Resummation, Evolution, Factorization with Massive Quarks in Drell-Yan at low q_T

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based on work with
Daniel Samitz, Anne Spiering, and Frank Tackmann

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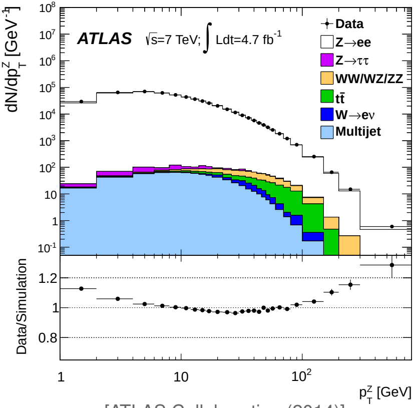
Drell-Yan at small q_T

- spectrum measured with high precision up to low q_T
- analytic high precision calculations, up to NNLL$'$+NNLO (soon N^{3}LL)
- no systematic description of b-mass effects at low q_T
  $\rightarrow$ e.g. important for m_W measurement
- in MC's: initial state splitting with massive quarks not well understood



[ATLAS Collaboration (2014)]

$\Rightarrow$ Goal: Factorization framework for massive quark effects (using EFTs), explicit results at NNLL$'$ for Z/γ^* (NNLL resummation with FO ingredients at $\mathcal{O}(\alpha_s^2)$)

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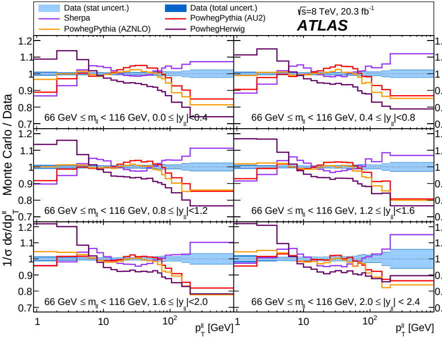

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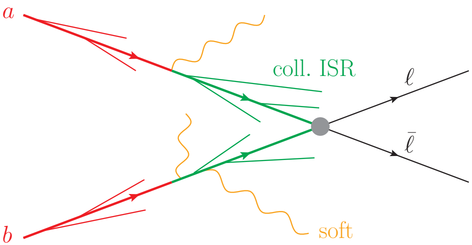
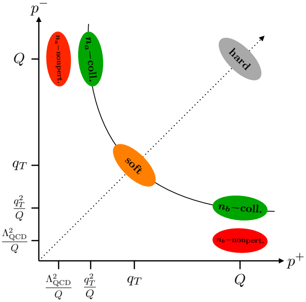
Outline

1. Factorization with massless quarks
2. Factorization with massive quarks
3. Resummation with massive quarks
4. Conclusions & Outlook

Outline

EFT modes for small q_T





DY with $q_T \ll Q = \sqrt{p_Z^2}$ is a multiscale process:

- hard process: $q\bar{q} \to Z/\gamma^* \to \ell\bar{\ell}$ at scale $\mu \sim Q$
- n_a-/n_b-collinear ISR at scale $\mu \sim q_T$
- wide-angle soft ISR at scale $\mu \sim q_T$
- nonperturbative collinear proton at scale $\mu \sim \Lambda_{\rm QCD}$

large scale hierarchies ⇒ large logs $\ln(q_T/Q)$, $\ln(\Lambda_{\rm QCD}/q_T)$ in perturbation theory

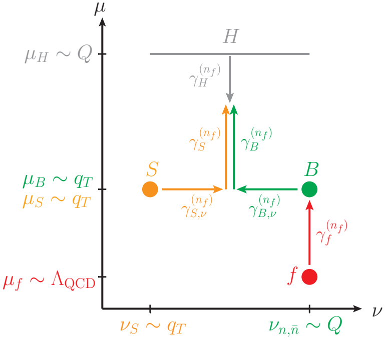
Factorization for small q_T

[Collins, Soper, Sterman (1985); Catani, de Florian, Grazzini (2001), Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2011); Chiu, Jain, Neill, Rothstein (2012); . . .]

