APFEL
A PDF Evolution Library

[V. Bertone, *et al.*, Comput. Phys. Commun. 185, 1647 (2014)]

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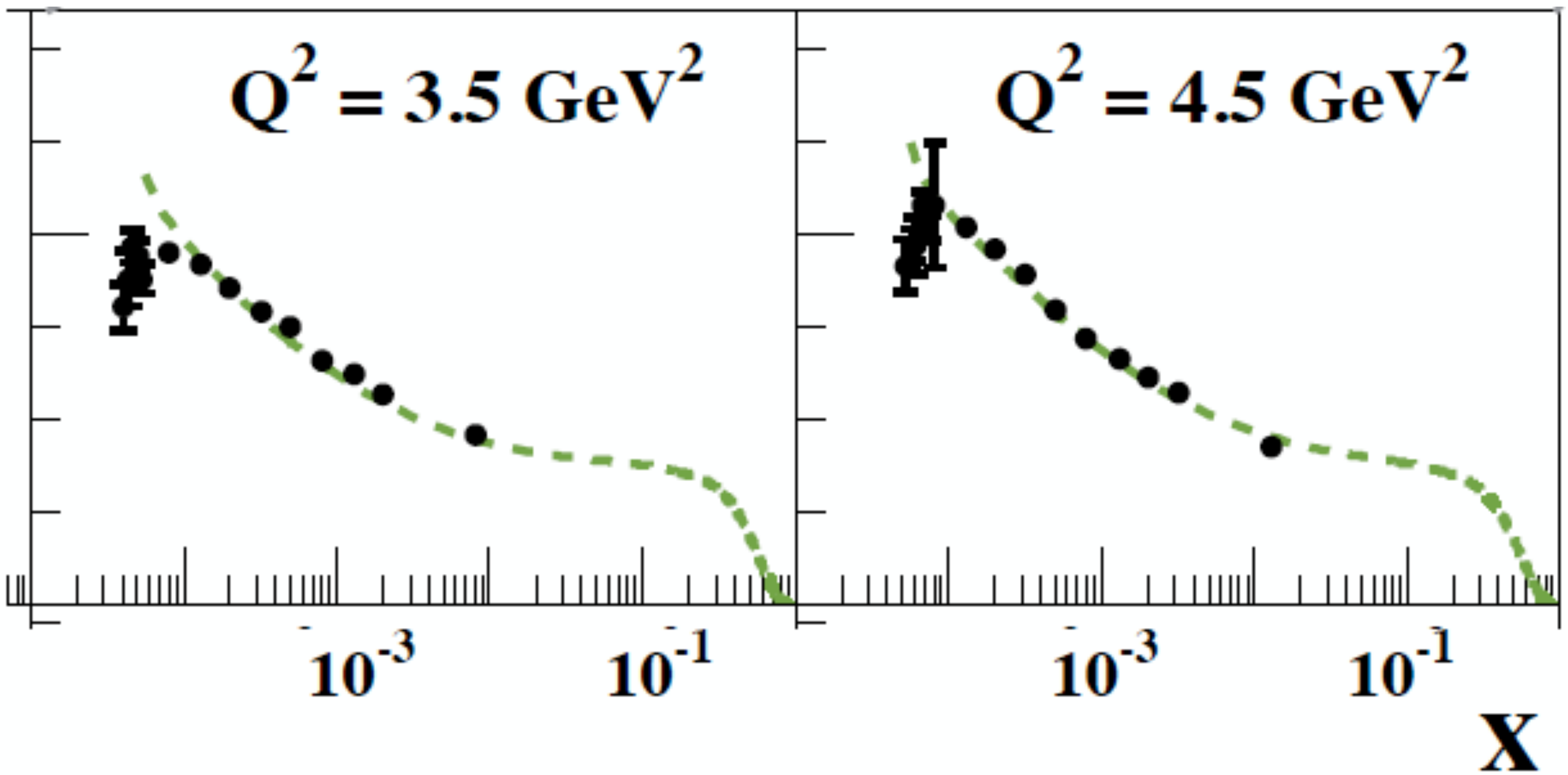
APFEL in a Nutshell

- APFEL is a **public** library for the computation of collinear PDF evolution and DIS structure functions:
  - up to NNLO in QCD combined to QED corrections up to NLO.
  - FFN and VFN schemes.
  - Pole and $\overline{\rm MS}$ heavy-quark masses.
  - fast computation of DIS NC and CC observables in different mass schemes (ZM-VFNS, FFNS and FONLL).
  - Interfaces to FORTRAN, C/C++ and Python.
  - Web interface available on http://apfel.mi.infn.it.
  - available from http://apfel.hepforge.org/.
- Interfaced to **xFitter** (see Ringaile's talk) and **Alpos**.
- Used for the next generation of the **NNPDF** fits (including FFs).

Small-x Resummation

- **Tension** between fixed-order predictions and data in the small-x region reached by HERA:



From Eram Rizvi talk at QCD@LHC14

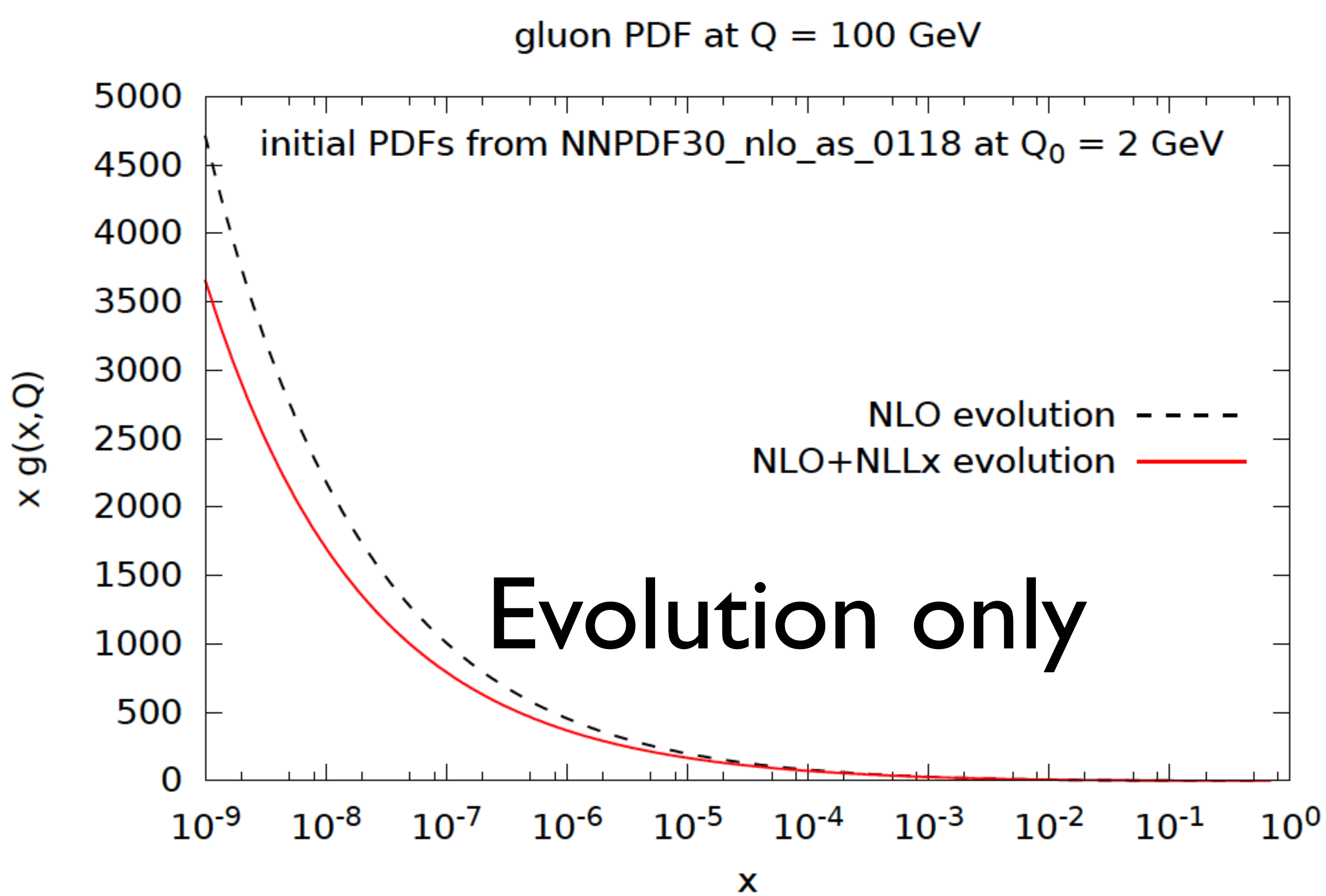
- A similar effect was observed some time ago in the NNPDF framework by F. Caola *et al.* [arXiv:1007.5405].

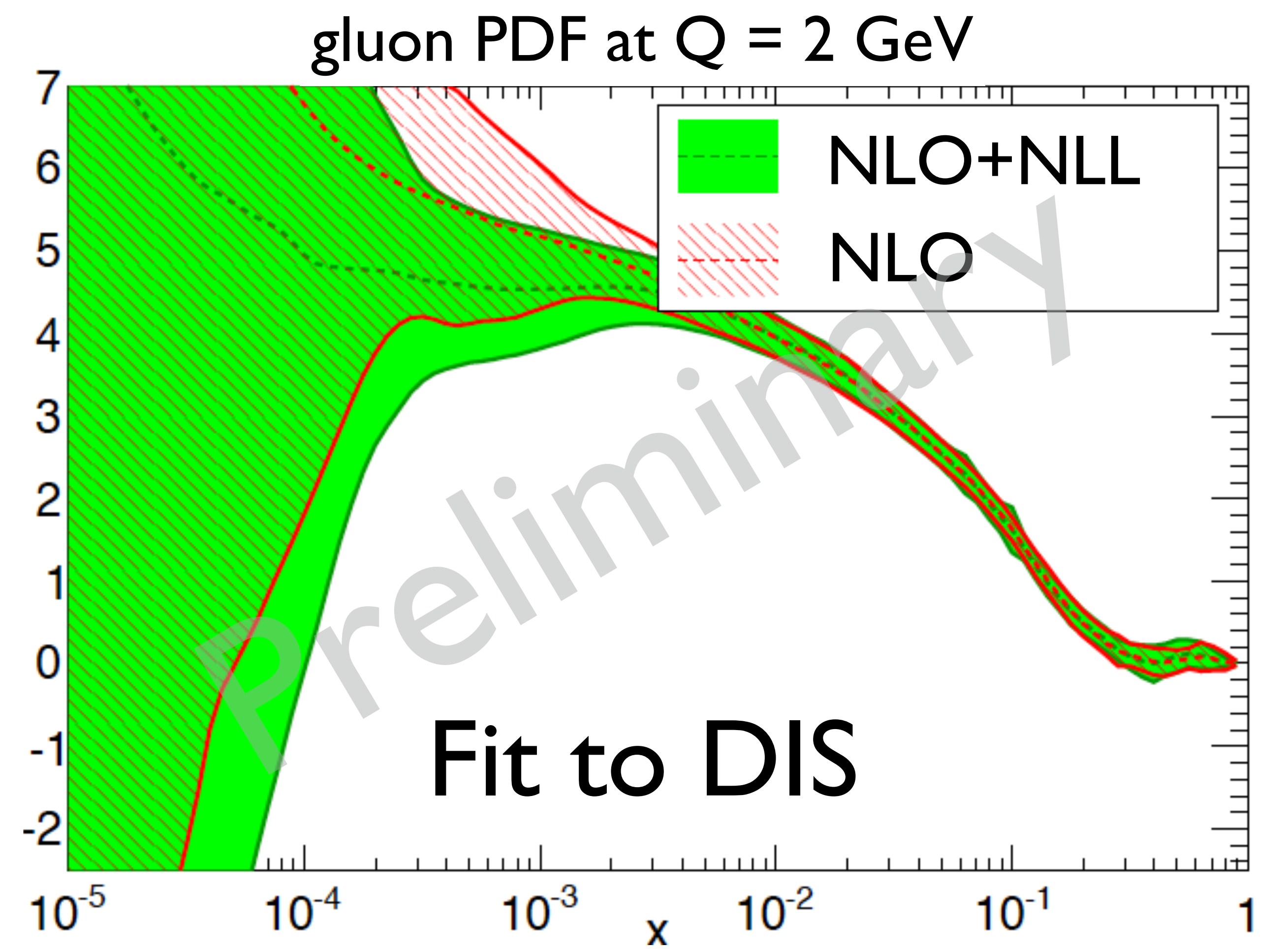
- Suggestion of the need for **small-x resummation**.

Small-x Resummation

- The **HELL** code [arXiv:1607.02153] has been interfaced to APFEL:
  - based on the ABF formalism (e.g. see [hep-ph/9501231]).
  - Small-x **resummed splitting functions** up to **NLL** accuracy,
  - Small-x **resummed DIS coefficient functions** up to **NLL**:
    - massless (already available from APFEL),
    - massive (since very recently ⇒ not interfaced to APFEL yet)
  - Resummed matching conditions (not implemented in HELL, to come).
- In NNPDF we have attempted DIS-only PDF fits with small-x resummation correction at NLL in the evolution and in the ZM sector of the DIS structure functions obtaining **encouraging results**.

