Small-x behavior of TMD gluon distributions in the Color Glass Condensate

Elena Petreska

Nikhef/VU Amsterdam

Kotko, Kutak, Marquet, EP, Sapeta and van Hameren (2015)

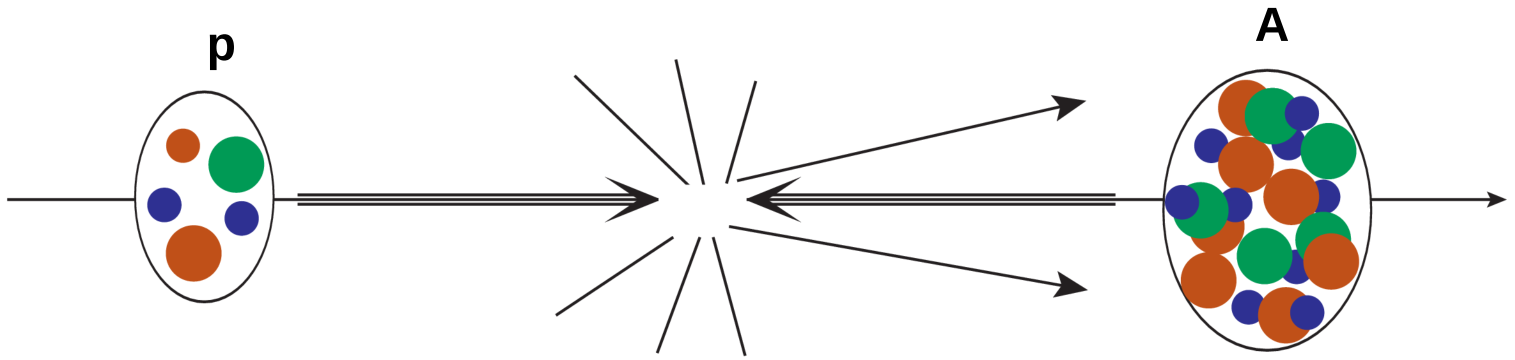
van Hameren, Kotko, Kutak, Marquet, EP and Sapeta (2016)

Marquet, EP and Roiesnel (2016)

REF workshop, Antwerp

November 10, 2016

Forward production of two jets in pA collisions



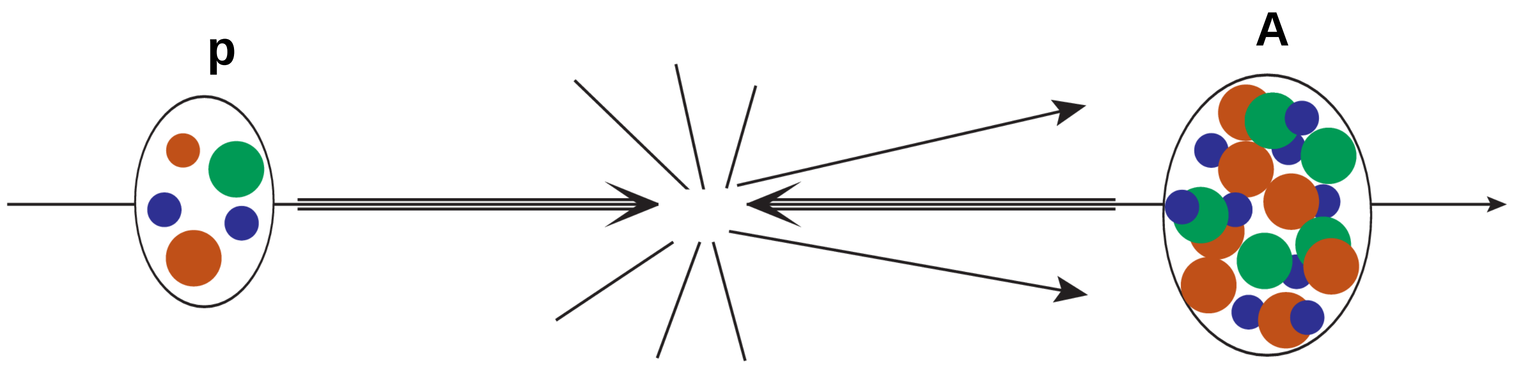
Dilute projectile
probed at large-*x*

Dense target
probed at small-*x*

Optimal for studying high-energy
(small-*x* saturation) QCD effects

x → Longitudinal momentum fraction

Forward production of two nearly back-to-back jets



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Color Glass Condensate (CGC)

L. McLerran and R. Venugopalan, 1994
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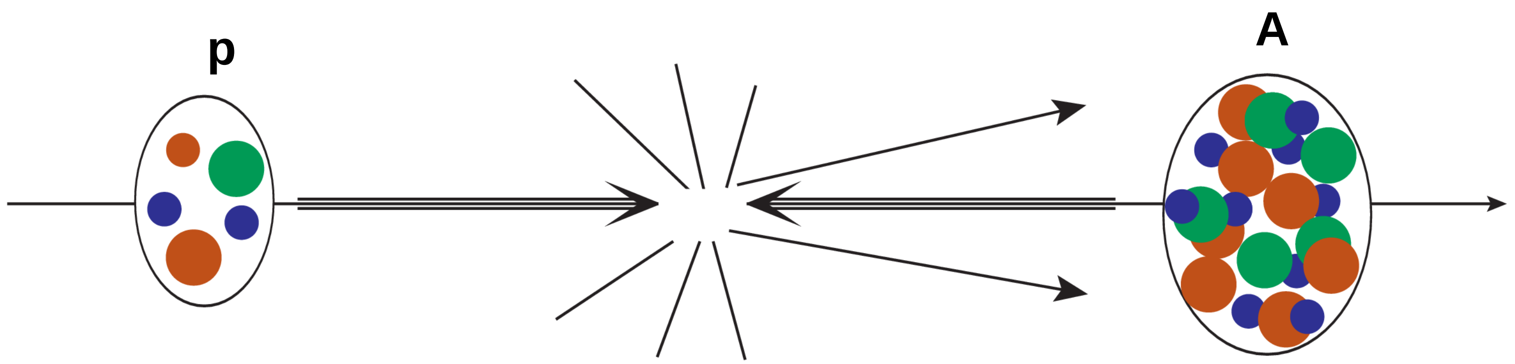

Dilute projectile probed at large-x

Dense target probed at small-x

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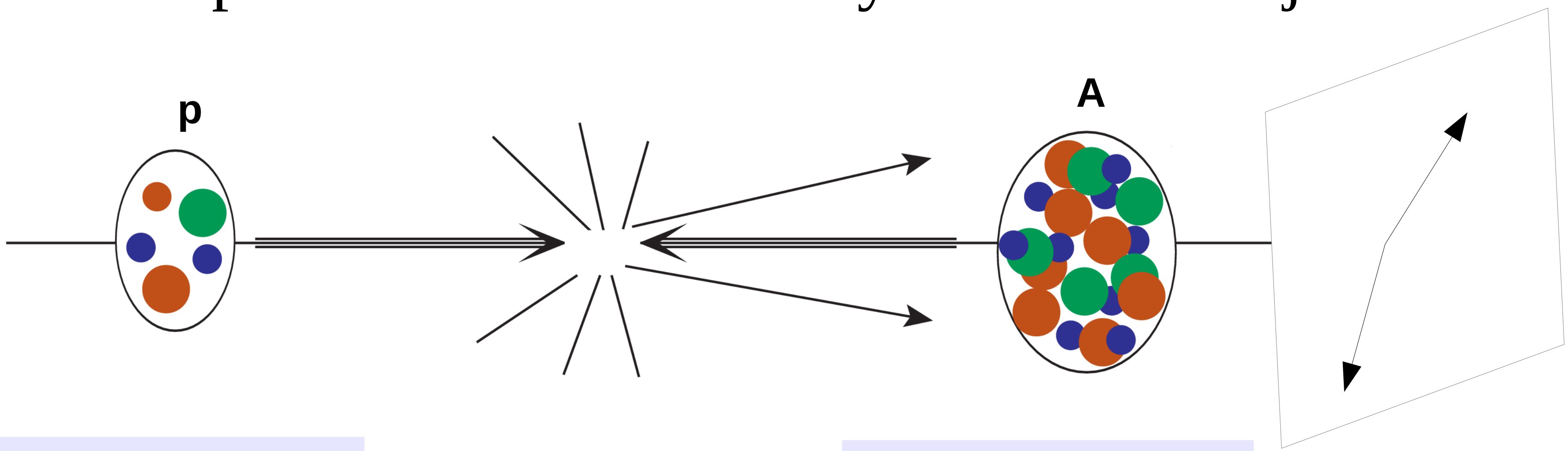
Color Glass Condensate (CGC)

- Non-linear QCD at small-x
- Strong classical gluon fields for the target
- No ordering of the momentum scales