Factorization theorem for n_f massless quarks: $(i, j, k \in \{q, \bar{q}, g\})$

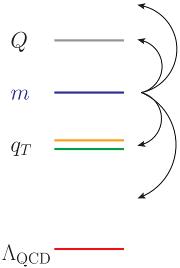
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_f)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_f)} \otimes f_k^{(n_f)}\Big]^2 \otimes S^{(n_f)} + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{q_T^2}\right)$$

- hard function $H_{ij}(Q)$: process dependence
- beam function $B_i = \sum_k \mathcal{I}_{ik} \otimes_x f_k$
  - $\to$ collinear ISR matching $\mathcal{I}_{ik}(\vec{q}_T, z)$
  - $\to$ nonperturbative PDF $f_k(\Lambda_{\mathrm{QCD}}, x)$
- soft function $S(\vec{q}_T)$: wide-angle soft radiation
- resummation via evolution factors (implicit)
- rapidity divergences in S and B_i
  - $\to$ associated rapidity logarithms
  - $\to$ resummed via rapidity RGE
    [Chiu, Jain, Neill, Rothstein (2012)]



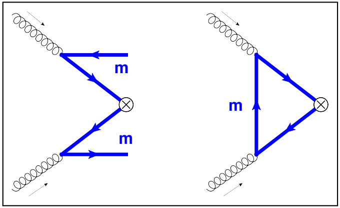
Outline

Massive quark effects

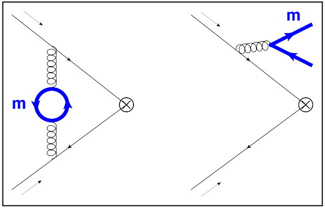


- additional scale m: different hierarchies possible
- required: multi-scale “Variable-flavor number scheme"
  for e^+e^- event shapes, DIS at threshold:
  [Gritschacher, Hoang, Jemos, Mateu, P.P. (2014); Hoang, P.P., Samitz (2015)]
- for Z/γ^*: $\mathcal{O}(\alpha_s^2)$ corrections
  $\rightarrow$ relevant at NNLL$'$ (evolution affected already at LL)

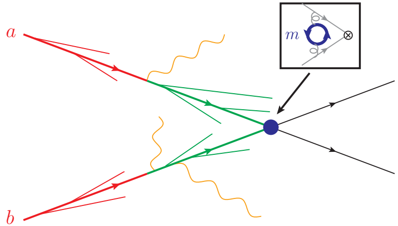
primary massive quark corrections:

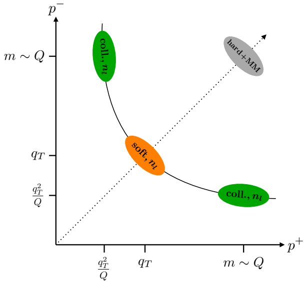


secondary massive quark corrections:



$\Lambda_{\rm QCD} \ll q_T \ll m \sim Q$



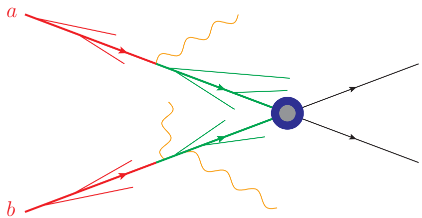


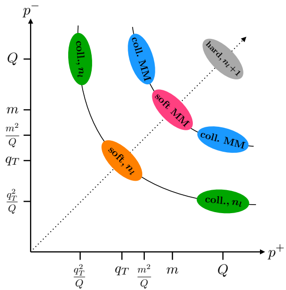
Factorization theorem: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

- virtual massive quark corrections to hard function
  $\rightarrow$ decouple for $m \gg Q$ (for conserved vector current)
- beam and soft function with n_l massless flavors