Small-x Resummation





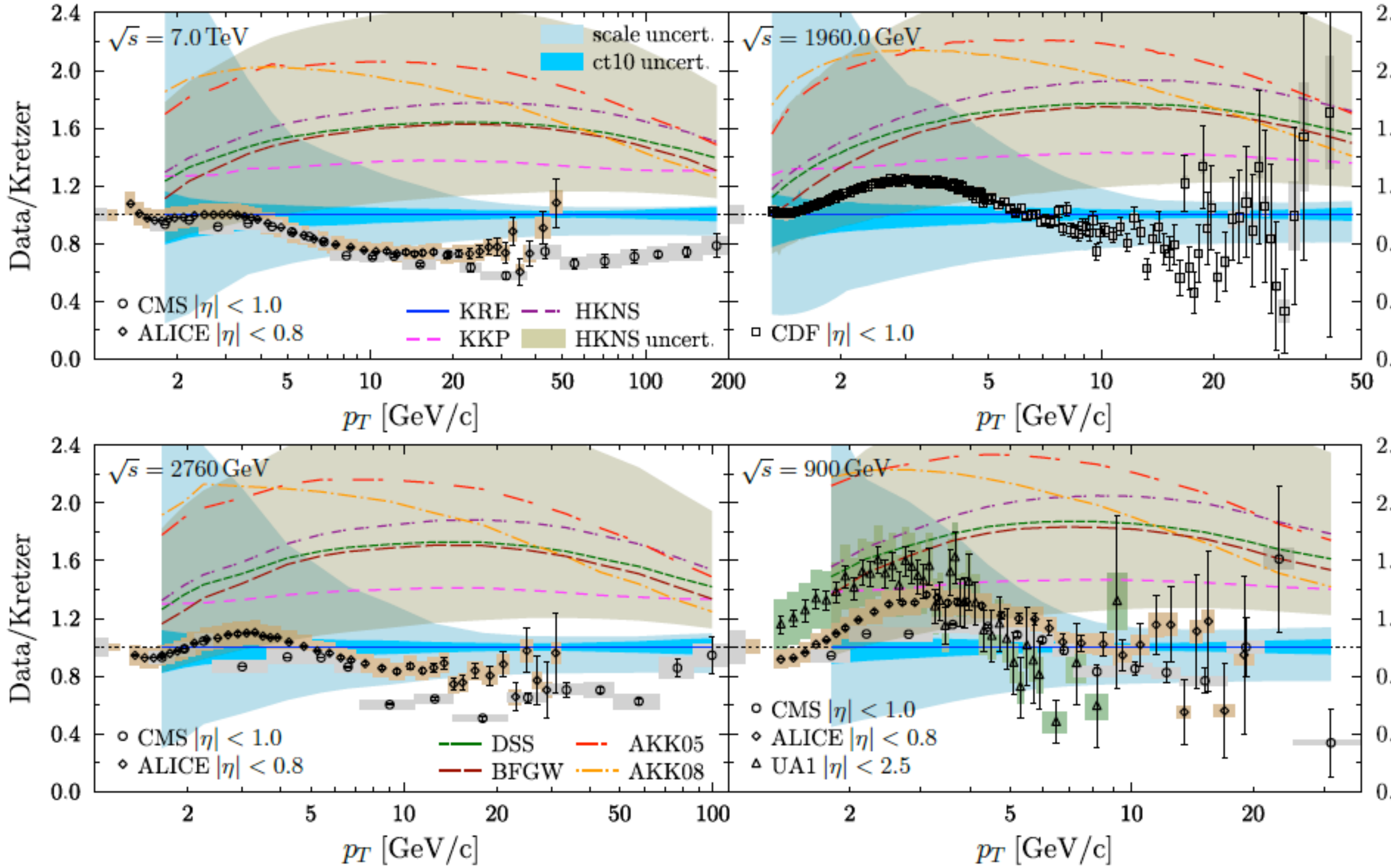
- Resummed evolution leads to a **suppression** of the **gluon PDF** at small values of x as compared to fixed order.
- **Compensation** when also resummed **coefficient functions** are introduced ⇒ effect on the small-x gluon PDF at the level of 1-σ.
- Other PDFs mostly unchanged.

Fragmentation Functions

- A faithful determination of fragmentation functions (FFs) is extremely important to study the universality of the QCD factorisation theorems.
- The inclusive hadron measurements at the LHC, sensibly extending the previous kinematical coverage, are particularly useful for studying the FFs.
- Moreover, a good knowledge of FFs is functional to the determination of the **polarised PDFs**.
- The **spread between the different FFs** present on the market is currently very large.
- In addition, none of the existing FF sets describes the recent LHC and Tevatron experimental data.

Fragmentation Functions

- Inclusive charged-hadron spectrum:



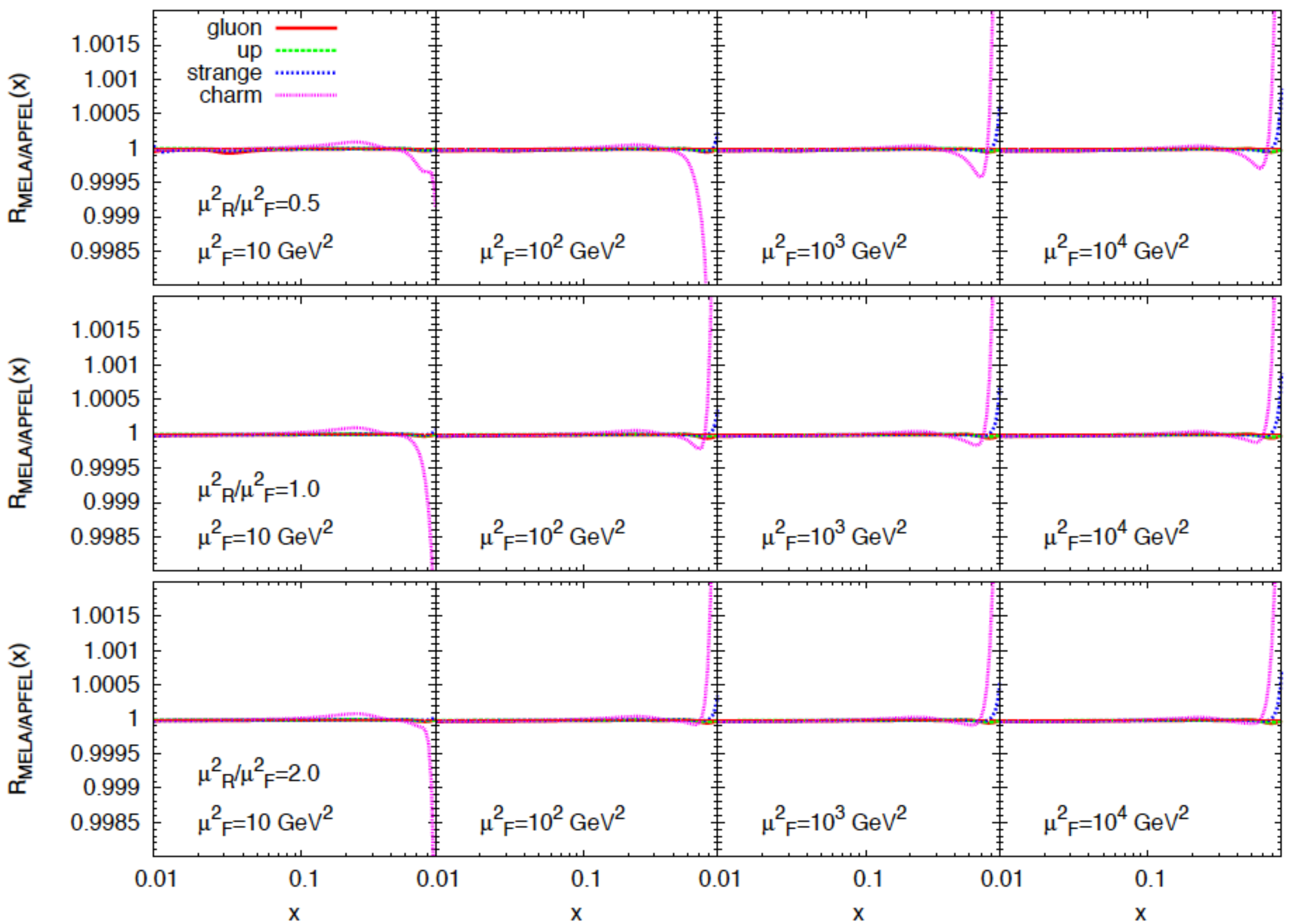
d'Enterria *et al.* [arXiv:1311.1415]

- Large discrepancies that need to be understood.

Fragmentation Functions

- APFEL implements the **time-like evolution**:
  - up to NLO in the VFNS,
  - up to NNLO in the FFNS (NNLO matching conditions unknown).

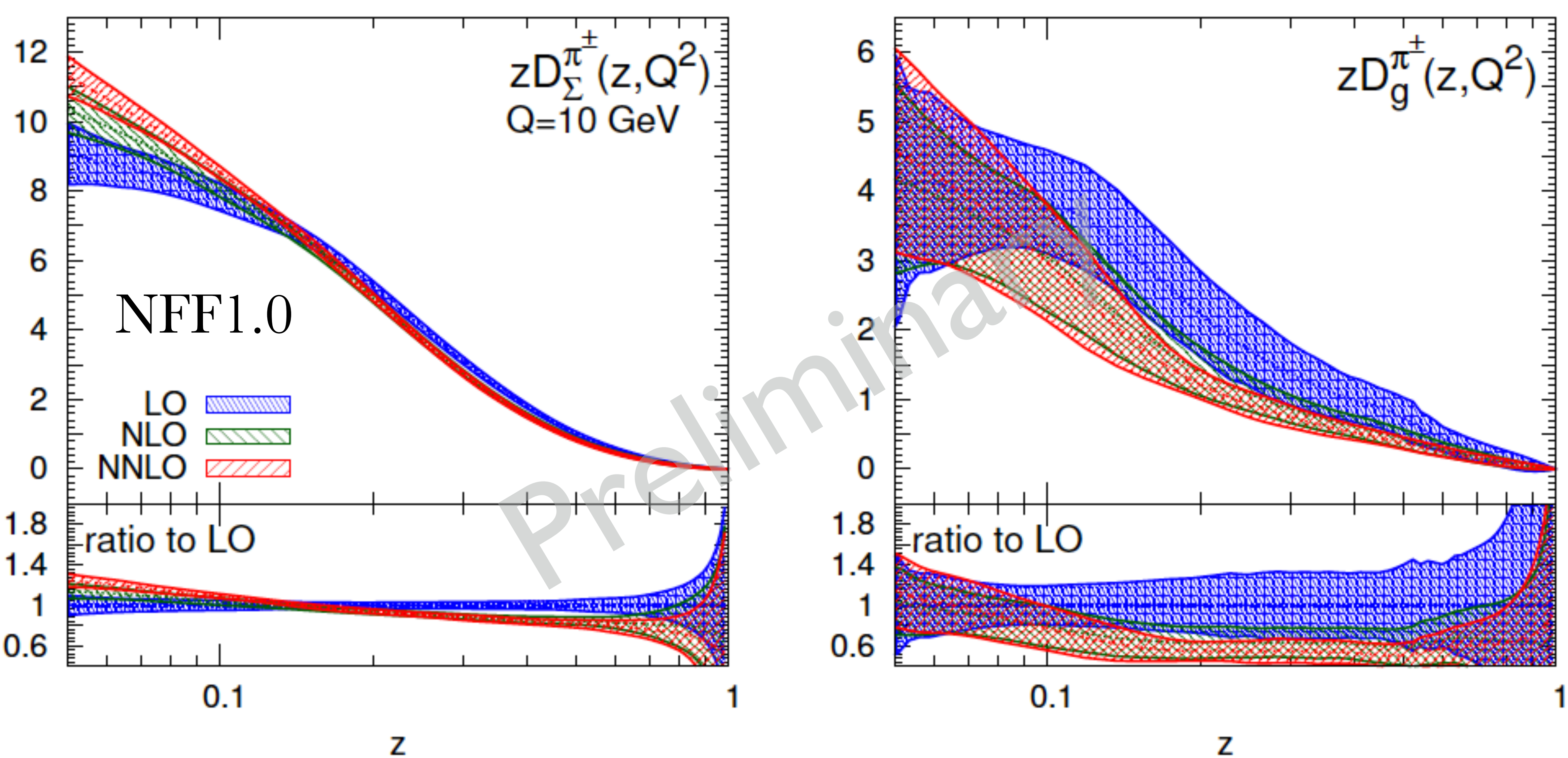
APFEL vs. MELA: VFNS at NLO



- Careful **benchmark** of the evolution against the MELA code [arXiv:1501.00494].
- *x*-space vs. *N*-space
- Excellent agreement at all perturbative orders.

Fragmentation Functions

- Single-inclusive e^+e^- annihilation (SIA) structure functions are also implemented in APFEL up to NNLO in QCD:
  - partial benchmark against DSS code (thanks to D.P. Anderle).
- APFEL can now be used to **determine FFs** from SIA data.



