

APFEL

A PDF Evolution Library

[V. Bertone, *et al.*, *Comput. Phys. Commun.* 185, 1647 (2014)]

Valerio Bertone

NIKHEF and VU Amsterdam



REF 2016

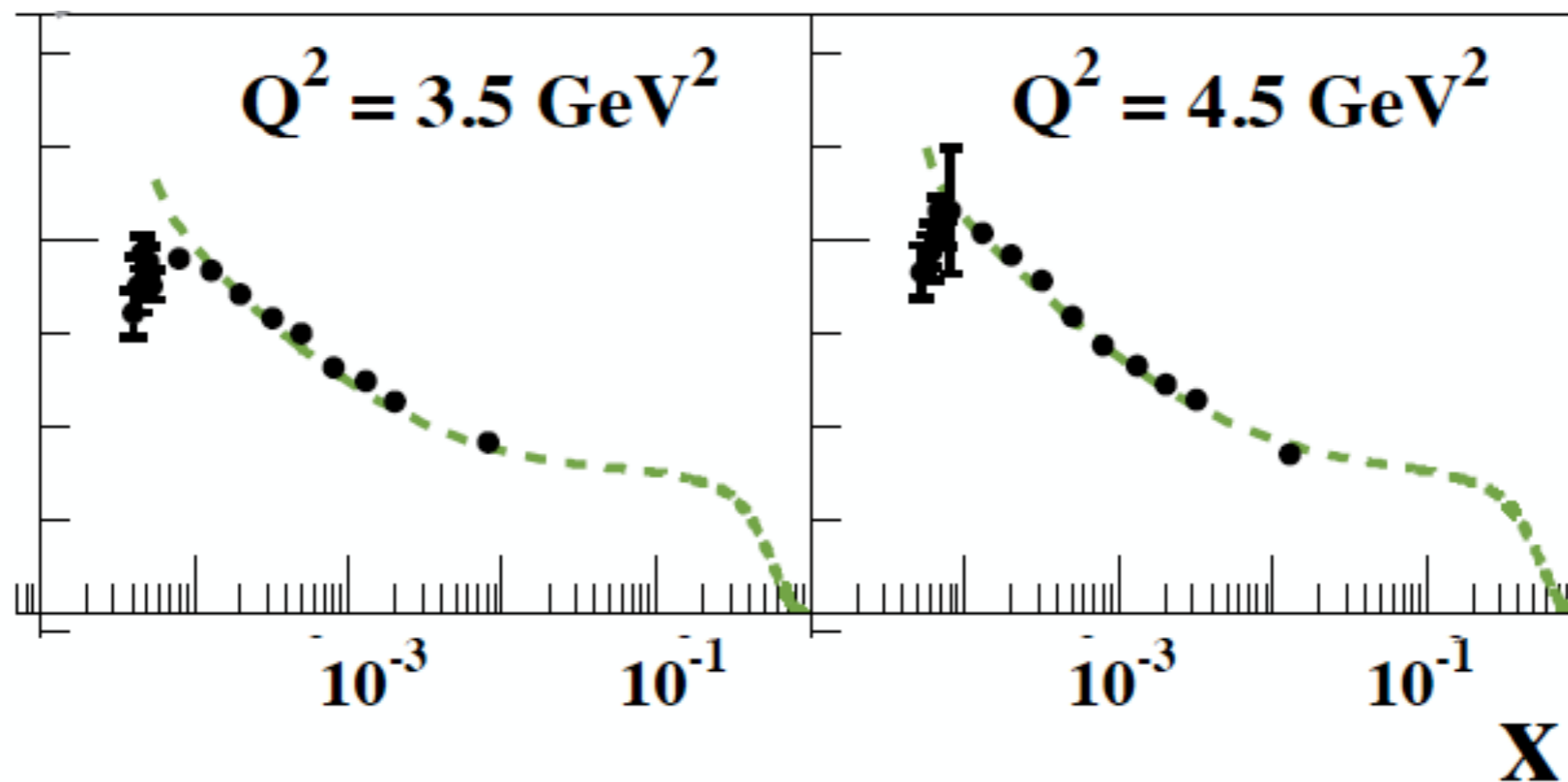
November 9, 2016, Antwerp

APFEL in a Nutshell

- 🍏 APFEL is a **public** library for the computation of collinear PDF evolution and DIS structure functions:
 - 🍏 up to NNLO in QCD combined to QED corrections up to NLO.
 - 🍏 FFN and VFN schemes.
 - 🍏 Pole and $\overline{\text{MS}}$ heavy-quark masses.
 - 🍏 fast computation of DIS NC and CC observables in different mass schemes (ZM-VFNS, FFNS and FONLL).
 - 🍏 Interfaces to FORTRAN, C/C++ and Python.
 - 🍏 Web interface available on <http://apfel.mi.infn.it>.
 - 🍏 available from <http://apfel.hepforge.org/>.
- 🍏 Interfaced to **xFitter** (see Ringaile's talk) and **Alpos**.
- 🍏 Used for the next generation of the **NNPDF** fits (including FFs).

Small- x Resummation

🍏 **Tension** between fixed-order predictions and data in the small- x region reached by HERA:



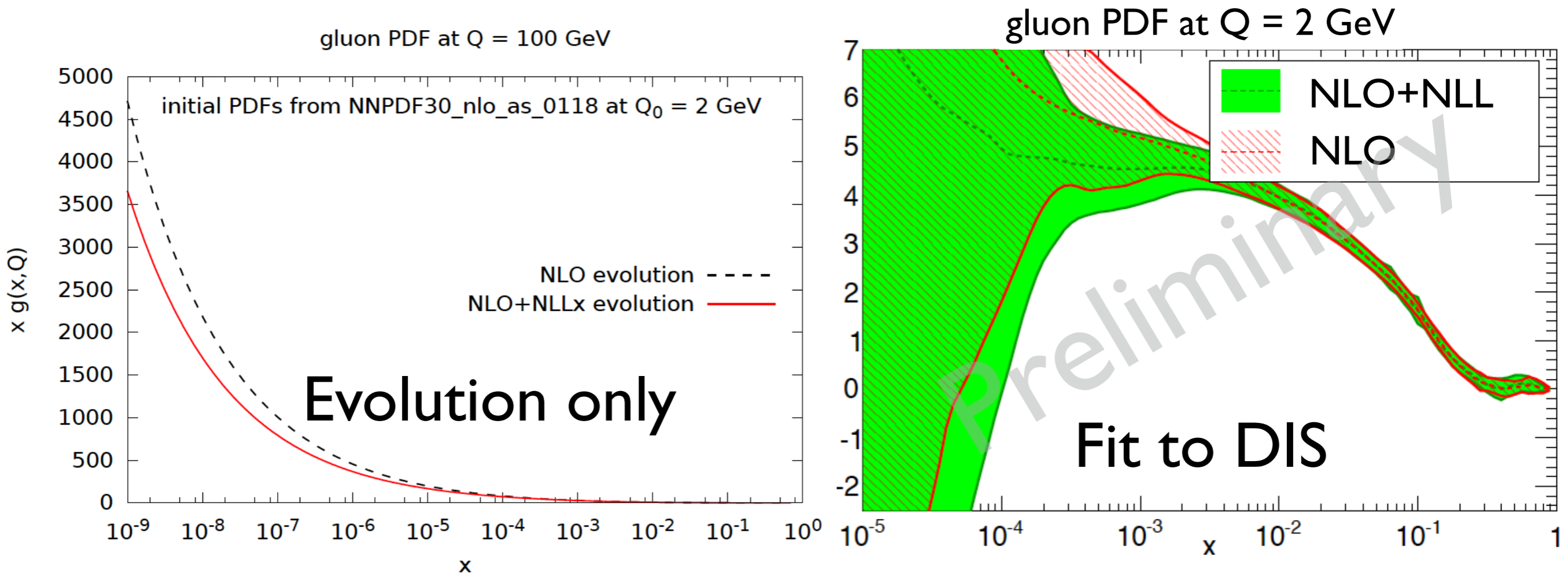
🍏 A similar effect was observed some time ago in the NNPDF framework by F. Caola *et al.* [[arXiv:1007.5405](https://arxiv.org/abs/1007.5405)].

🍏 Suggestion of the need for **small- x resummation**.

Small- x Resummation

- 🍏 The **HELL** code [[arXiv:1607.02153](#)] has been interfaced to APFEL:
 - 🍏 based on the ABF formalism (e.g. see [[hep-ph/9501231](#)]).
 - 🍏 Small- x **resummed splitting functions** up to **NLL** accuracy,
 - 🍏 Small- x **resummed DIS coefficient functions** up to **NLL**:
 - 🍏 massless (already available from APFEL),
 - 🍏 massive (since very recently \Rightarrow not interfaced to APFEL yet)
 - 🍏 Resummed matching conditions (not implemented in HELL, to come).
- 🍏 In NNPDF we have attempted DIS-only PDF fits with small- x resummation correction at NLL in the evolution and in the ZM sector of the DIS structure functions obtaining **encouraging results**.

Small- x Resummation



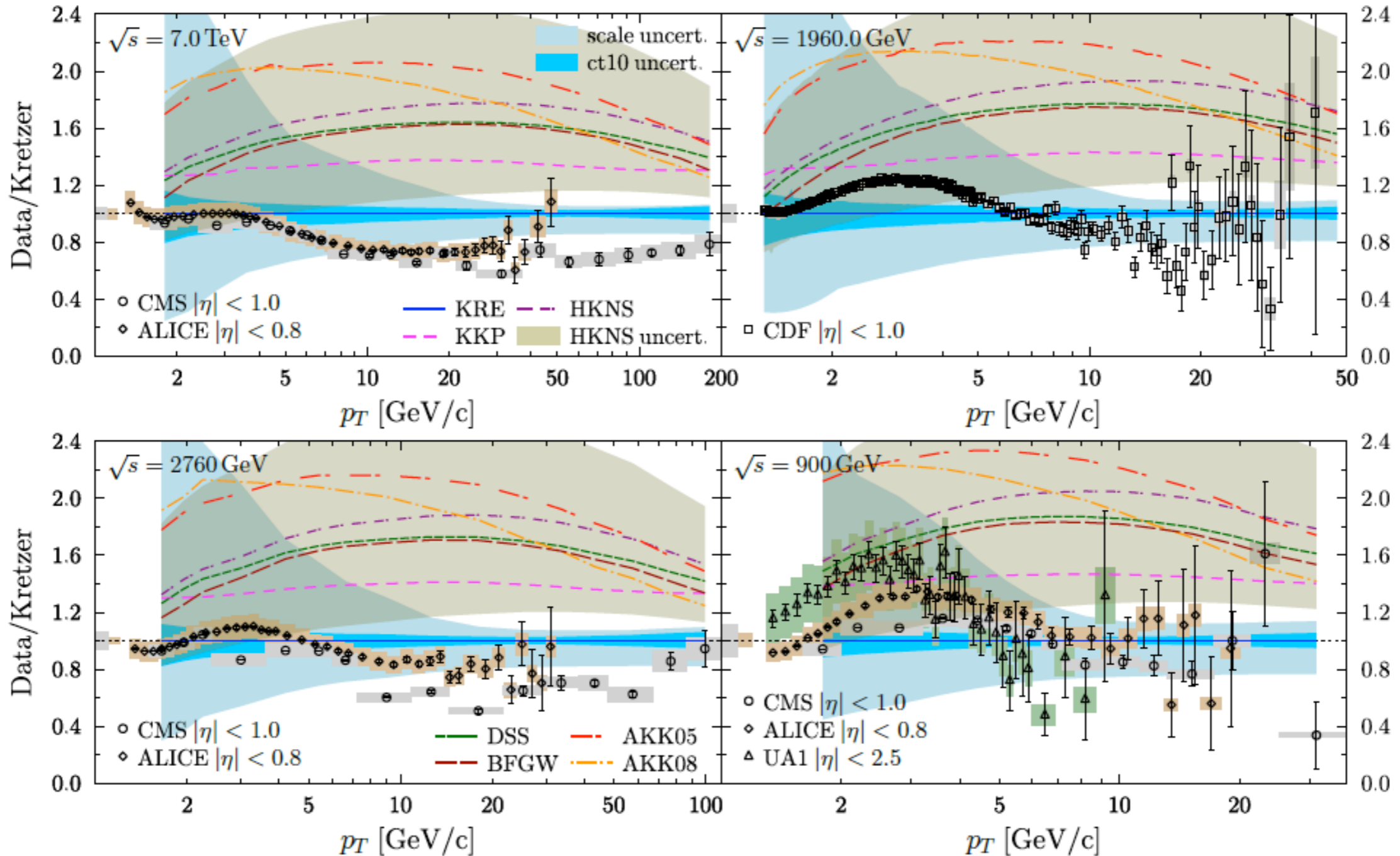
- 🍏 Resummed evolution leads to a **suppression** of the **gluon PDF** at small values of x as compared to fixed order.
- 🍏 **Compensation** when also resummed **coefficient functions** are introduced \Rightarrow effect on the small- x gluon PDF at the level of $1-\sigma$.
- 🍏 Other PDFs mostly unchanged.