$\Lambda_{\mathrm{QCD}} \ll q_T \ll m \ll Q$



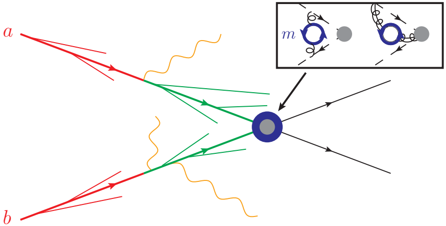


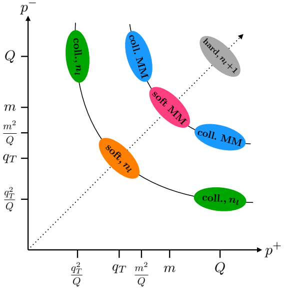
Factorization theorem: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

- two step matching: (1) QCD $\to$ SCET$^{(n_l+1)}$, (2) SCET$^{(n_l+1)}$ $\to$ SCET$^{(n_l)}$
- hard function with $n_l + 1$ massless quarks
- beam and soft function with n_l massless quarks

$\Lambda_{\rm QCD} \ll q_T \ll m \ll Q$





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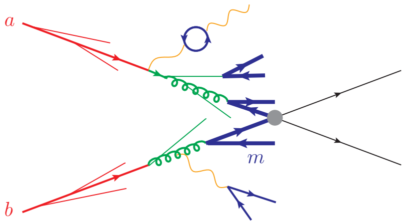
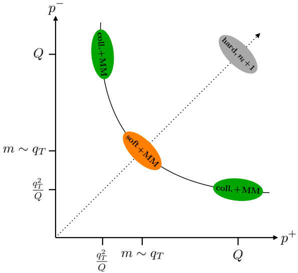
- collinear & soft massive quark matching factors H_c, H_s (at $\mu_m \sim m$)
  - $\rightarrow$ only secondary virtual corrections at $\mathcal{O}(\alpha_s^2)$
  - $\rightarrow$ yield together rapidity logarithm $\ln(Q/m)$
  - $\rightarrow$ independent of hard process (for 0 jets) and measurement

see also [Gritschacher, Hoang, Jemos, Mateu, P.P. (2014); Hoang, P.P., Samitz (2015); Hoang, Pathak, P.P, Stewart (2015)]

$\Lambda_{\rm QCD} \ll m \sim q_T \ll Q$





Factorization theorem: $(i,j \in \{q,\bar q, Q, \bar Q, g\}, k \in \{q, \bar q, g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S(m) + \mathcal{O}\left(\frac{m}{Q}, \frac{\Lambda^2_{\rm QCD}}{m^2}\right)$$

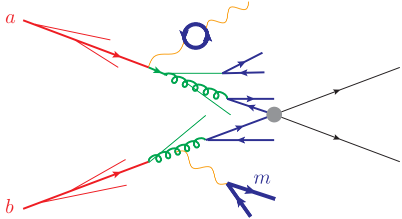
- primary and secondary massive quark corrections to beam functions/TMDs:

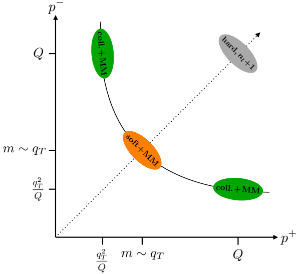
$$B_i^{(n_l+1)}(\vec q_T, x, m) = \sum_{k\in\{q,\bar q, g\}} \mathcal{I}_{ik}(\vec q_T, x, m) \otimes f_k^{(n_l)}(x)$$

$\to \mathcal{O}(\alpha_s)$ primary massive: $\mathcal{I}_{Qg}^{(1)}(\vec q_T, x, m)$ ✓ [→ massive quark TMD]

$\to \mathcal{O}(\alpha_s^2)$ secondary massive: $\mathcal{I}_{qq}^{(2)}(\vec q_T, x, m)$ ✓