Forward production of two nearly back-to-back jets



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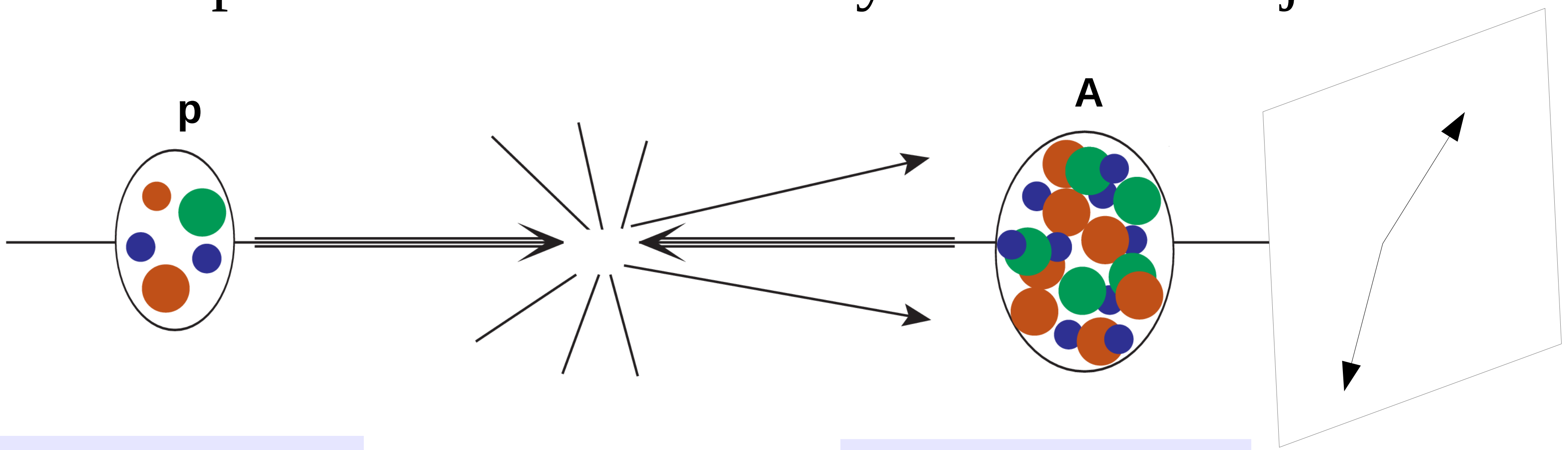

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$x \rightarrow$ Longitudinal momentum fraction

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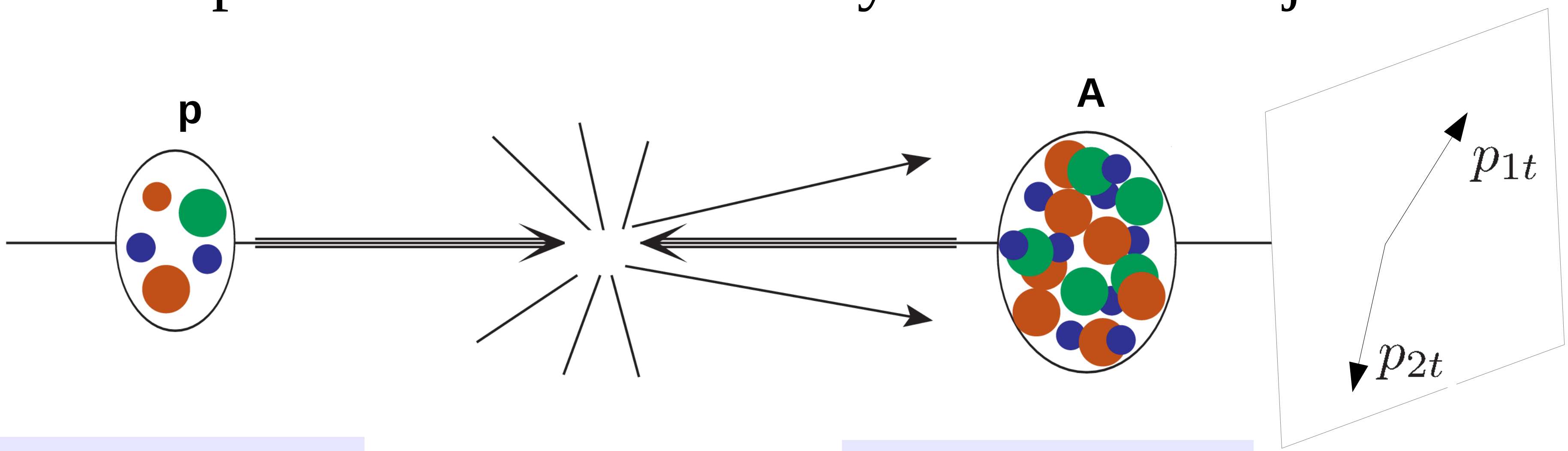
Transverse Momentum Dependent (TMD) factorization

C. J. Bomhof, P. J. Mulders and F. Pijlman 2006
F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan, 2011

Forward production of two nearly back-to-back jets



Dilute projectile probed at large-*x*

Dense target probed at small-*x*

x → Longitudinal momentum fraction

$|k_t|^2 = |p_{1t} + p_{2t}|^2$ → Transverse momentum imbalance

Color Glass Condensate (CGC)

- Non-linear QCD at small-x
- Strong classical gluon fields for the target
- No ordering of the momentum scales

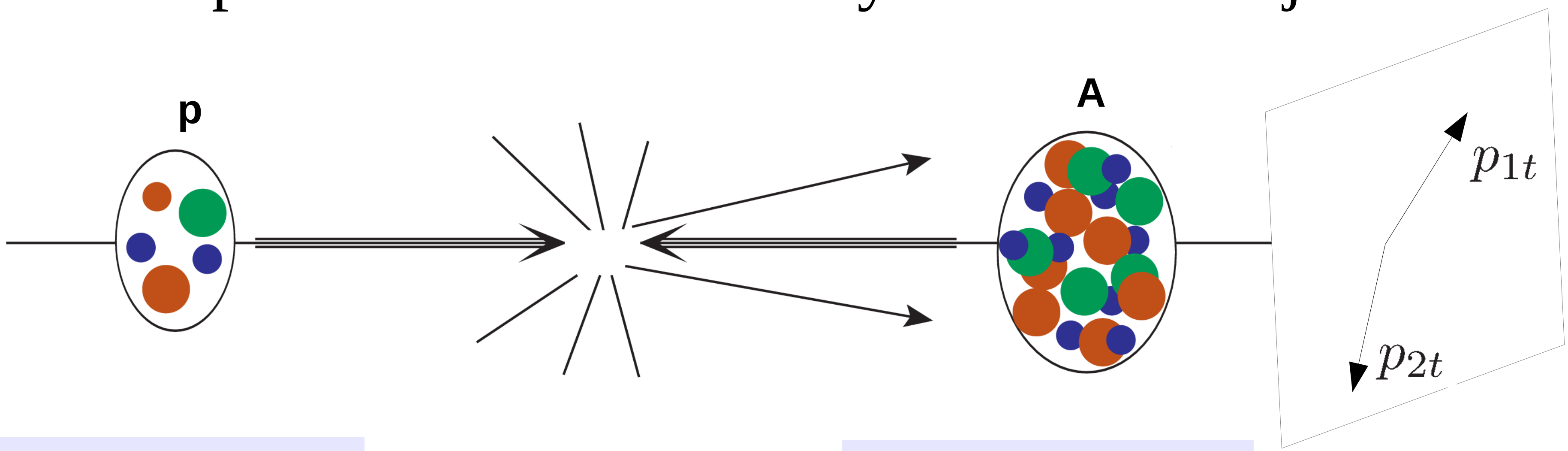
Transverse Momentum Dependent (TMD) factorization

- Correlation (nearly back-to-back) limit

$$k_t \ll P_t$$

- Several gluon TMDs for the target
- Loss of universality
- Small-x is not assumed

Forward production of two nearly back-to-back jets



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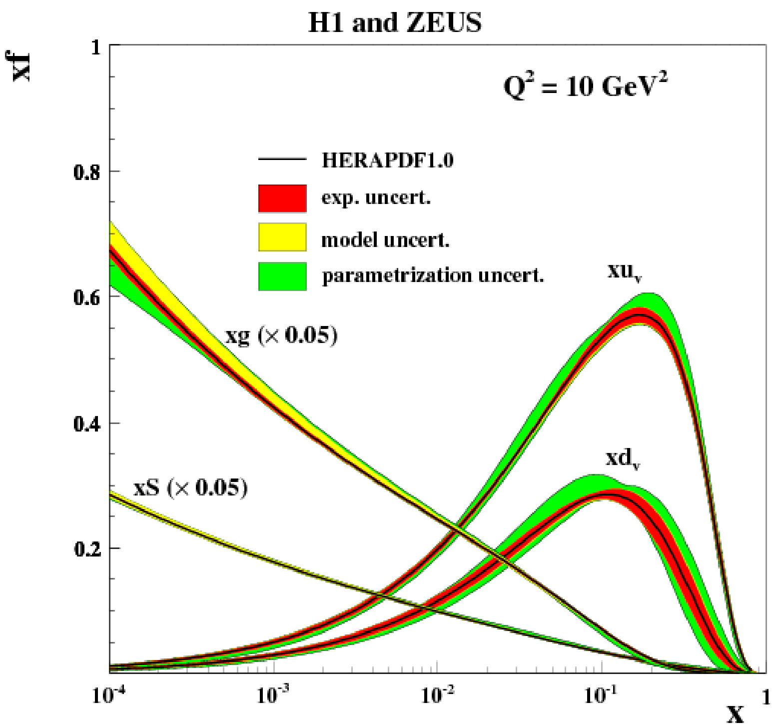
Color Glass Condensate (CGC)