Fragmentation Functions

- 🍏 A faithful determination of fragmentation functions (FFs) is extremely important to study the universality of the QCD factorisation theorems.
- 🍏 The inclusive hadron measurements at the LHC, sensibly extending the previous kinematical coverage, are particularly useful for studying the FFs.
- 🍏 Moreover, a good knowledge of FFs is functional to the determination of the **polarised PDFs**.
- 🍏 The **spread between the different FFs** present on the market is currently very large.
- 🍏 In addition, none of the existing FF sets describes the recent LHC and Tevatron experimental data.

Fragmentation Functions

🍏 Inclusive charged-hadron spectrum:



🍏 Large discrepancies that need to be understood.

Fragmentation Functions

🍏 APFEL implements the **time-like evolution**:

🍏 up to NLO in the VFNS,

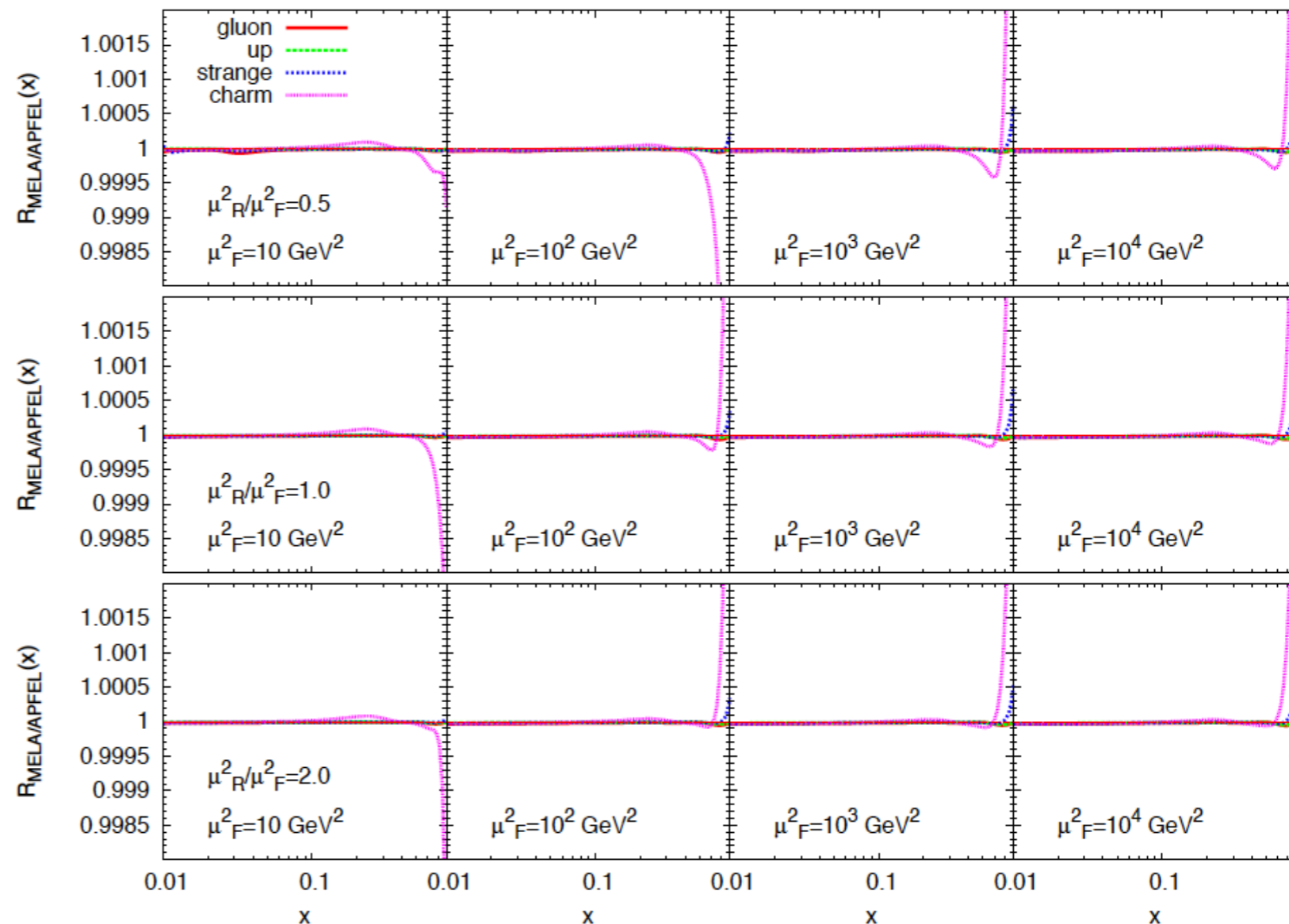
🍏 up to NNLO in the FFNS (NNLO matching conditions unknown).

APFEL vs. MELA: VFNS at NLO

🍏 Careful **benchmark** of the evolution against the MELA code
[arXiv:1501.00494].

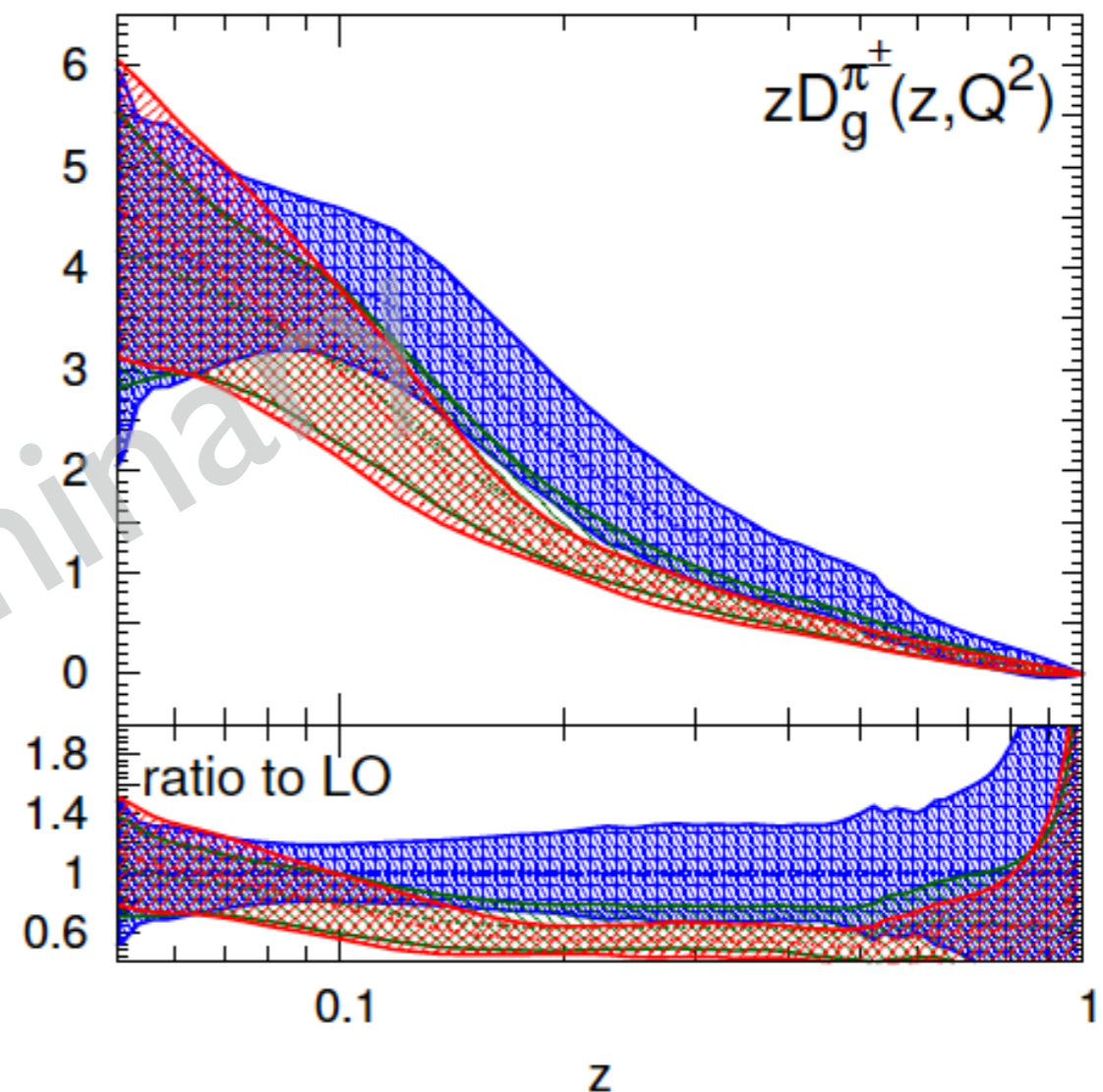
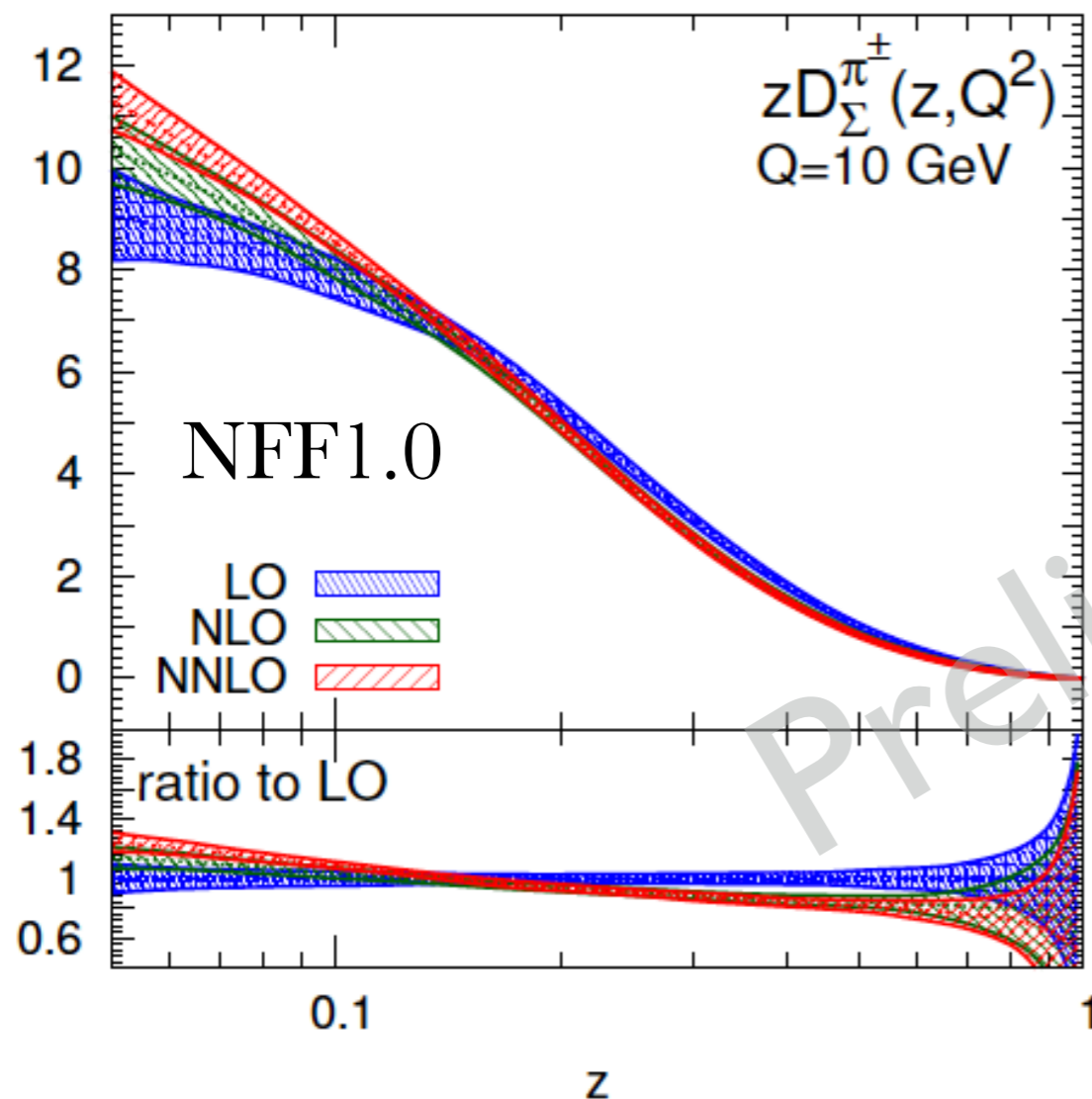
🍏 x -space vs. \mathcal{N} -space

🍏 Excellent agreement at all perturbative orders.