$\Lambda_{\mathrm{QCD}} \ll m \sim q_T \ll Q$



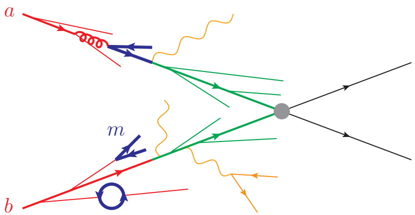


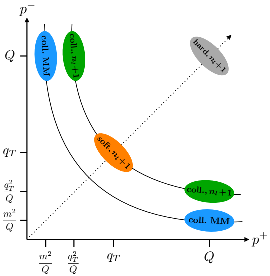
Factorization theorem: $(i, j \in \{q, \bar{q}, Q, \bar{Q}, g\}, k \in \{q, \bar{q}, g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S(m) + \mathcal{O}\left(\frac{m}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

- $\mathcal{O}(\alpha_s^2)$ secondary massive quark corrections to soft function: $S^{(2)}(\vec{q}_T, m)$ ✓
- mass dependent rapidity divergences (and logs)

$\Lambda_{\mathrm{QCD}} \ll m \ll q_T \ll Q$



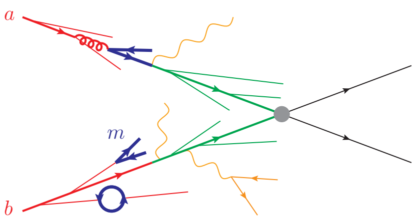


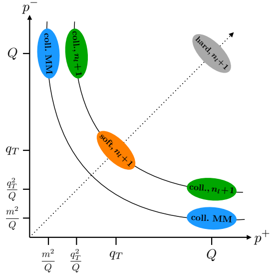
Factorization theorem: $(i,j,m \in \{q,\bar{q},Q,\bar{Q},g\}, k \in \{q,\bar{q},g\})$

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

- hard fct, coll. ISR matching and soft fct with n_l+1 massless flavors

$\Lambda_{\mathrm{QCD}} \ll m \ll q_T \ll Q$





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$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2} \right)$$

- primary and secondary massive quark corrections in PDF matching

$$f_i^{(n_l+1)}(x, m) = \sum_{k \in \{q, \bar{q}, g\}} \mathcal{M}_{ik}(x, m) \otimes f_k^{(n_l)}(x)$$

$\rightarrow \mathcal{O}(\alpha_s)$ primary massive: $\mathcal{M}_{Qg}^{(1)}(x, m)$ [$\rightarrow$ massive quark PDF]

$\rightarrow \mathcal{O}(\alpha_s^2)$ secondary massive: $\mathcal{M}_{qq}^{(2)}(x, m)$ [Buza, Matiounine, Smith, van Neerven (1998)]

Relations between factorization theorems

Factorization theorem for $m \sim q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S(m)$$

Factorization theorem for $m \ll q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

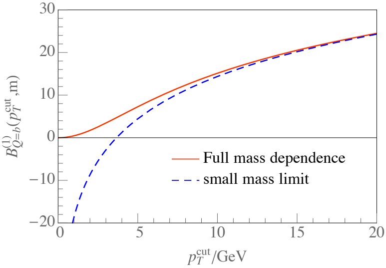
$\Rightarrow$ Relations between ingredients:

$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right], \quad S(m) = S^{(n_l+1)} \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right]$$

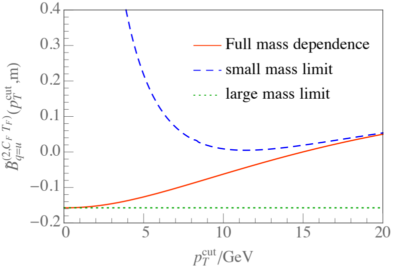
- checked explicitly at $\mathcal{O}(\alpha_s^2)$ using known massless results for $\mathcal{I}_{ij}^{(n_f)}$ and $S^{(n_f)}$ $\surd$
  [Gehrmann, Luebbert, Yang (2012); Luebbert, Oredsson, Stahlhofen (2016)]
- resummation of $\ln(m^2/q_T^2)$ can be combined with power corrections of $\mathcal{O}(m^2/q_T^2)$
- similar relations between other hierarchies
  $\Rightarrow$ continuous description over complete spectrum = (GM-)VFNS

