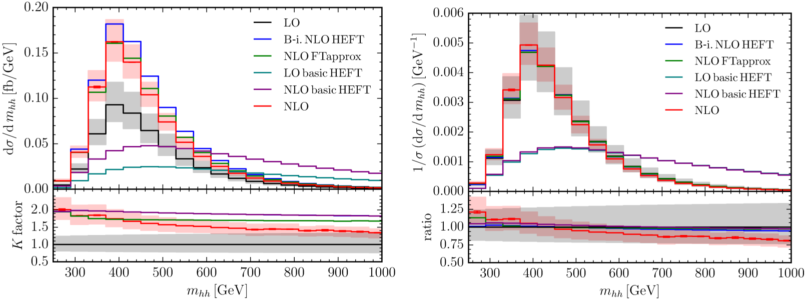
Full QCD and effective theory approximation

HH produced with an invariant mass $M_{HH} > M_t$: HEFT description breaks down.
Approximations to include finite M_t effects beyond LO:

- "Born-improved HEFT approx." (NLO HEFT reweighted by exact Born B_{FT}/B_{HEFT});
- "FT approx." at NLO (real emission correction calculated in FT, virtual amplitude computed in the "Born-improved HEFT") [Maltoni et al.('14)];
- HEFT at NLO and NNLO improved by an expansion in $1/M_t^2$.



HH invariant mass distribution at LHC $\sqrt{s} = 14$ TeV,
absolute (left) and normalised (right) results, from [Borowka et al.('16)].

Theoretical predictions for SM Higgs pair production via gluon fusion.

LO (FT) and NLO (HEFT) results known since a long time:

- LO result in FT: [Glover,van der Bij('88)],[Eboli,Marques,Novaes,Natale('87)].
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Recent activity: NNLO + higher-order QCD corrections and finite top-mass effects.

- NNLO result in HEFT: [de Florian,Mazzitelli('13)], [Grigo,Hoff,Melnikov,Steinhauser('14)].
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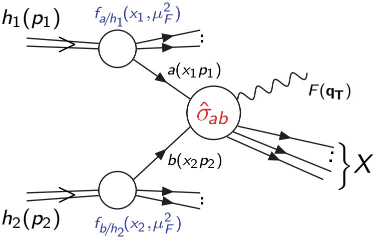
Transverse-momentum resummation

$\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; F + X}$$

where F system of non-QCD partons

QCD *collinear* factorization formula:

$$\frac{d\sigma}{dq_T^2} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \overset{q_T \ll M}{\sim} 1 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \cdots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

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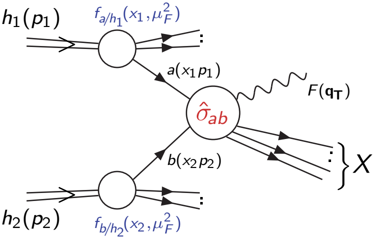

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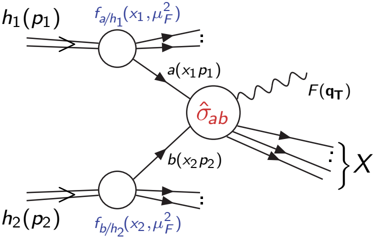

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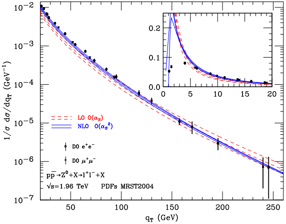

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State of the art: q_T resummation

- Method to resum large q_T logarithms is known [Dokshitzer,Diakonov,Troian('78)], [Parisi,Petronzio('79)],[Curci,Greco,Srivastava('79)],[Kodaira,Trentadue('82)], [Collins,Soper('81,'82)],[Collins,Soper,Sterman('85)],[Catani,de Florian, Grazzini('01)],[Bozzi et al.('06,'08)],[Catani,Grazzini('11)],[Catani et al.('13)].
- Phenomenological studies [Altarelli et al.('84)],[ResBos:Balazs et al.('95,'97)], [Guzzi et al.('13)],[Ellis et al.('97,'98)],[Qiu et al.('01)],[Kulesza et al. ('01,'02)],[Berger et al.('02,'03)],[Landry et al.('03)],[Banfi et al.('12)].
- Results for q_T resummation by using Soft Collinear Effective Theory methods and transverse-momentum dependent (TMD) factorization [Gao et al.('05)],[Idilbi et al.('05)],[Mantry,Petriello('10,'11)],[Becher et al.('11)],[Echevarria et al. ('12,'13,'15)],[Chiu et al.('12)], [Roger,Mulders('10)],[Collins('11)], [Collins, Rogers('13)],[D'Alesio et al.('14)].
- Effective q_T-resummation can be obtained with Parton Shower algorithms. Results for NNLO predictions matched with PS obtained [Hoeche,Li,Prestel('14)], [Karlberg,Re,Zanderighi('14)], [Alioli,Bauer,Berggren,Tackmann,Walsh('14)].

