Heavy quark production in non polarized eA & pA collisions: the role of gluon polarization

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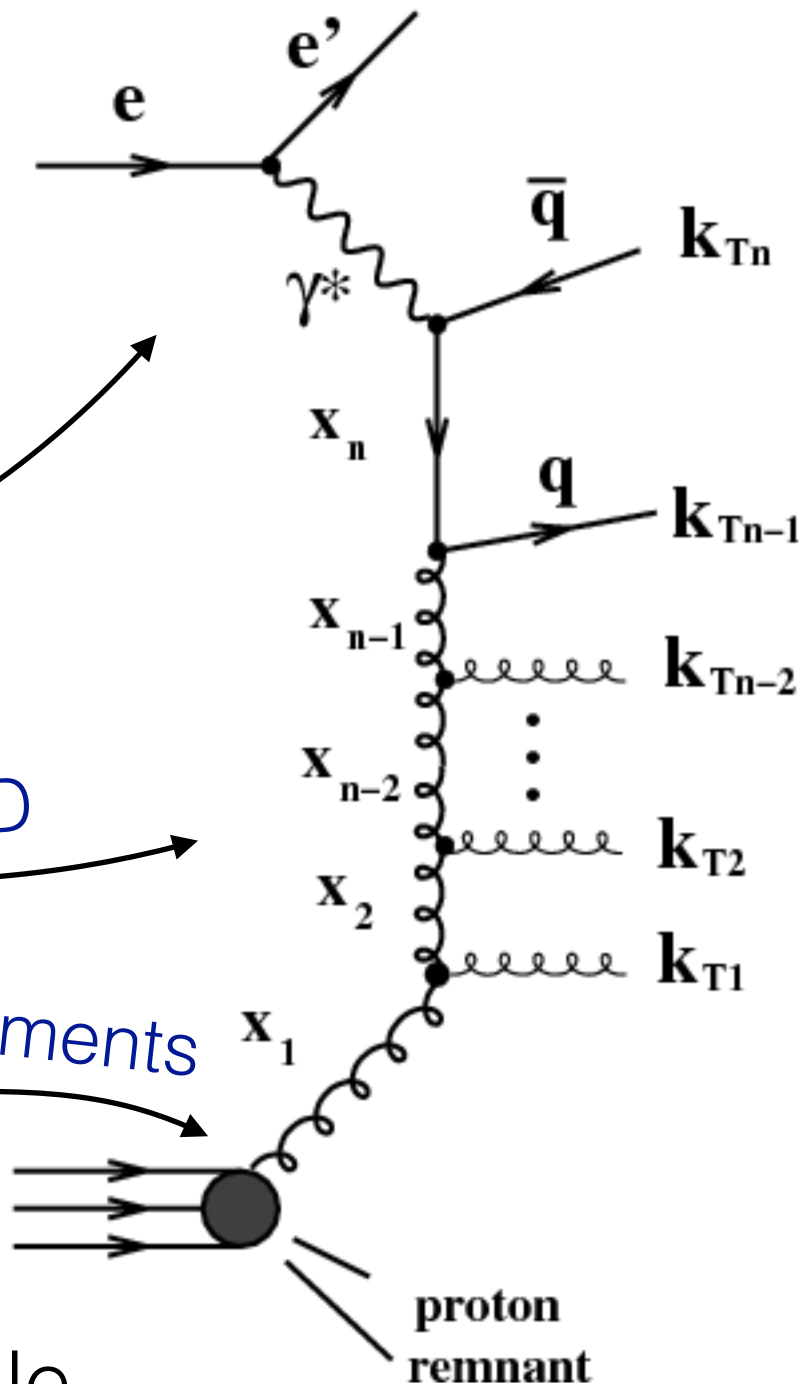
Factorization in QCD

Need to separate the physics we can calculate pertubatively: **hard part**, from the physics that we cannot obtain from pQCD: **soft part**

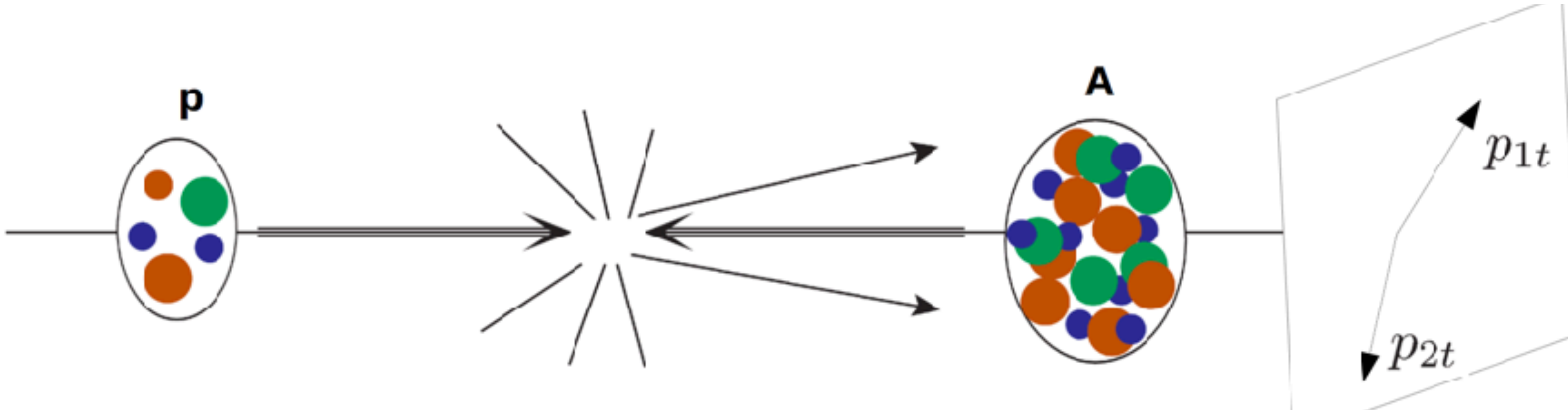
Collinear factorization works for inclusive processes like DIS:

$$\sigma_{\gamma^* p} = \sigma_{hard} \otimes \mathrm{DGLAP} \otimes \mathrm{pdf's}$$



Parton Density Functions (pdf's) are function of the transverse momentum scale and the longitudinal momentum fraction