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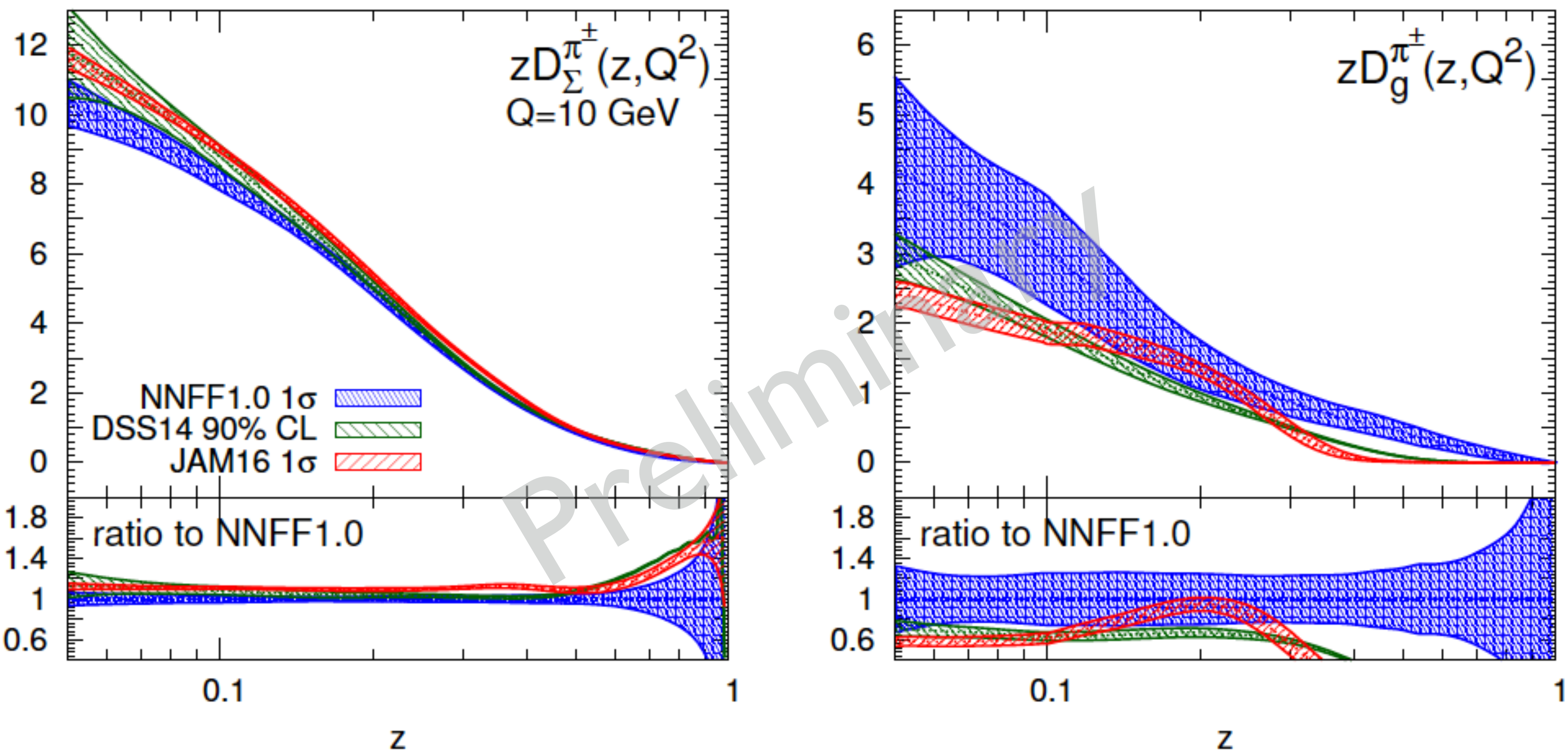


NLO QCD+QED Corrections
Evolution

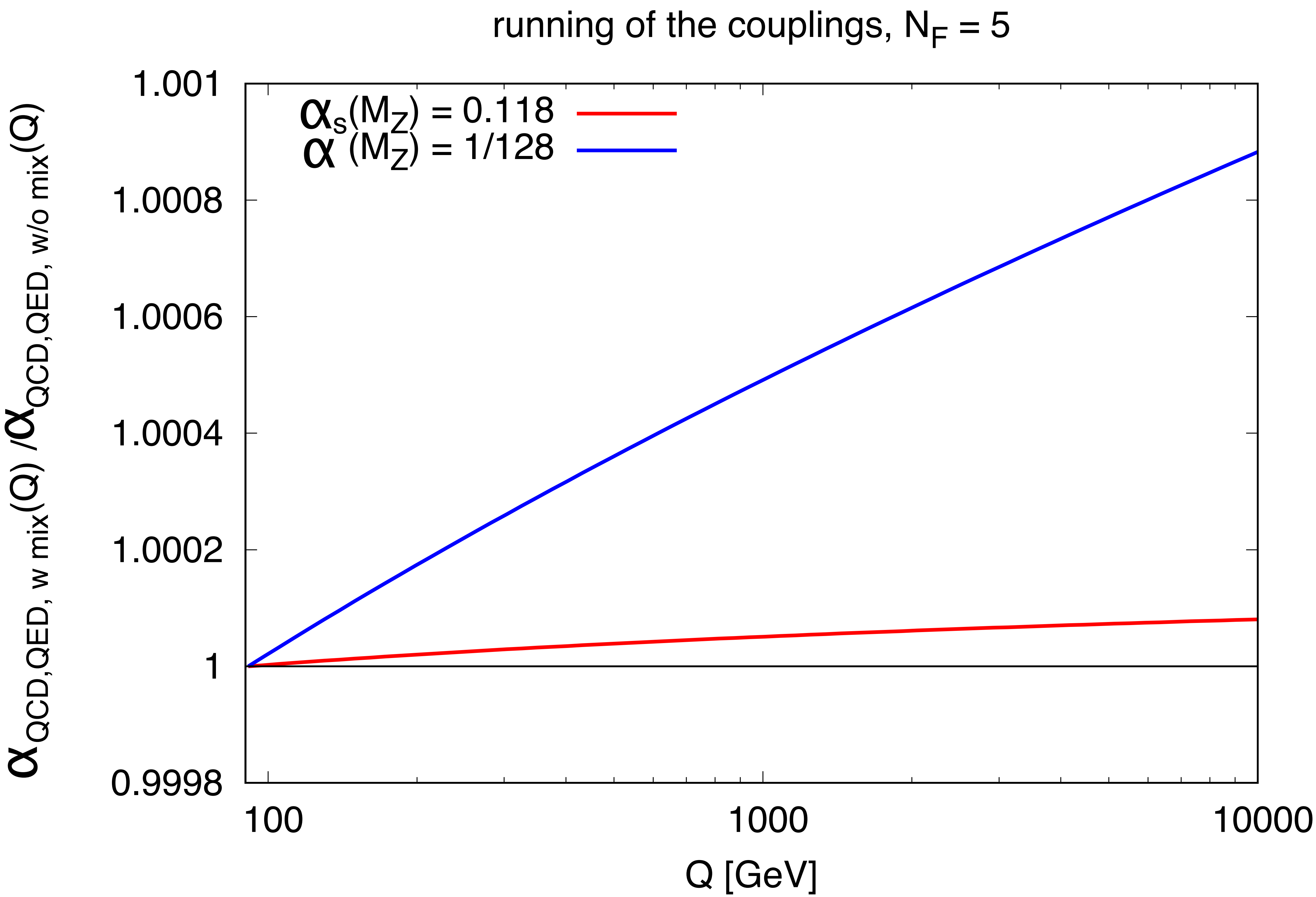
In order to implement the full NLO QCD+QED corrections in the DGLAP evolution two main steps are required:

1. Implementing the $O(\alpha_s^2\alpha)$, $O(\alpha^3)$, $O(\alpha^2\alpha_s)$ corrections to the **β-functions**:

   - running of α_s and α is coupled $\Rightarrow$ solve of a coupled ODE,

   - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for α $\Rightarrow$ **unneeded complication**.

NLO QCD+QED Corrections
Coupling Evolution



- Mixed terms in the β-functions lead to negligible effects.

NLO QCD+QED Corrections
Evolution

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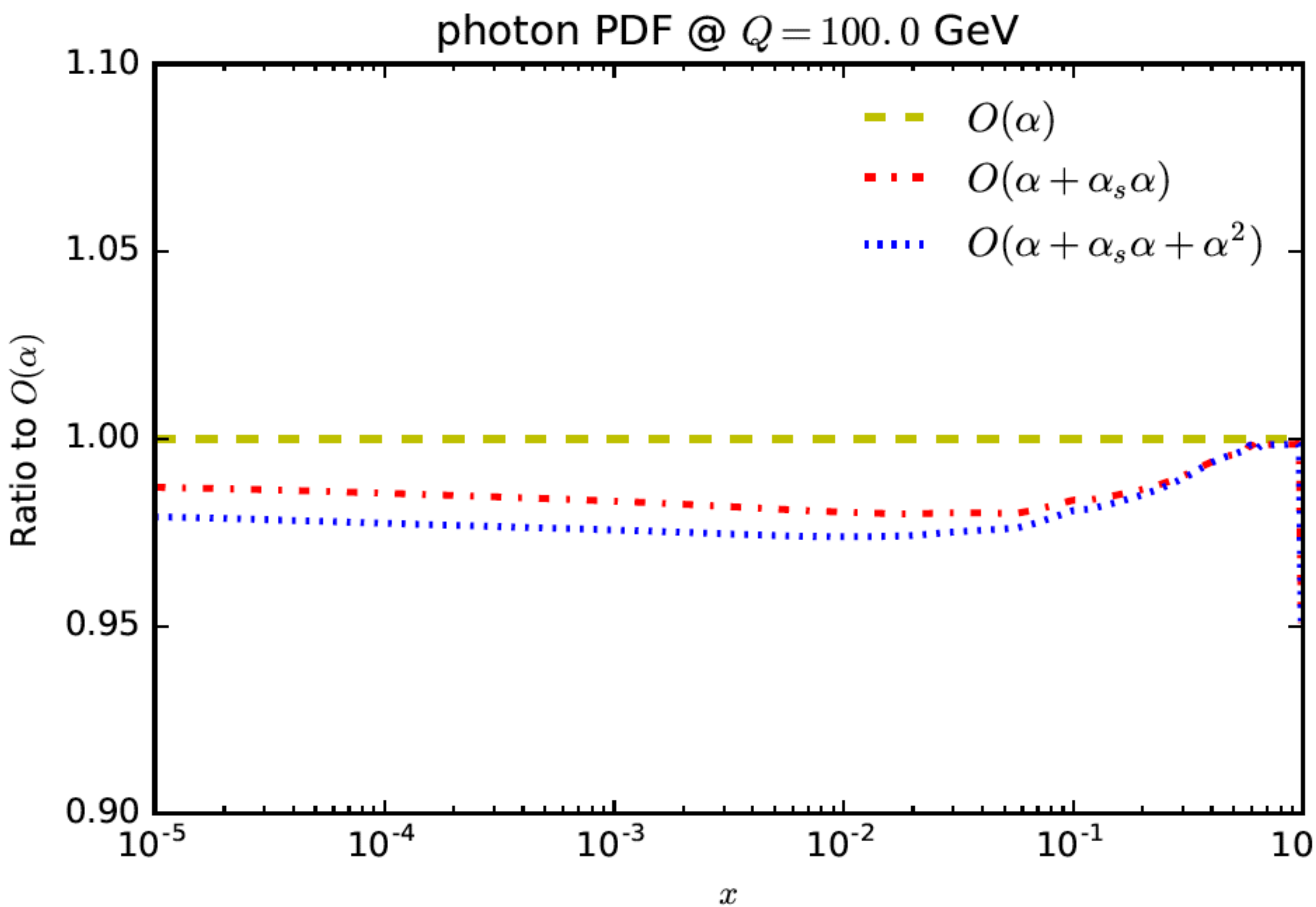
    - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for α $\Rightarrow$ **unneeded complication**.