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution of DY process it holds in the impact parameter space (Fourier transform).

$$\int d^2\mathbf{q_T}\, \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta\left(\mathbf{q_T} - \sum_{j=1}^{n} \mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space.

$\mathbf{q_T}$ resummation: $\mathbf{q\bar{q}}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

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$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2} \; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;\; ,$$

[Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}$, $L \equiv \ln Mb$

$$S_q(M,b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left[A_q(\alpha_S(q^2)) \;\ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2))\right]\right\} \;\; .$$

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[Collins,Soper,Sterman('85)],

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $\quad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $\quad L \equiv \ln Mb$ [Catani,de Florian,Grazzini('01)]

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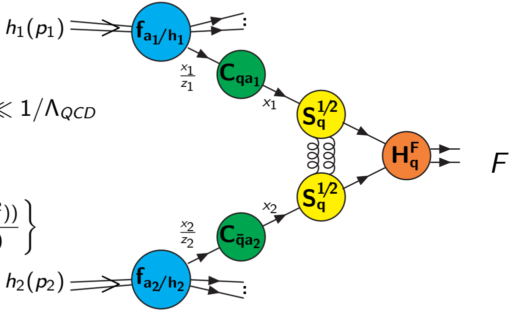
Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD} \; , \quad b \gg 1/M \; , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2)) \times \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \, \beta(\alpha_S(q^2)) \, \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} \, e^{i\mathbf{b}\cdot\mathbf{q_T}} \, S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1} \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2} \, C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z} \sqrt{S_q(M,b)} C_{q_f\,a}(z; \alpha_S(b_0^2/b^2)) \, f_{a/h}(x/z, b_0^2/b^2)$$

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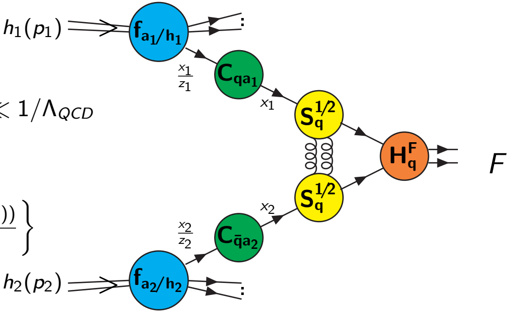

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$\mathbf{q_T}$ resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \to F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani,Grazzini('11)].

$$\left[H^F C_1 C_2\right]_{gg;a_1 a_2} = H^F_{g;\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \alpha_S(M^2))$$
$$\times \; C^{\mu_1\nu_1}_{ga_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \, C^{\mu_2\nu_2}_{ga_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \, .$$

where $\quad H_g^{F\,\mu_1\nu_1,\mu_2\nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2} \, ,$

$$C^{\mu\nu}_{ga}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\,\mu\nu}(p_1, p_2)\; C_{ga}(z; \alpha_S) + D^{\,\mu\,\nu}(p_1, p_2; \mathbf{b})\; G_{ga}(z; \alpha_S) \;\; ,$$

$$d^{\,\mu\nu}(p_1, p_2) = -\,g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \; , \quad D^{\,\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\,\mu\nu}(p_1, p_2) - 2\,\frac{b^\mu\, b^\nu}{\mathbf{b}^2} \;\; ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga}\;\; \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C^{(n)}_{ga}(z) \, , \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n G^{(n)}_{ga}(z) \;\; .$$

- Unlike $q\bar{q}$ annih. $\left[H^F C_1 C_2\right]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(\mathbf{q_T})$ (consistent with [Mulders,Rodrigues('00)],[Henneman et al.('02)]).
- Small-q_T cross section expressed in terms of $\phi(\mathbf{q_T})$-independent plus $\cos(2\phi(\mathbf{q_T}))$, $\sin(2\phi(\mathbf{q_T}))$, $\cos(4\phi(\mathbf{q_T}))$ and $\sin(4\phi(\mathbf{q_T}))$ dependent contributions.

