Transverse momentum dependent splitting functions from k_T factorization

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Martin Hentschinski

Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla
Puebla 1152, Mexico

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México
Apartado Postal 70-543, CDMX 04510, Mexico

martin.hentschinski@gmail.com

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based on

M.H., A. Kusina, K. Kutak; arXiv:1607.01507

O. Gituliar, M.H. , K. Kutak; arXiv:1511.08439 (JHEP 1601 (2016) 181)

Outline

TMDs in the low x region

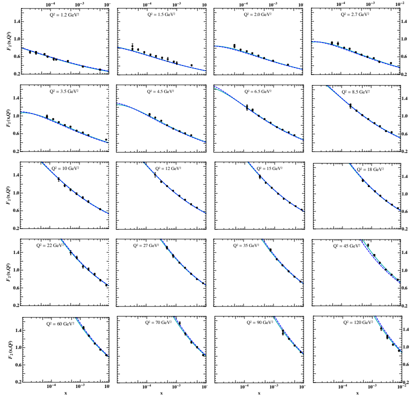
high energy limit $x \to 0$: factorization in terms of TMD coefficients (= impact factors) and (unintegrated) parton distributions

e.g. $F_2(x, Q^2) =$

$$= \int_0^\infty d\boldsymbol{k}^2 C_{2g^*}\left(Q^2, \boldsymbol{k}^2\right) \mathcal{G}(x, \boldsymbol{k}^2)$$

unintegrated gluon $\mathcal{G}$ subject to BFKL equation (up to NLL):

$$\partial_{\ln 1/x} \mathcal{G}(x, \boldsymbol{k}^2) = K \otimes \mathcal{G}(x, \boldsymbol{k}^2)$$

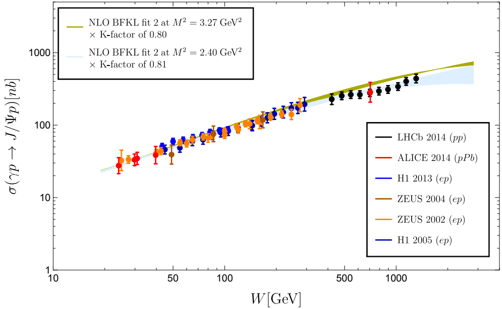


NLO BFKL fit with collinear resummation of BFKL kernel

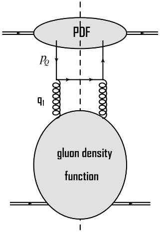
[MH, Sabio Vera, Salas; arXiv:1209.1353, 1301.5283]

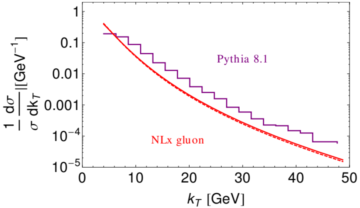
Main idea: evolve observables to lower values of x

e.g. exclusive photo-production of J/Ψ in ultra-peripheral collisions at LHC:
probe gluon down to $x = 10^{-5}$



[Bautista, Fernandez-Tellez, MH; arXiv:1607.05203]

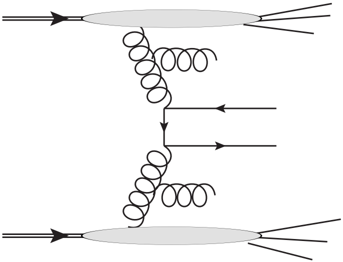




... but can also probe k_T dependence (generated by initial condition × BFKL evolution)

[Chachamis, Deak, MH, Sabio Vera, Rodrigo; arXiv:1507.05778]

Essential limitations: valid in low $x < 10^{-2}$ region

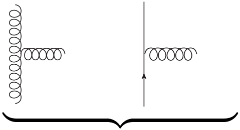


- limited to exclusive observables which allow to fix x of both gluons
- problematic in hadron-hadron collisions, combination with fragmentation functions ($\rightarrow$ convolution over intial x), ...