“Nonsingular" bottom mass corrections

- most relevant hierarchies for m_b effects: $m_b \ll q_T$, $m_b \sim q_T$
- here: full m_b-dependent TMD cumulants vs. small mass limit at FO
- note: secondary corrections are rather $\mathcal{O}\left(\frac{(2m_b)^2}{q_T^2}\right)$



$\mathcal{O}(\alpha_s T_F)$ primary

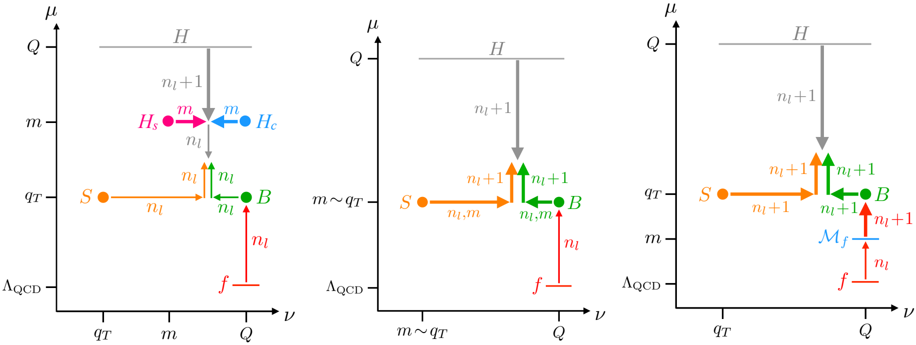


$\mathcal{O}(\alpha_s^2 C_F T_F)$ secondary

$[\tilde{B} = B \times \sqrt{S}]$

Outline

Resummation of logs



$q_T \ll m \ll Q$ $\quad$ $m \sim q_T \ll Q$ $\quad$ $m \ll q_T \ll Q$

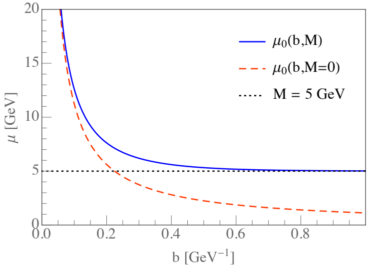
- μ-evolution with $n_l = 4$ quark flavors below the mass scale
- μ-evolution with $n_l + 1 = 5$ quark flavors above the mass scale
- for $q_T \ll m \ll Q$: additional ν-evolution at $\mu_m \sim m \rightarrow$ straightforward solution
- for $q_T \sim m$: ν-evolution modified by quark mass (due to secondary effects)

Resummation of rapidity logs for $q_T \sim m$

- convenient in impact parameter (= Fourier) space ($\vec{p}_T \leftrightarrow \vec{b}$)
- avoid (or otherwise resum) large logarithms in anomalous dimension $\gamma_\nu(b,\mu)$
- illustration at one-loop for $\gamma_\nu \equiv \gamma_{S,\nu}$ with massless/massive gluon:

$$M=0\text{:}\ \gamma_\nu(b,\mu) = -\frac{2\alpha_s(\mu)C_F}{\pi}\ln\Big(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\Big) \qquad \Rightarrow \mu \sim \mu_0(b) \equiv \frac{2e^{-\gamma_E}}{b}$$

$$M\neq 0\text{:}\ \gamma_\nu(b,M,\mu) = \frac{2\alpha_s(\mu)C_F}{\pi}\Big(\ln\frac{M^2}{\mu^2} + 2K_0(b\,M)\Big) \quad \Rightarrow \mu \sim \mu_0(b,M) \equiv M e^{K_0(b\,M)}$$



$$\mu_0(b,M) \xrightarrow{b\to 0} \frac{2e^{-\gamma_E}}{b}$$
$$\mu_0(b,M) \xrightarrow{b\to\infty} M$$