Transverse Momentum Dependent (TMD) factorization

- Show the equivalence in the overlapping region at finite *Nc*
- Express TMDs as CGC correlators of Wilson lines
- Study the behavior of TMDs and the loss of universality at small *x*

High-energy limit of QCD



H1 and ZEUS Collaborations, (2010)

High-energy QCD is the domain of high gluon densities

high energy s <=> small x

- Linear evolution of gluon densities with decreasing x: BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation

E.A. Kuraev, L.N. Lipatov and V.S. Fadin, (1977)
Ya.Ya. Balitsky and L.N. Lipatov, (1978)

↓

- Violation of unitarity

x → Longitudinal momentum fraction

Q → Virtuality of the photon

- Non-linear evolution

↓

- Gluon saturation at small x

↓

- Saturation scale: $Q_s^2 \sim A^{\frac{1}{3}} \left(\frac{1}{x}\right)^{0.25}$

Color Glass Condensate (CGC) theory

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Basic ideas

- $\alpha_s(Q_s^2) \ll 1$
- High occupation number of gluons => Classical gluon field at small x, $A_\mu \sim \frac{1}{g}$
- Larger-*x* partons, ρ, act as classical sources for the smaller-x gluons
- Solve classical YM equations for the gluon filed with current generated by the sources:

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J_a^\mu = \delta^{\mu+}\delta(x^-)\rho_a(x_t)$$

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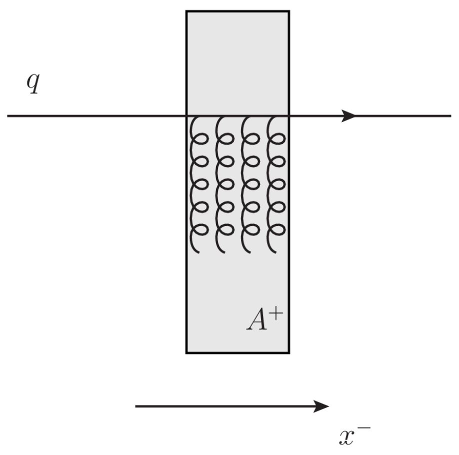
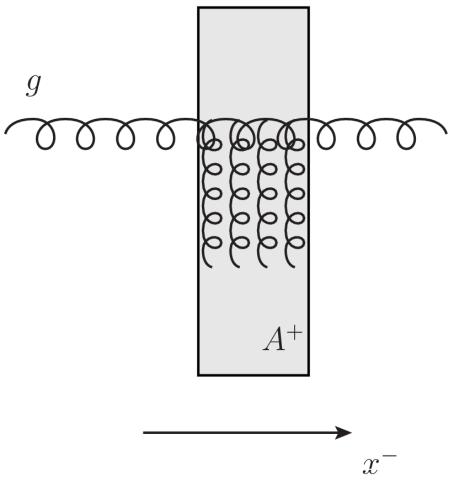
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(x_t)$$

- The color sources are random, described by a distribution functional $W_x[\rho]$
- Quantum corrections enter through small-x evolution of the distribution functional

Scattering off the CGC





$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^- A_a^+(x^-,\mathbf{x})t^a\right] \qquad V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^- A_a^+(x^-,\mathbf{x})T^a\right]$$

Fundamental/Adjoint eikonal Wilson line

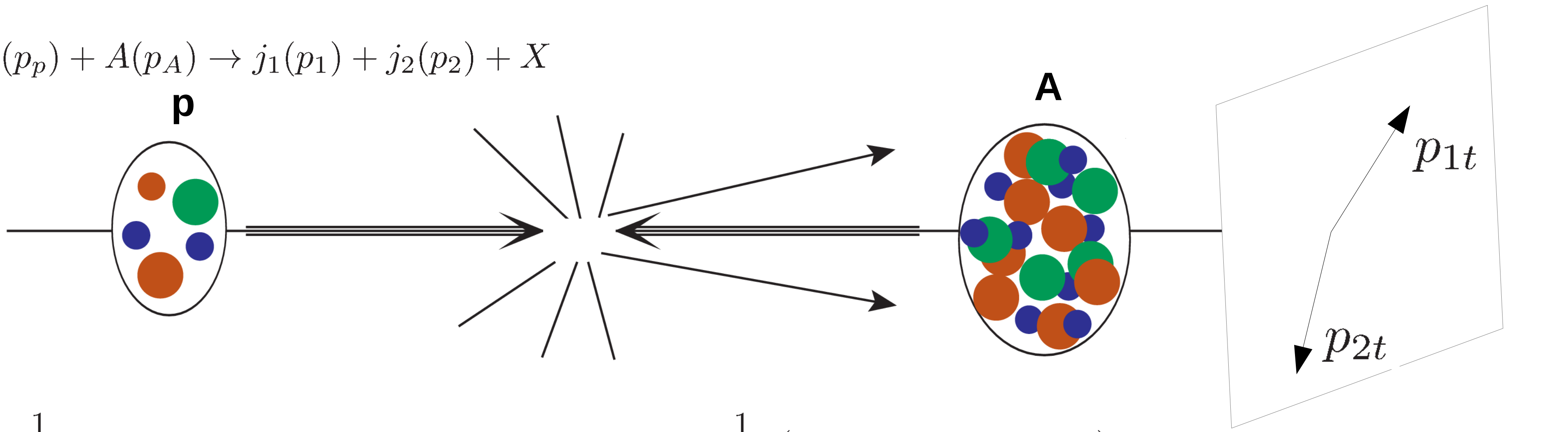
- On the level of the cross section → Correlators of Wilson lines

Eg:

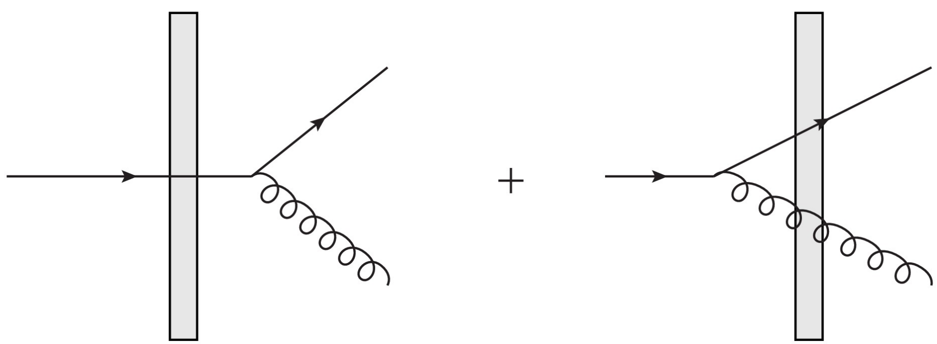
$$\left\langle \mathrm{Tr}\left(U(\mathbf{b})U^\dagger(\mathbf{b}')t^d t^c\right)\left[V(\mathbf{x})V^\dagger(\mathbf{x}')\right]^{cd}\right\rangle$$

Forward dijet production in the CGC framework

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



$$x_1 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \sim 1 \qquad x_2 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \ll 1$$



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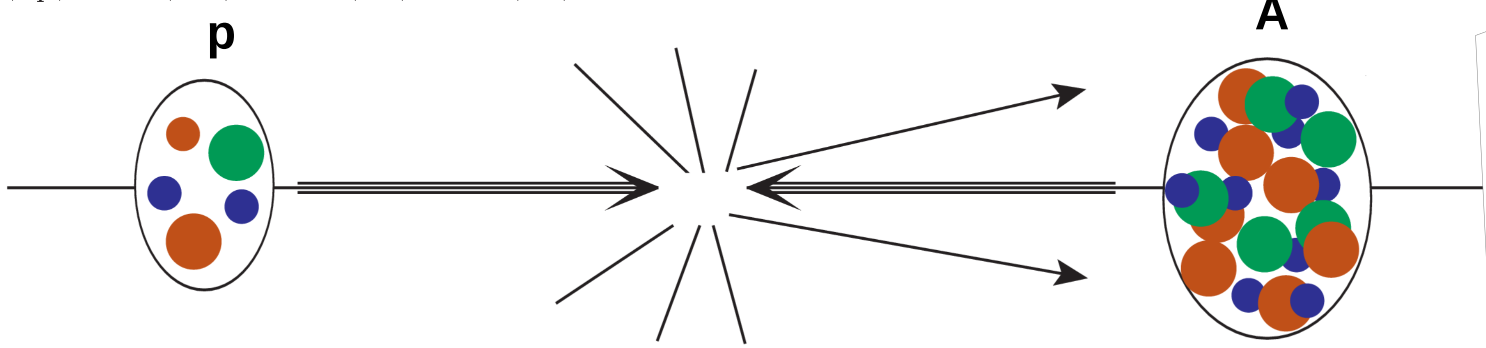