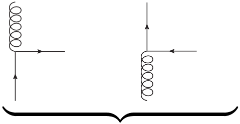
a first step: use CCFM evolution instead of BFKL evolution ...

- based on QCD coherence ➡ includes also resummation of soft logarithms
- can be used to fit F_2 data, but also remain limited to low $x < 0.5 \cdot 10^{-2}$ [Hautmann, Jung; arXiv:1312.7875]
- cannot achieve a fit for $x \in (0, 1]$

CCFM evolution based on QCD coherence → only gluonic emissions

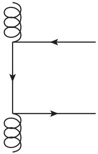


contained$\rightarrow \frac{1}{z}, \frac{1}{1-z}$



not contained→collinear logs

CCFM allows to include valence quarks, but matrix valued evolution essential for complete formulation of DGLAP evolution ....



collinear logarithms (including those associated with quarks) appear as elements of NLO BFKL
→ not a correction, need to be resummed to all orders!

[Salam; hep-ph/9806482], [MH, Sabio Vera, Salas; arXiv:1209.1352], many more wishlist:

- resum low x logarithms
- make use of resulting TMD/unintegrated parton distribution
- continue in a smooth way to the large x region

first task: implement complete collinear evolution into the framework
→ extension into large x region at the very least for inclusive observables

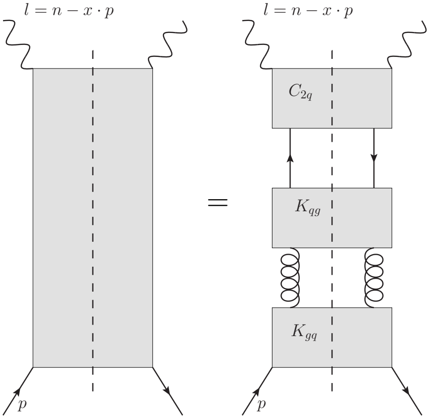
goal:

coupled system of evolution equations for updfs
→ k_T dependent splitting kernels

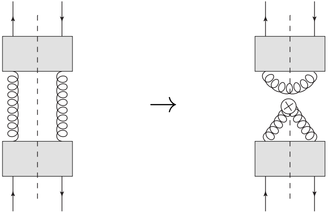
A possible way to achieve this ....

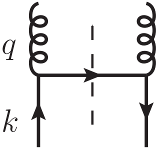
use formulation of DGLAP evolution/collinear factorization in terms of 2 particle irreducible expansion, *e.g.* DIS process

[Curci, Furmanski, Petronzio, Nucl.Phys. B 175 (1980) 27]



- axial, light-cone gauge: collinear singularities only form propagator which connect sub-amplitudes
- to isolate coefficient of collinear singularities use projectors in spinor/Lorentz space
- calculate DGLAP splitting functions as expansion in α_s





"upper" (outgoing) projectors:

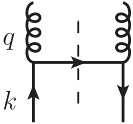
$$\mathbb{P}^{\mu\nu}_{\text{gluon, out}} = -g^{\mu\nu}, \qquad \mathbb{P}_{\text{quark, out}} = \frac{\not{n}}{2q\cdot n}$$

"lower" (incoming) projectors:

$$\mathbb{P}^{\mu\nu}_{\text{gluon, in}} = \frac{1}{d-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n}\right), \qquad \mathbb{P}_{\text{quark, in}} = \frac{\not{k}}{2}$$

Calculating a splitting function ...

$$\hat{K}_{gq}^{(0)}(q,k) \equiv$$



- contains propagator of out-going parton
- incoming propagators amputated + incoming on-shell $k^2 = 0$

$$\hat{K}_{gq}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 - q^2)\mathbb{P}_{q,\text{in}} \otimes \hat{K}_{gq}^{(0)}(q,k) \otimes \mathbb{P}_{g,\text{out}}$$