$\Rightarrow$ mass introduces IR cutoff

$\Rightarrow$ no Landau pole for $b \to \infty$

- similar for massive quarks at $\mathcal{O}(\alpha_s^2)$ (with correct choices for α_s-scheme)

Outline

Summary

Conclusions:

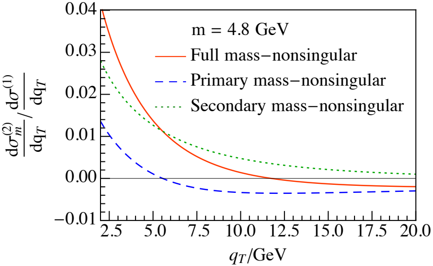
- factorization with massive quarks for Drell-Yan + 0 jets at low q_T ✓
- required ingredients for resummation of m_b-logs at NNLL$'$ accuracy ✓
- setup & ingredients also for beam thrust ✓

Outlook:

- phenomenological analysis of m_b effects for q_T-spectrum

quark mass corrections at $\mathcal{O}(\alpha_s^2)$ to Z-spectrum at FO

$\Rightarrow$ percent level effect for $q_T \sim m_b$



Summary

Conclusions:

- factorization with massive quarks for Drell-Yan + 0 jets at low q_T ✓
- required ingredients for resummation of m_b-logs at NNLL$'$ accuracy ✓
- setup & ingredients also for beam thrust ✓

Outlook:

- phenomenological analysis of m_b effects for q_T-spectrum
- charm quark effects for W-production at NNLL$'$ (need $\mathcal{O}(\alpha_s^2)$ primary corrections)
- application to other processes, e.g. H-production
  $\rightarrow$ due to Casimir scaling of γ_ν all ingredients known for NNLL resummation

Outline

Massless factorization theorem (explicit)

Factorization for $q_T \ll Q$: (quark initiated channels)

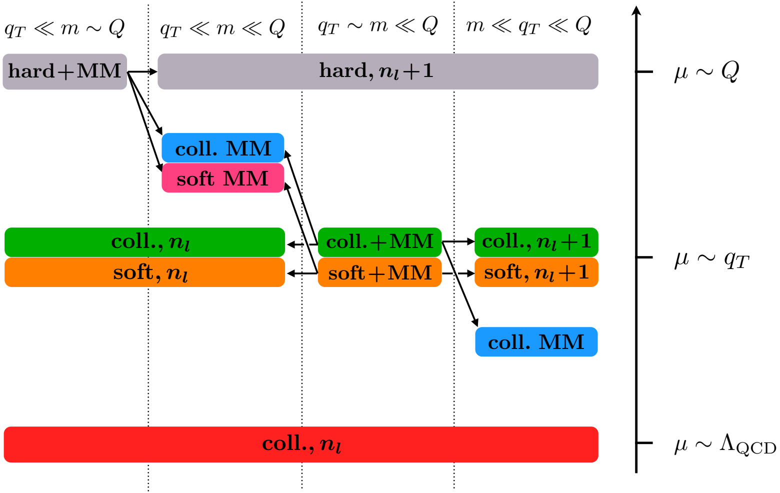
$$\frac{d\sigma}{dq_T^2\, dQ^2\, dY} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_f)}(Q,\mu) \int d^2p_{T,a}\, d^2p_{T,b}\, d^2p_{T,s}\, \delta(q_T^2 - |\vec{p}_{T,a} + \vec{p}_{T,b} + \vec{p}_{T,s}|^2)$$
$$\times B_i^{(n_f)}\Big(\vec{p}_{T,a}, x_a, \mu, \frac{\nu}{\omega_a}\Big)\, B_j^{(n_f)}\Big(\vec{p}_{T,b}, x_b, \mu, \frac{\nu}{\omega_b}\Big)\, S^{(n_f)}(\vec{p}_{T,s}, \mu, \nu)$$
$$\times \Big[1 + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{\Lambda_{\rm QCD}^2}{q_T^2}\Big)\Big]$$