The q_T resummation formalism

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) = \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \to \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

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  - recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) = \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \to \widetilde{L} \equiv \ln(Q^2 b^2 + 1) \quad \Rightarrow \quad \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1$$

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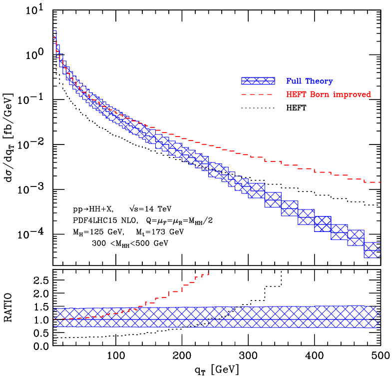
HH resummed q_T distribution with M_t effects at the LHC

HH q_T spectrum at the LHC: fixed-order predictions

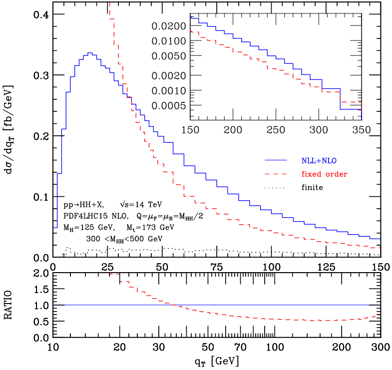
- One loop amplitudes in the FT generated with GoSam [Cullen et al.('12),('14)].
- Scale variation band in FT: $M_{HH}/4 \leq \{\mu_R, \mu_F\} \leq M_{HH}$ (with $1/2 \leq \mu_R/\mu_F \leq 2$). It is $\sim \pm 35\%$ at small q_T, $\sim \pm 40\%$ at $q_T \sim 400 - 500$ GeV. Band is rather large and flat (driven by the overall factor $\alpha_S(\mu_R)^3$).
- HEFT is a poor approximation. Born-improved HEFT works well for $q_T \lesssim 50$ GeV (where both results diverge logarithmically). For $q_T \gtrsim 50$ GeV the agreement rapidly deteriorates: $\sim 20 - 25\%$ at $q_T \sim 100$ GeV and $\sim 80 - 100\%$ at $q_T \sim 175$ GeV.



HH q_T spectrum at the LHC (14 TeV). Fixed-order prediction at $\mathcal{O}(\alpha_S^3)$) in the full theory (blue solid), HEFT (black dotted) and Born-improved reweighted HEFT (red dashed).

HH q_T spectrum at the LHC: NLL+NLO predictions

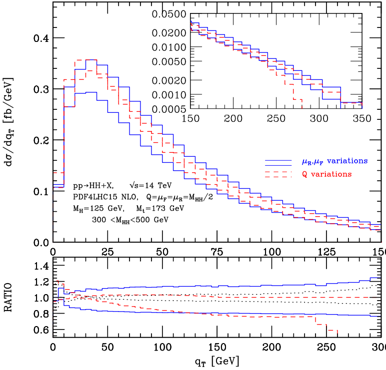
- q_T resummation performed at full NLL+NLO in the FT. Process dependent coeff. $H_g^{HH\,(1)}$ with full M_t effects extracted from the virtual amplitude computed by [Borowka et al.('16)].
- Resummation leads to a well-behaved distribution: vanishes as $q_T \to 0$, kinematical peak at $q_T \sim 18$ GeV and reproduce FO for $q_T \sim M_{HH}$.
- Finite component: vanishes as $q_T \to 0$, $\sim 4\%$ in the peak region, $\sim 15\%$ at $q_T \sim 125$ GeV, $\sim 30\%$ at $q_T \sim 200$ GeV.
- Resummation important from small to intermediate-q_T region: resummation effects $\sim 40 - 50\%$ for $80 \lesssim q_T \lesssim 250$ GeV.
- Integral over q_T of the NLL+NLO spectrum in agreement with the NLO FT total cross section at percent level.