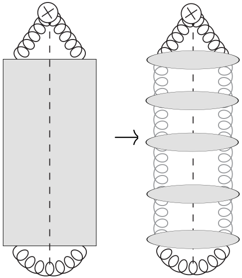
$$= \frac{\alpha_s}{2\pi\Gamma(1+\epsilon)}\, z \int_0^{\mu_F^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\left(\frac{e^{-\gamma_E}\boldsymbol{q}^2}{\mu^2}\right)^{\epsilon} P_{gq}^{(0)}(z;\epsilon)$$

allows to extract splitting function $P_{gq}^{(0)}(z;\epsilon)$

k_T factorization I

High energy/low x resummation of splitting functions

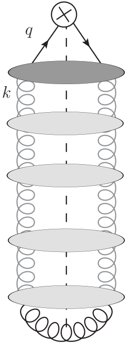
[Catani, Hautmann; NPB 427 (1994) 475]



- essentially the BFKL Green's function ➡ low x resummation of gluon splitting function
- use off-shell extension of incoming projector $\mathbb{P}^{\mu\nu}_{\text{gluon, in}} \to \frac{\boldsymbol{k}^\mu \boldsymbol{k}^\nu}{\boldsymbol{k}^2}$
- derived within high energy factorization + reduces to conventional projector in on-shell limit

obtain: all order P_{gg} with $(\alpha_s \ln 1/x)^n$
however:
all order P_{qg} requires $\alpha_s \, (\alpha_s \ln 1/x)^n$ (starts at NLL ➡ finite coefficient)

TMD gluon-to-quark splitting



upper blob: no low x logarithm; finite ➞ defines a TMD quark-to-gluon splitting function

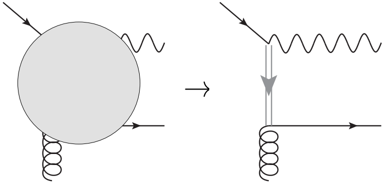
$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}, \epsilon\right) = \mathrm{Tr}\left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2 \cdot \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]$$

$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

so far: take into account off-shellness of incoming gluon (most upper gluon in ladder); quark standard collinear factorization ⇒ transverse momentum integrated over

towards a complete (double) off-shell splitting

task: off-shell factorization of a partonic process (with off-shell initial gluon) [Hautmann, MH, Jung; arXiv:1205.1759]



collinear limit: trivial ✓ but provides usual on-shell factorization ....

to achieve off-shellness: use high-energy factorization
t-channel quark exchange: reggeized quark formalism (high energy quark)
➡ generalization of gluon/singlet channel treatment to quarks ...

Modern formulation: high energy effective action

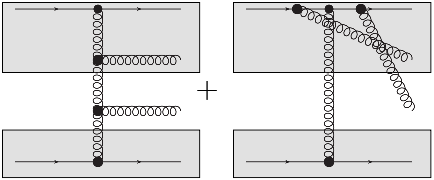
[Lipatov; hep-ph/9502308]

divide final state particles into clusters of particles "local in rapidity"

for each cluster

- integrate out specific details of fast $+/-$ fields
- dynamics in local cluster: QCD Lagrangian + universal eikonal factor

(up to power suppressed corrections)



→ effective field theory for **each cluster** of particles local in rapidity to reconstruct QCD scattering amplitudes: new (scalar) field $A_\pm$ properties:

- $\delta A_\pm = 0$ — invariant w.r.t. local gauge transformation, but charged under $SU(N_c)$
- $\partial_- A_+ = 0 = \partial_+ A_-$ — strong ordering in light-cone momenta $k^\pm$ between different clusters

$$S_{\text{eff}} = S_{\text{QCD}}[v_\mu, \psi] + S_{\text{ind}}[v_\mu, A_\pm]$$

$$S_{\text{ind}}[v_\mu, A_\pm] = \int d^4x \mathrm{tr}\left[v_+ U_+[v]\partial_\perp^2 A_- - A_+ \partial_\perp^2 A_-\right] + \{(+) \leftrightarrow (-)\}$$

$$U_\pm[v] = \frac{1}{\partial_\pm + g v_\pm(x)}\partial_\pm$$

tested in a number of NLO and 2-loop calculations

[MH, Sabio Vera; arXiv:1110.6741],[Chachamis, MH, Madrigal, Sabio Vera; arXiv:1202.0649, 1212.4992, 1307.2591] ...