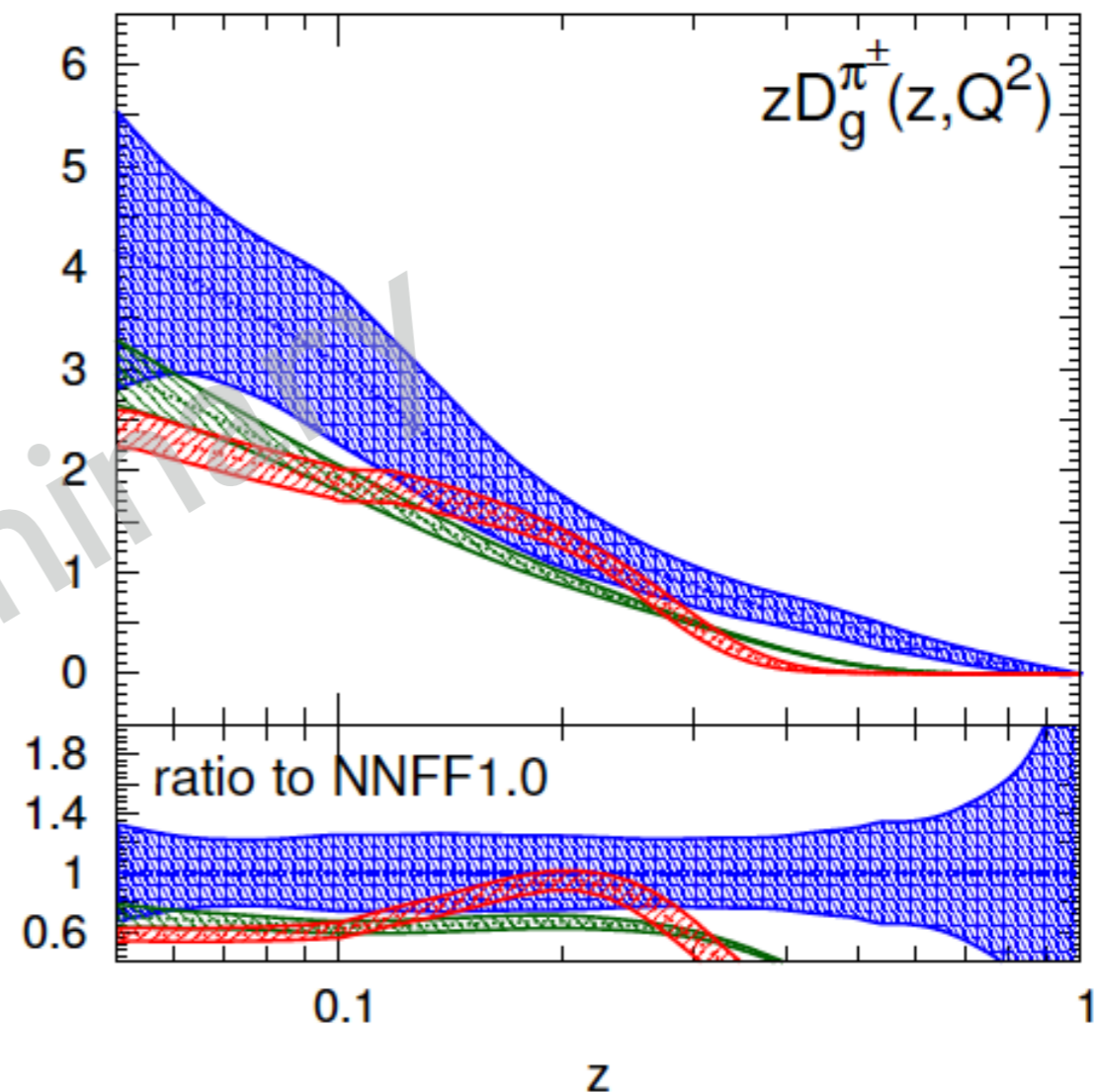
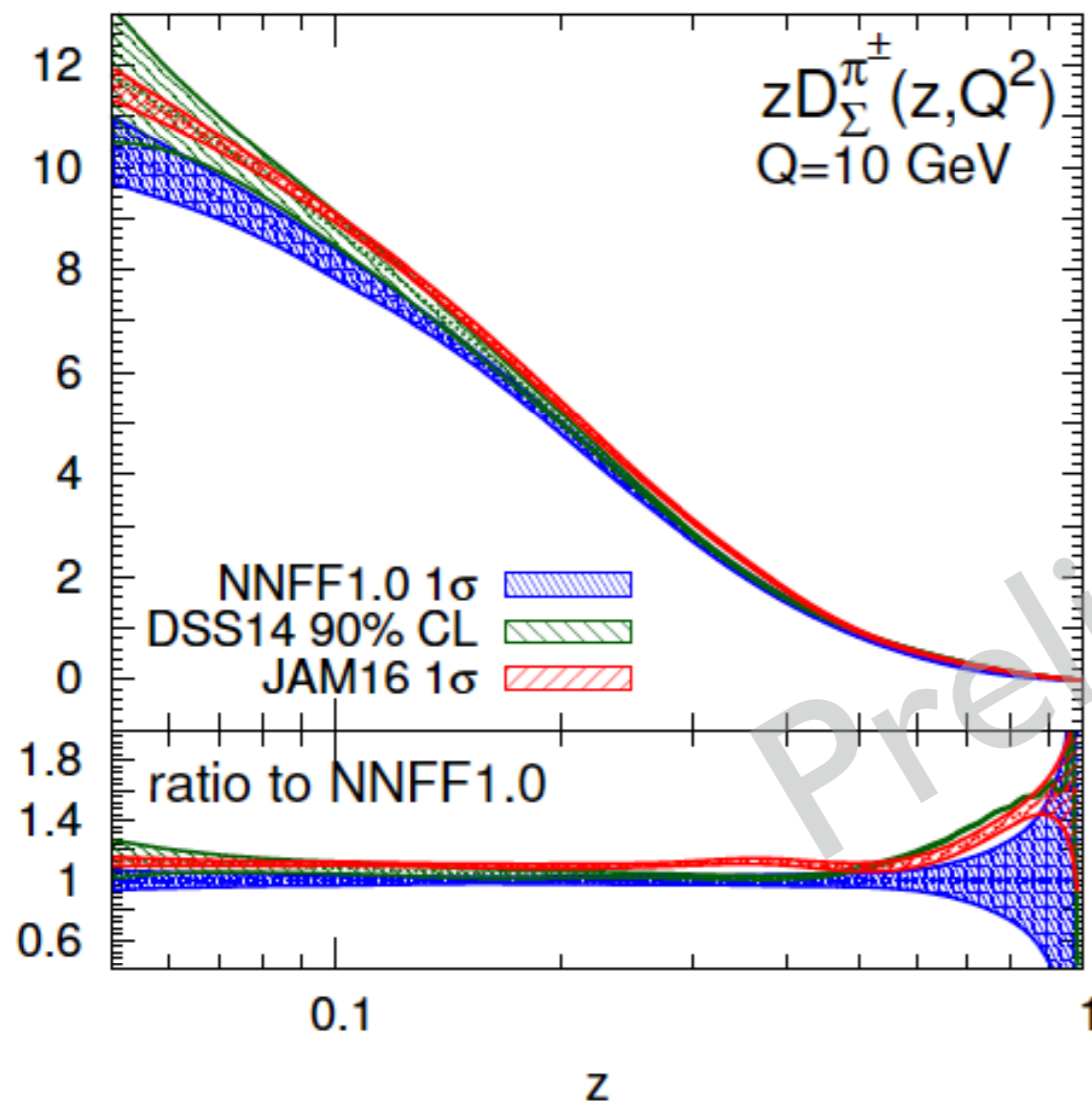
Fragmentation Functions

- Single-inclusive e^+e^- annihilation (SIA) structure functions are also implemented in APFEL up to NNLO in QCD:
 - partial benchmark against DSS code (thanks to D.P. Anderle).
- APFEL can now be used to **determine FFs** from SIA data.



Fragmentation Functions

- Single-inclusive e^+e^- annihilation (SIA) structure functions are also implemented in APFEL up to NNLO in QCD:
 - partial benchmark against DSS code (thanks to D.P. Anderle).
- APFEL can now be used to **determine FFs** from SIA data.



NLO QCD+QED Corrections

Evolution

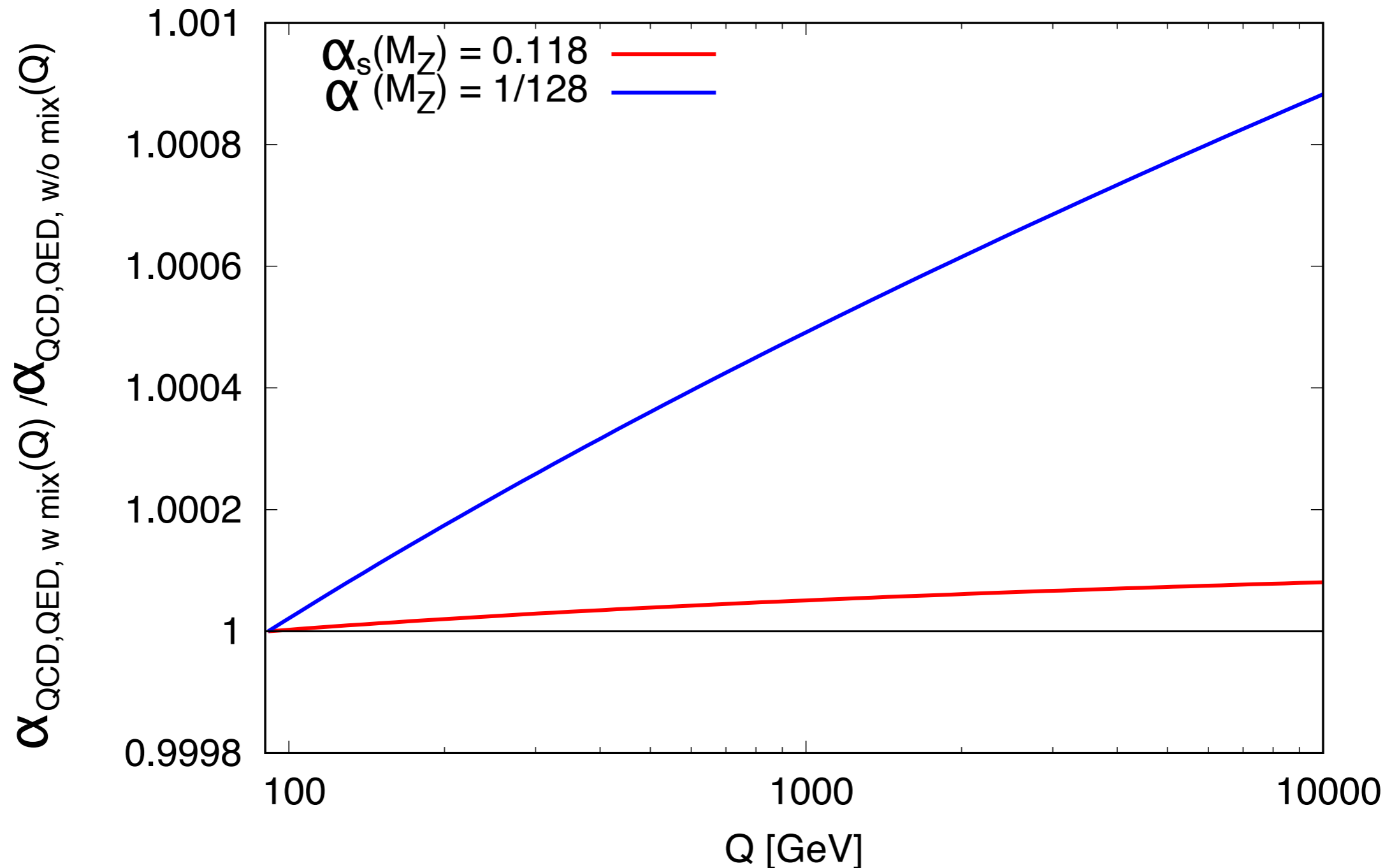
In order to implement the full NLO QCD+QED corrections in the DGLAP evolution two main steps are required:

1. Implementing the $O(\alpha_s^2\alpha)$, $O(\alpha^3)$, $O(\alpha^2\alpha_s)$ corrections to the **β -functions**:
 - running of α_s and α is coupled \Rightarrow solve of a coupled ODE,
 - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for $\alpha \Rightarrow$ **unneeded complication.**

NLO QCD+QED Corrections

Coupling Evolution

running of the couplings, $N_F = 5$



🍏 Mixed terms in the β -functions lead to negligible effects.