Transverse Momentum Dependent pdf's



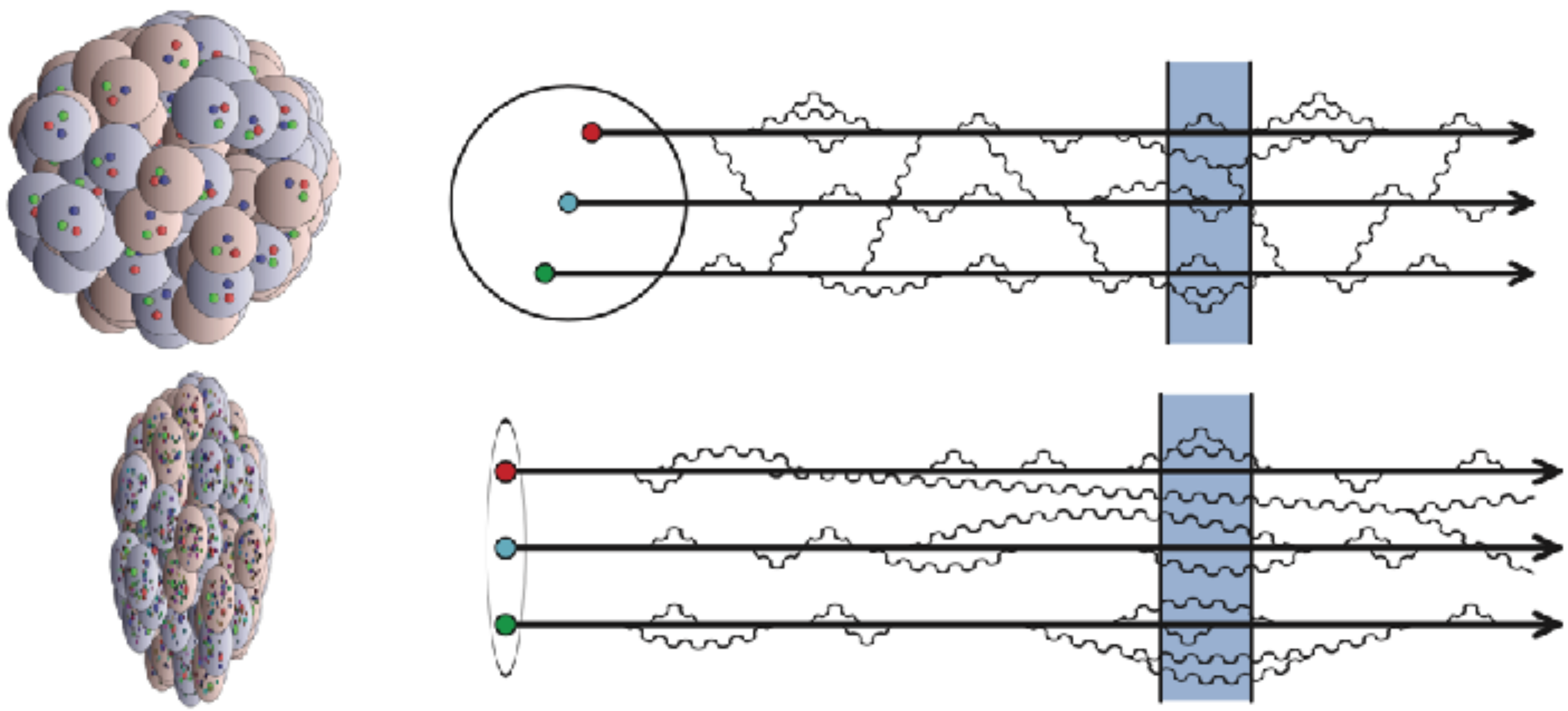
In more exclusive processes (such as Drell-Yan, SIDIS) or processes that involve the spin content of the hadron (Sivers effect), the need for information about the transverse hadron structure arises

Dijet production -> two ordered scales involved

'Effective' TMD - factorization

Kotko, Kutak, Marquet, Petreska, Sapeta & van Hameren (2015)

McLerran-Venugopalan (MV) Model



Gelis, F. 1009.0093

Model for a large nucleus in IMF based on kinematic separation of partons (in rapidity):

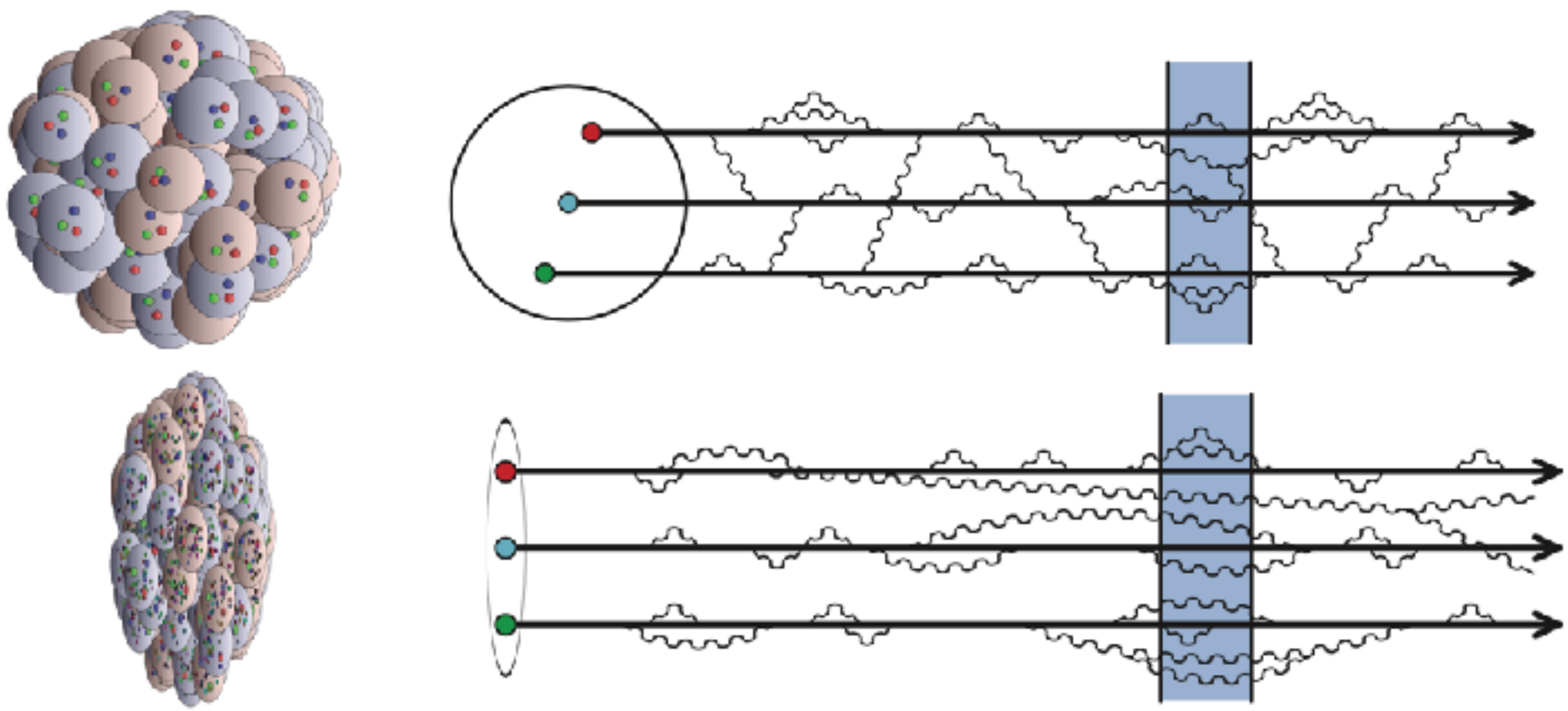
$$\text{soft} \leftarrow \Lambda^+ \equiv xP^+ \ll P^+ \rightarrow \text{valence}$$