Effective action for the quark sector [Lipatov, Vyazovsky; hep-ph/0009340]

quark exchange suppressed w.r.t. gluon exchange $\to$ not contained in the (general) high energy effective action; but can be equally formulated

$$S_{\mathsf{Q;eff}} = S_{\mathsf{QCD}}[v_\mu, \psi] + S_{\mathsf{Q;ind}}[v_\mu, a_\pm, \bar{a}_\pm]$$

$$S_{\mathsf{Q;ind}}[v_\mu, A_\pm] = \int d^4x \bigg\{ \bar{a}_- i\not{\partial} \left(a_+ - U_+[v]\psi\right) + \left(\bar{a}_+ - \bar{\psi}U_+[v]\right) a_- + \{(+) \leftrightarrow (-)\} \bigg\}, \qquad U_\pm[v] = \frac{1}{\partial_\pm + g v_\pm(x)} \partial_\pm$$

reggeized quark fields $a_\pm, \bar{a}_\pm$

$$\partial_+ a_- = 0 = \partial_- a_+ \qquad \partial_+ \bar{a}_- = 0 = \partial_- \bar{a}_+$$

$$\not{n}_- a_+ = 0 = \not{n}_+ a_- \qquad \bar{a}_+ \not{n}_- = 0 = \bar{a}_- \not{n}_+$$

$$\delta \bar{a}_\pm = 0 = \delta a_\pm$$

Feynman rules: t-channel propagators (factorized)

reggeized quark propagator: $p \cdot q = 0 = n \cdot q$

$$q \quad = \frac{(\not{n}\not{p})_{\beta\alpha}}{2p \cdot n} \cdot \frac{i \cdot \not{q}}{q^2 + i\epsilon}$$

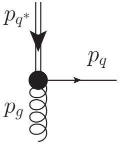
$$q \quad = \frac{(\not{p}\not{n})_{\alpha\beta}}{2p \cdot n} \cdot \frac{i \cdot \not{q}}{q^2 + i\epsilon}$$

useful property:

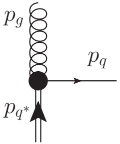
$$\frac{(\not{n}\not{p})_{\beta_1\alpha_1}(\not{p}\not{n})_{\alpha_2\beta_2}}{p \cdot n} = \underbrace{\not{p}_{\alpha_1\alpha_2}\not{n}_{\beta_1\beta_2}}_{\text{helicity independent input}} + (\gamma_5\not{p})_{\alpha_1\alpha_2}(\gamma_5\not{n})_{\beta_1\beta_2}.$$

→ recover (up to normalization) projectors of collinear factorization

Feynman rules: effective vertex = QCD + induced

$\Gamma_-^\mu =$ 

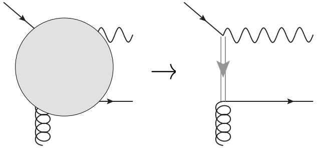
$= igt^a \left(\gamma^\mu + \frac{n^\mu}{n \cdot p_g} \not{p}_{q^*} \right)$ with $p_{q^*} \cdot n = 0$

$\Gamma_+^\mu =$ 

$= igt^a \left(\gamma^\mu + \frac{p^\mu}{p \cdot p_g} \not{p}_{q^*} \right)$ with $p_{q^*} \cdot p = 0$

vertices fulfill Ward identities: $\bar{u}(p)_q \Gamma_\pm^\mu \cdot p_{g,\mu} = 0$

First application: $g^* q \to Zq$



- allows for off-shell factorization, but high energy limit ($z \to 0$) yields a constant splitting function $P_{qg}^{\text{h.e.f.}} = \mathrm{Tr}$

however: at this order in g, possible to relax condition $p_{q^*} \cdot n = 0$ AND maintain current conservation property of vertex

re-obtain Catani-Hautmann splitting function (and off-shell extension of coefficient $qq^* \to Z$) [Hautmann, MH, Jung; arXiv:1205.1759]

Remaining splitting functions ...