HH q_T spectrum at the LHC (14 TeV). Resummed prediction at NLL+NLO in the FT (blue solid) compared with FO (red dashed) and finite component (black dotted).

NLL+NLO predictions: scale dependence

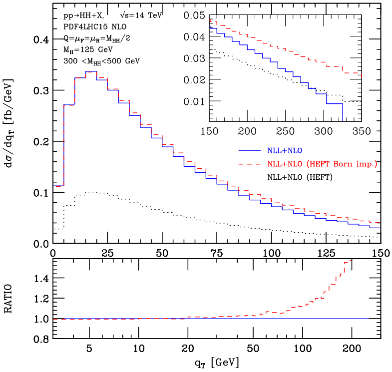
- NLL+NLO scale dependence: $M_{HH}/4 \leq \{\mu_R, \mu_F\} \leq M_{HH}$ (with $1/2 \leq \mu_R/\mu_F \leq 2$) at fixed $Q = M_{HH}/2$; $M_{HH}/4 \leq Q \leq M_{HH}$ at fixed $\mu_R = \mu_F = M_{HH}/2$.
- μ_R and μ_F dependence band $\sim \pm 10\%$ at the peak, $\sim \pm 12\%$ at $q_T \sim 30$ GeV, $\sim \pm 17\%$ at $q_T \sim 100$ GeV, $\sim \pm 20\%$ at $q_T \gtrsim 250$ GeV.
- Q dependence $\sim \pm 5\%$ at the peak, $\sim \pm 2\%$ at $q_T \sim 30$ GeV, $\sim \pm 12\%$ at $q_T \sim 200$ GeV. For $q_T \gtrsim M_{HH}$ the resummation looses predictivity.
  Resummed scale dependence band is *not* flat and is smaller than FO one for $q_T \lesssim 250$ GeV.
- μ_R and μ_F variation substantially reduced considering *normalised* q_T spectrum, $1/\sigma \times d\sigma/dq_T$.



HH q_T spectrum at the LHC (14 TeV). scale variation bands for NLL+NLO result in the FT. Q variations (red dashed), μ_R, μ_F variations (blue solid) and μ_R, μ_F variations for the *normalised* spectrum (black dotted).

NLL+NLO predictions: $\mathbf{M_t}$ effects

- Born-improved HEFT gives a good approximation (within 5% accuracy) of the FT result for $q_T \lesssim 70$ GeV and extremely well (within 1% accuracy) around the peak.
- However this agreement is not general. It depends on the particular M_{HH} window (for $350 < M_{HH} < 400$ GeV the agreement around the peak is $\sim 7\%$.).
- At higher values of q_T, M_t effects are large and have a strong q_T dependence. The effect is about 12% at $q_T \sim 100$ GeV, about 60% at $q_T \sim 200$ GeV and larger than 200% for $q_T \gtrsim 250$ GeV.
- M_t effects are important over a wide region from intermediate to large q_T.



HH q_T spectrum at the LHC (14 TeV). Resummed prediction at NLL+NLO in FT (blue solid), HEFT (black dotted) and Born-improved reweighted HEFT (red dashed).

Conclusions

- We have calculated the q_T spectrum for HH production in gluon fusion taking into account finite top-quark mass (M_t) effects.
- We have performed q_T resummation at NLL+NLO (i.e. matching with $\mathcal{O}(\alpha_S^3)$ at large q_T and including NLO virtual contributions at small q_T). Our calculation exactly reproduces the NLO total cross section with the full M_t dependence upon integration over q_T.
- We have presented illustrative numerical results for LHC at $\sqrt{s} = 14$ TeV, with an estimate of the perturbative uncertainties through the study of the scale μ_R, μ_F and Q dependence.
- We have shown that resummation is essential at small q_T and give an important contribution ($\gtrsim 40 - 50\%$) in a wide region of intermediate values of q_T ($q_T \lesssim 250$ GeV).
- We have quantified the size of the finite M_t effects which turn out to be large ($\gtrsim 60\%$) for $q_T \gtrsim 200$ GeV and very large ($\gtrsim 200\%$) for $q_T \gtrsim 250$ GeV.
- In conclusion: q_T resummation and finite top-quark mass effects are necessary to get reliable predictions for the HH q_T spectrum over the full q_T range.