## TMD fragmentation in jets

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REF 2016 Antwerp

## Outline

1. Fragmentation in jets
2. TMD fragmentation in jets
3. First results
4. Conclusions and outlook

In collaboration with D. Neill and I. Scimemi

## 1. Fragmentation in jets

## Introduction: What is fragmentation?



- Parton $i=q, g$ radiates and hadronizes $\rightarrow$ produces hadron $h$
- Described by fragmentation function $D_{i \rightarrow h}\left(z_{h}, \mu\right)$ [colins, Soper]
- E.g. $e^{+} e^{-} \rightarrow h X$

$$
\frac{d \sigma}{d z_{h}}=\sigma_{0} \sum_{i} \int_{z_{h}}^{1} \frac{d z}{z} \hat{\sigma}_{i}(z, Q, \mu) D_{i \rightarrow h}\left(\frac{z_{h}}{z}, \mu\right)
$$

[Collins, Soper, Sterman]

## Case study: Fragmentation with a cut on thrust

- Motivation: study light-quark fragmentation at Belle
- Dominant $b$-quark contribution in on-resonance data removed by cut on thrust $T>0.8$



## Case study: Fragmentation with a cut on thrust

- Motivation: study light-quark fragmentation at Belle
- Dominant $b$-quark contribution in on-resonance data removed by cut on thrust $T>0.8$
- Thrust cut modifies shape of fragmentation spectrum



[Jain, Procura, WW]


## Key ingredient: Fragmenting jet function



$$
\mathcal{G}_{i \rightarrow h}\left(s, z_{h}, \mu\right)
$$

- Collinear radiation contributes s to thrust and produces hadron


## Key ingredient: Fragmenting jet function

$$
\begin{aligned}
& \mathcal{G}_{i \rightarrow h}\left(s, z_{h}, \mu\right)=\sum_{j} \int_{z_{h}}^{1} \frac{d z}{z} \mathcal{J}_{i j}(s, z, \mu) D_{j \rightarrow h}\left(\frac{z_{h}}{z}, \mu\right)
\end{aligned}
$$

- Collinear radiation contributes $s$ to thrust and produces hadron
- OPE in $\Lambda_{\mathrm{QCD}}^{2} / s$ with perturbatively calculable coefficients

$$
\mathcal{J}_{i j}(s, z, \mu)=\delta_{i j} \delta(s) \delta(1-z)+\mathcal{O}\left(\alpha_{s}\right)
$$

- Fragmentation function probed at $\mu \sim \sqrt{s} \ll Q$


## Developments

- Hemisphere jets [Procura, Stewart; Liu; Jain, Procura, WW; Bauer, Mereghetti; Ritzmann, WW]
- Exclusive jet production [Procura, ww; Chien, Kang, Ringer, Vitev, Xing; Baumgart, Leibovich, Mehen, Rothstein; Bain, Dai, Hornig, Leibovich, Makris, Mehen]
- Inclusive jet production [Kaufmann, Mukherjee, Vogelsang; Dai, Kim, Leibovich; Kang, Ringer, Vitev]
- Jet charge [Krohn, Schwartz, Lin, ww; ww]




## Challenges

x Event shapes: susceptible to spectator-spectator interactions (Glaubers) in pp collisions [Gaunt; Zeng; Rothstein, Stewart]
x Exclusive jet production: non-global logarithms from different restrictions in regions of phase-space [Dasgupta, salam, ...]
$\checkmark$ Inclusive jet production: insensitive to soft radiation


Glauber exchange


Soft emission giving rise to non-global logarithm

## 2. TMD fragmentation in jets



## Removing soft recoil



- Fragmentation in inclusive jet production is purely collinear
- TMD measurement introduces soft sensitivity through axis
$\checkmark$ Choose a recoil free axis, e.g. winner-takes-all [Larkoski, Neill, Thaler]