NLO QCD+QED Corrections

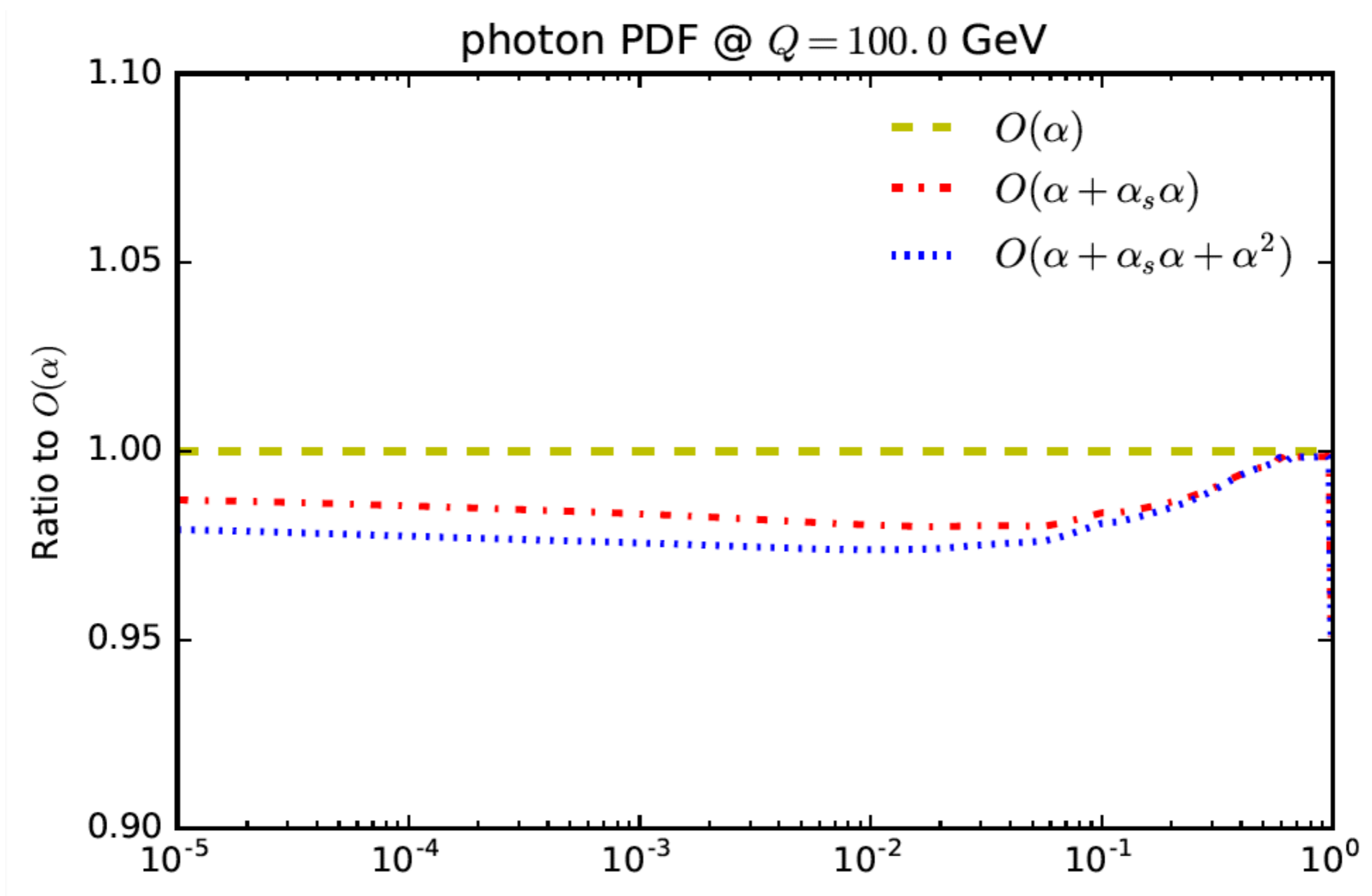
Evolution

In order to implement the full NLO QCD+QED corrections in the DGLAP evolution two main steps are required:

1. Implementing the $O(\alpha_s^2\alpha)$, $O(\alpha^3)$, $O(\alpha^2\alpha_s)$ corrections to the **β -functions**:
 - running of α_s and α is coupled \Rightarrow solve of a coupled ODE,
 - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for $\alpha \Rightarrow$ **unneeded complication**.
2. Implementing the $O(\alpha_s\alpha)$ and the $O(\alpha^2)$ corrections to the DGLAP **splitting functions** on top of the $O(\alpha)$ ones:
 - complication of the flavour structure due to the presence of terms promotional to e_q^2 and e_q^4 that break the isospin symmetry,
 - need for a more optimal evolution basis as compared to pure QCD.

NLO QCD+QED Corrections

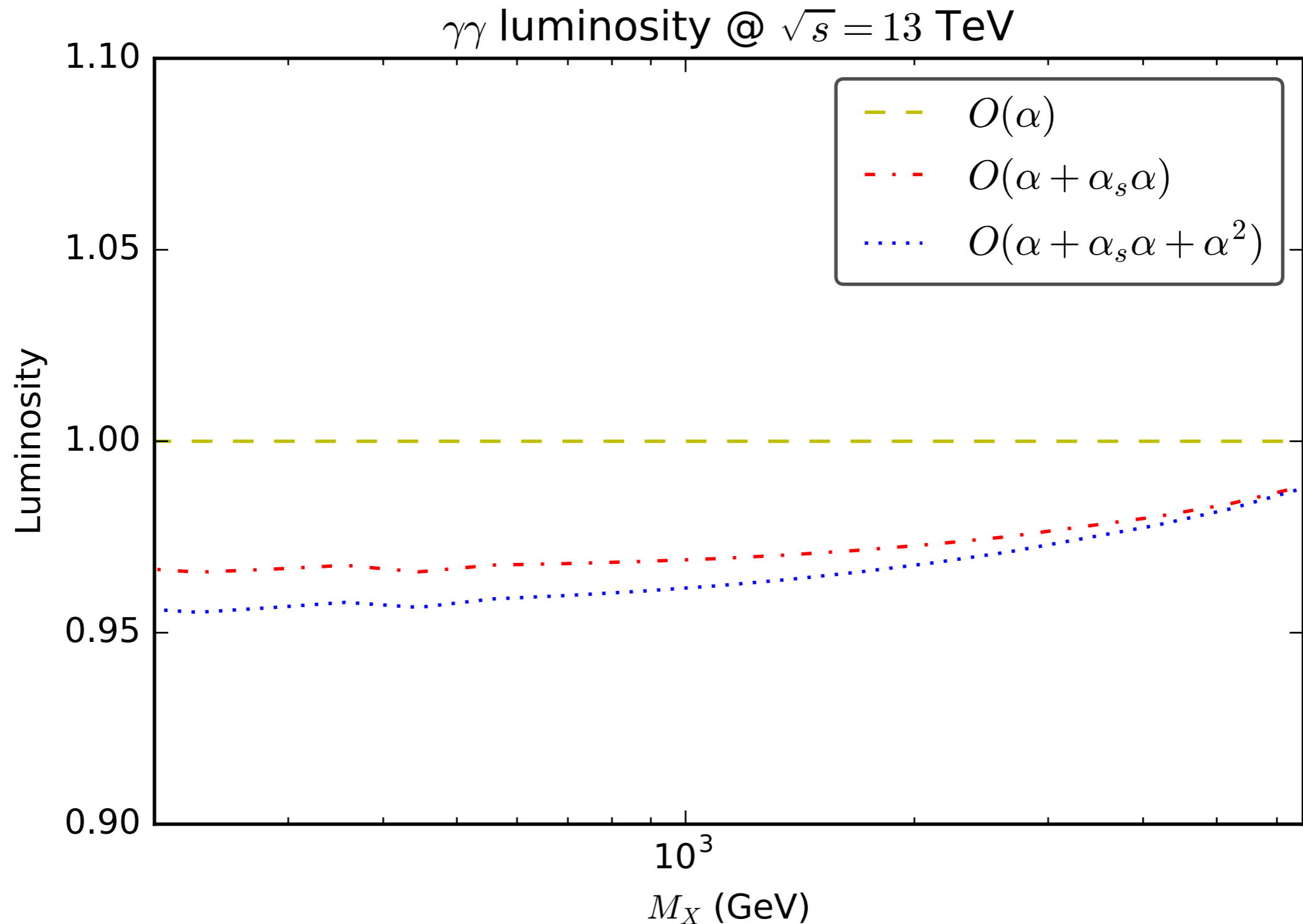
DGLAP Evolution



🍏 Effect on the photon PDF of the NLO corrns. at the level of 1-2%.

NLO QCD+QED Corrections

DGLAP Evolution



🍏 Slightly more sizeable effect on the $\gamma\gamma$ luminosity.

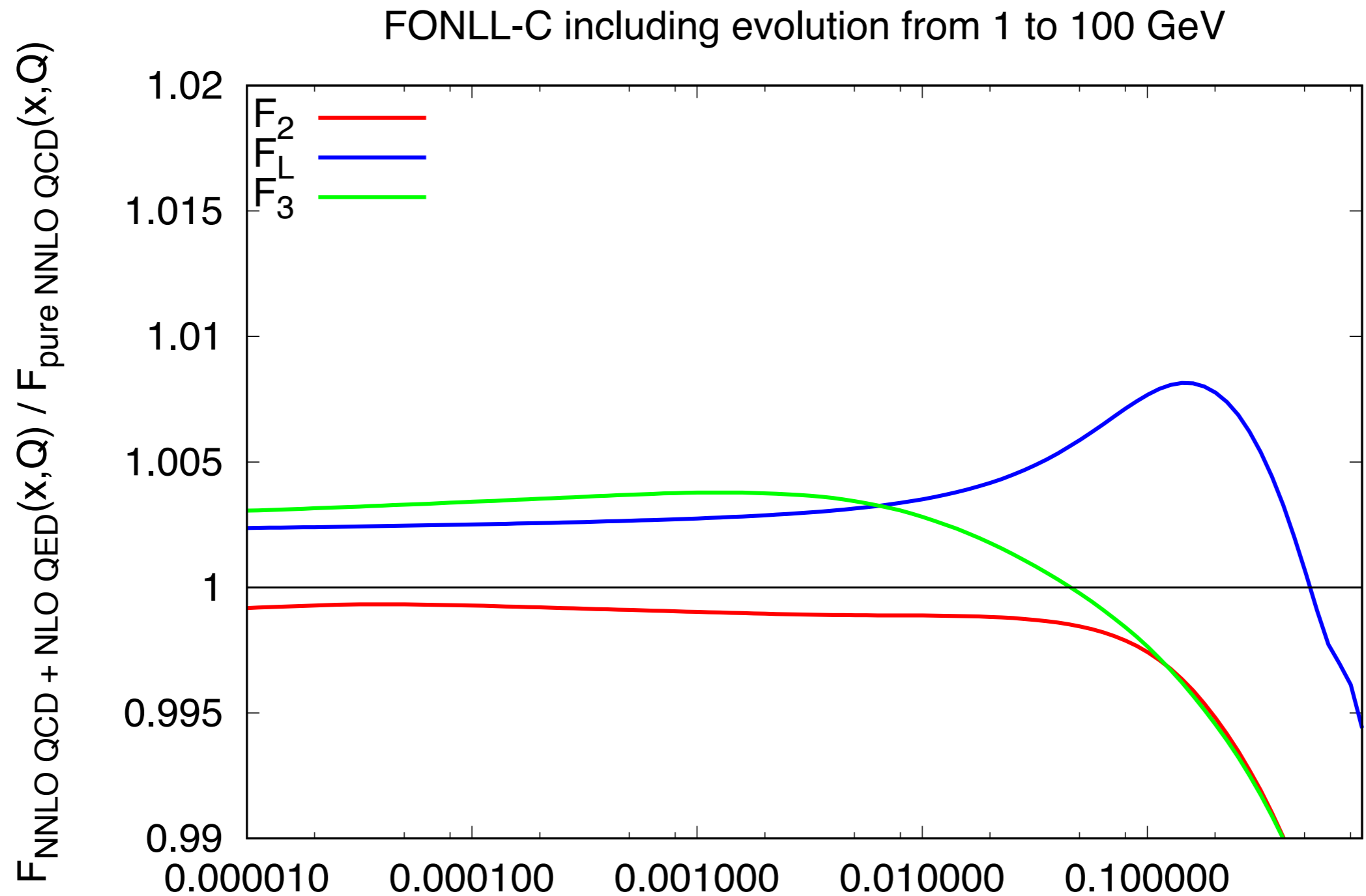
NLO QCD+QED Corrections

DIS Structure Functions

- While at LO in QED no corrections to the DIS structure functions are required ($\gamma^* q \rightarrow q$ itself is the LO), at NLO in QED $O(\alpha)$ corrections need to be taken into account:
 - **new diagrams:** $\gamma^* \gamma \rightarrow q\bar{q}$ and $\gamma^* q \rightarrow q\gamma$,
 - easily derivable from the corresponding QCD diagrams.
- The additional diagrams offer a **direct handle** on the photon PDF in DIS observables:
 - at LO in QED the photon PDF was entirely driven by the evolution.
- Small contribution proportional to $\alpha\gamma \sim O(\alpha^2)$ but can be relevant in some kinematic regions:
 - typically at large x and large Q^2 .