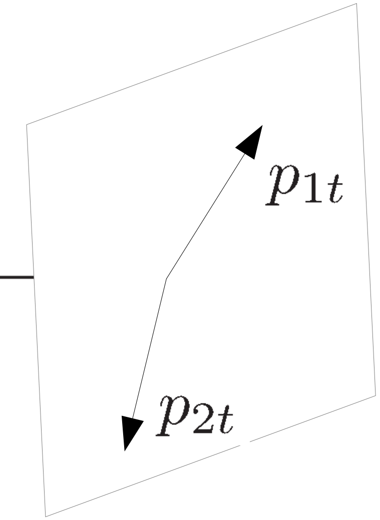

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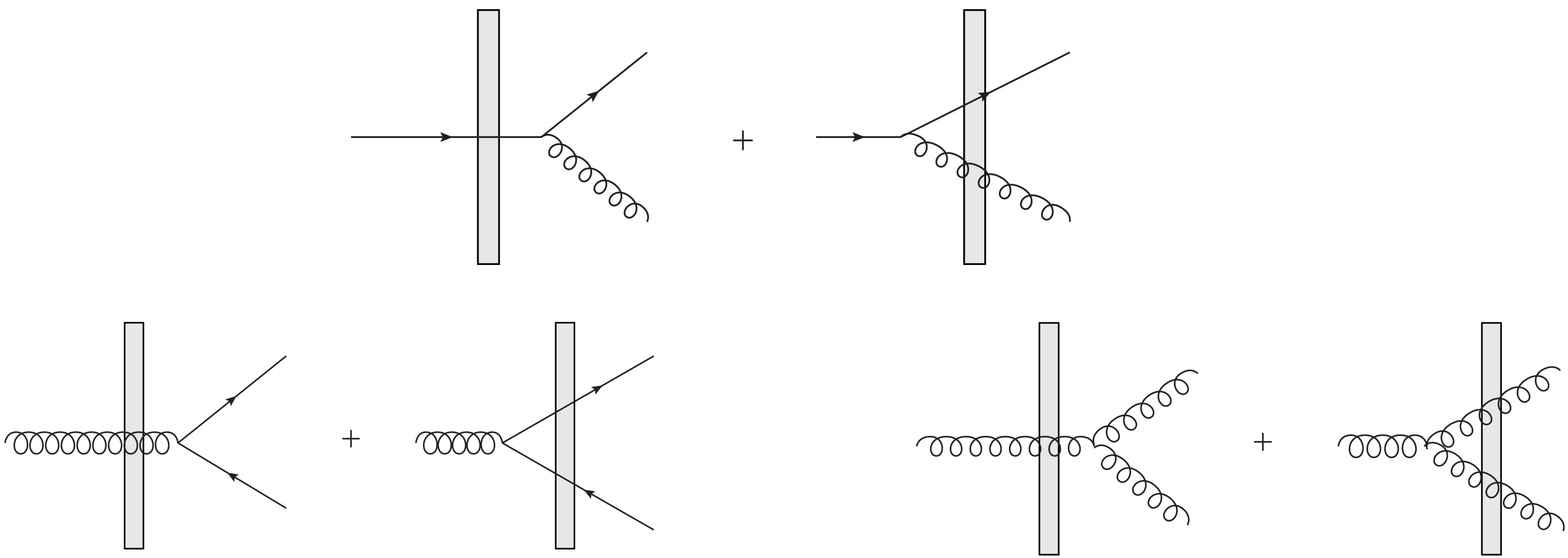

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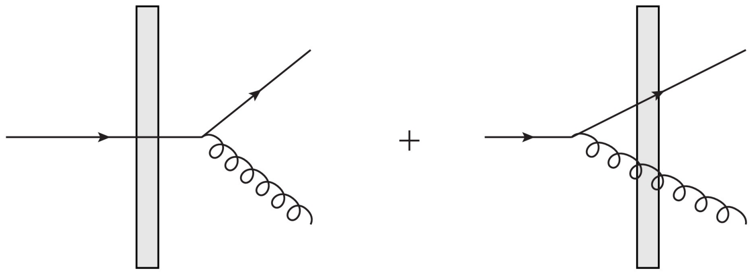

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

$$\times \Big\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]$$

$$-S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\Big\}$$

C. Marquet (2007)
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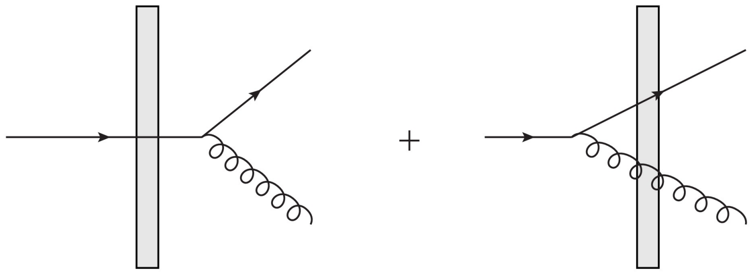
Forward dijet production in the CGC framework



$$S^{(2)}_{q\bar{q}}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{z}') = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^\dagger(\mathbf{z}')t^c U(\mathbf{b})t^d\right) V^{cd}(\mathbf{x})\right\rangle$$

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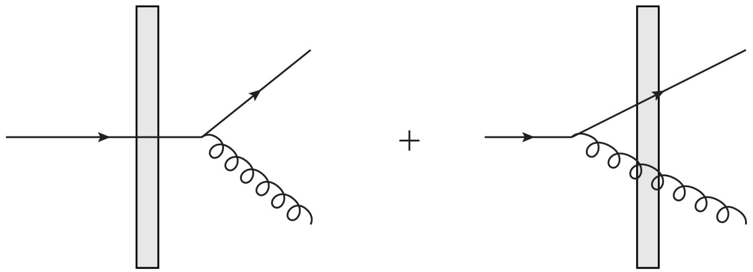

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Valid at small x, no ordering of momentum scales

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

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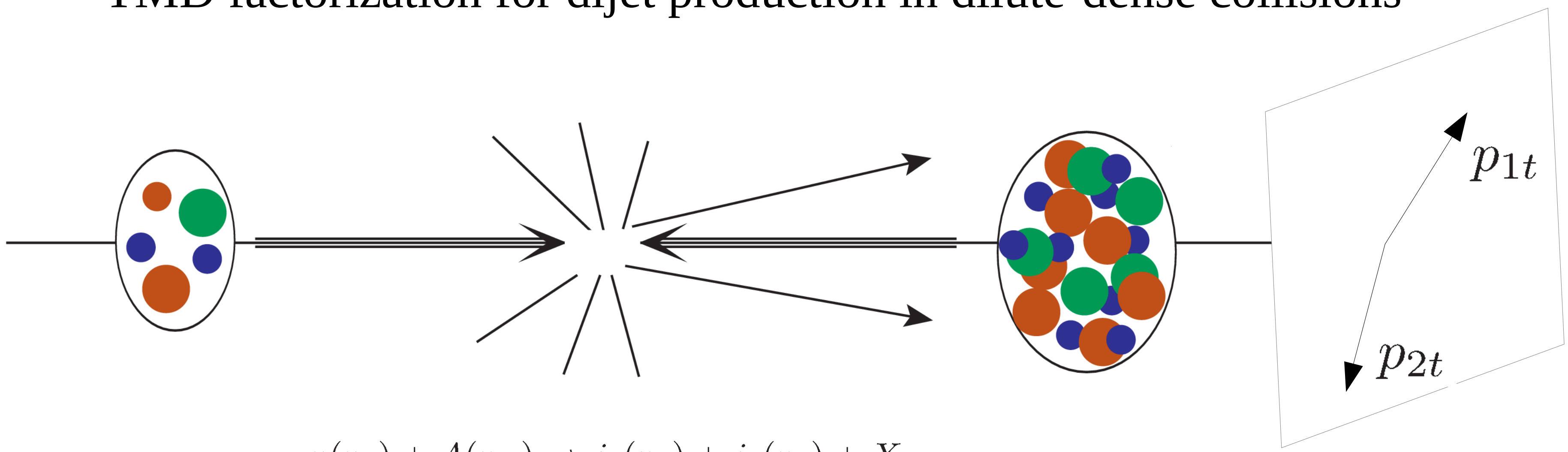
TMD factorization for dijet production in dilute-dense collisions