2. Implementing the $O(\alpha_s\alpha)$ and the $O(\alpha^2)$ corrections to the DGLAP **splitting functions** on top of the $O(\alpha)$ ones:

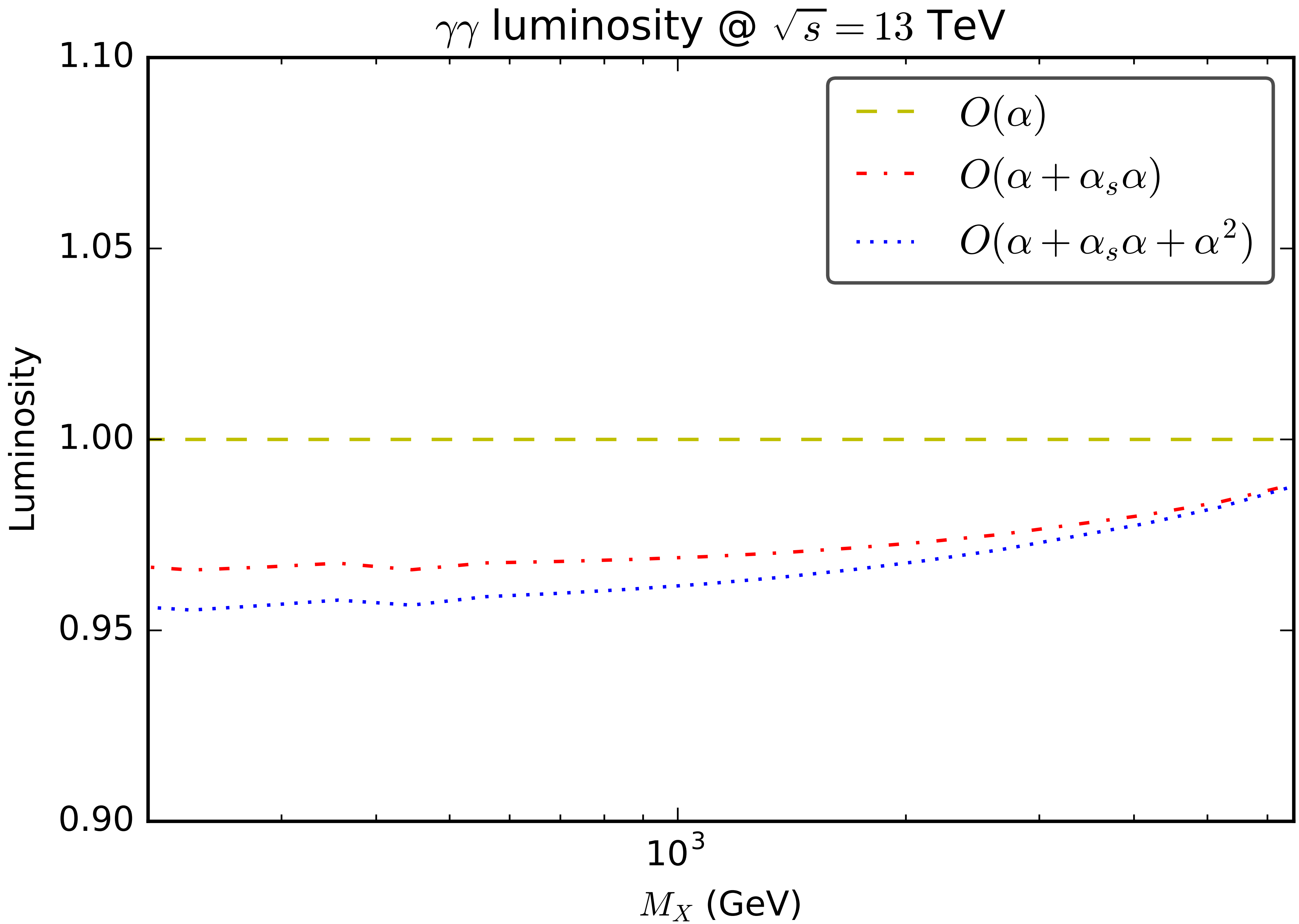
    - complication of the flavour structure due to the presence of terms promotional to e_q^2 and e_q^4 that break the isospin symmetry,
    - need for a more optimal evolution basis as compared to pure QCD.

NLO QCD+QED Corrections
DGLAP Evolution



- Effect on the photon PDF of the NLO corrns. at the level of 1-2%.

NLO QCD+QED Corrections
DGLAP Evolution

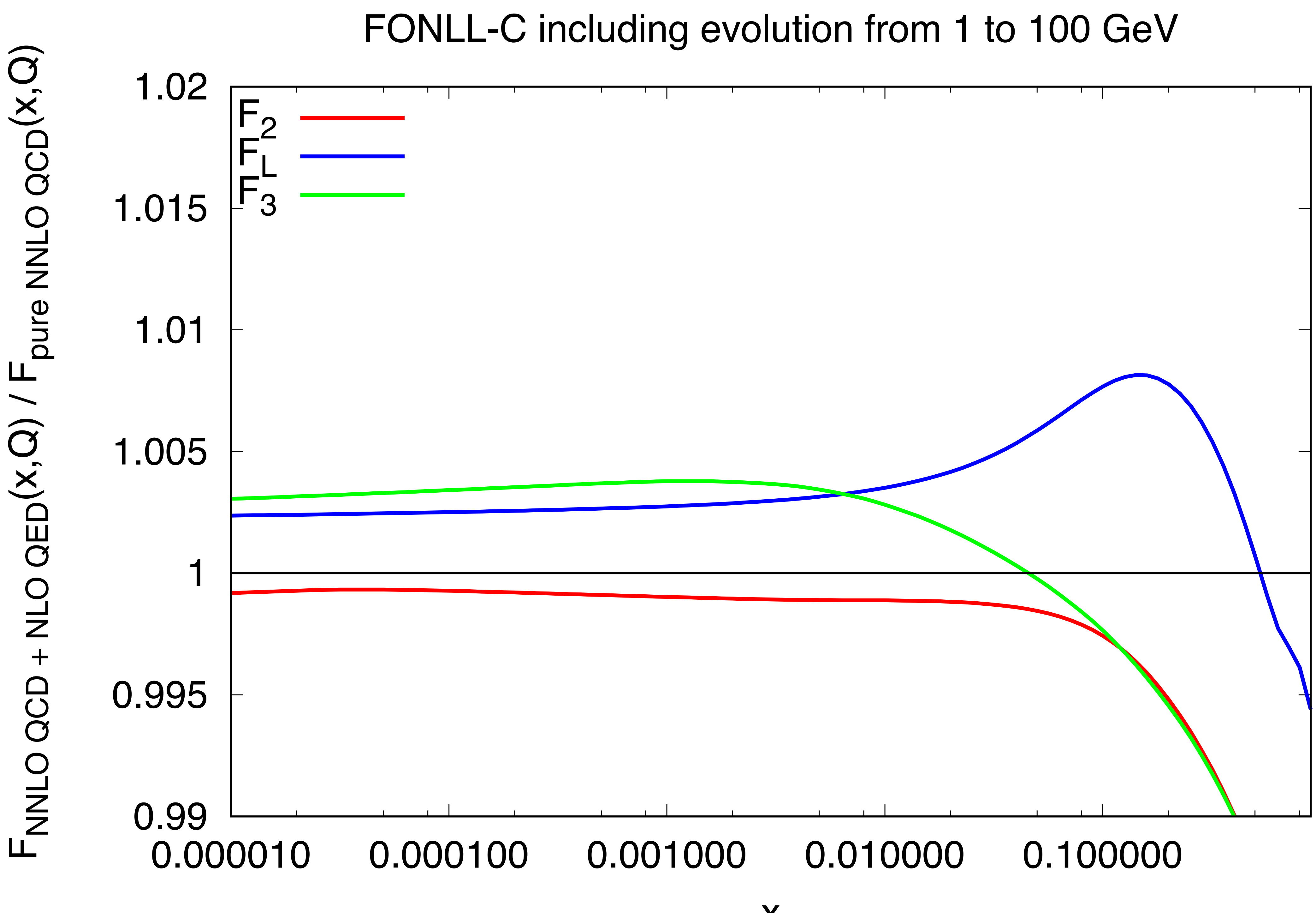


- Slightly more sizeable effect on the $\gamma\gamma$ luminosity.

NLO QCD+QED Corrections
DIS Structure Functions

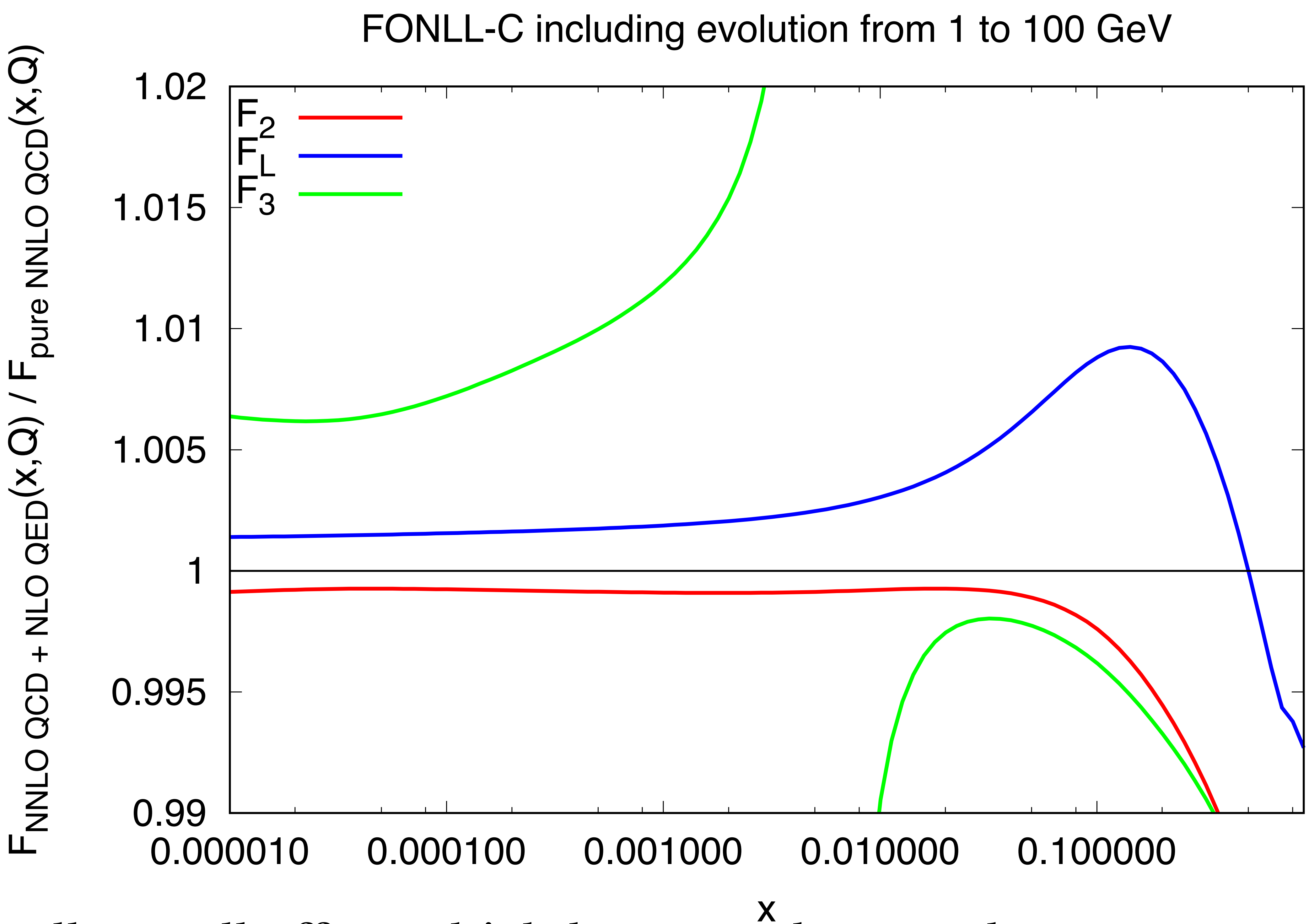
- While at LO in QED no corrections to the DIS structure functions are required ($\gamma^* q \to q$ itself is the LO), at NLO in QED $O(\alpha)$ corrections need to be taken into account:
    - **new diagrams**: $\gamma^*\gamma \to q\overline{q}$ and $\gamma^* q \to q\gamma$,
    - easily derivable from the corresponding QCD diagrams.
- The additional diagrams offer a **direct handle** on the photon PDF in DIS observables:
    - at LO in QED the photon PDF was entirely driven by the evolution.
- Small contribution proportional to $\alpha\gamma \sim O(\alpha^2)$ but can be relevant in some kinematic regions:
    - typically at large x and large Q^2.

NLO QCD+QED Corrections
DIS Structure Functions (NC)



- Generally small effect which becomes large at large *x*.