NLO QCD+QED Corrections

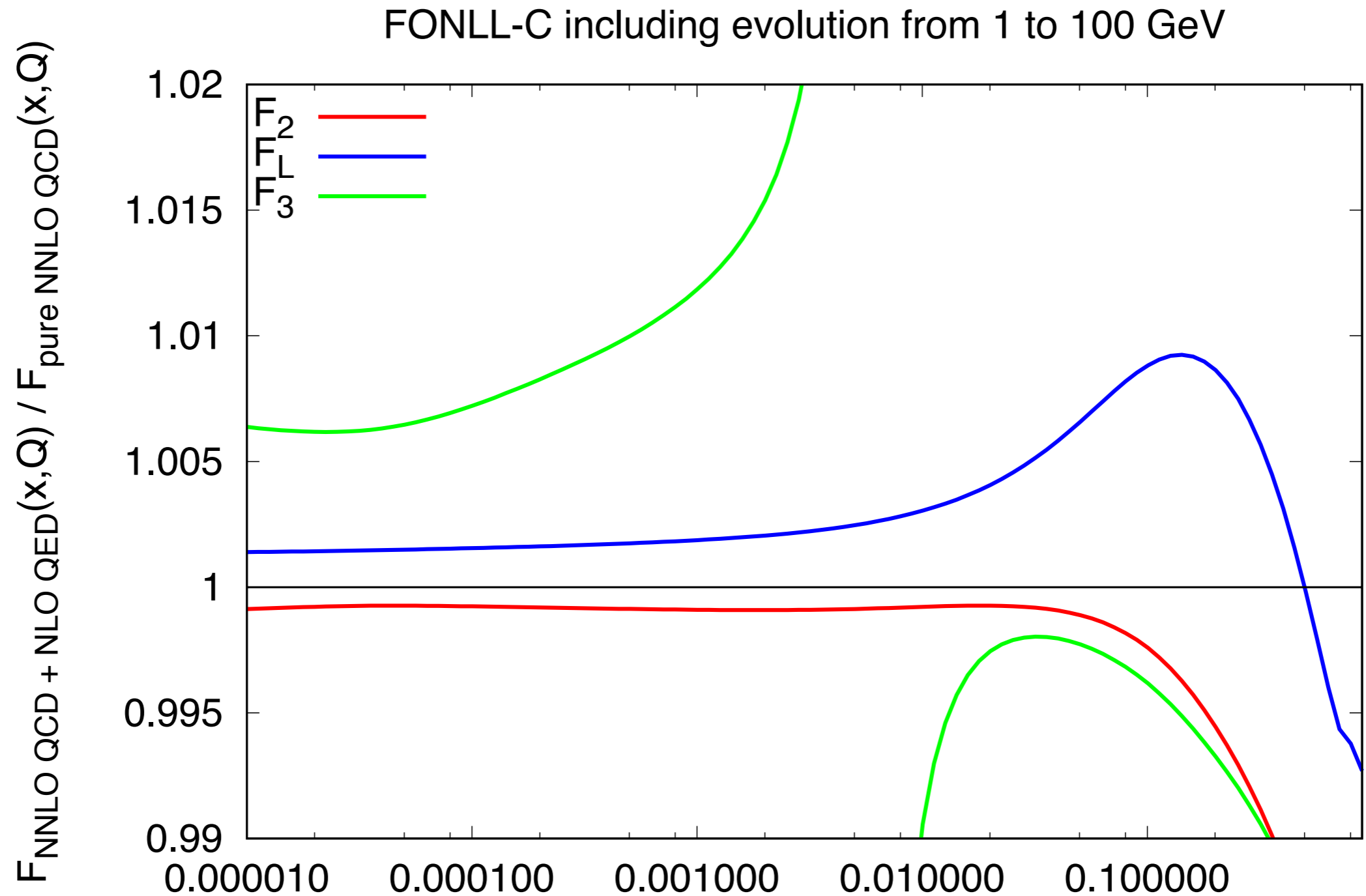
DIS Structure Functions (NC)



🍏 Generally small effect which becomes large at large x .

NLO QCD+QED Corrections

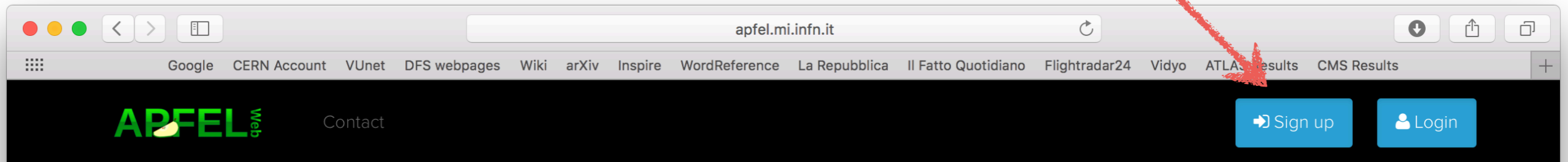
DIS Structure Functions (CC)



🍏 Generally small effect which becomes large at large x .

APFEL Web

Go to <http://apfel.mi.infn.it> and sign up



Welcome to APFEL online cluster!

This web-application is a tool designed for High Energy Physics by providing a simple and intuitive interface to plot and compute the most common observables with Parton Distribution Functions (PDFs).

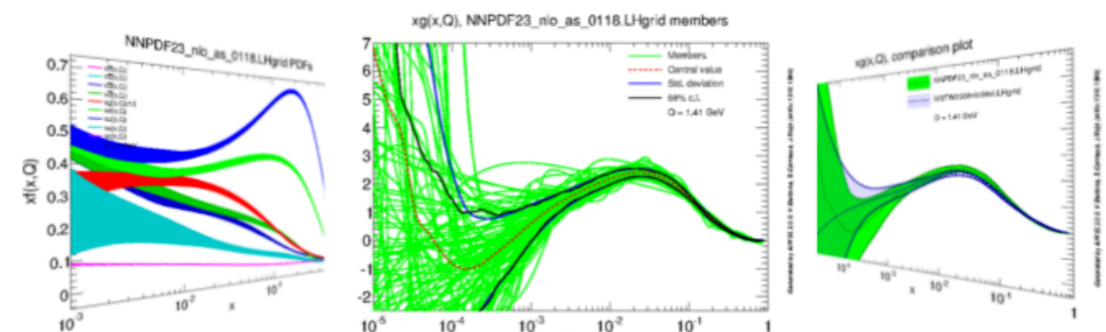
To begin to produce on-line plots, please [register](#) and [login](#)!

The APFEL library

APFEL, a PDF evolution library, is a computer library specialized in the solution of DGLAP evolution equations up to NNLO in QCD and to LO in QED, both with Pole and $\overline{\text{MS}}$ masses. With APFEL you can replace the evolution of LHAPDF sets and check the impact on the choice of evolution parameters. APFEL also computes deep-inelastic scattering processes using multiple schemes.

If you use the APFEL library or the online cluster in a scientific publication, please cite:
V. Bertone, S. Carrazza and J. Rojo, "APFEL: A PDF Evolution Library with QED corrections", Comput. Phys. Commun. 185, 1647 (2014), [arXiv:1310.1394](#).

S. Carrazza et al., "APFEL Web: a web-based application for the graphical visualization of parton distribution functions", J. Phys. G: Nucl. Part. Phys. 42 057001, [arXiv:1410.5456](#).
[Labtalk](#).



Web developers: D. Palazzo, S. Carrazza, A. Ferrara
APFEL developers: V. Bertone, S. Carrazza, J. Rojo. ([Contact](#))

APFEL Web

Go to <http://apfel.mi.infn.it> and sign up

apfel.mi.infn.it

Google CERN Account VUnet DFS webpages Wiki arXiv Inspire WordReference La Repubblica Il Fatto Quotidiano Flightradar24 Vidyo ATLAS Results CMS Results

APFEL Web Contact

Sign up Login

apfel.mi.infn.it

Google CERN Account VUnet DFS webpages Wiki arXiv Inspire WordReference La Repubblica Il Fatto Quotidiano Flightradar24 Vidyo ATLAS Results CMS Results

APFEL Web Contact Home Sign up

APFEL registration

Please complete the form below to access to the APFEL website:

Enter your e-mail address

Enter your password

Confirm your password

Spin of the Standard Model Higgs?

Register

Mobile website:

ANDROID APP ON Google play

APFEL Web

Workspace

The screenshot shows the APFEL Web Workspace interface. A red arrow points from the 'Workspace' header to the left sidebar menu. The main content area shows a welcome message, a tweet with a plot, and a status table.

Workspace

- Home
- My Profile

PDF MANAGER

- My PDF sets
- Add PDF set
- Import a LHAPDF grid

TOOLS

- Plotting Tools

DOWNLOAD RESULTS

- View jobs

Welcome to APFEL Web

APFEL Web News & Documentation

Tweets by @apfelwebteam

01 Jun

APFEL Web @apfelwebteam
Check out the updated graphics interface in

[Embed](#) [View on Twitter](#)

APFEL Web Status

Component	Status
Cores	100%
APFEL library	2.4.0
LHAPDF6	6.1.5
LHAPDF5	5.9.1
ROOT	5.34/21
APPLgrid	1.4.56
Overall	running

APFEL Web Workflow

APFEL Web

Workspace



The screenshot displays the APFEL Web Workspace interface. At the top, there is a navigation bar with the APFEL logo and a 'Logout' button. Below this is a sidebar menu with options: 'Workspace', 'Home', 'My Profile', 'PDF MANAGER', 'My PDF sets', 'Add PDF set', 'Import a LHAPDF', 'TOOLS', 'Plotting Tools', and 'DOWNLOAD RESULTS', 'View jobs'. The main content area is titled 'My APFEL Web Gallery' and contains a grid of 20 plots. Each plot shows a 'Ratio' on the y-axis and 'x' on the x-axis (log scale). The plots are arranged in a 5x4 grid. The top row contains plots for 'xy(x,Q), comparison' and 'xu(x,Q), comparison'. The second row contains 'xu(x,Q), comparison' and 'xg(x,Q), comparison'. The third row contains 'xu(x,Q), comparison' and 'xg(x,Q), comparison'. The fourth row contains 'xg(x,Q), comparison' and 'xg(x,Q), comparison'. The fifth row contains 'xg(x,Q), comparison' and 'xg(x,Q), comparison'. Each plot includes a legend with parameters like 'NNPDF3.0_LHAPDF', 'CT14ns', and 'Q = 9.12e+01 GeV'. A red arrow points from the 'Workspace' text above to the 'Workspace' menu item in the sidebar.

APFEL Web

Add a new PDF set

The screenshot shows the APFEL Web interface in a browser window. The address bar displays `apfel.mi.infn.it`. The top navigation bar includes the APFEL Web logo, a 'Contact' link, and a 'Logout' button. The left sidebar contains a 'Workspace' section with 'Home' and 'My Profile' links, a 'PDF MANAGER' section with 'My PDF sets' and 'Add PDF set' (highlighted in blue), and 'TOOLS' and 'DOWNLOAD RESULTS' sections. The main content area is titled 'Set parameters for PDF' and features a 'LHAPDF grid setup:' section. This section includes a 'PDF Set:' dropdown menu with 'NNPDF30_nlo_as_0118' selected, a 'PDF Error:' dropdown menu with 'Montecarlo - Standard Deviation' selected, a 'Select member:' dropdown menu with '0' selected, and a 'Library for evolution:' dropdown menu with 'LHAPDF' selected. A blue 'Confirm' button is located below these options. A red arrow points from the 'Add PDF set' option in the sidebar to the 'LHAPDF' dropdown menu. Another red arrow points from the bottom of the page to the 'LHAPDF' dropdown menu.