## Winner-takes-all axis



- Run jet algorithm with following recombination scheme

$$
\begin{aligned}
& E_{r}=E_{1}+E_{2} \\
& \hat{n}_{r}= \begin{cases}\hat{n}_{1} & \text { if } E_{1}>E_{2} \\
\hat{n}_{2} & \text { if } E_{2}>E_{1}\end{cases}
\end{aligned}
$$

- Axis tracks energetic radiation, along direction of a particle


## Factorization of the jet



- Factor hard scattering from jet production for jet radius $R \ll 1$
$\frac{d \sigma_{h}}{d p_{T} d \eta d \vec{p}_{h \perp}^{2} d z_{h}}=\sum_{i} \int \frac{d x}{x} \hat{\sigma}_{i}\left(\frac{p_{T}}{x}, \eta, \mu\right) \mathcal{G}_{i \rightarrow h}\left(x, p_{T} R, \vec{p}_{h \perp}^{2}, z_{h}, \mu\right)$
- Transverse momentum $p_{T}$ and rapidity $\eta$ of jet
$\checkmark \mathcal{G}$ is universal because measurement is purely collinear, i.e. same for ee, ep and pp and independent of other jets


## Factorization of TMD fragmentation

- For $r \equiv\left|\vec{p}_{h \perp}\right| / p_{T} \ll R$ factor jet from TMD fragmentation
$\mathcal{G}_{i \rightarrow h}\left(x, p_{T} R, \vec{p}_{h \perp}^{2}, z_{h}, \mu\right)=\sum_{k} \int \frac{d z}{z} B_{i k}\left(x, p_{T} R, z, \mu\right) D_{k \rightarrow h}\left(\frac{\vec{p}_{h \perp}^{2}}{z^{2}}, \frac{z_{h}}{z}, \mu\right)$
- $B$ and $D$ describe emissions at angular scales $R$ and $r$


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- $B$ and $D$ describe emissions at angular scales $R$ and $r$

Factorization of the measurement:

- B identifies pixel of size $r$ containing axis
- D determines axis in pixel $\checkmark$ Ok for Cambridge/Aachen with winner-takes-all axis



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- $B$ and $D$ describe emissions at angular scales $R$ and $r$

Factorization of the amplitude:
$\checkmark$ Winner-takes-all axis guarantees that $B$ produces one energetic parton near axis (not true for standard axis)

- Hadron must fragment from this to be $1 / \vec{p}_{h \perp}^{2}$ enhanced


## Factorization of fragmentation



- For perturbative $\left|\vec{p}_{h \perp}\right| \gg \Lambda_{Q C D}$

$$
D_{i}^{k}\left(\vec{p}_{h \perp}^{2}, z_{h}, \mu\right)=\sum_{k} \int \frac{d z}{z} C_{i k}\left(\frac{\vec{p}_{h \perp}^{2}}{z^{2}}, \frac{z_{h}}{z}, \mu\right) D_{k \rightarrow h}(z, \mu)
$$

## Evolution and resummation

TMD fragmentation $D\left(\vec{p}_{h \perp}^{2}\right)$

$$
\begin{aligned}
& \frac{d \sigma_{h}}{d p_{T} d \eta d \vec{p}_{h \perp}^{2} d z_{h}}=\hat{\sigma}\left(p_{T}, \eta\right) \otimes B\left(p_{T} R\right) \otimes C\left(\vec{p}_{h \perp}^{2}\right) \otimes D\left(\Lambda_{\mathrm{QCD}}\right) \\
& \mathcal{J}\left(p_{T} R, \vec{p}_{h \perp}^{2}\right)
\end{aligned}
$$

$$
\mathcal{G}\left(p_{T} R, \vec{p}_{h \perp}^{2}\right)
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- Factorization separates physics at disparate scales