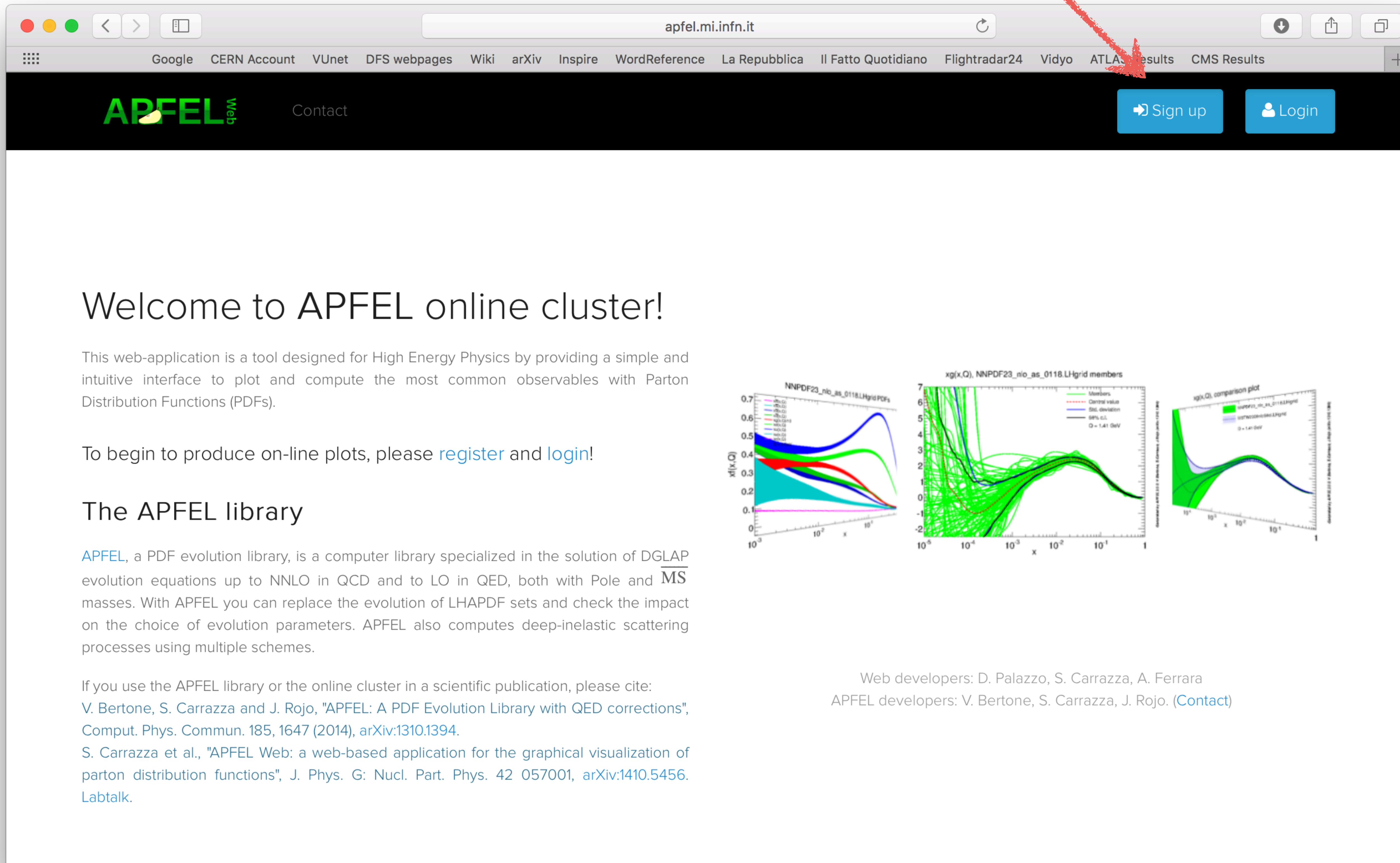
NLO QCD+QED Corrections
DIS Structure Functions (CC)



- Generally small effect which becomes large at large x.

APFEL Web

Go to http://apfel.mi.infn.it and sign up



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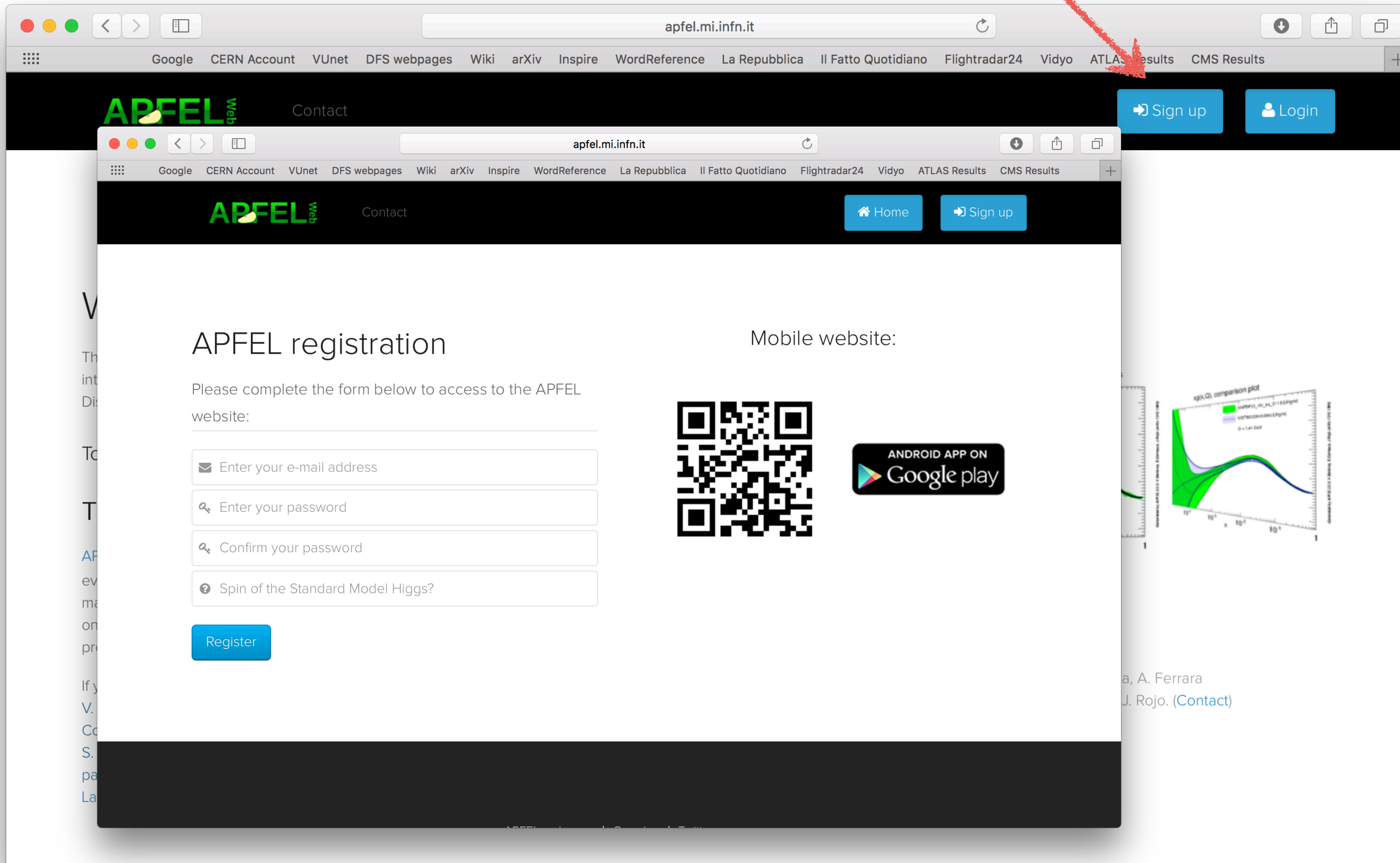
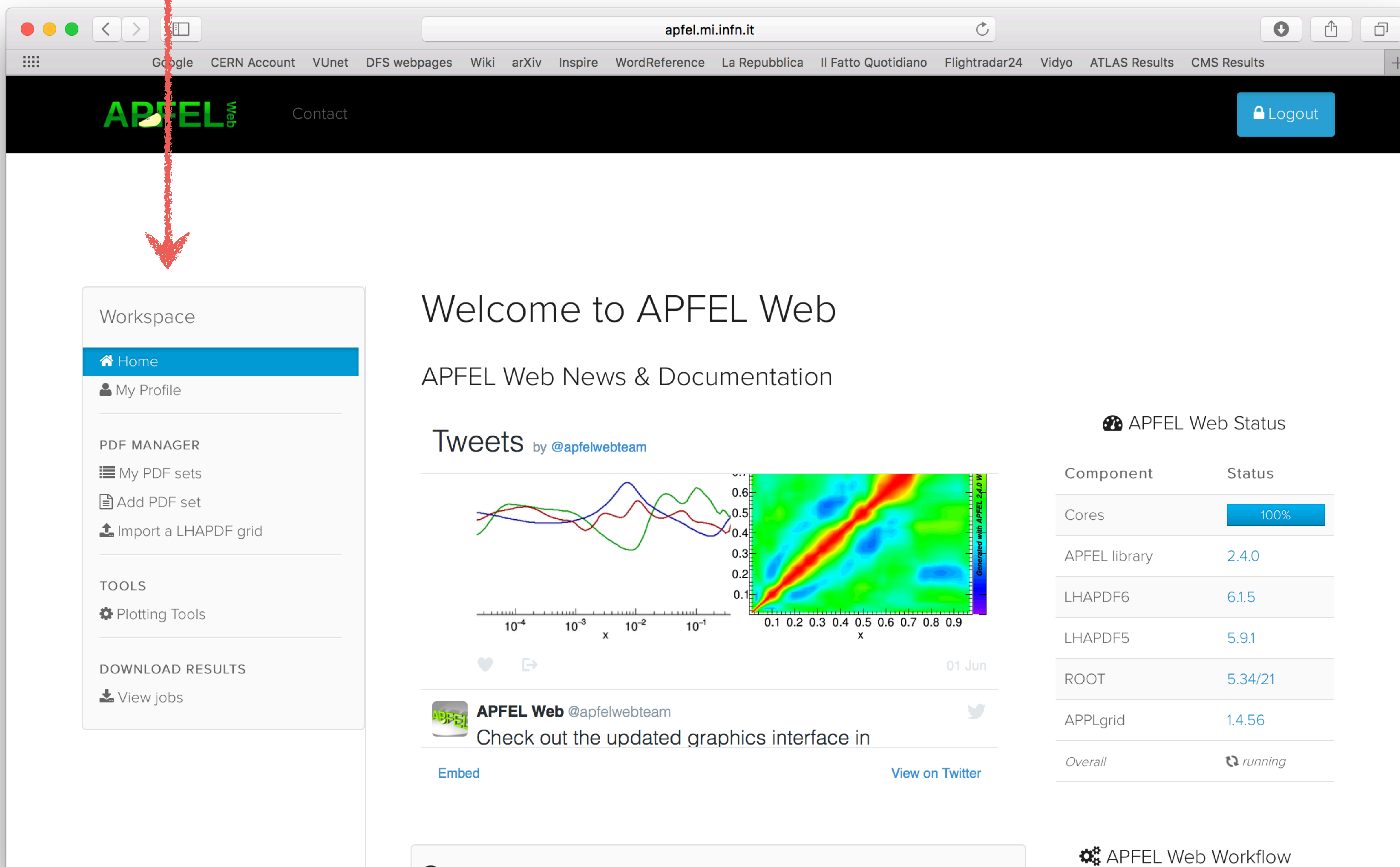