The soft partons in the nucleus 'see' the valence partons as *frozen* and *localized:*

$$\Delta x^-_{valence} \ll \Delta x^-_{soft}$$

$$\Delta x^+_{valence} \gg \Delta x^+_{soft}$$

McLerran-Venugopalan (MV) Model



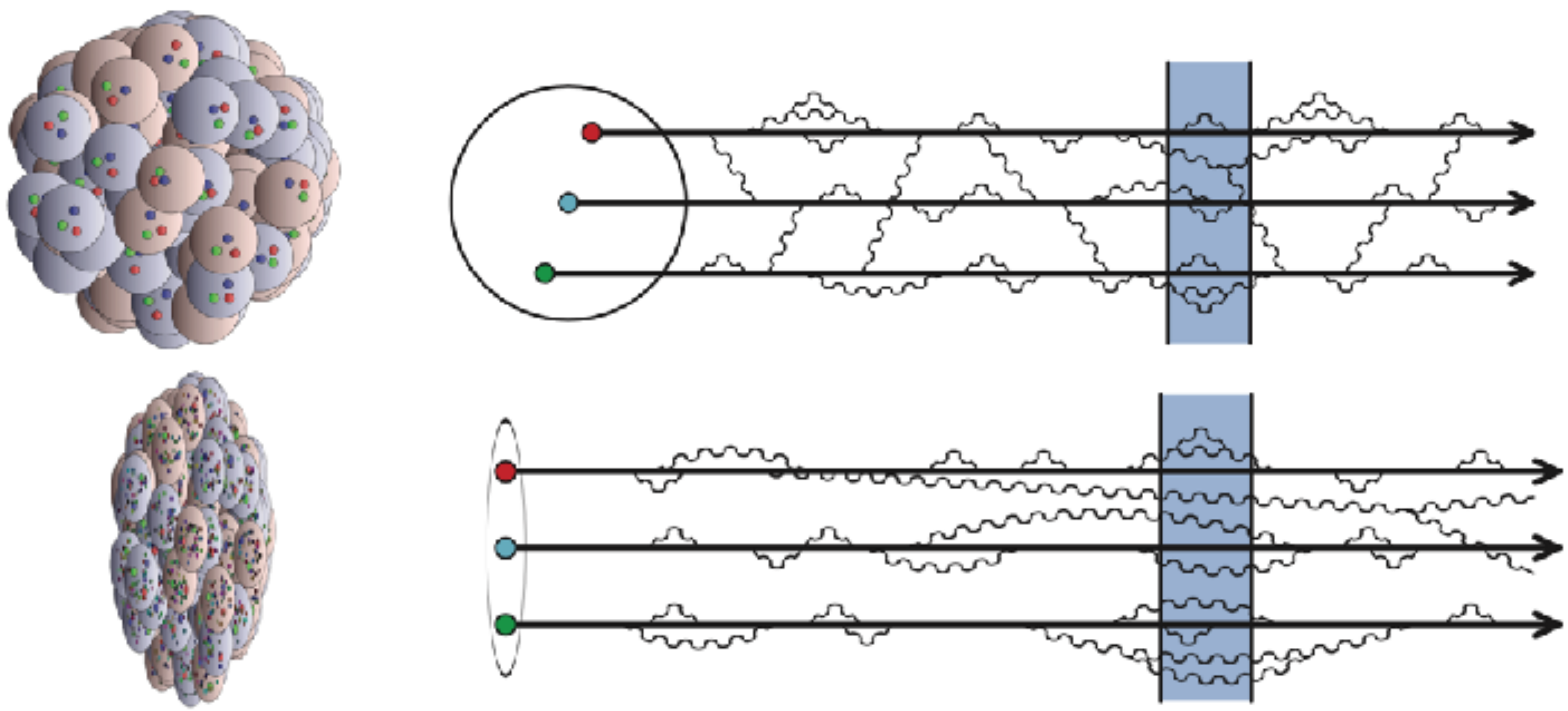
Gelis, F. 1009.0093

Treat the valence partons as static sources of color charge, that generate the soft partons according to the Yang-Mills equation: $(D_\nu F^{\nu\mu})_a(x^-,\mathbf{x}) = \delta^{\mu+}\rho_a(x^-,\mathbf{x})$

The sources are assumed to follow Gaussian distribution: $\mathcal{W}_{\Lambda_+}[\rho]$ with only local 2-point correlations:

$$\langle\rho_a(x^-,\mathbf{x})\rho_b(y^-,\mathbf{y})\rangle_A = \delta_{ab}\delta^{(2)}(\mathbf{x}-\mathbf{y})\delta(x^- - y^-)\lambda_A(x^-)$$

McLerran-Venugopalan (MV) Model

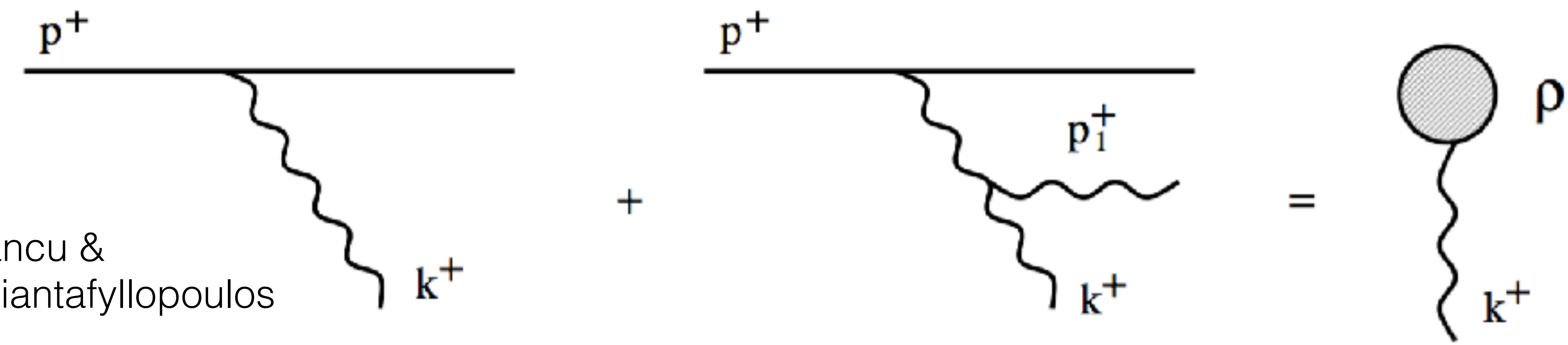


Gelis, F. 1009.0093

Emergence of the *saturation momentum scale* below which recombination effects play a role

$$Q_s^2(A) \simeq A^{1/3} \ln A^{1/3}$$

Color Glass Condensate (CGC)



Iancu & Triantafyllopoulos

Vary the scale that distinguishes soft and hard partons

$$\frac{\partial \mathcal{W}_{\Lambda_+}}{\partial \mathrm{ln}\Lambda_+} = \mathcal{H}_{JIMWLK}\mathcal{W}_{\Lambda_+}$$