$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

$$Q_s \sim k_t \ll P_t$$

From the generic TMD factorization framework:
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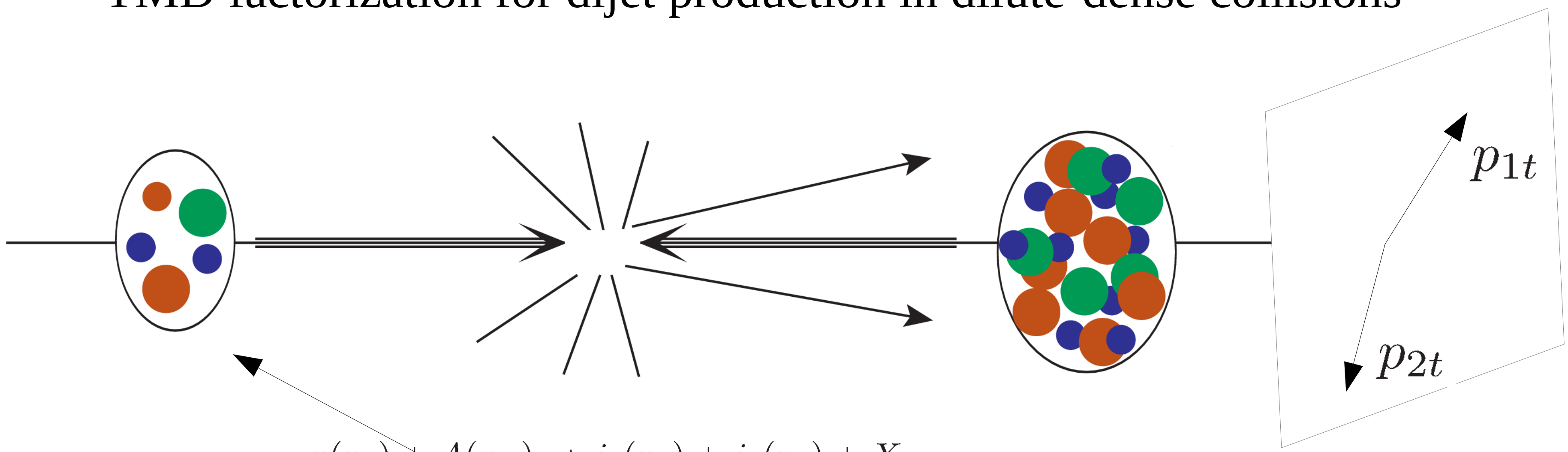

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Collinear PDFs for the projectile

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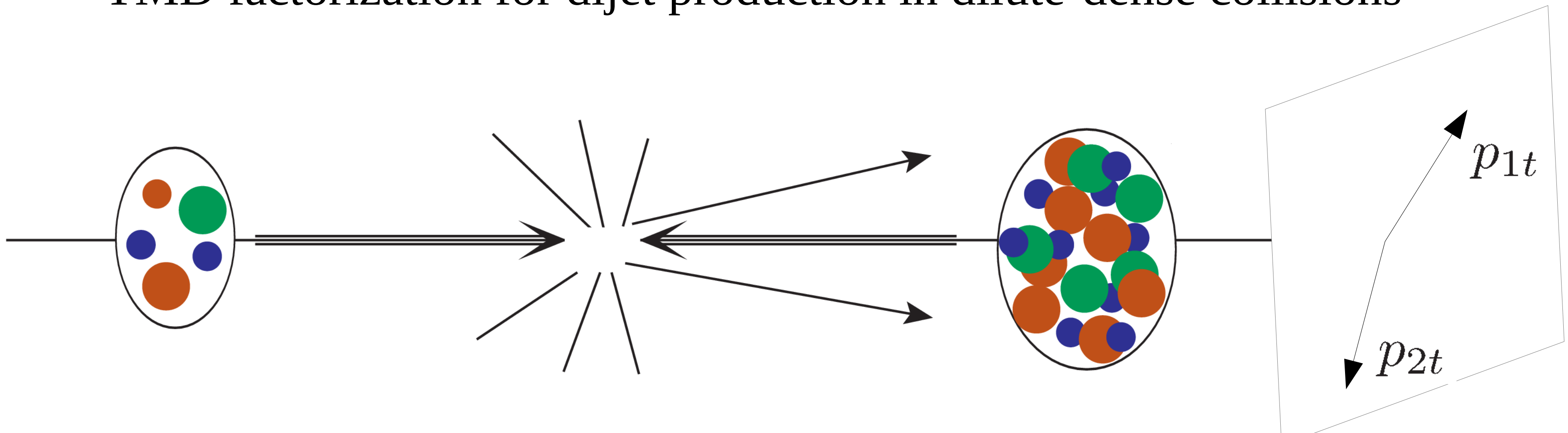

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Hard factors

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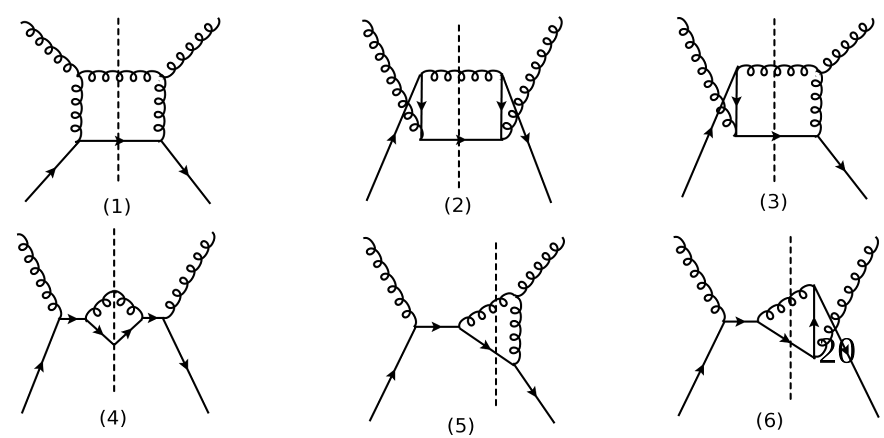

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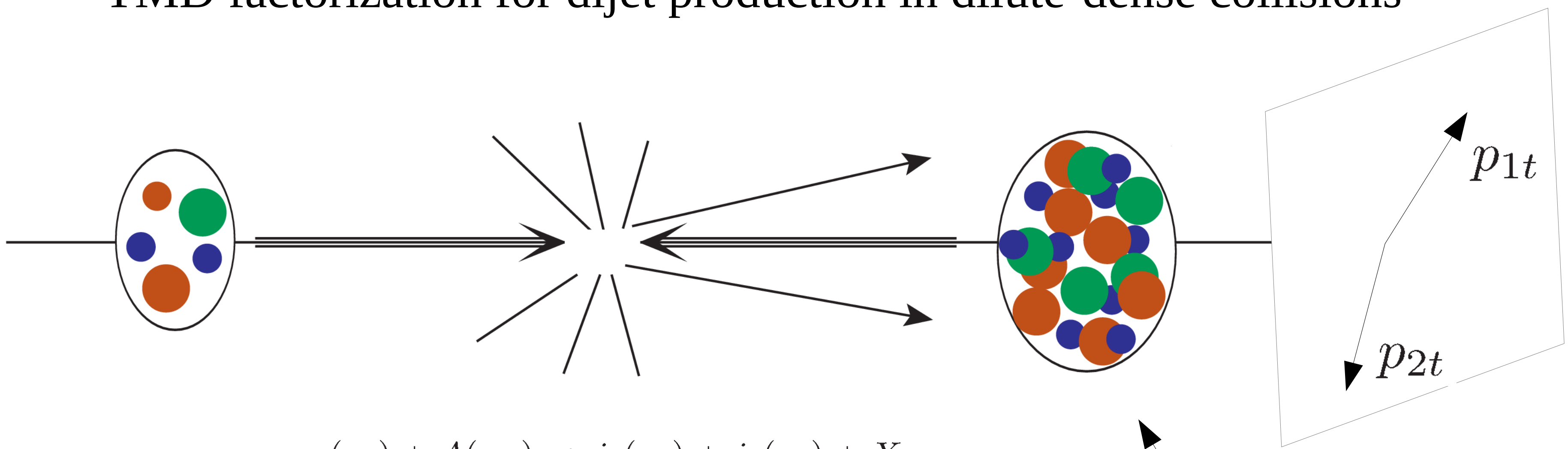

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Eight transverse momentum dependent gluon distributions for the target