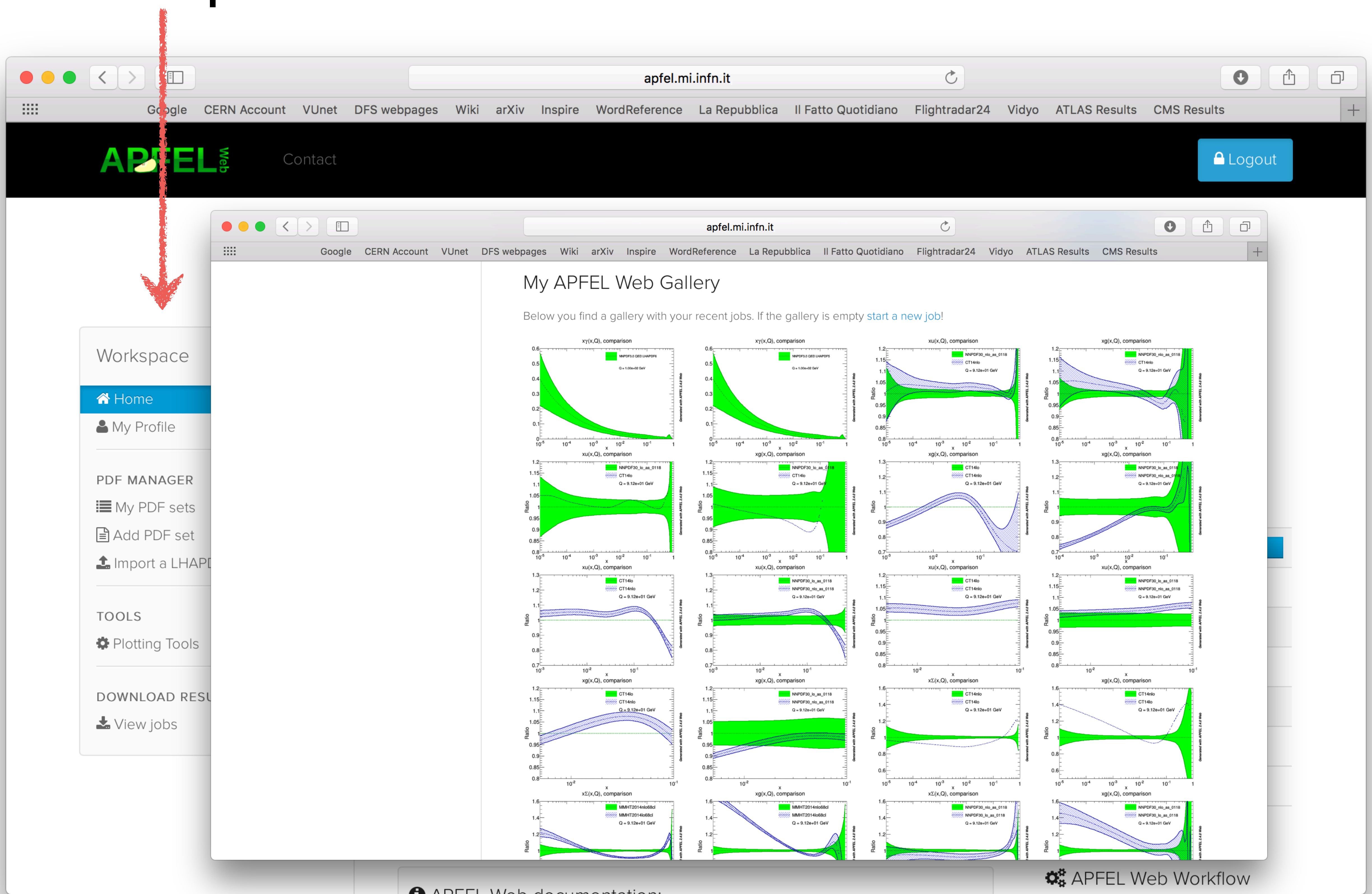
APFEL Web

Workspace



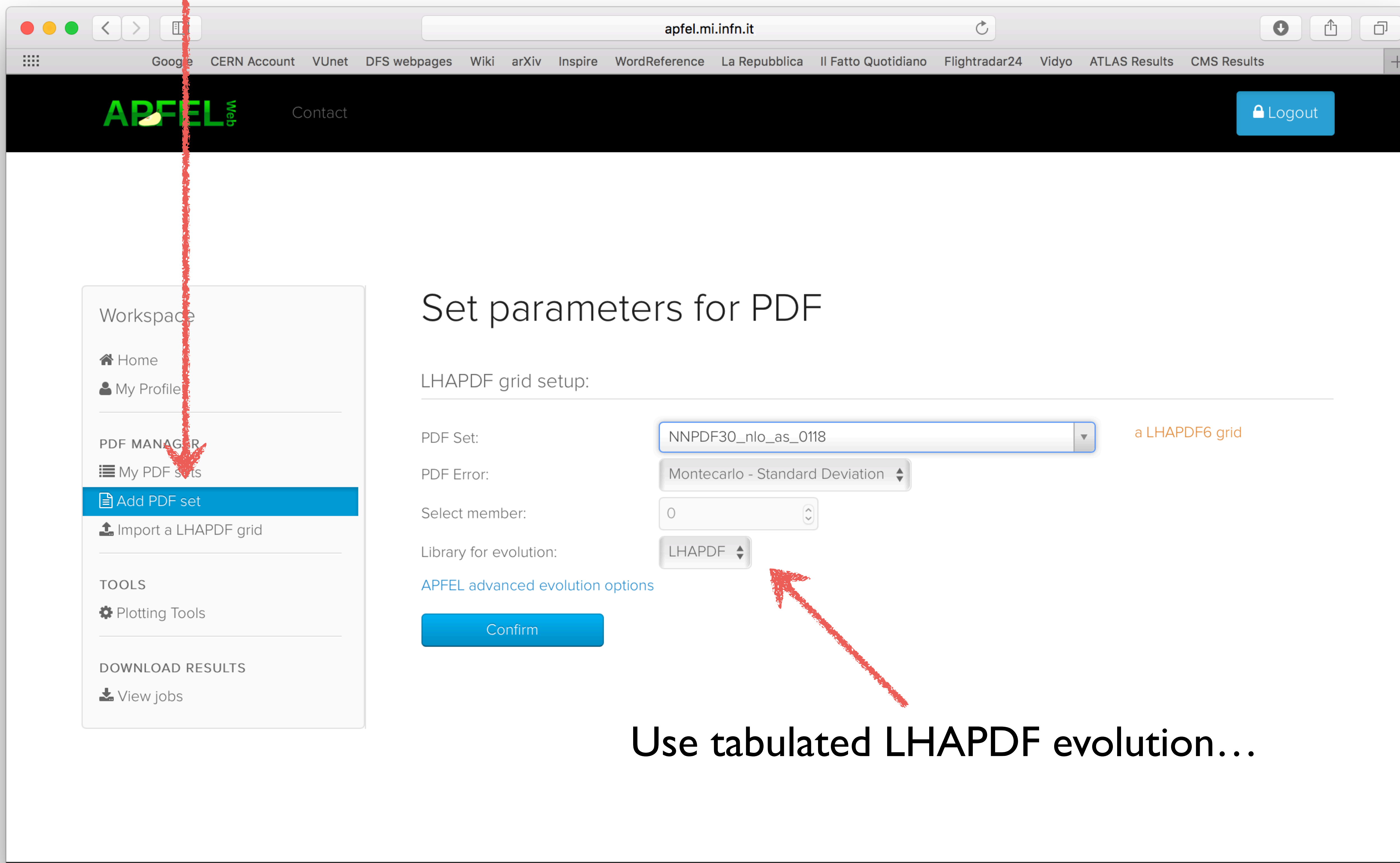
APFEL Web

Workspace



APFEL Web

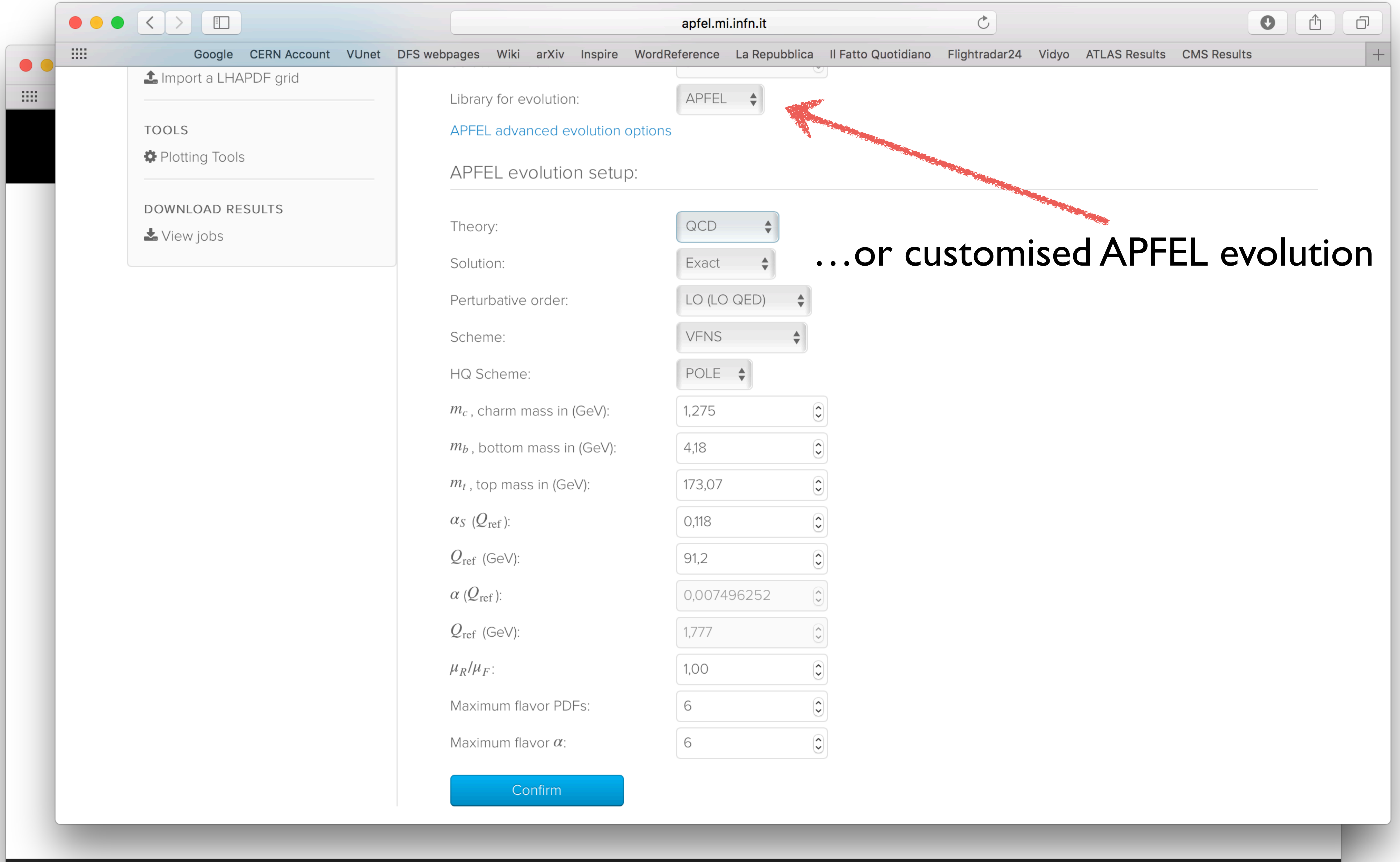
Add a new PDF set



Use tabulated LHAPDF evolution…

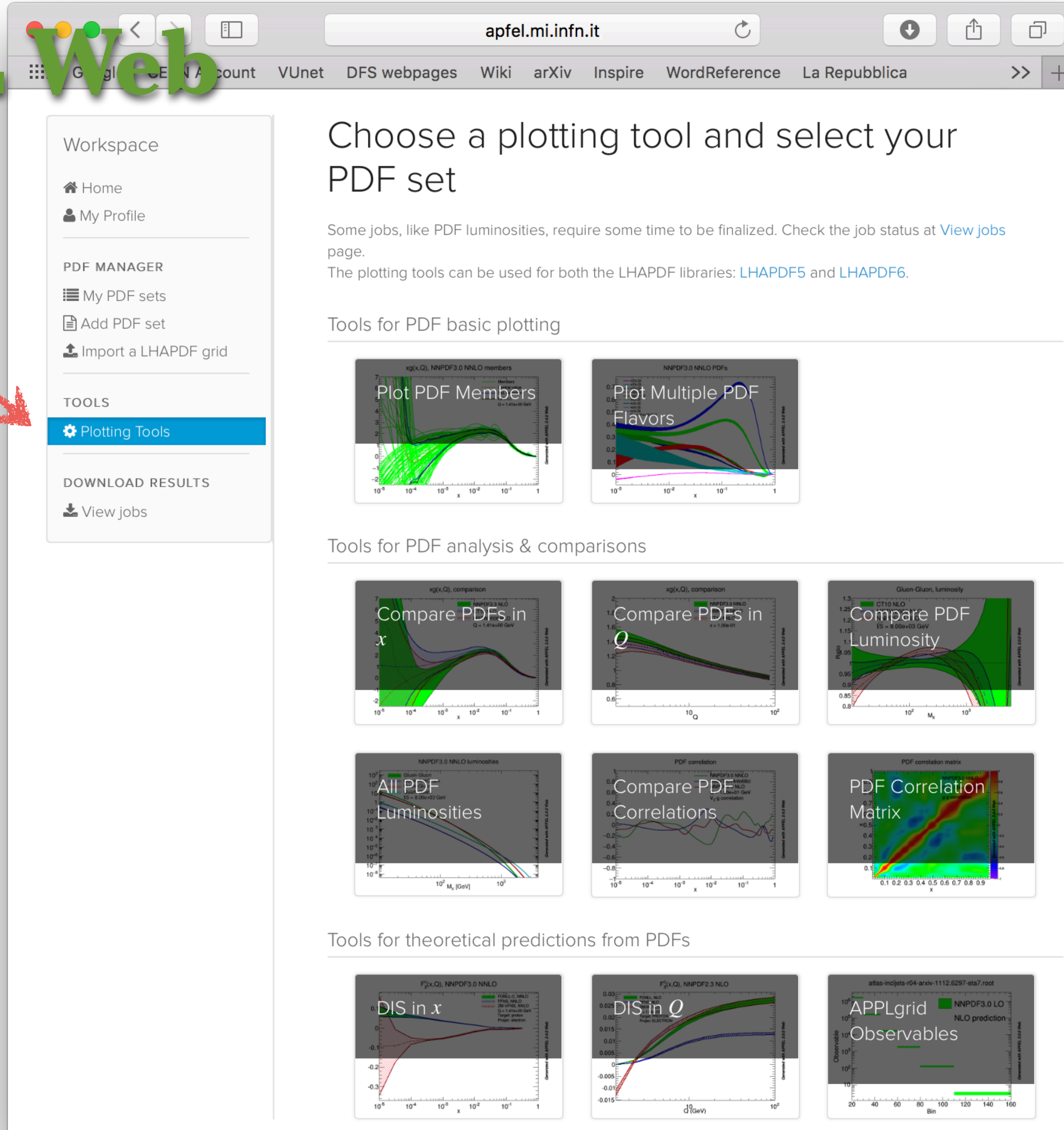
APFEL Web

Add a new PDF set



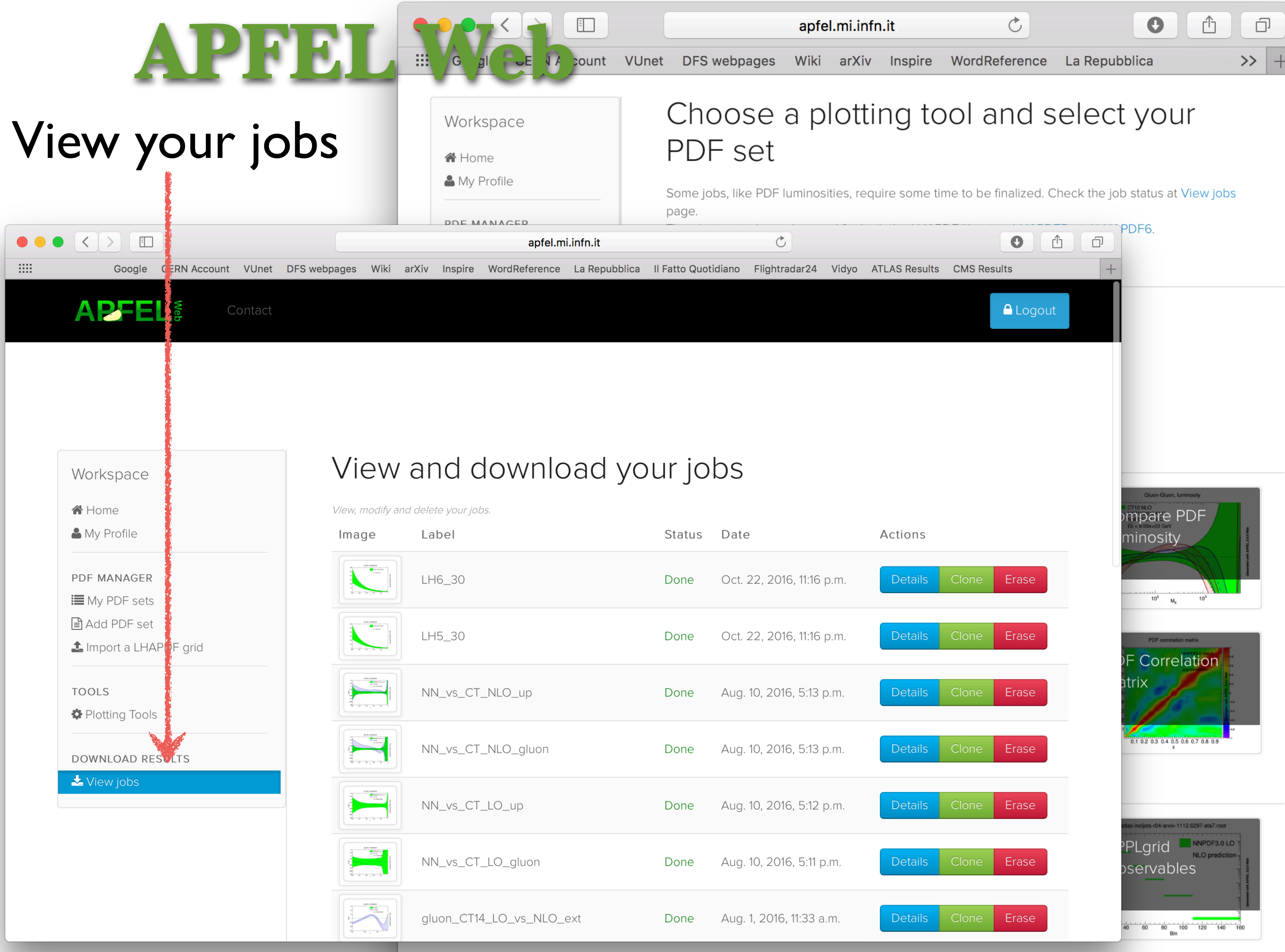
APFEL Web

Plotting tools



APFEL Web

View your jobs



TMD Evolution (PDFs)

$$
\begin{aligned}
F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
&\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
&\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C
\end{aligned}
$$

TMD Evolution (PDFs)

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \quad : A$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \quad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \quad : C$$

- $b_T \ll 1/\Lambda_{QCD}$
- matching to the collinear region
- factorises as hard and non-perturbative
- numerically cumbersome
- precompute using the APFEL technology

- CS evolution
- perturbative

- matching between the small and large b_T
- non perturbative
- parametrised and fitted to data

TMDs in SIDIS

- In SIDIS, what enters the computation of the cross sections is:

$$\mathcal{L}_{\text{SIDIS}} = \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{q}_T\cdot\mathbf{b}_T} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F) D_{H/f}(x, \mathbf{b}_T; \mu, \zeta_D)$$

Fourier transform PDFs FFs

- The ingredients are:
  - a set of evolved TMD-PDFs,
  - a set of evolved TMD-FFs,
  - the Fourier transform of its product.
- Complex set of tasks that have to be performed optimally
- APFEL provides the ideal environment for this computation:
  - fast and accurate interpolation techniques,