## Evolution and resummation

TMD fragmentation $D\left(\vec{p}_{h \perp}^{2}\right)$

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- Factorization separates physics at disparate scales
- Logarithms are resummed by renormalization group evolution


$$
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\end{aligned}
$$



## Matching coefficients for $p_{T} R \sim \vec{p}_{h \perp}^{2}$

- We calculated all matching coefficients at next-to-leading order

$$
\begin{aligned}
\mathcal{J}_{q q}\left(x, p_{T} R, \vec{p}_{h \perp}^{2}, z\right)= & \delta\left(\vec{p}_{h \perp}^{2}\right) \delta(x-1) \delta(z-1) \\
& +\frac{\alpha_{s} C_{F}}{2 \pi}\left[\frac{1}{\mu^{2}} \frac{1}{\left(\vec{p}_{h \perp}^{2} / \mu^{2}\right)} \delta(x-1) \theta\left(\frac{1}{2} \geq z \geq \frac{\left|\vec{p}_{h \perp}\right|}{p_{T} R}\right) \frac{1+z^{2}}{1-z}\right. \\
& \left.+\delta\left(\vec{p}_{h \perp}^{2}\right)(\ldots)\right]+\mathcal{O}\left(\alpha_{s}^{2}\right)
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- At this order only two partons and winner-takes-all axis along most energetic one, so $\vec{p}_{h \perp}=0$ for $z>1 / 2$
- No rapidity divergences


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- No rapidity divergences
- Interplay between jet boundary and TMD measurement (more complicated at higher orders)


## Matching coefficients for $p_{T} R \gg \vec{p}_{h \perp}^{2}$

- Jet boundary restriction and TMD measurement factorize

$$
\begin{aligned}
B_{q q}\left(x, p_{T} R, z, \mu\right)= & \delta(x-1) \delta(z-1)+\frac{\alpha_{s} C_{F}}{2 \pi}\left\{-\ln \frac{p_{T}^{2} R^{2}}{\mu^{2}}\left[\frac{1+x^{2}}{(1-x)} \delta(z-1)\right.\right. \\
& \left.\left.-\delta(x-1) \theta\left(z \geq \frac{1}{2}\right) \frac{1+z^{2}}{(1-z)_{+}}\right]+\ldots\right\} \\
C_{q q}\left(\vec{p}_{h \perp}^{2}, z\right)= & \delta\left(\vec{p}_{h \perp}^{2}\right) \delta(1-z)+\frac{\alpha_{s} C_{F}}{2 \pi}\left[\frac{1}{\mu^{2}} \frac{1}{\left(\vec{p}_{h \perp}^{2} / \mu^{2}\right)_{+}} \theta\left(\frac{1}{2} \geq z\right) \frac{1+z^{2}}{1-z}\right. \\
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- $B$ describes parton along axis and thus vanishes for $z<1 / 2$


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- $B$ describes parton along axis and thus vanishes for $z<1 / 2$
- Between $B$ and $C$ the DGLAP evolution in $z$ is modified


## Transverse momentum dependence

- TMD fragmentation matching coefficients $C$ give $1 / \vec{p}_{h \perp}^{2}$
- Difference between RG evolution above and below $C$ modify

$$
1 / \vec{p}_{h \perp}^{2} \quad \rightarrow \quad 1 / \vec{p}_{h \perp}^{2-\Delta}
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where $\Delta$ follows from anomalous dimensions

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$$

where $\Delta$ follows from anomalous dimensions

- For $N$-th moment and $\alpha_{s}=0.1$ this gives




## 4. Conclusions and outlook



## Conclusions and outlook

- TMD fragmentation in a jet with winner-takes-all axis:
- Purely collinear observable, so universal
- Jet and TMD fragmentation factorize for Cambridge/Aachen
- No rapidity divergences
- Applications (work in progress)
- Extend to spin-dependent fragmentation
- Medium modification of TMD fragmentation
- Jet substructure for boosted analyses
- ...


## Application: medium modifications



- Medium modification of fragmentation function is studied
- Can now be extended to TMD fragmentation
$\checkmark$ Observable must be insensitive to overload of soft "crap"


## Application: jet substructure

- Jet substructure is key to tag heavy particles at high energies

[ATLAS-CONF-2013-052]
- Exploits that subjet splittings differ between e.g. top and gluon
- TMD fragmentation can be extended from hadrons to subjets
$\checkmark$ Provides direct measure of subjet energies and angle


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