Nonlinear evolution equation in rapidity known as JIMWLK

Can be used to evolve observables in rapidity

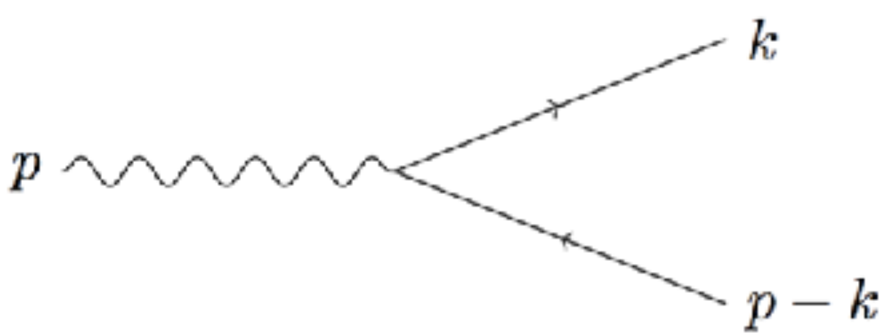
If applied to the probe instead of the target = Balitsky hierarchy

DIS dijet production in the CGC

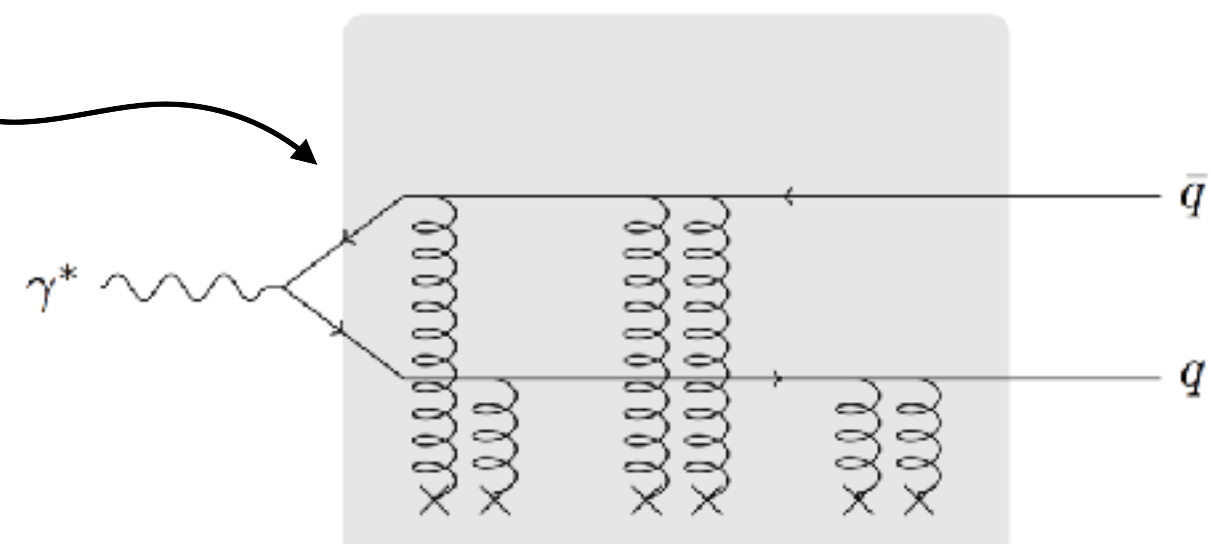
Dominguez, Marquet, Xiao & Yuan 1101.0715

$$\frac{\mathrm{d}\sigma^{\gamma^* A\to q\bar{q}X}}{\mathrm{d}^3k_1\mathrm{d}^3k_2} = N_c\alpha_{em}e_q^2\delta\left(p^+ - k_1^+ - k_2^+\right)\int \frac{\mathrm{d}^2x_1}{(2\pi)^2}\frac{\mathrm{d}^2x_1'}{(2\pi)^2}\frac{\mathrm{d}^2x_2}{(2\pi)^2}\frac{\mathrm{d}^2x_2'}{(2\pi)^2}$$
$$\times\, e^{-i\mathbf{k}_{1\perp}(\mathbf{x}_1-\mathbf{x}_1')}e^{-i\mathbf{k}_{2\perp}(\mathbf{x}_2-\mathbf{x}_2')}\sum_{\lambda\alpha\beta}\psi_{\alpha\beta}^{T,L\lambda}(x_1-x_2)\,\psi_{\alpha\beta}^{T,L\lambda*}(x_1'-x_2')$$
$$\times\left[1+S_{x_g}^{(4)}(x_1,x_2;x_2',x_1') - S_{x_g}^{(2)}(x_1,x_2) - S_{x_g}^{(2)}(x_2',x_1')\right]$$

photon to quark-antiquark wave functions



eikonal partons interact with the CGC



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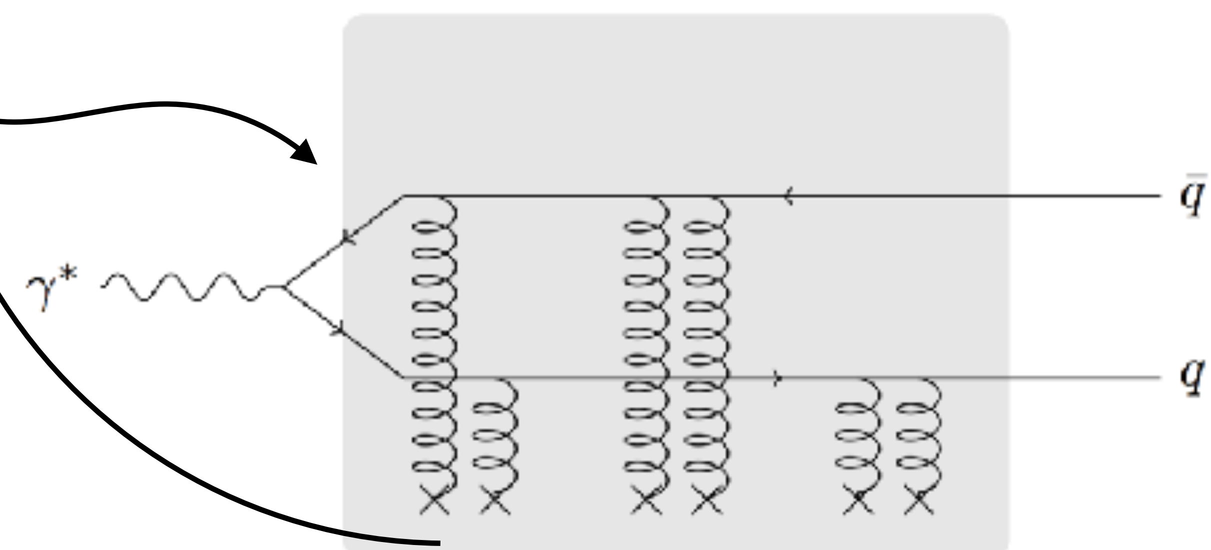
$$\times\, e^{-i\mathbf{k}_{1\perp}\left(\mathbf{x}_1-\mathbf{x}_1'\right)}e^{-i\mathbf{k}_{2\perp}\left(\mathbf{x}_2-\mathbf{x}_2'\right)}\sum_{\lambda\alpha\beta}\psi_{\alpha\beta}^{T,L\lambda}\left(x_1-x_2\right)\psi_{\alpha\beta}^{T,L\lambda*}\left(x_1'-x_2'\right)$$
$$\times\left[1+S_{x_g}^{(4)}\left(x_1,x_2;x_2',x_1'\right)-S_{x_g}^{(2)}\left(x_1,x_2\right)-S_{x_g}^{(2)}\left(x_2',x_1'\right)\right]$$

multiple scatterings are resummed in Wilson lines

$$U(x) = \mathcal{P}\exp\left\{ig\int\mathrm{d}x^+A_a^-\left(x^+,\mathbf{x}\right)T^a\right\}$$

eikonal partons interact with the CGC



dipoles and quadrupoles averaged over CGC:

$$S_{x_g}^{(2)}\left(x_1,x_2\right) = \frac{1}{N_c}\mathrm{tr}\left\langle U\left(x_1\right)U^\dagger\left(x_2\right)\right\rangle_{x_g}$$
$$S_{x_g}^{(4)}\left(x_1,x_2;x_2',x_1'\right) = \frac{1}{N_c}\mathrm{tr}\left\langle U\left(x_1\right)U^\dagger\left(x_1'\right)U\left(x_2'\right)U^\dagger\left(x_2\right)\right\rangle_{x_g}$$