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From the CGC framework:
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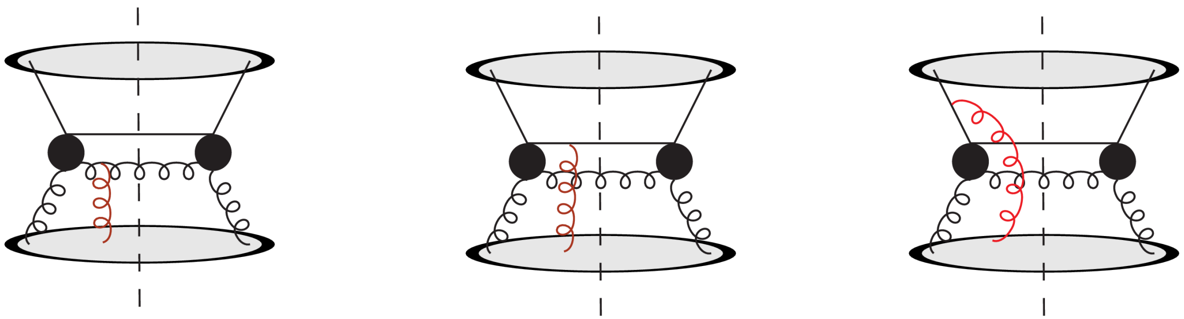
TMD gluon distributions

$$\mathcal{F}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \langle A|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}\right)F^{i-}(0)\right]|A\rangle$$



Gauge links

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;\mathbf{0})U(\pm\infty,\xi^+;\boldsymbol{\xi})$$

$$U(a,b;\mathbf{x}) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

$$\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

- TMD gluon distributions are gauge invariant, but process dependent.

TMD gluon distributions for dijet production

$$k_t \ll P_t$$

$$\mathcal{F}_{qg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger} F^{i-}(0)\ \mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[-]\dagger} F^{i-}(0)\ \mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}_\xi^{[\square]\dagger}\right] \mathrm{Tr}\left[F^{i-}(0)\ \mathcal{U}_0^{[\square]}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger} F^{i-}(0)\ \mathcal{U}^{[+]}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[-]\dagger} F^{i-}(0)\ \mathcal{U}^{[-]}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}_\xi^{[\square]\dagger}\mathcal{U}^{[+]\dagger} F^{i-}(0)\ \mathcal{U}_0^{[\square]}\mathcal{U}^{[+]}\right]\middle| A\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c^2} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger} F^{i-}(0)\ \mathcal{U}^{[+]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]}\right] \mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]\middle| A\right\rangle$$

Bomhof, Mulders and Pijlman (2006)

TMD gluon distributions for dijet production

$k_t \ll P_t$

$$\mathcal{F}_{qg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \mathrm{Tr}\left[F\left(\xi\right)\mathcal{U}^{[-]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right]\right\rangle \longrightarrow$$ Dipole gluon distribution

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger}F^{i-}(0)\ \mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]\middle|A\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[-]\dagger}F^{i-}(0)\ \mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]\middle|A\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}_\xi^{[\Box]\dagger}\right]\mathrm{Tr}\left[F^{i-}(0)\ \mathcal{U}_0^{[\Box]}\right]\middle|A\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger}F^{i-}(0)\ \mathcal{U}^{[+]}\right]\middle|A\right\rangle \longrightarrow$$ Weizsacker-Williams gluon distribution

$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[-]\dagger}F^{i-}(0)\ \mathcal{U}^{[-]}\right]\middle|A\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}_\xi^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}(0)\ \mathcal{U}_0^{[\Box]}\mathcal{U}^{[+]}\right]\middle|A\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}}\ \frac{1}{N_c^2}\left\langle A\middle|\mathrm{Tr}\left[F^{i-}(\xi)\ \mathcal{U}^{[+]\dagger}F^{i-}(0)\ \mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]\middle|A\right\rangle$$

TMD gluon distributions for dijet production as CGC correlators

$k_t \ll P_t$ + Small x

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = \frac{N_c k_t^2}{2\pi^2\alpha_s}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^2}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

→ Dipole gluon distribution

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{y})(\partial_i U_\mathbf{x}^\dagger)\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

→ Weizsacker-Williams gluon distribution

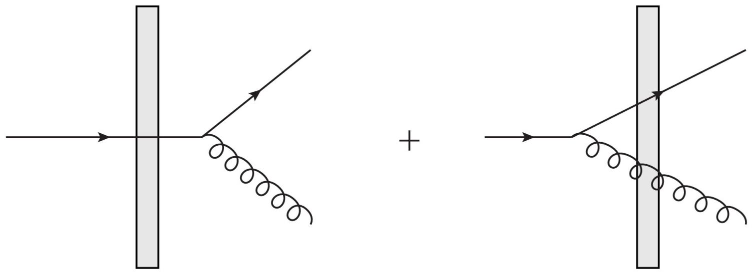
$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{x}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger U_\mathbf{x}U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c^2}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

Marquet, EP, Roiesnel (2016)

Forward dijet production in the CGC framework



$$S^{(2)}_{q\bar{q}}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{z}') = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^\dagger(\mathbf{z}')t^c U(\mathbf{b})t^d\right) V^{cd}(\mathbf{x})\right\rangle$$

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{b})U^\dagger(\mathbf{b}')t^d t^c\right)\left[V(\mathbf{x})V^\dagger(\mathbf{x}')\right]^{cd}\right\rangle$$

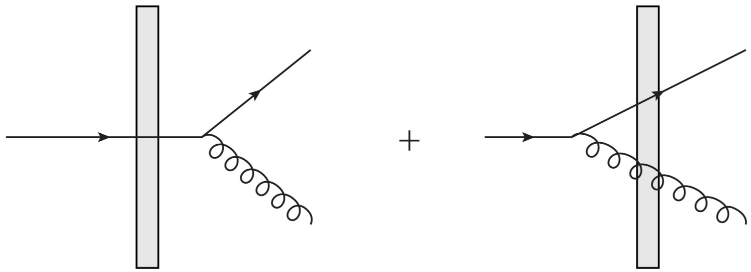
Valid at small x, no ordering of momentum scales

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')}e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')}\sum_{\lambda\alpha\beta}\phi^{\lambda^*}_{\alpha\beta}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

$$\times\Big\{S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]$$

$$-S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\Big\}$$

Forward dijet production in the CGC framework



$$S_{q\bar{q}}^{(2)}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

$$S_{qg\bar{q}}^{(3)}(\mathbf{b},\mathbf{x},\mathbf{z}') = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^\dagger(\mathbf{z}')t^c U(\mathbf{b})t^d\right) V^{cd}(\mathbf{x})\right\rangle$$

$$S_{qg\bar{q}g}^{(4)}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{b})U^\dagger(\mathbf{b}')t^d t^c\right)\left[V(\mathbf{x})V^\dagger(\mathbf{x}')\right]^{cd}\right\rangle$$

Small x + $k_t \ll P_t$

$$\frac{d\sigma(pA\to qgX)}{d^2P_t d^2k_t dy_1 dy_2} = \frac{N_c\alpha_s}{C_F}\frac{z(1-z)}{P_t^4}x_1 q(x_1,\mu^2)P_{gq}(z)\int\frac{d^2\mathbf{v}}{(2\pi)^2}\frac{d^2\mathbf{v}'}{(2\pi)^2}e^{ik_t\cdot(\mathbf{v}'-\mathbf{v})}$$
$$\times\left\{\left[(1-z)^2-\frac{z^2}{N_c^2}\right]\partial_v^i\partial_{v'}^j\left\langle D(\mathbf{v},\mathbf{v}')\right\rangle_{x_2} - \left\langle D(\mathbf{v},\mathbf{v}')\partial_x^i\partial_{v'}^j Q(\mathbf{x},\mathbf{y},\mathbf{v}',\mathbf{v})\right\rangle_{x_2}\Big|_{\substack{\mathbf{x}=\mathbf{v}\\ \mathbf{y}=\mathbf{v}'}}\right\}$$

Equivalent to the TMD formula
(all channels)

- The CGC framework contains the TMD factorization as a leading power

Large Nc: Dominguez, Marquet, Xiao and Yuan (2011)
Finite Nc: Marquet, EP, Roiesnel (2016)

TMD gluon distributions for dijet production as CGC correlators

$k_t \ll P_t$ + Small x

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = \frac{N_c k_t^2}{2\pi^2\alpha_s}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^2}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

→ Dipole gluon distribution

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{y})(\partial_i U_\mathbf{x}^\dagger)\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

→ Weizsacker-Williams gluon distribution

$$\mathcal{F}_{gg}^{(4)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{x}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{y}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(5)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger U_\mathbf{x}U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = -\frac{4}{g^2}\int\frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\,e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c^2}\left\langle \mathrm{Tr}\left[(\partial_i U_\mathbf{x})U_\mathbf{y}^\dagger(\partial_i U_\mathbf{y})U_\mathbf{x}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{x}U_\mathbf{y}^\dagger\right]\mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

Marquet, EP, Roiesnel (2016)