  - precomputation of the time consuming bits.

Summary

- Recent developments in APFEL:
    - **small-*x* resummation** in PDF evolution and structure functions,
    - framework for the **determination of FFs**,
    - **NLO QED corrections** to evolution and structure functions.

Other Recent Developments

- **Intrinsic-charm** in DIS *a la* FONLL.
- **Polarised DGLAP evolution** up to NNLO.
- Independent factorisation and renormalisation **scale variations** both in the DIS structure functions and in the evolution.

In the Pipeline

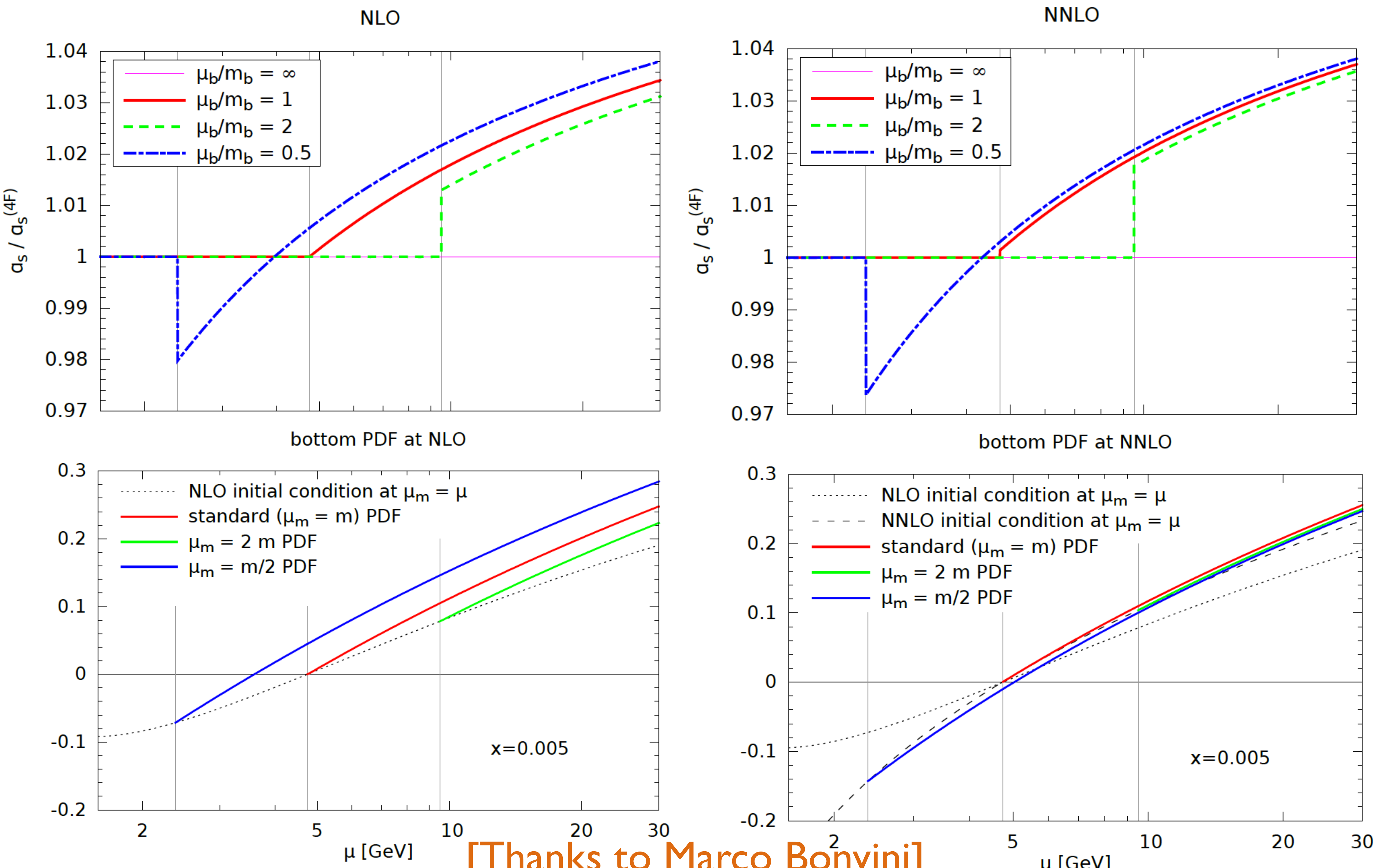
- Implementation of **TMD evolution** and **SIDIS cross sections**,
- Implementation of the **polarised structure functions**,
- **mass corrections** to SIA structure functions.

Backup Slides

Displaced Heavy-Quark Thresholds

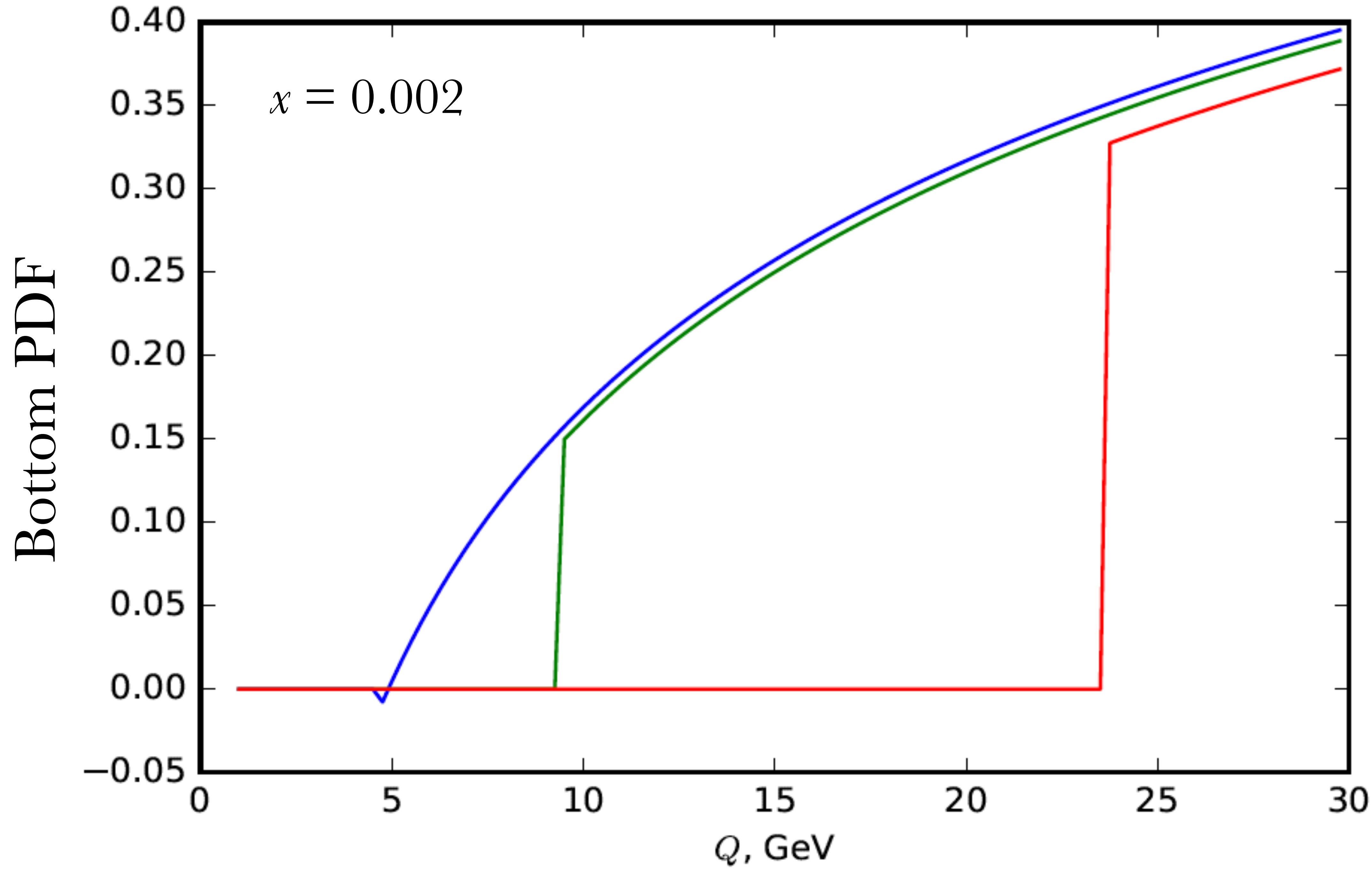
- The implementation of the VFNS evolution both for PDFs and α_s requires **matching** factorization schemes differing in the number of active flavours:
  - the scale at which two consecutive factorization schemes are matched are usually referred to as **heavy-quark thresholds**.
  - Heavy-quark thresholds are usually (and for convenience) identified with the heavy quark masses by means of the so-called **matching conditions** presently know up to $O(\alpha_s^2)$ [hep-ph/9612398].
  - However, heavy-quark thresholds are actually free parameters and can be chosen **arbitrarily**.
  - If masses and thresholds are taken to be different, the matching conditions need to be "generalized" including **logarithmic terms**.
- APFEL now implements the possibility to set masses and thresholds to different values in a consistent way both in the pole mass and in the $\overline{\text{MS}}$ renormalization schemes.