DIS dijet production in the CGC

Study the *correlation limit* in which the two forward jets are almost back-to-back

$$q_\perp^2 \sim Q_s^2(x_2, A) \ll \tilde{P}_\perp^2$$

total transverse momentum of jet pair

momentum difference stems from interaction with CGC

transverse momentum of individual jet

Appearance of a small and a large momentum scale, just like in TMD factorization

Results turns out to be the same as in effective TMD factorization approach

Dominguez, Marquet, Xiao & Yuan 1101.0715

DIS dijet production in the CGC

$$\frac{\mathrm{d}\sigma^{\gamma^T A\to q\bar{q}X}}{\mathrm{d}\mathcal{P}.\mathcal{S}.} = \alpha_s\alpha_{em}e_q^2\delta\left(x_{\gamma^*}-1\right)z\bar{z}$$

transverse polarized photon

$$\times\left[\left(\frac{\delta_{ij}}{\left(\tilde{P}_\perp^2+\epsilon_f^2\right)^2}-\frac{4\epsilon_f^2\tilde{P}_i\tilde{P}_j}{\left(\tilde{P}_\perp^2+\epsilon_f^2\right)^4}\right)\left(z^2+\bar{z}^2\right)+\frac{4m^2\tilde{P}_i\tilde{P}_j}{\left(\tilde{P}_\perp^2+\epsilon_f^2\right)^4}\right]\left(16\pi^3\right)$$

$$\times\int\frac{\mathrm{d}^3v}{(2\pi)^3}\frac{\mathrm{d}^3v'}{(2\pi)^3}e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\mathrm{tr}\left\langle F^{i-}\left(v^+,v\right)U^{[+]\dagger}F^{j-}\left(v'^+,v'\right)U^{[+]}\right\rangle_{x_g}$$

$$\epsilon_f^2 = m^2 + z\bar{z}Q^2$$

$z =$ longitudinal momentum fraction

$\bar{z}\equiv 1-z$

$$U_{v'v}^{[+]\dagger}\equiv U\left(v^+,+\infty;v\right)U\left(+\infty,v'^+;v'\right)$$

$$\frac{\mathrm{d}\sigma^{\gamma^L A\to q\bar{q}X}}{\mathrm{d}\mathcal{P}.\mathcal{S}.} = \alpha_s\alpha_{em}e_q^2\delta\left(x_{\gamma^*}-1\right)4z^2\bar{z}^2\frac{4Q^2z\bar{z}\tilde{P}_i\tilde{P}_j}{\left(\tilde{P}_\perp^2+\epsilon_f^2\right)^4}\left(16\pi^3\right)$$

$$\times\int\frac{\mathrm{d}^3v}{(2\pi)^3}\frac{\mathrm{d}^3v'}{(2\pi)^3}e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\mathrm{tr}\left\langle F^{i-}\left(v^+,v\right)U^{[+]\dagger}F^{j-}\left(v'^+,v'\right)U^{[+]}\right\rangle_{x_g}$$

longitudinally polarized photon

DIS dijet production in the CGC

To unravel the physics, let us decompose the operator expression that appears in the result:

$$4\,(2\pi)^3 \int \frac{\mathrm{d}^3 v}{(2\pi)^3}\frac{\mathrm{d}^3 v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\mathrm{tr}\left\langle F^{i-}\left(\vec{v}\right)U^{[+]\dagger}F^{j-}\left(\vec{v}'\right)U^{[+]}\right\rangle_{x_g}$$

$$= \frac{1}{2}\delta^{ij} xG^{(1)}\left(x,q_\perp\right) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) xh^{(1)}\left(x,q_\perp\right).$$

the Weizsäcker-Williams gluon distribution

... its linearly polarized partner

$$\frac{1}{2}\left(\frac{2\tilde{P}_i q_\perp^i \tilde{P}_j q_\perp^j}{q_\perp^2} - \tilde{P}_\perp^2\right) = \frac{1}{2}\tilde{P}_\perp^2 \cos\left(2\phi\right)$$

$$\frac{\mathrm{d}\sigma^{\gamma^T A\to q\bar{q}X}}{\mathrm{d}\mathcal{P}.\mathcal{S}.} = \frac{1}{2}\alpha_s\alpha_{em}e_q^2\delta\left(x_{\gamma^*}-1\right)z\left(1-z\right)\frac{1}{\left(\tilde{P}_\perp^2+m^2\right)^4}$$

$$\times\left\{\left(\left(\tilde{P}_\perp^4+m^4\right)\left(z^2+\left(1-z\right)^2\right)+2m^2\tilde{P}_\perp^2\right)xG^{(1)}\left(x,q_\perp\right)\right.$$

$$\left.+z\left(1-z\right)4m^2\tilde{P}_\perp^2\cos\left(2\phi\right)xh^{(1)}\left(x,q_\perp\right)\right\}$$

disappears in massless case!

pA dijet production in the CGC

Marquet, Petreska, Roiesnel, 1608.02577

$$\frac{\mathrm{d}\sigma^{pA\to q\bar{q}X}}{\mathrm{d}^3k_1\mathrm{d}^3k_2} = \alpha_s \frac{1}{p^+} x_1 g\left(x_1,\mu^2\right) T_R \int \frac{\mathrm{d}^2x_1}{(2\pi)^2}\frac{\mathrm{d}^2x_1'}{(2\pi)^2}\frac{\mathrm{d}^2x_2}{(2\pi)^2}\frac{\mathrm{d}^2x_2'}{(2\pi)^2}$$
$$\times e^{-i\mathbf{k}_{1\perp}\left(\mathbf{x}_1-\mathbf{x}_1'\right)} e^{-i\mathbf{k}_{2\perp}\left(\mathbf{x}_2-\mathbf{x}_2'\right)} \sum_{\lambda\alpha\beta} \psi^{\lambda}_{\alpha\beta}\left(x_1-x_2\right)\psi^{\lambda *}_{\alpha\beta}\left(x_1'-x_2'\right)$$
$$\times \left[C_{x_g}\left(x_1,x_2;x_2',x_1'\right) + S^A_{x_g}\left(zx_1+(1-z)\,x_2; zx_1'+(1-z)\,x_2'\right)\right.$$
$$\left. - S^{(3)}_{x_g}\left(x_1, zx_1'+(1-z)\,x_2', x_2\right) - S^{(3)}_{x_g}\left(x_2', zx_1+(1-z)\,x_2, x_1'\right)\right],$$