Workspace

- Home
- My Profile

PDF MANAGER

- My PDF sets
- Add PDF set**
- Import a LHAPDF grid

TOOLS

- Plotting Tools

DOWNLOAD RESULTS

- View jobs

Set parameters for PDF

LHAPDF grid setup:

PDF Set: NNPDF30_nlo_as_0118 a LHAPDF6 grid

PDF Error: Montecarlo - Standard Deviation

Select member: 0

Library for evolution: LHAPDF

[APFEL advanced evolution options](#)

Confirm

Use tabulated LHAPDF evolution...

APFEL Web

Add a new PDF set

Library for evolution: **APFEL**

[APFEL advanced evolution options](#)

APFEL evolution setup:

Theory: QCD

Solution: Exact

Perturbative order: LO (LO QED)

Scheme: VFNS

HQ Scheme: POLE

m_c , charm mass in (GeV): 1,275

m_b , bottom mass in (GeV): 4,18

m_t , top mass in (GeV): 173,07

$\alpha_S(Q_{\text{ref}})$: 0,118

Q_{ref} (GeV): 91,2

$\alpha(Q_{\text{ref}})$: 0,007496252

Q_{ref} (GeV): 1,777

μ_R/μ_F : 1,00

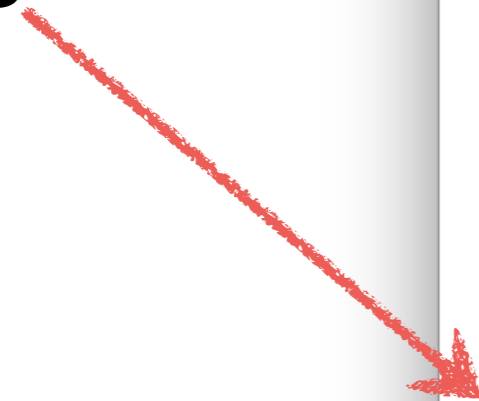
Maximum flavor PDFs: 6

Maximum flavor α : 6

Confirm

...or customised APFEL evolution

Plotting tools



Browser address bar: apfel.mi.infn.it

Navigation: Home, My Profile, PDF MANAGER, TOOLS, DOWNLOAD RESULTS

Plotting Tools

Choose a plotting tool and select your PDF set

Some jobs, like PDF luminosities, require some time to be finalized. Check the job status at [View jobs](#) page.

The plotting tools can be used for both the LHAPDF libraries: [LHAPDF5](#) and [LHAPDF6](#).

Tools for PDF basic plotting

- Plot PDF Members
- Plot Multiple PDF Flavors

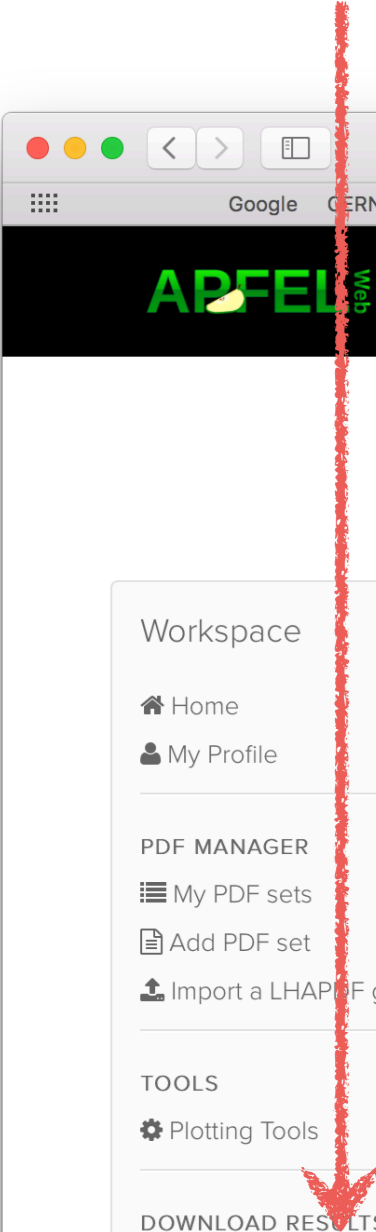
Tools for PDF analysis & comparisons

- Compare PDFs in x
- Compare PDFs in Q
- Compare PDF Luminosity
- All PDF Luminosities
- Compare PDF Correlations
- PDF Correlation Matrix

Tools for theoretical predictions from PDFs

- DIS in x
- DIS in Q
- APPLgrid Observables

View your jobs



Browser window: apfel.mi.infn.it

Workspace

- Home
- My Profile

Choose a plotting tool and select your PDF set

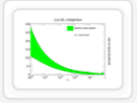
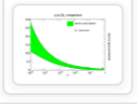
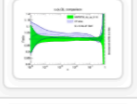
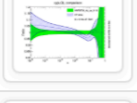
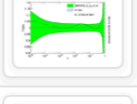

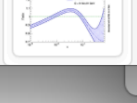
Some jobs, like PDF luminosities, require some time to be finalized. Check the job status at [View jobs](#) page.

Browser window: apfel.mi.infn.it

APFEL Web Contact [Logout](#)

View and download your jobs

View, modify and delete your jobs.

Image	Label	Status	Date	Actions
	LH6_30	Done	Oct. 22, 2016, 11:16 p.m.	Details Clone Erase
	LH5_30	Done	Oct. 22, 2016, 11:16 p.m.	Details Clone Erase
	NN_vs_CT_NLO_up	Done	Aug. 10, 2016, 5:13 p.m.	Details Clone Erase
	NN_vs_CT_NLO_gluon	Done	Aug. 10, 2016, 5:13 p.m.	Details Clone Erase
	NN_vs_CT_LO_up	Done	Aug. 10, 2016, 5:12 p.m.	Details Clone Erase
	NN_vs_CT_LO_gluon	Done	Aug. 10, 2016, 5:11 p.m.	Details Clone Erase
	gluon_CT14_LO_vs_NLO_ext	Done	Aug. 1, 2016, 11:33 a.m.	Details Clone Erase

TMD Evolution (PDFs)

$$\begin{aligned} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\ &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\ &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C \end{aligned}$$

TMD Evolution (PDFs)

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C
 \end{aligned}$$

- $b_T \ll 1/\Lambda_{\text{QCD}}$
- matching to the collinear region
- factorises as hard and non-perturbative
- numerically cumbersome
- precompute using the APFEL technology

- CS evolution
- perturbative

- matching between the small and large b_T
- non perturbative
- parametrised and fitted to data

TMDs in SIDIS

🍏 In SIDIS, what enters the computation of the cross sections is:

$$\mathcal{L}_{\text{SIDIS}} = \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{q}_T \cdot \mathbf{b}_T} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F) D_{H/f}(x, \mathbf{b}_T; \mu, \zeta_D)$$

Fourier transform

PDFs

FFs

🍏 The ingredients are:

🍏 a set of evolved TMD-PDFs,

🍏 a set of evolved TMD-FFs,

🍏 the Fourier transform of its product.

🍏 Complex set of tasks that have to be performed optimally

🍏 APFEL provides the ideal environment for this computation:

🍏 fast and accurate interpolation techniques,

🍏 precomputation of the time consuming bits.

Summary

- Recent developments in APFEL:
 - small- x resummation** in PDF evolution and structure functions,
 - framework for the **determination of FFs**,
 - NLO QED corrections** to evolution and structure functions.

Other Recent Developments

- Intrinsic-charm** in DIS *a la* FONLL.
- Polarised DGLAP evolution** up to NNLO.
- Independent factorisation and renormalisation **scale variations** both in the DIS structure functions and in the evolution.

In the Pipeline

- Implementation of **TMD evolution** and **SIDIS cross sections**,
- Implementation of the **polarised structure functions**,
- mass corrections** to SIA structure functions.

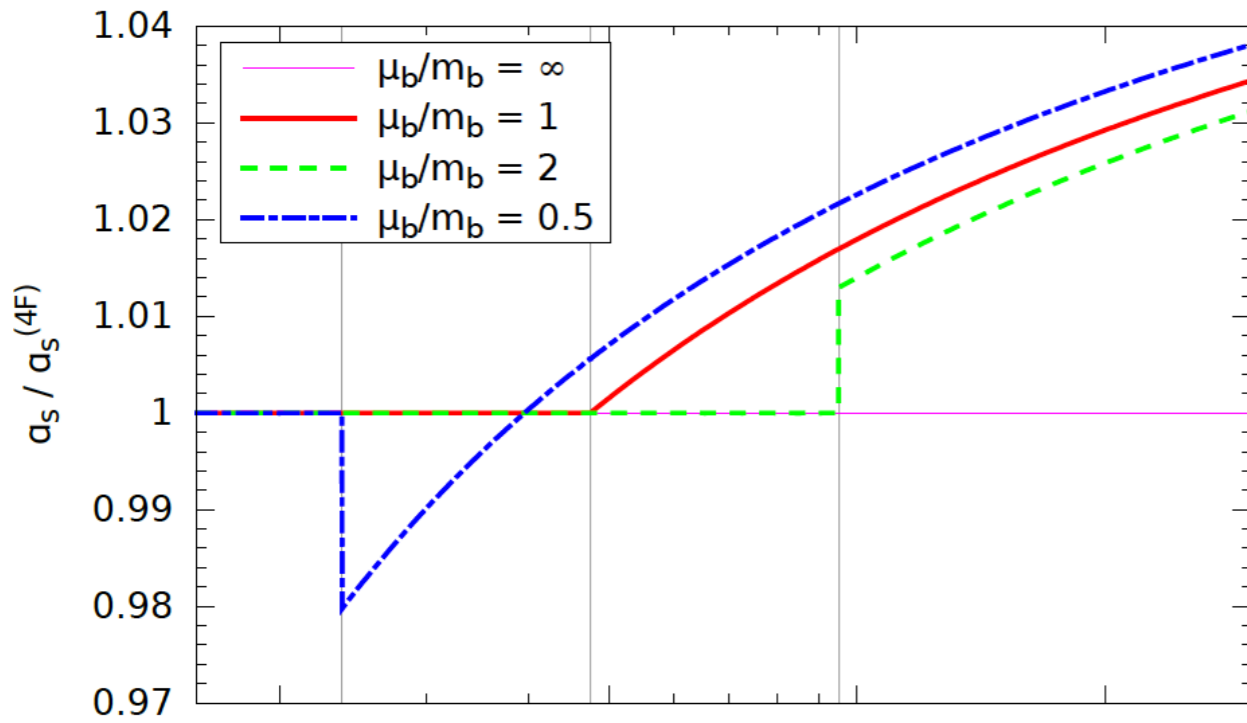
Backup Slides

Displaced Heavy-Quark Thresholds

- 🍏 The implementation of the VFNS evolution both for PDFs and α_s requires **matching** factorization schemes differing in the number of active flavours:
 - 🍏 the scale at which two consecutive factorization schemes are matched are usually referred to as **heavy-quark thresholds**.
 - 🍏 Heavy-quark thresholds are usually (and for convenience) identified with the heavy quark masses by means of the so-called **matching conditions** presently known up to $O(\alpha_s^2)$ [[hep-ph/9612398](https://arxiv.org/abs/hep-ph/9612398)].
 - 🍏 However, heavy-quark thresholds are actually free parameters and can be chosen **arbitrarily**.
 - 🍏 If masses and thresholds are taken to be different, the matching conditions need to be “generalized” including **logarithmic terms**.
- 🍏 APFEL now implements the possibility to set masses and thresholds to different values in a consistent way both in the pole mass and in the $\overline{\text{MS}}$ renormalization schemes.