Displaced Heavy-Quark Thresholds



[Thanks to Marco Bonvini]

Displaced Heavy-Quark Thresholds



[Thanks to Sasha Glazov]

APFELgrid

A fast(er) interface for PDF fits

- While being an extremely useful tool, **APPLgrid** might not be appropriate to be directly employed in a global PDF fit where usually **thousands of iterations** are needed:
    - Need to calculate PDF and α_s **evolution in real time**.
    - **Not particularly fast** convolution.
    - many tables need to be loaded with the concrete risk of **exceeding the memory limit** (pretty common on clusters).
- In the NNPDF collaboration we developed **APFELgrid** which, starting from an existing APPLgrid, combines PDF evolution from APFEL to the hard cross sections producing *derived* interpolation tables (FK tables):

Observable	APPLGRID	FK	optimized FK
W^+ production	1.03 ms	0.41 ms (2.5x)	0.32 ms (3.2x)
Inclusive jet production	2.45 ms	20.1 μs (120x)	6.57 μs (370x)

- APFELgrid will soon be made **public in APFEL.** [thanks to N. Harthland]

Improvements

A New Fast Evolution

- In the previous versions of APFEL the DGLAP evolution equations were written in terms of the **evolution operator**:

$$\mu^2 \frac{\partial}{\partial \mu^2} M_{ij}(\mu, \mu_0) = P_{ik}(\mu) \otimes M_{kj}(\mu, \mu_0) \quad \text{with} \quad f_i(\mu) = M_{ij}(\mu, \mu_0) \otimes f_j(\mu_0)$$

- This may be convenient because the evolution operator can be evaluated once and for all and convoluted with any initial PDF set.

- On the other hand, this requires solving numerically a big coupled system of ODEs, therefore it can be slow.

- Alternatively, one can directly solve the DGLAP equations in terms of **PDFs**:

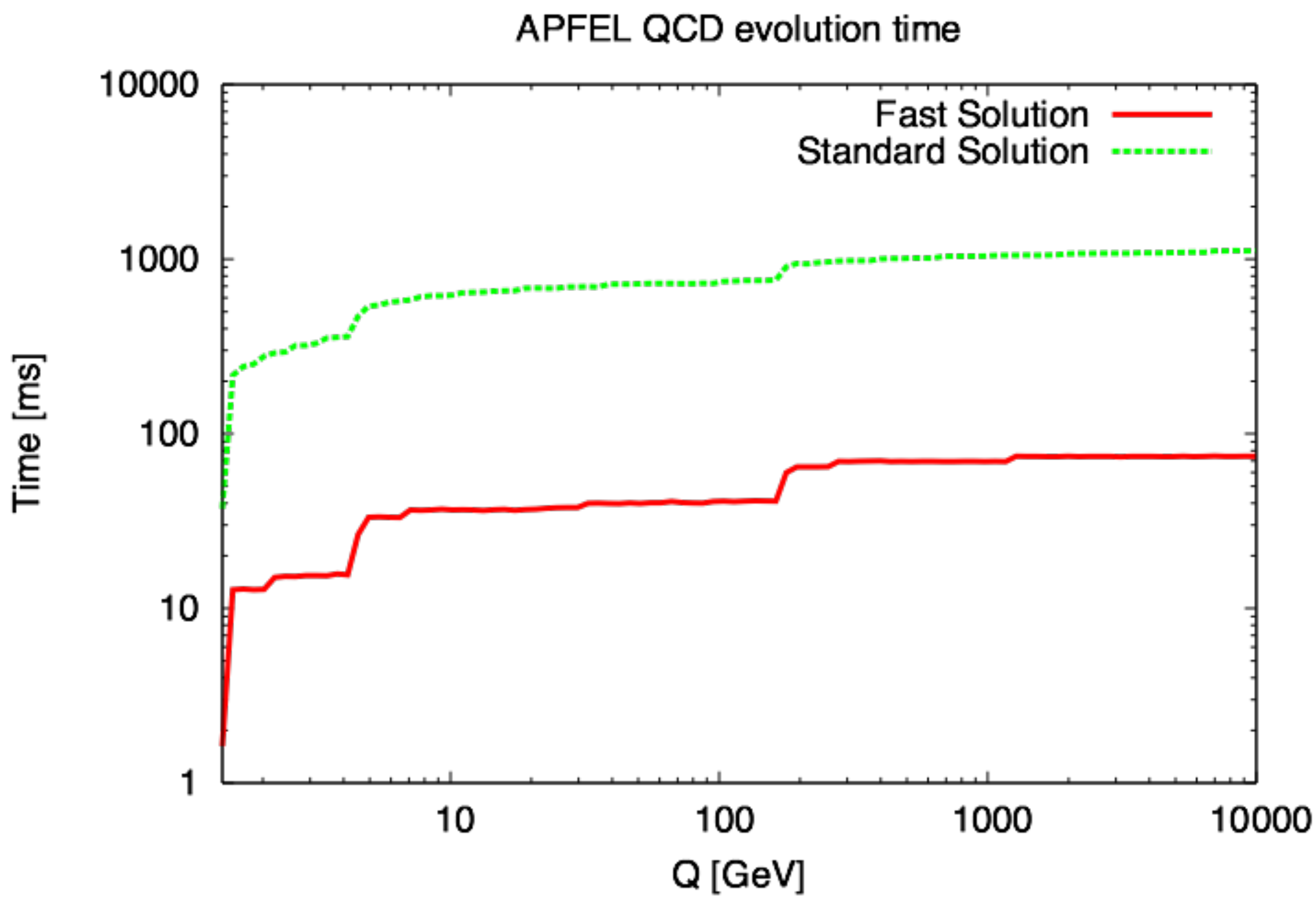
$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = P_{ij}(\mu) \otimes f_j(\mu)$$

- This requires the solution of a much smaller system of equations and is consequently much faster.

Improvements

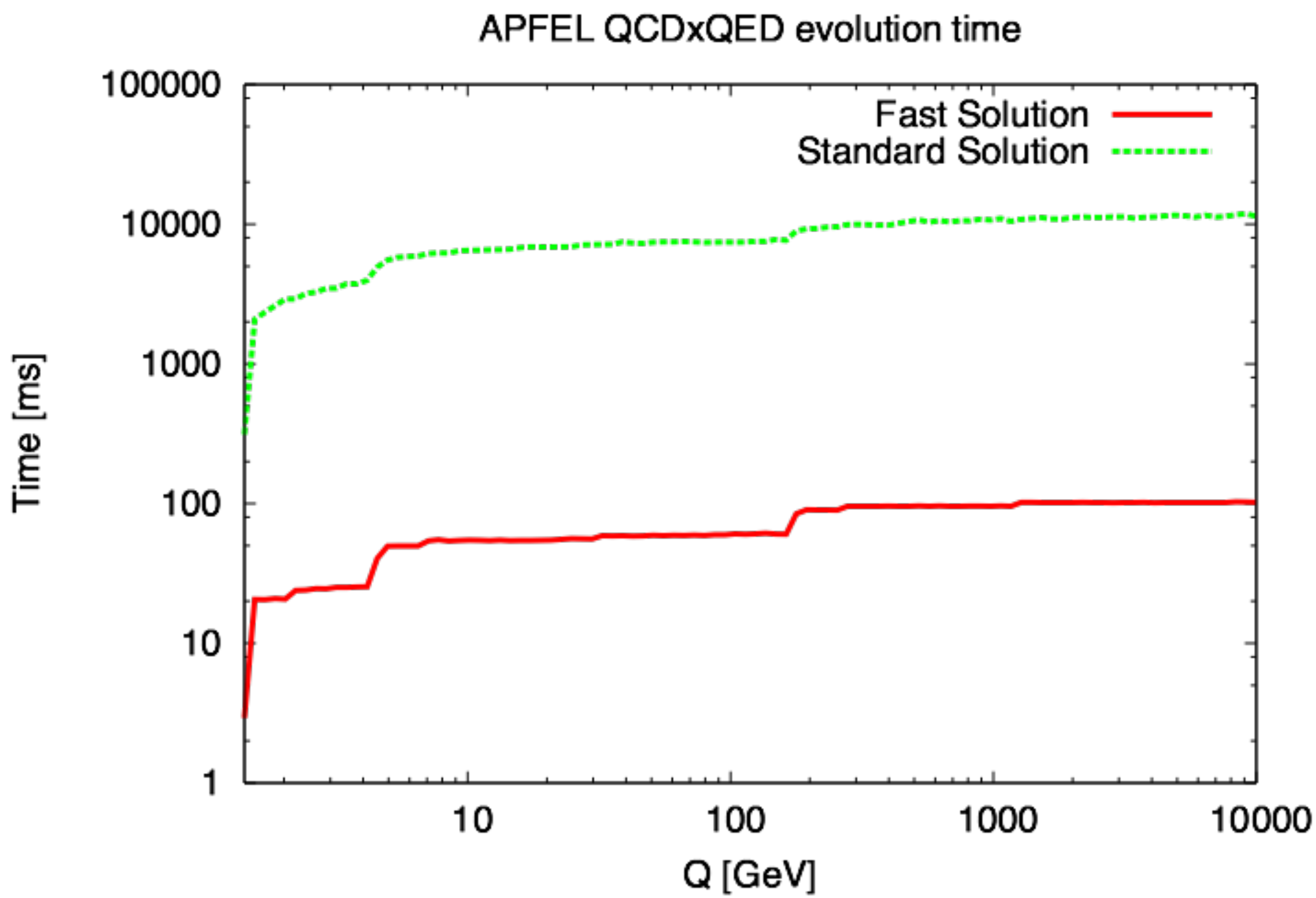
A New Fast Evolution

- Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD evolution:



Improvements
A New Fast Evolution

- Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD+QED evolution:



Improvements

A New QCD+QED Evolution

- In the previous versions of APFEL the QCD+QED evolution was performed by combining the **separate** QCD and QED evolution:
  - we showed that the differences, of a few % at most, with the standard implementations which evolve contemporaneously in QCD and QED were due to **subleading terms in α**.

- We have now implemented a new evolution basis which allows a **simultaneous diagonalization** of the QCD+QED evolution matrix:

1) g
2) γ
3) $\Sigma = \Sigma_u + \Sigma_d$
4) $\Delta_\Sigma = \Sigma_u - \Sigma_d$
5) $T_1^u = u^+ - c^+$
6) $T_2^u = u^+ + c^+ - 2t^+$
7) $T_1^d = d^+ - s^+$
8) $T_2^d = d^+ + s^+ - 2b^+$
9) $V = V_u + V_d$
10) $\Delta_V = V_u - V_d$
11) $V_1^u = u^- - c^-$
12) $V_2^u = u^- + c^- - 2t^-$
13) $V_1^d = d^- - s^-$
14) $V_2^d = d^- + s^- - 2b^-$

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Coupled

1) g
2) γ
3) $\Sigma = \Sigma_u + \Sigma_d$
4) $\Delta_\Sigma = \Sigma_u - \Sigma_d$

9) $V = V_u + V_d$
10) $\Delta_V = V_u - V_d$

Decoupled

5) $T_1^u = u^+ - c^+$
6) $T_2^u = u^+ + c^+ - 2t^+$
7) $T_1^d = d^+ - s^+$
8) $T_2^d = d^+ + s^+ - 2b^+$

11) $V_1^u = u^- - c^-$
12) $V_2^u = u^- + c^- - 2t^-$
13) $V_1^d = d^- - s^-$
14) $V_2^d = d^- + s^- - 2b^-$

- This new basis is also suitable for an easy implementation of the mixed **higher order corrections** to the evolution.

Intrinsic Charm

- Introducing an intrinsic charm (IC) component in the context of a GM-VFNS like FONLL (or ACOT, or TR) requires some care:
  - **relax** the assumption of **pure perturbative generation** of heavy quarks at the thresholds,
  - take into account **charm-initiated diagrams** both in the **massive** and in the massless sectors [arXiv:1510.00009].
- A full formulation of the FONLL scheme in the presence of IC has **recently** been **achieved** [arXiv:1510.02491]:
  - interestingly, it has been found that **FONLL** with IC is **equivalent** to full **ACOT** to all orders, while the standard FONLL (w/o IC) is instead equivalent to S-ACOT.
  - Implemented in APFEL up to NLO both in the **NC** and **CC** sector and **benchmarked** against the public `massiveDISsFuntion` code (`https://www.ge.infn.it/~bonvini/massivedis/`).

Intrinsic Charm

- Consider realistic models:
  - BHPS model:

$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = Ax^2\left[6x(1+x)\ln x + (1-x)(1+10x+x^2)\right]$$

  - SEA model:

$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = Ax^{-1.25}(1-x)^3$$

- A determined requiring the charm to carry 0.5% of the momentum