The gauge structures are now more complicated since there is initial state radiation:

$$C_{x_g}\left(x_1,x_2;x_2',x_1'\right) = \frac{1}{C_F N_c}\mathrm{Tr}\left\langle U^\dagger\left(x_2\right)t^c U\left(x_1\right)U^\dagger\left(x_1'\right)t^c U\left(x_2'\right)\right\rangle_{x_g}$$

$$S^A_{x_g}\left(v;v'\right) = \frac{1}{N_c^2-1}\mathrm{Tr}\left\langle W\left(v\right)W^\dagger\left(v'\right)\right\rangle_{x_g}$$

$$S^{(3)}_{x_g}\left(u,v,w\right) = \frac{1}{2C_F N_c}\left\langle \mathrm{Tr}U\left(u\right)U^\dagger\left(v\right)\mathrm{Tr}U\left(v\right)U^\dagger\left(w\right) - \frac{1}{N_c}\mathrm{Tr}U\left(u\right)U^\dagger\left(w\right)\right\rangle_{x_g}$$

pA dijet production in the CGC

Again, in the correlation limit

$$\frac{\mathrm{d}\sigma^{pA\to q\bar{q}X}}{\mathrm{d}\mathcal{P.S.}} = \frac{\alpha_s^2}{2C_F}\frac{z(1-z)}{\left(\tilde{P}_\perp^2+m^2\right)^2}x_1 g\left(x_1,\mu^2\right)$$

$$\times\left\{\left(P_{qg}(z)+z(1-z)\frac{2m^2\tilde{P}_\perp^2}{\left(\tilde{P}_\perp^2+m^2\right)^2}\right)\right.$$

$$\times\left(2z(1-z)\mathcal{F}_{gg}^{(2)}(x,q_\perp)+2P_{qg}(z)\mathcal{F}_{gg}^{(1)}(x,q_\perp)-\frac{1}{N_c^2}\mathcal{F}_{gg}^{(3)}(x,q_\perp)\right)$$

$$+\cos(2\phi)z(1-z)\frac{2m^2\tilde{P}_\perp^2}{\left(\tilde{P}_\perp^2+m^2\right)^2}$$

$$\left.\times\left(2z(1-z)\mathcal{H}_{gg}^{(2)}(x,q_\perp)+2P_{qg}(z)\mathcal{H}_{gg}^{(1)}(x,q_\perp)-\frac{1}{N_c^2}\mathcal{H}_{gg}^{(3)}(x,q_\perp)\right)\right\}$$

$$xG^{(1)} = \mathcal{F}_{gg}^{(3)}$$

$$xh^{(1)} = \mathcal{H}_{gg}^{(3)}$$

→ disappears again in massless case!

Apart from the Weizsäcker-Williams gluon distr. and its linearly polarized partner, two other gluon TMDs and their linearly polarized counterparts appear!

pA dijet production in the CGC

The six gluon TMDs that appear in heavy dijet production in pA:

$$\mathcal{F}_{gg}^{(1)}(x,q_\perp) \equiv \frac{4}{g^2}\frac{1}{N_c}\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\left\langle \mathrm{Tr}\left([\partial_i U(v)]\left[\partial_i U^\dagger(v')\right]\right)\mathrm{Tr}\left(U(v')U^\dagger(v)\right)\right\rangle_{x_g},$$

$$\mathcal{F}_{gg}^{(2)}(x,q_\perp) \equiv -\frac{4}{g^2}\frac{1}{N_c}\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\mathrm{Re}\left\langle \mathrm{Tr}\left([\partial_i U(v)]U^\dagger(v')\right)\mathrm{Tr}\left([\partial_i U(v')]U^\dagger(v)\right)\right\rangle_{x_g},$$

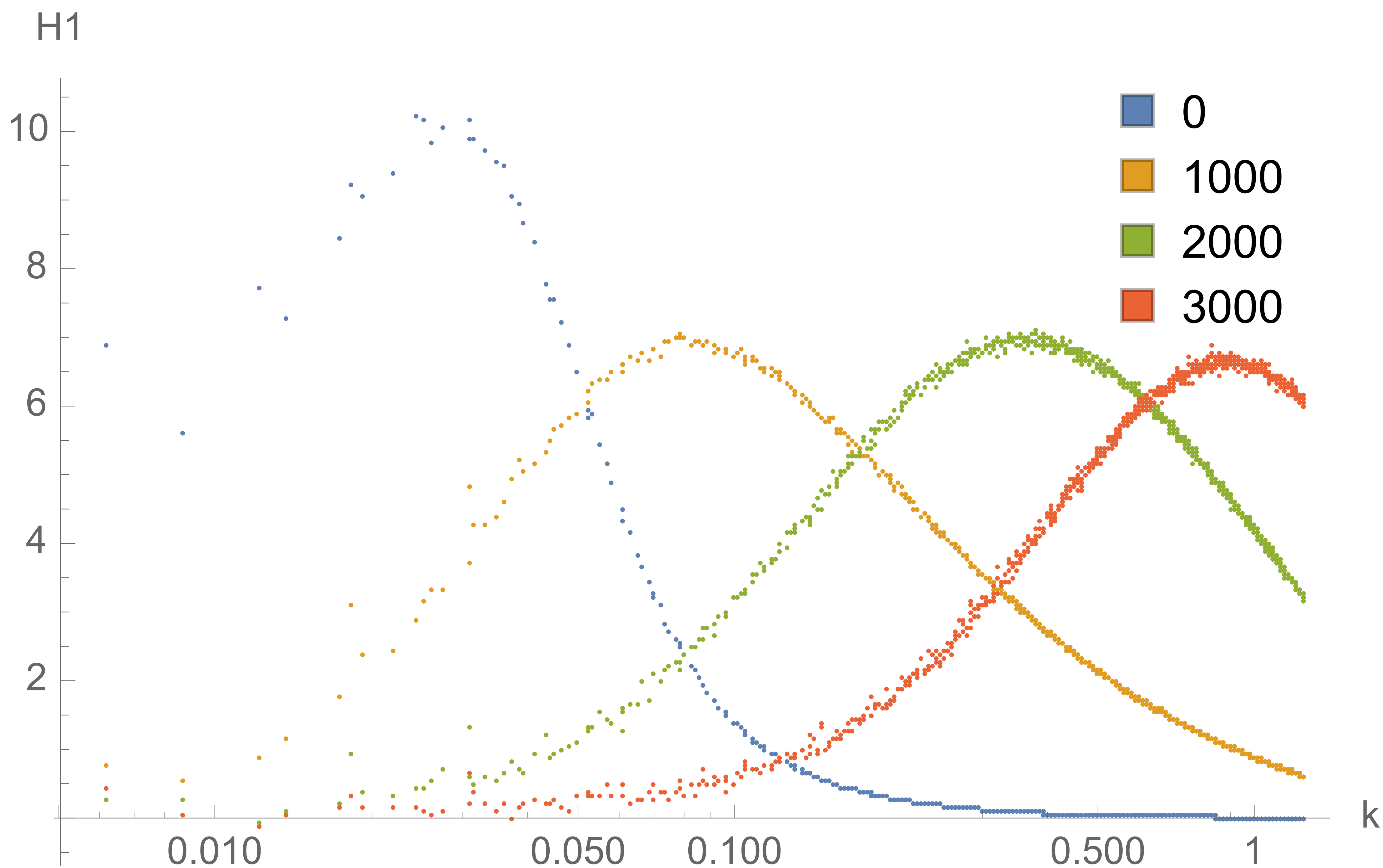
$$\mathcal{F}_{gg}^{(3)}(x,q_\perp) \equiv -\frac{4}{g^2}\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\left\langle \mathrm{Tr}\left([\partial_i U(v)]U^\dagger(v')[\partial_i U(v')]U^\dagger(v)\right)\right\rangle_{x_g},$$