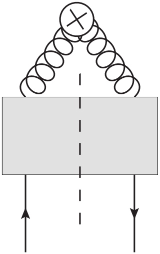
- cannot be defined/determined as coefficient of high energy resummation of DGLAP splitting function (unlike P_{qg})
- not well defined at first

HERE: attempt to fix/constrain them using

1. gauge invariance/current conservation of vertices
2. correct collinear limit
3. correct high energy limit

point 1. faces a challenge: Curci-Furmanski-Petronzi formalism based on light-cone/axial gauge ➡ what about gauge invariance?

Problem arrises in splittings with out-going gluons



- incoming gluons safe from high energy factorization
- outgoing gluons: both projector and amplitude at first limited to light-cone gauge

a) define new (upper) projector: $\Delta^{\mu\nu}(q,n) = -g^{\mu\nu} + \frac{q^\mu n^\nu + n^\mu q^\nu}{q \cdot n}$

$$\tilde{\mathbb{P}}^{\mu\nu}_{g,\text{out}}(q,n) \equiv \Delta_{\mu\mu'}(q)\mathbb{P}^{\mu'\nu'}_{g,\text{out}}\Delta_{\nu'\nu}(q) = -g_{\mu\nu} + \frac{q^\mu n^\nu + n^\mu q^\nu}{q \cdot n} - q^2 \frac{n^\mu n^\nu}{(q \cdot n)^2},$$

properties:

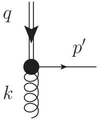
$$0 = \tilde{\mathbb{P}}^{\mu\nu}_{g,\text{out}} \cdot q_\mu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\text{out}} \cdot q_\nu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\text{out}} \cdot n_\mu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\text{out}} \cdot n_\nu$$

kills gauge dependent terms of adjacent propagators

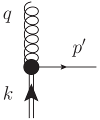
Current conservation for the amplitude

general definition (all orders etc.) to be achieved; for leading order splittings sufficient to use the following (generalized) vertices

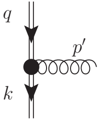
➡ fulfill Ward identities for $q^2 \neq 0$ & ${p'}^2 = 0$ & $k = xp + \boldsymbol{k}$



$$= igt^a \left(\gamma^\mu - \frac{n^\mu}{k \cdot n} \not{q} \right)$$

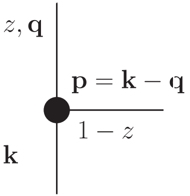


$$= igt^a \left(\gamma^\mu - \frac{p^\mu}{k \cdot p} \not{k} \right)$$



$$= igt^a \left(\gamma^\mu - \frac{p^\mu}{p \cdot p'} \not{k} + \frac{n^\mu}{n \cdot p'} \not{q} \right)$$

Results for splitting functions



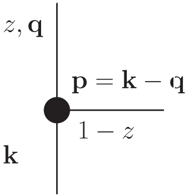
$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

$$\hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \frac{\alpha_s}{2\pi} z \int_0^{(1-z)(\mu_F^2 - z\boldsymbol{k}^2)} \frac{d\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2} \left(\frac{\boldsymbol{\Delta}^2}{\mu^2}\right)^{\epsilon} \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P_{ij}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right),$$

a) reproduce Catani-Hautmann splitting functions

$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right) = \mathrm{Tr}\left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]$$