TMD gluon distributions in the GBW model

K. Golec-Biernat and M. Wusthoff (1998)

$$F(x_2,k_t) = \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{-ik_t\cdot\mathbf{r}} S_F(x_2,\mathbf{r})$$

→ Fourier transform of the fundamental dipole

$$S^{(2)}(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) == \frac{N_c\ k_t^2\ S_\perp}{2\pi^2\alpha_s} F(x_2,k_t)$$

→ Dipole gluon distribution

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = \frac{1}{2}\int_{k_t^2}^{\infty} dk_t'^2 \ln\left(\frac{k_t'^2}{k_t^2}\right)\int \frac{d^2q_t}{q_t^2}\ \mathcal{F}_{qg}^{(1)}(x_2,q_t)F(x_2,k_t'-q_t)$$

→ Weizsacker-Williams gluon distribution

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = \int d^2q_t\,\mathcal{F}_{gg}^{(3)}(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \int d^2q_t\,\mathcal{F}_{qg}^{(1)}(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\int d^2q_t \frac{(k_t-q_t)\cdot q_t}{q_t^2}\mathcal{F}_{qg}^{(1)}(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = \int d^2q_t d^2q_t'\,\mathcal{F}_{gg}^{(3)}(x_2,q_t)F(x_2,q_t')F(x_2,k_t-q_t-q_t')$$

F. Dominguez, C. Marquet, B. Xiao and F. Yuan (2011)

van Hameren, Kotko, Kutak, Marquet, EP and Sapeta (2016)

TMD gluon distributions in the GBW model

K. Golec-Biernat and M. Wusthoff (1998)

$$F(x_2,k_t) = \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{-ik_t\cdot\mathbf{r}} S_F(x_2,\mathbf{r})$$

→ Fourier transform of the fundamental dipole

$$S^{(2)}(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\right]$$

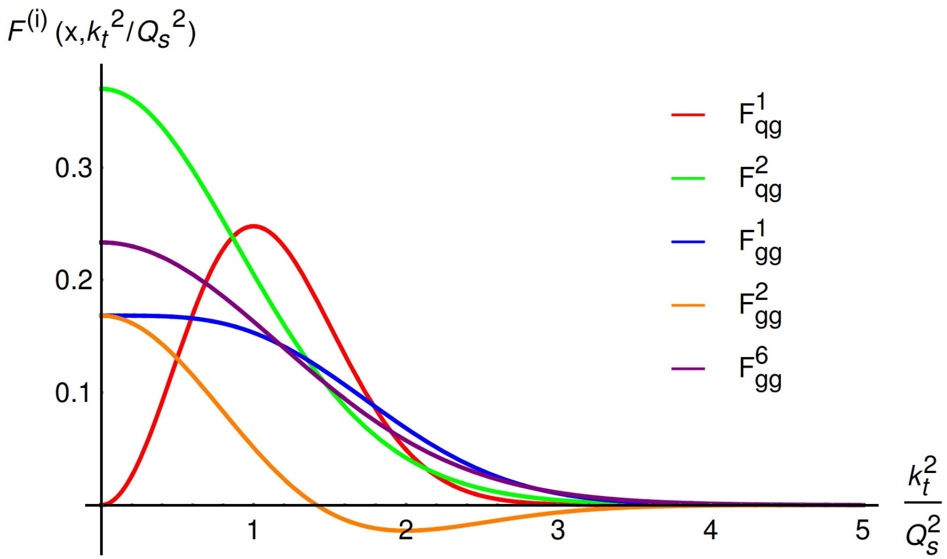
$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = 2\gamma\frac{S_\perp}{Q_s^2(x_2)}k_t^2 \exp\left[-\frac{k_t^2}{Q_s^2(x_2)}\right]$$

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = \gamma\left[\mathrm{Ei}\left(-\frac{k_t^2}{Q_s^2(x)}\right) - \mathrm{Ei}\left(-\frac{k_t^2}{3Q_s^2(x)}\right)\right]$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \frac{\gamma}{4}e^{-\frac{k_t^2}{2Q_s^2(x_2)}}\left(2+\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = \frac{\gamma}{4}e^{-\frac{k_t^2}{2Q_s^2(x_2)}}\left(2-\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = \gamma\left[\mathrm{Ei}\left(-\frac{k_t^2}{2Q_s^2(x)}\right) - \mathrm{Ei}\left(-\frac{k_t^2}{4Q_s^2(x)}\right)\right]$$



van Hameren, Kotko, Kutak, Marquet, EP and Sapeta (2016)

TMD gluon distributions in the MV model

L. D. McLerran and R. Venugopalan, (1994)

$$F(x_2,k_t) = \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{-ik_t\cdot\mathbf{r}} S_F(x_2,\mathbf{r})$$

→ Fourier transform of the fundamental dipole

$$S^{(2)}(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2Q_s^2}{4}\log\frac{1}{\Lambda r}\right]$$

- High-kt behavior in the McLerran-Venugopalan model

$$\mathcal{F}^{(i)} \simeq \gamma \frac{Q_s^2(x_2)}{k_t^2} + \mathcal{O}\left(\frac{Q_s^4(x_2)}{k_t^4}\log\frac{k_t^2}{\Lambda^2}\right) \qquad \gamma = N_c S_\perp / 4\pi^3\alpha_s$$

van Hameren, Kotko, Kutak, Marquet, EP and Sapeta (2016)

JIMWLK small-x evolution

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = \frac{N_c k_t^2}{2\pi^2\alpha_s}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^2}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U_\mathbf{y}U_\mathbf{x}^\dagger\right]\right\rangle_{x_2}$$

→ Averages over color field configurations in the dense target

$$\langle O\rangle_{x_2} = \int DA^-|\phi_{x_2}[A^-]|^2 O[A^-]$$

JIMWLK equation: $$\frac{d}{d\log(1/x_2)}\langle O\rangle_{x_2} = \langle H_{JIMWLK}\ O\rangle_{x_2}$$

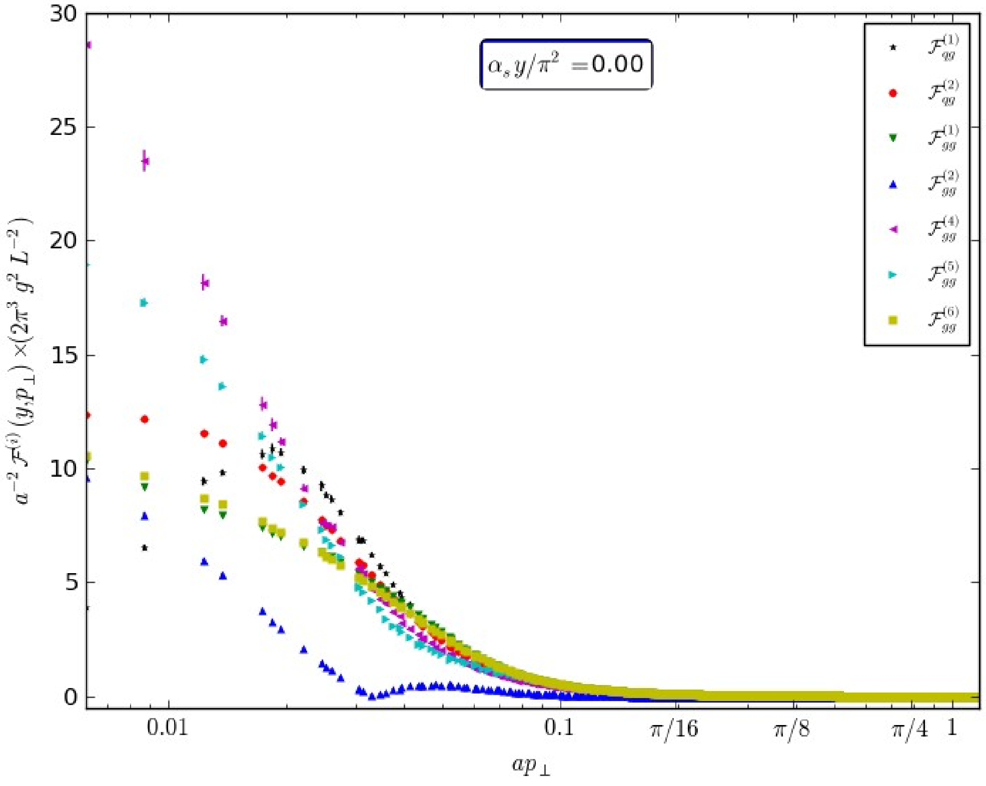
$$H_{JIMWLK}|\phi_{x_2}[A^-]|^2 = \int \frac{d^2\mathbf{x}}{2\pi}\frac{d^2\mathbf{y}}{2\pi}\frac{d^2\mathbf{z}}{2\pi}\frac{(\mathbf{x}-\mathbf{z})\cdot(\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\frac{\delta}{\delta A_c^-(\mathbf{x})}\left[1+V_\mathbf{x}^\dagger V_\mathbf{y} - V_\mathbf{x}^\dagger V_\mathbf{z} - V_\mathbf{z}^\dagger V_\mathbf{y}\right]^{cd}\frac{\delta}{\delta A_d^-(\mathbf{y})}$$