$$\mathcal{H}_{gg}^{(1)}(x,q_\perp) \equiv \left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2}-\delta^{ij}\right)\frac{4}{g^2}\frac{1}{N_c}\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\left\langle \mathrm{Tr}\left([\partial_i U(v)]\left[\partial_j U^\dagger(v')\right]\right)\mathrm{Tr}\left(U(v')U^\dagger(v)\right)\right\rangle_{x_g},$$

$$\mathcal{H}_{gg}^{(2)}(x,q_\perp) \equiv \left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2}-\delta^{ij}\right)\left(-\frac{4}{g^2}\right)\frac{1}{N_c}\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\mathrm{Re}\left\langle \mathrm{Tr}\left([\partial_i U(v)]U^\dagger(v')\right)\mathrm{Tr}\left([\partial_j U(v')]U^\dagger(v)\right)\right\rangle_{x_g},$$

$$\mathcal{H}_{gg}^{(3)}(x,q_\perp) \equiv \left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2}-\delta^{ij}\right)\left(-\frac{4}{g^2}\right)\int \frac{\mathrm{d}^2v\mathrm{d}^2v'}{(2\pi)^3} e^{-i\mathbf{q}_\perp(\mathbf{v}-\mathbf{v}')}\left\langle \frac{1}{N_c}\mathrm{Tr}\left([\partial_i U(v)]U^\dagger(v')[\partial_j U(v')]U^\dagger(v)\right)\right\rangle_{x_g}.$$

Numerically: MV + JIMWLK



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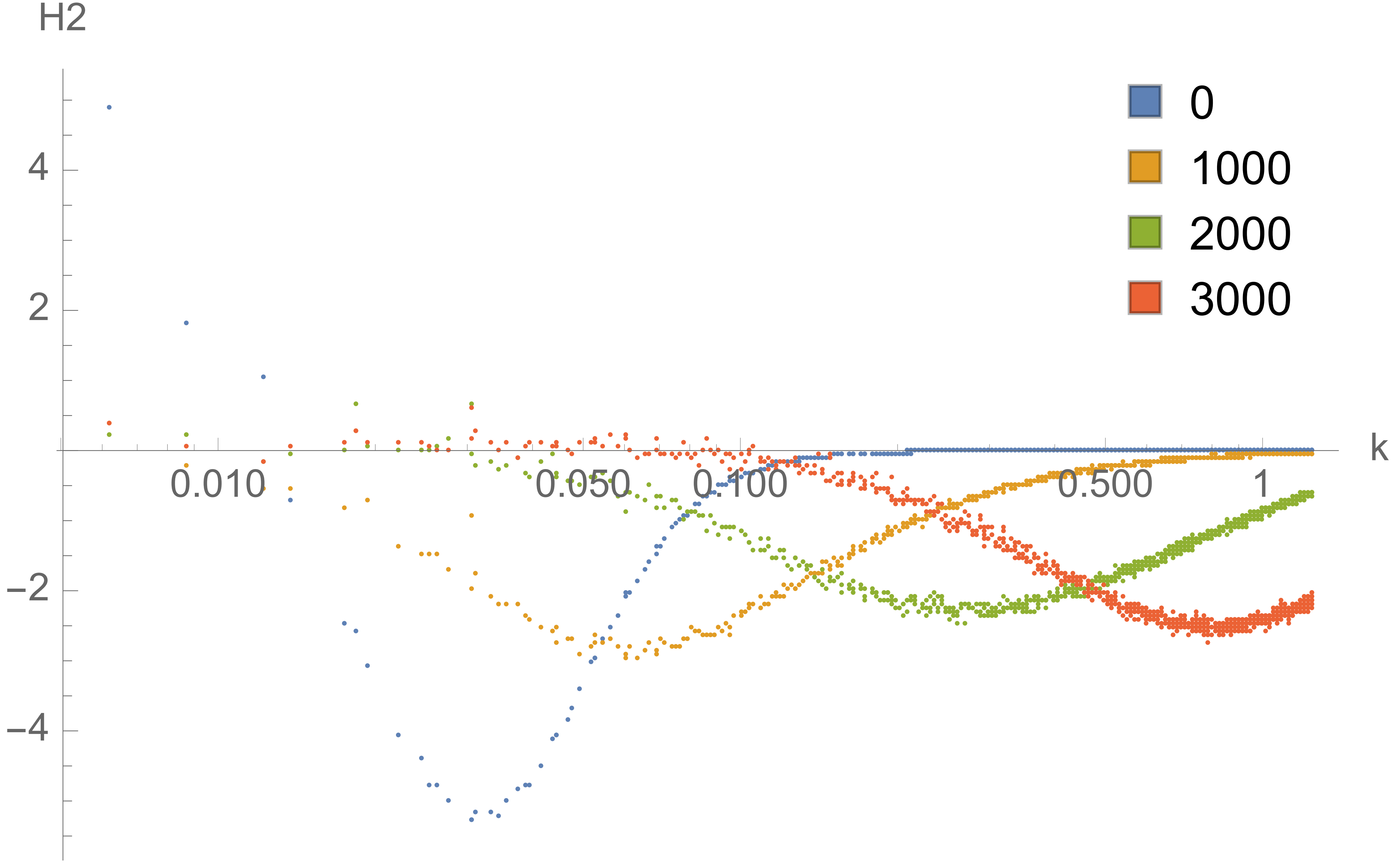