with

$$\omega_a = Qe^Y\,, \quad \omega_b = Qe^{-Y}\,, \quad x_{a,b} = \frac{\omega_{a,b}}{E_{\rm cm}}\,.$$

$$B_i^{(n_f)}\Big(\vec{p}_T, x, \mu, \frac{\nu}{\omega}\Big) = \sum_k \underbrace{\int_x^1 \frac{dz}{z}\, \mathcal{I}_{ik}^{(n_f)}\Big(\vec{p}_T, \frac{x}{z}, \mu, \frac{\nu}{\omega}\Big)\, f_k^{(n_f)}(z,\mu)}_{\equiv\, \mathcal{I}_{ik}^{(n_f)}(\vec{p}_T, x, \mu, \frac{\nu}{\omega}) \otimes f_k^{(n_f)}(x,\mu)} \Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^2}{p_T^2}\Big)\Big],$$

Relations between modes



Relations between matrix elements:

Factorization theorem for $q_T \ll m \sim Q$: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

Factorization theorem for $q_T \ll m \ll Q$: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

$\Rightarrow$ relation between hard functions:

$$H_{ij}(m) = H_{ij}^{(n_l+1)}\, |H_c(m)|^2\, H_s(m) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Relations between matrix elements:

Factorization theorem for $q_T \ll m \ll Q$: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

Factorization theorem: $(i,j \in \{q,\bar{q},Q,\bar{Q},g\}, k \in \{q,\bar{q},g\})$

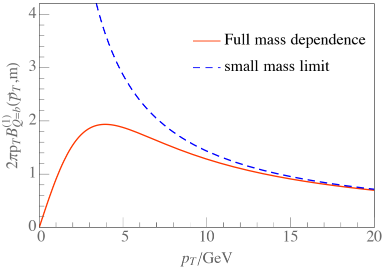
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S(m) + \mathcal{O}\left(\frac{m}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$\Rightarrow$ relations:

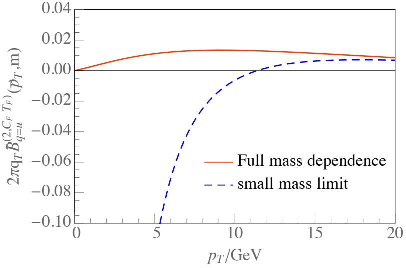
$$\mathcal{I}_{ik}(m) = H_c(m)\,\mathcal{I}_{ik}^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right), \quad S(m) = H_s(m)\,S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

"Nonsingular" mass corrections

m_b-dependent TMD beam functions (for $m_b \sim q_T$) vs. small mass limit at FO



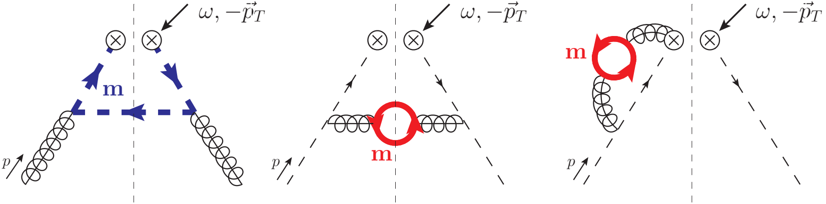
$\mathcal{O}(\alpha_s T_F)$ primary



$\mathcal{O}(\alpha_s^2 C_F T_F)$ secondary
$[\tilde{B} = B \times \sqrt{S}]$

Typical diagrams

beam function diagrams: primary and secondary (virtual and real)



soft function diagrams: only secondary (virtual and real)