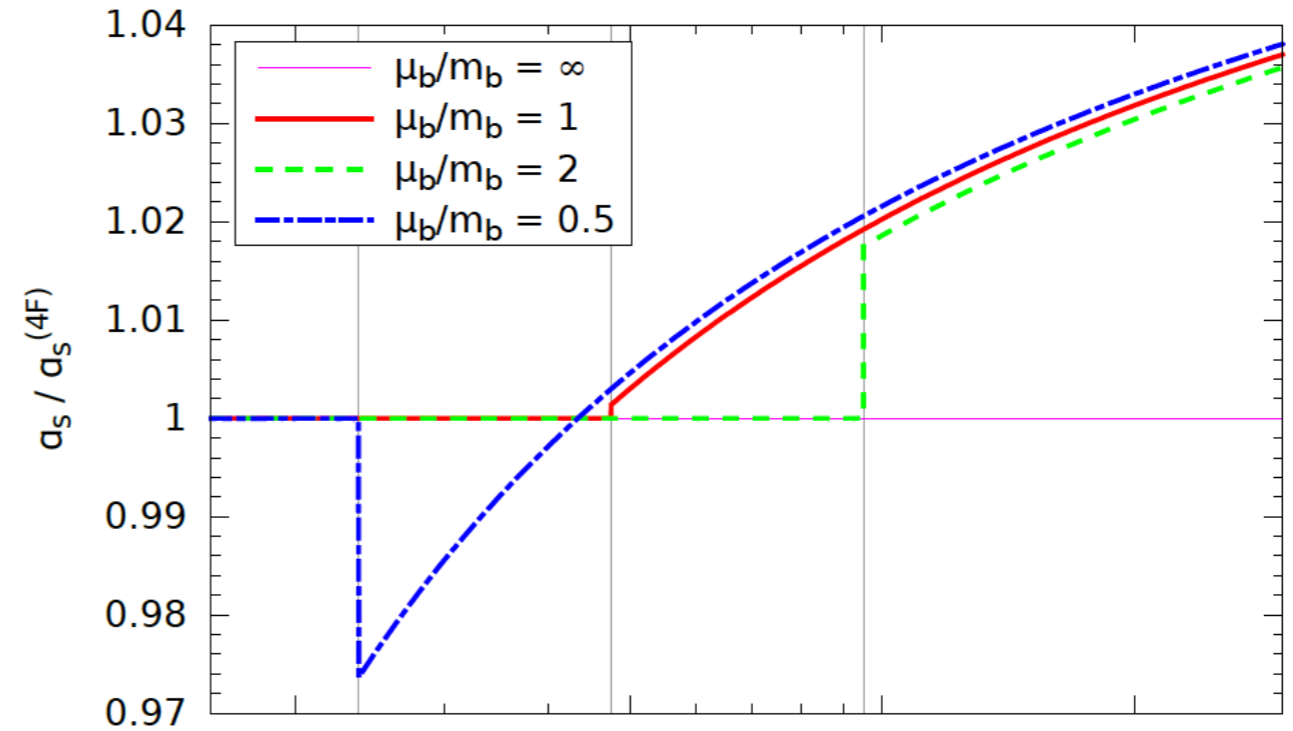
Displaced Heavy-Quark Thresholds

NLO

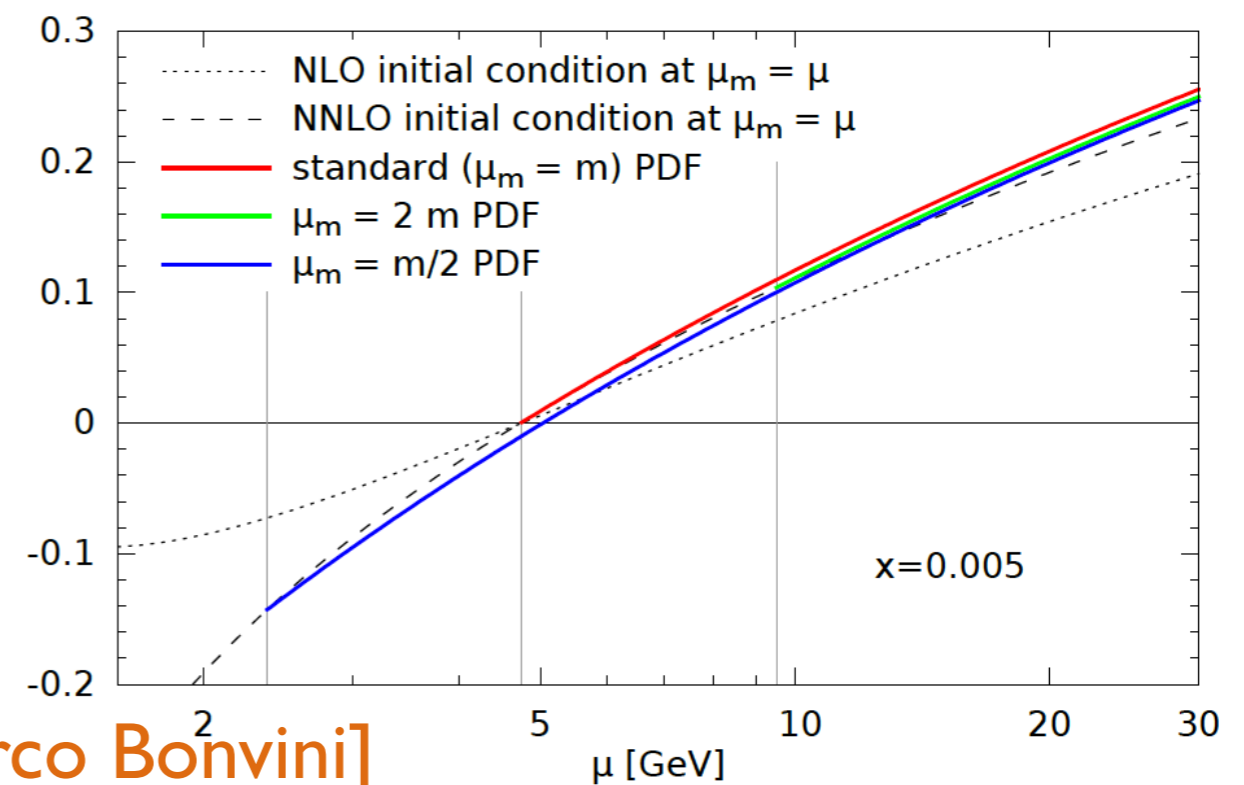
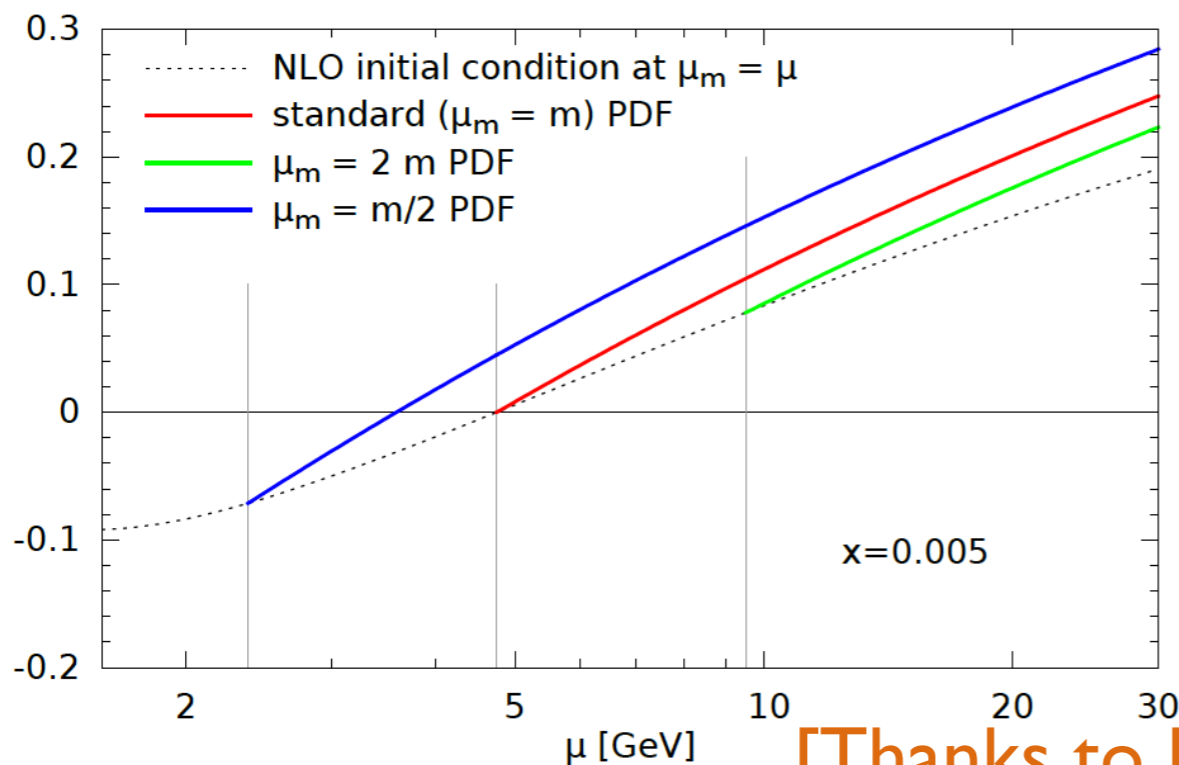


bottom PDF at NLO

NNLO

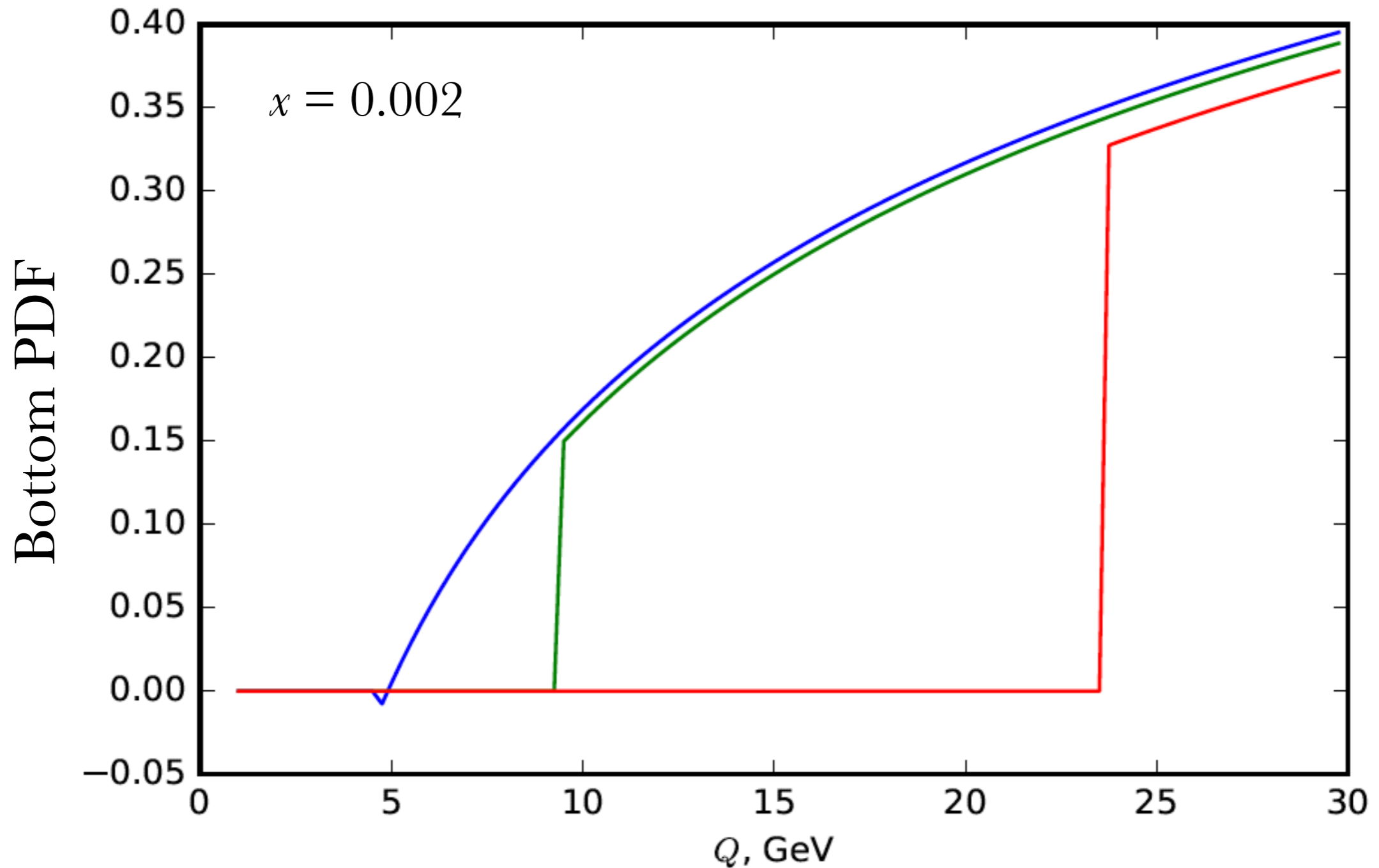


bottom PDF at NNLO



[Thanks to Marco Bonvini]

Displaced Heavy-Quark Thresholds



[Thanks to Sasha Glazov]

APFELgrid

A fast(er) interface for PDF fits

- While being an extremely useful tool, **APPLgrid** might not be appropriate to be directly employed in a global PDF fit where usually **thousands of iterations** are needed:
 - Need to calculate PDF and α_s **evolution in real time**.
 - Not particularly fast** convolution.
 - many tables need to be loaded with the concrete risk of **exceeding the memory limit** (pretty common on clusters).
- In the NNPDF collaboration we developed **APFELgrid** which, starting from an existing APPLgrid, combines PDF evolution from APFEL to the hard cross sections producing *derived* interpolation tables (FK tables):