New Results: quark induced splitting functions

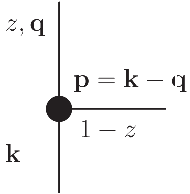


$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

$$P_{gq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right) = C_f \left[\frac{2\boldsymbol{\Delta}^2}{z|\boldsymbol{\Delta}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{\boldsymbol{\Delta}^2(\boldsymbol{\Delta}^2(2-z) + \boldsymbol{k}^2 z(1-z^2)) - \epsilon z(\boldsymbol{\Delta}^2 + (1-z)^2\boldsymbol{k}^2)}{(\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2)^2} \right],$$

$$P_{qq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right) = C_f \left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2}\right) \left[\frac{\boldsymbol{\Delta}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\boldsymbol{\Delta}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\boldsymbol{\Delta}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2 + (1-z)^2\epsilon(\boldsymbol{\Delta}^2 + z^2\boldsymbol{k}^2)}{(1-z)(\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2)} \right].$$

Analysis



$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

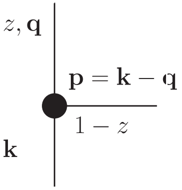
in the collinear limit $\boldsymbol{k}^2/\boldsymbol{\Delta}^2 \to 0$: find real part of DGLAP splitting functions ✓

singularities:

- $\lim\limits_{z\to 1} P_{qq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right) = \frac{2 \cdot C_f}{1-z}$ – coincides with singularity of DGLAP case
- both splitting functions: singularity for $\boldsymbol{p} \to 0$

Singularities: $p \to 0$

- best analysed for kernels NOT averaged over azimuthal angle
- additional benefit: appropiate kernels to describe *i.e.* angular decorrelation of jets, see *e.g* [Chachamis, Deak, Sabio Vera, Stephens; arXiv:1102.1890]



$\tilde{\boldsymbol{p}} = \frac{\boldsymbol{p}}{1-z}$

$$\hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu_F^2}, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \int \frac{d^{2+2\epsilon}\tilde{\boldsymbol{p}}}{\pi^{1+\epsilon}(\mu^2 e^{\gamma_E})^\epsilon} \Theta\left(\mu_F^2 - q^2\right) \frac{\tilde{P}_{ij}\left(z, \tilde{\boldsymbol{p}}, \boldsymbol{k}, \epsilon\right)}{\tilde{\boldsymbol{p}}^2}$$

the coefficients of the singularities:

$$\lim_{\tilde{\boldsymbol{p}}^2 \to 0} \tilde{P}_{qq} = \frac{2 \cdot C_f}{(1-z)^{1-2\epsilon}} \qquad \lim_{\tilde{\boldsymbol{p}}^2 \to 0} \tilde{P}_{gq} = \frac{2 \cdot C_f (1-z)^{2\epsilon}}{z}$$

Coincides with high energy ($z \to 0$ for P_{gq}) and soft ($z \to 1$ for P_{qq}) poles!

A first study of the $p \to 0$ limit

real part of P_{qq} to be complemented by virtual corrections $\to$ can expect cancelation of (some?) of the singularities – to be done (on our list)

P_{gq} expect at first no virtual corrections ➡ can test whether one can formulate evolution equations with this splitting functions which 'make sense'

to start: evolution equation for unintegrated gluon $\mathcal{F}$ (BFKL equation)

$$\mathcal{F}(x, \boldsymbol{q}^2) = \mathcal{F}^0(x, \boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \left[\mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) - \theta(\boldsymbol{q}^2 - \boldsymbol{p}^2) \mathcal{F}\left(\frac{x}{z}, \boldsymbol{q}^2\right) \right]$$

& add quark induced contribution

$$+ \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z, \boldsymbol{p}, \boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

A (common) reformulation of the BFKL equation

a) introduce phase space slicing parameter $\mu \to 0$ to separate hard from soft ($\boldsymbol{p} \to 0$) emissions b) for $|\boldsymbol{p}| < \mu \Rightarrow |\boldsymbol{p} + \boldsymbol{q}| \to |\boldsymbol{q}|$ ➡ allows for cominbation with virtual corrections

$$\mathcal{F}(x, \boldsymbol{q}^2) = \mathcal{F}^0(x, \boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \left[\int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) - \ln \frac{\boldsymbol{q}^2}{\mu^2} \mathcal{F}\left(\frac{x}{z}, \boldsymbol{q}^2\right) \right]$$

b) using Mellin transform $(\omega \leftrightarrow x)$ this can be re-written as

$$\mathcal{F}(x, \boldsymbol{q}^2) = \underbrace{\tilde{\mathcal{F}}^0(x, \boldsymbol{q}^2)}_{\text{modified}} + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \underbrace{\Delta_R(z, \boldsymbol{q}^2, \mu^2)}_{\exp(-\overline{\alpha}_s \ln 1/z \ln \boldsymbol{q}^2/\mu^2)} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right)$$

stable in the limit $\mu \to 0$!