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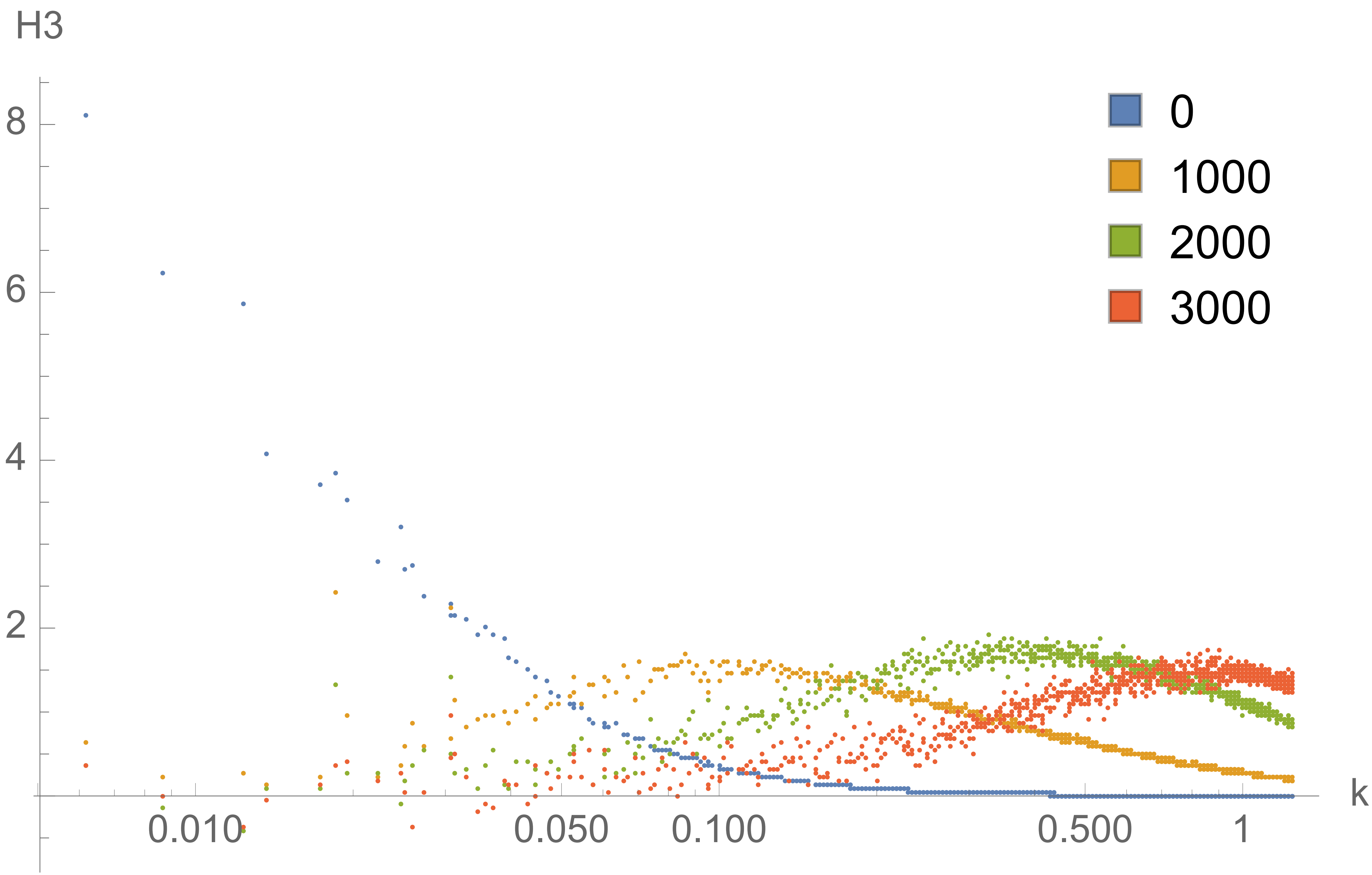

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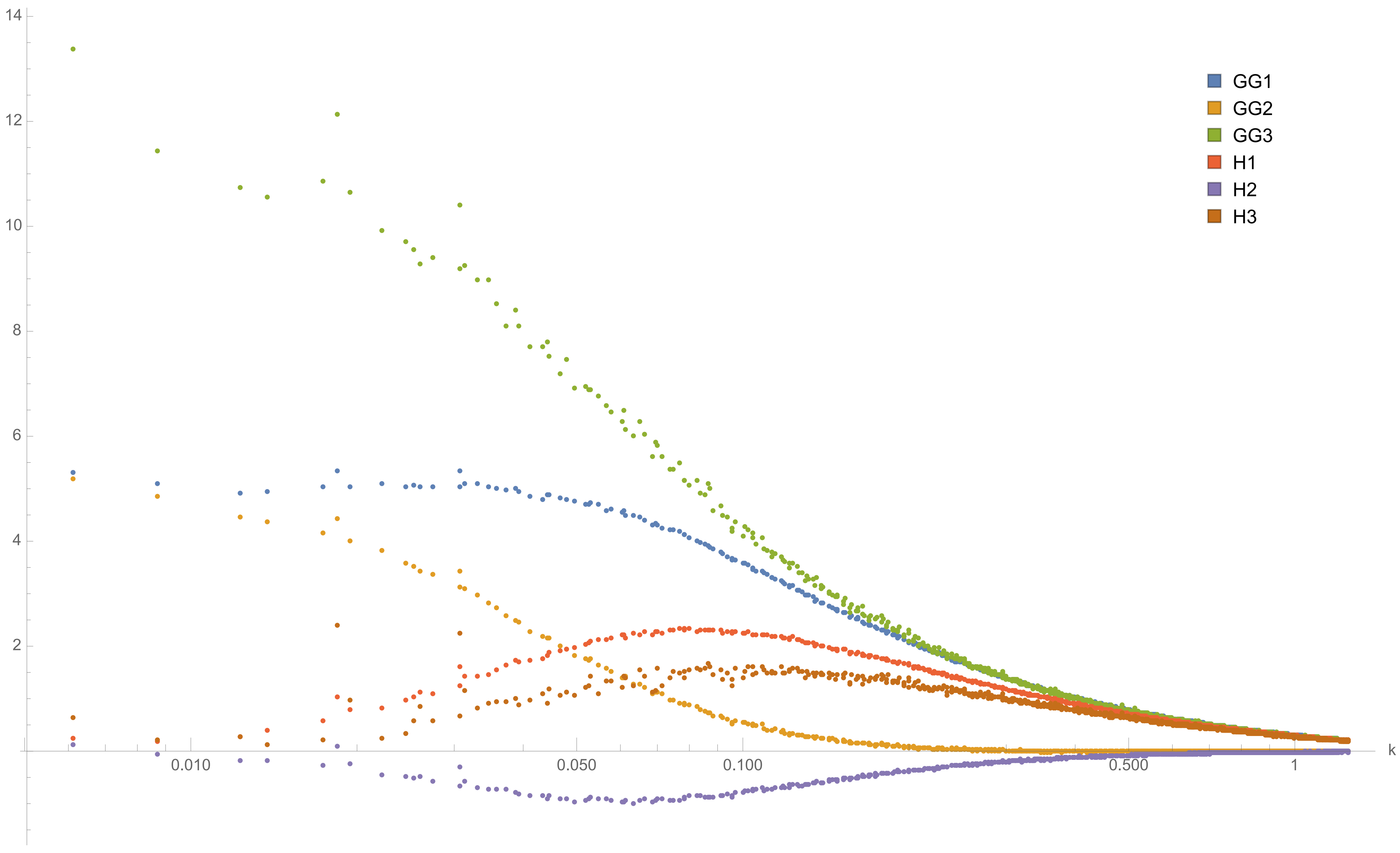

Conclusions & Outlook

We look at the overlap between hybrid TMD factorization and the CGC formalism in the small-x regime

In (heavy) dijet production in eA and pA collisions, the CGC calculation contains the hybrid TMD factorization result in the correlation limit

For eA and pA, we obtain one resp. three gluon TMDs along with their linearly polarized partner, the latter accessible if we study heavy quarks

These TMDs can be numerically calculated within CGC

Thanks for your attention!