Observable	APPLGRID	FK	optimized FK
W^+ production	1.03 ms	0.41 ms (2.5x)	0.32 ms (3.2x)
Inclusive jet production	2.45 ms	20.1 μ s (120x)	6.57 μ s (370x)

- APFELgrid will soon be made **public in APFEL**. [thanks to N. Harthland]

Improvements

A New Fast Evolution

- 🍏 In the previous versions of APFEL the DGLAP evolution equations were written in terms of the **evolution operator**:

$$\mu^2 \frac{\partial}{\partial \mu^2} M_{ij}(\mu, \mu_0) = P_{ik}(\mu) \otimes M_{kj}(\mu, \mu_0) \quad \text{with} \quad f_i(\mu) = M_{ij}(\mu, \mu_0) \otimes f_j(\mu_0)$$

- 🍏 This may be convenient because the evolution operator can be evaluated once and for all and convoluted with any initial PDF set.
- 🍏 On the other hand, this requires solving numerically a big coupled system of ODEs, therefore it can be slow.
- 🍏 Alternatively, one can directly solve the DGLAP equations in terms of **PDFs**:

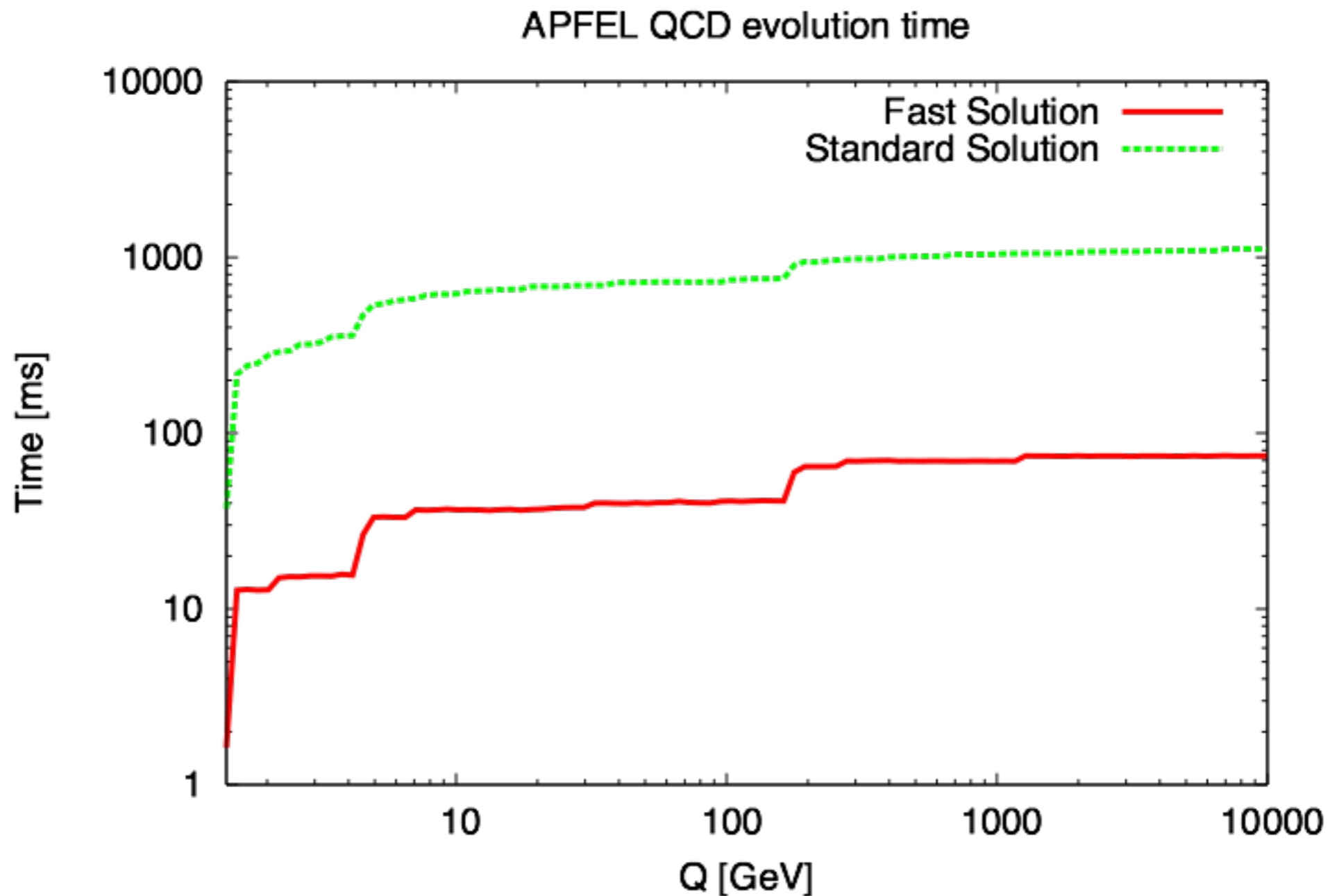
$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = P_{ij}(\mu) \otimes f_j(\mu)$$

- 🍏 This requires the solution of a much smaller system of equations and is consequently much faster.

Improvements

A New Fast Evolution

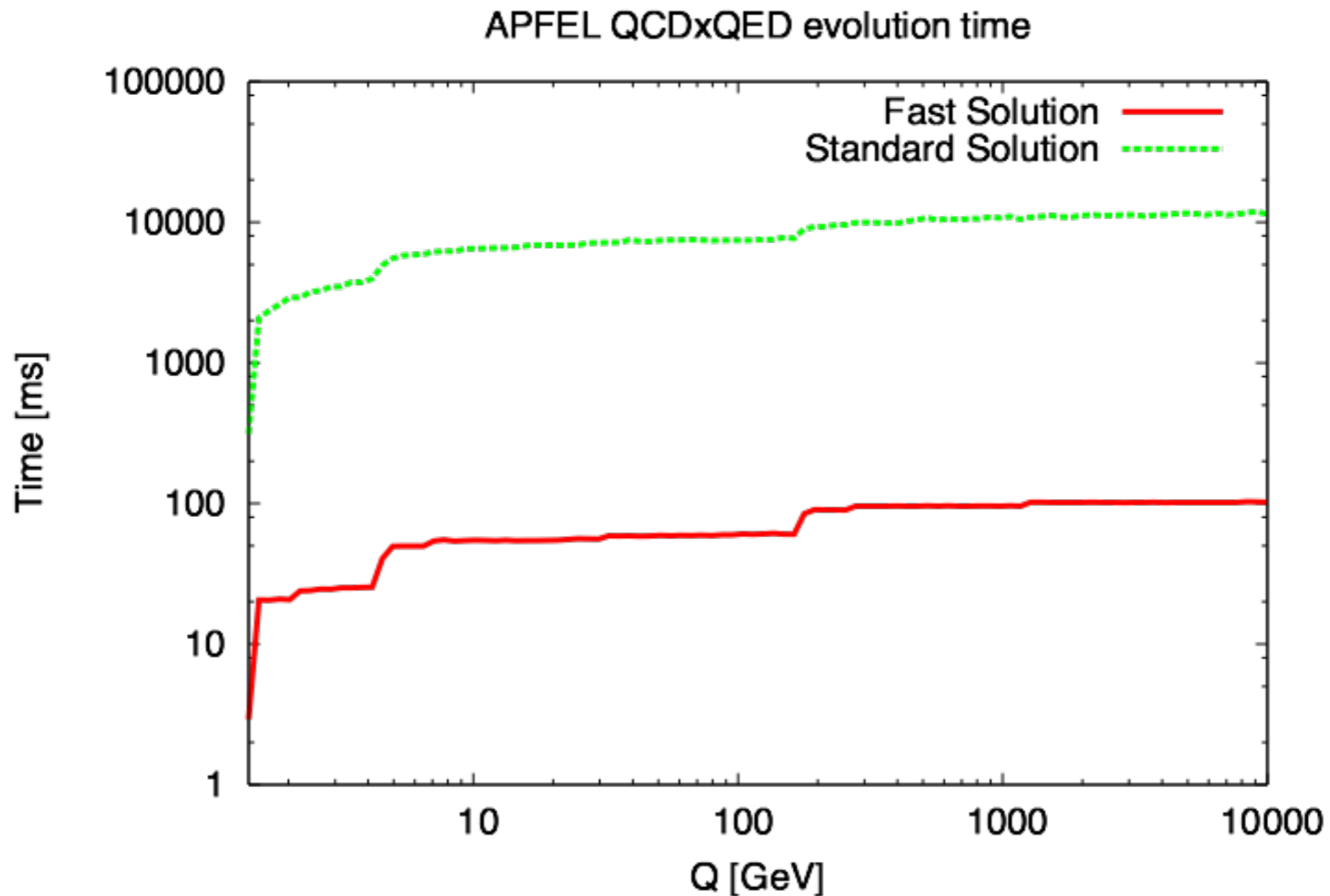
- 🍏 Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD evolution:



Improvements

A New Fast Evolution

- 🍏 Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD+QED evolution:



Improvements

A New QCD+QED Evolution

🍏 In the previous versions of APFEL the QCD+QED evolution was performed by combining the **separate** QCD and QED evolution:

🍏 we showed that the differences, of a few % at most, with the standard implementations which evolve contemporaneously in QCD and QED were due to **subleading terms in α** .

🍏 We have now implemented a new evolution basis which allows a **simultaneous diagonalization** of the QCD+QED evolution matrix:

1) g

2) γ

3) $\Sigma = \Sigma_u + \Sigma_d$

4) $\Delta_\Sigma = \Sigma_u - \Sigma_d$

5) $T_1^u = u^+ - c^+$

6) $T_2^u = u^+ + c^+ - 2t^+$

7) $T_1^d = d^+ - s^+$

8) $T_2^d = d^+ + s^+ - 2b^+$

9) $V = V_u + V_d$

10) $\Delta_V = V_u - V_d$

11) $V_1^u = u^- - c^-$

12) $V_2^u = u^- + c^- - 2t^-$

13) $V_1^d = d^- - s^-$

14) $V_2^d = d^- + s^- - 2b^-$

Improvements

A New QCD+QED Evolution

🍏 In the previous versions of APFEL the QCD+QED evolution was performed by combining the **separate** QCD and QED evolution:

🍏 we showed that the differences, of a few % at most, with the standard implementations which evolve contemporaneously in QCD and QED were due to **subleading terms in α** .

🍏 We have now implemented a new evolution basis which allows a **simultaneous diagonalization** of the QCD+QED evolution matrix:

1) g

2) γ

3) $\Sigma = \Sigma_u + \Sigma_d$

4) $\Delta_\Sigma = \Sigma_u - \Sigma_d$

5) $T_1^u = u^+ - c^+$

6) $T_2^u = u^+ + c^+ - 2t^+$

7) $T_1^d = d^+ - s^+$

8) $T_2^d = d^+ + s^+ - 2b^+$

9) $V = V_u + V_d$

10) $\Delta_V = V_u - V_d$

11) $V_1^u = u^- - c^-$

12) $V_2^u = u^- + c^- - 2t^-$

13) $V_1^d = d^- - s^-$

14) $V_2^d = d^- + s^- - 2b^-$

Coupled

Decoupled

🍏 This new basis is also suitable for an easy implementation of the mixed **higher order corrections** to the evolution.

Intrinsic Charm

- 🍏 Introducing an intrinsic charm (IC) component in the context of a GM-VFNS like FONLL (or ACOT, or TR) requires some care:
 - 🍏 **relax** the assumption of **pure perturbative generation** of heavy quarks at the thresholds,
 - 🍏 take into account **charm-initiated diagrams** both in the **massive** and in the massless sectors [[arXiv:1510.00009](https://arxiv.org/abs/1510.00009)].
- 🍏 A full formulation of the FONLL scheme in the presence of IC has **recently** been **achieved** [[arXiv:1510.02491](https://arxiv.org/abs/1510.02491)]:
 - 🍏 interestingly, it has been found that **FONLL** with IC is **equivalent** to full **ACOT** to all orders, while the standard FONLL (w/o IC) is instead equivalent to S-ACOT.
 - 🍏 Implemented in APFEL up to NLO both in the **NC** and **CC** sector and **benchmarked** against the public massiveDISsFuntion code (<https://www.ge.infn.it/~bonvini/massivedis/>).

Intrinsic Charm

🍏 Consider realistic models:

🍏 BHPS model:

$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = Ax^2 [6x(1+x) \ln x + (1-x)(1+10x+x^2)]$$

🍏 SEA model:

$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = Ax^{-1.25}(1-x)^3$$

🍏 A determined requiring the charm to carry 0.5% of the momentum