Can do the same with the quark included ...

the crucial difference: no virtual corrections
Necessarily $\mu \to 0$ ... stable?

$\longrightarrow$ $\int \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2} \to \int_{\mu^2} \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2}$

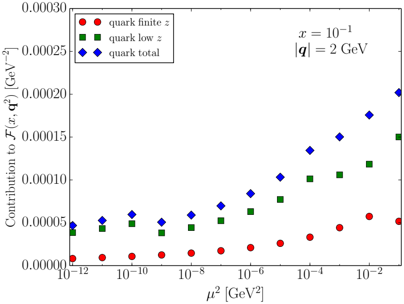
$$\text{'BFKL'} + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z, \boldsymbol{p}, \boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

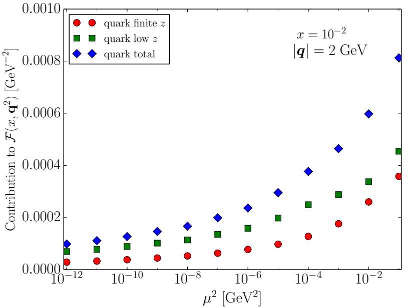
using again Mellin transform $(\omega \leftrightarrow x)$ this can be rewritten in the following general form (with $P_{gq} = \tilde{P}_{gq}/z$):

$$\mathcal{F}(x, \boldsymbol{q}^2) = \tilde{\mathcal{F}}^0(x, \boldsymbol{q}^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \theta(\boldsymbol{p}^2 - \mu^2) \Bigg[\Delta_R(z, \boldsymbol{q}^2, \mu^2) \left(2C_A \mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) + C_F \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) \right) - \int_z^1 \frac{dz_1}{z_1} \Delta_R(z_1, \boldsymbol{q}^2, \mu^2) \left[\tilde{P}'_{gq}\left(\frac{z}{z_1}, \boldsymbol{p}, \boldsymbol{q}\right) \frac{z}{z_1} \right] \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) \Bigg]$$

A first study

use for $\mathcal{F}, \mathcal{Q}$ the DLC 2016 set of parton densities [Kutak, Maciula, Serino, Szczurek, van Hameren; arXiv:1602.06814] ($\sim$ modified KMR updf set)





- quark finite z: $\sim \boldsymbol{p}^2$ ➡ finite
- mechanism: resummation of $\ln \boldsymbol{q}^2/\mu^2$ in $\Delta_R = \left(\frac{\mu^2}{\boldsymbol{q}^2}\right)^{\overline{\alpha}_s \ln 1/z}$ cuts off $\mu \to 0$ region ➡ finiteness

It's just the beginning it's not the end....

What did we do? .... k_T dependent splitting function with correct collinear and correct high energy limits ✓

$\rightarrow$ contain terms fixed by none of the two limits ("interpolation + gauge invariance")

additional result (not mentioned): the gluon evolution equation can be extended to included non-linear (saturation) correction
➡ see arXiv:1607.01507

It's just the beginning it's not the end

Comments:

- a better understanding of $\boldsymbol{p} \to 0$ limit desireable $\leftrightarrow$ resummation of gluon (quark?) virtual correction gives already numerical stability

- long term: systematic formulation (operator definition, prescription how to calculate in principle higher order corrections etc.)

- for the moment: an extension of low x updfs to complete x range + missing virtual corrections + P_{gg} in the same frame

work in progress ...