Resums: $\alpha_s \ln\frac{1}{x}$, $\quad g_s A$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

TMD gluon distributions in the MV model

• Initial conditions in the McLerran-Venugopalan model

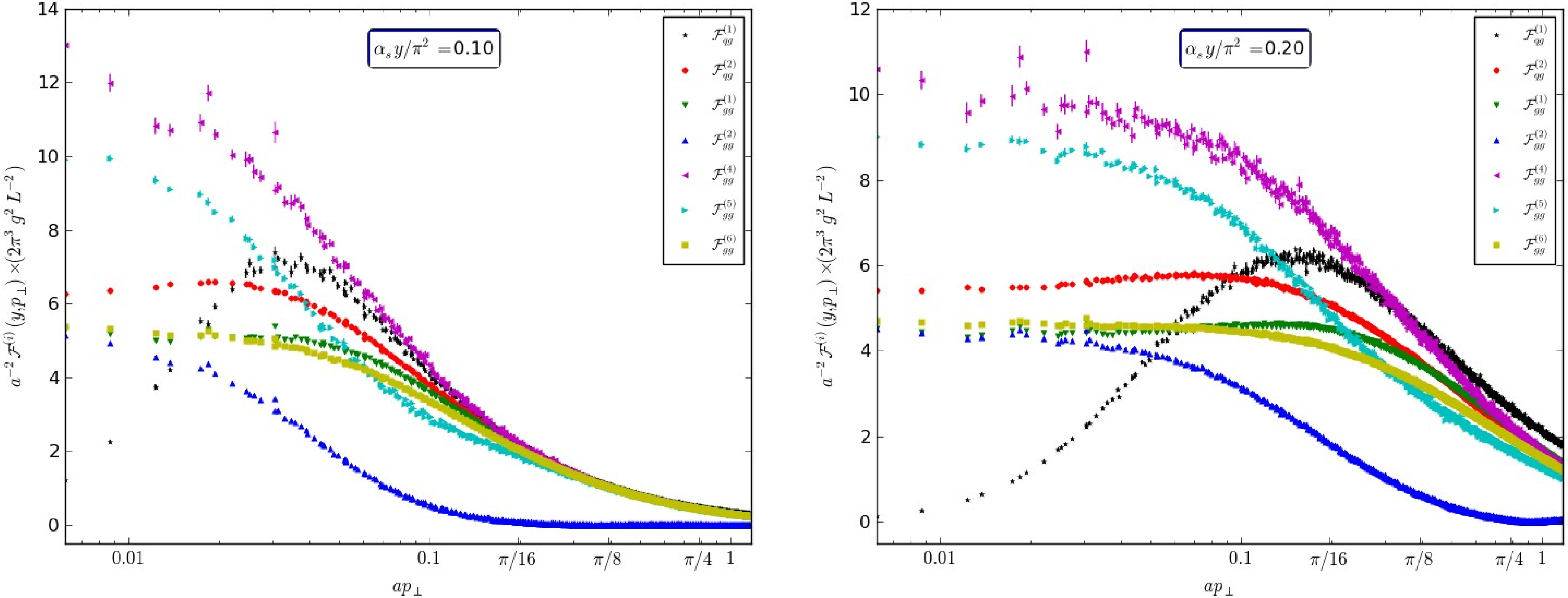


Marquet, EP, Roiesnel (2016)

JIMWLK small-x evolution

code written by Claude Roiesnel

- McLerran-Venugopalan model as initial condition at *y=0*



Different small-x behavior for different TMDs → Loss of universality at low transverse momentum

Marquet, EP, Roiesnel (2016)

Summary

• We identify the operator definition of each TMD gluon distribution, at small *x* and finite *Nc*, as a CGC correlator of Wilson lines

• The nearly back-to-back limit of CGC is equivalent to the small-*x* limit of the effective TMD factorization for forward dijet production

• Small-*x* evolution shows universality of gluon TMDs at high transverse momenta, and non-universality at low transverse momenta

Outlook

• Look for scattering processes that will probe each of the TMD distributions individually and that will separate them experimentally.

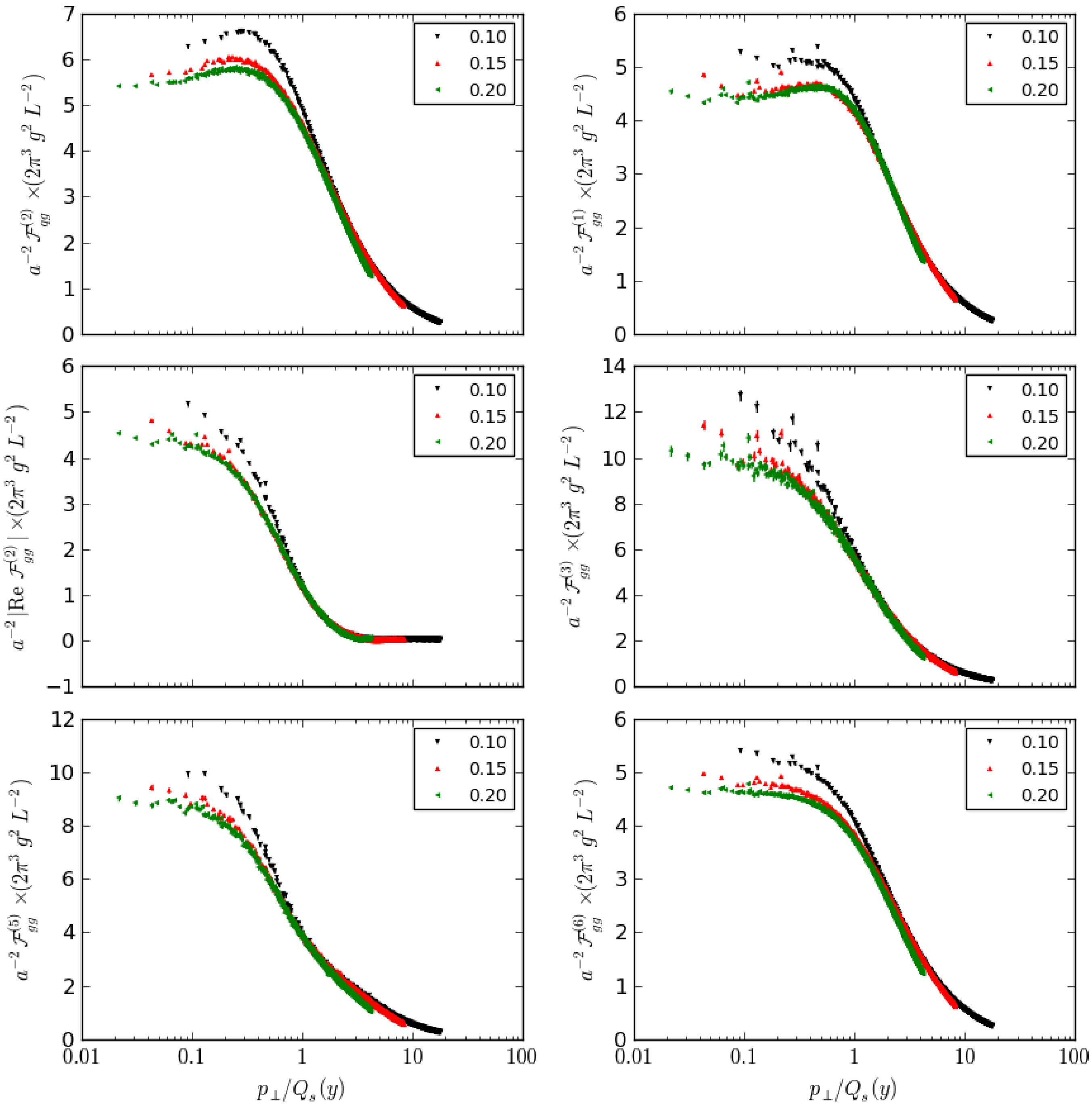
Thank you

$$\langle p|p'\rangle = (2\pi)^3\ 2p^-\delta(p^- - p'^-)\delta^{(2)}(p_t - p'_t)$$

$$\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \langle A|O(0,\xi)|A\rangle = \frac{2}{\langle A|A\rangle} \int \frac{d^3\xi d^3\xi'}{(2\pi)^3} e^{ix_2 p_A^-(\xi^+ - \xi^{'+}) - ik_t\cdot(\boldsymbol{\xi}-\boldsymbol{\xi}')} \langle A|O(\xi',\xi)|A\rangle$$

$$\exp[ix_2 p_A^-(\xi^+ - \xi^{'+})] = 1$$

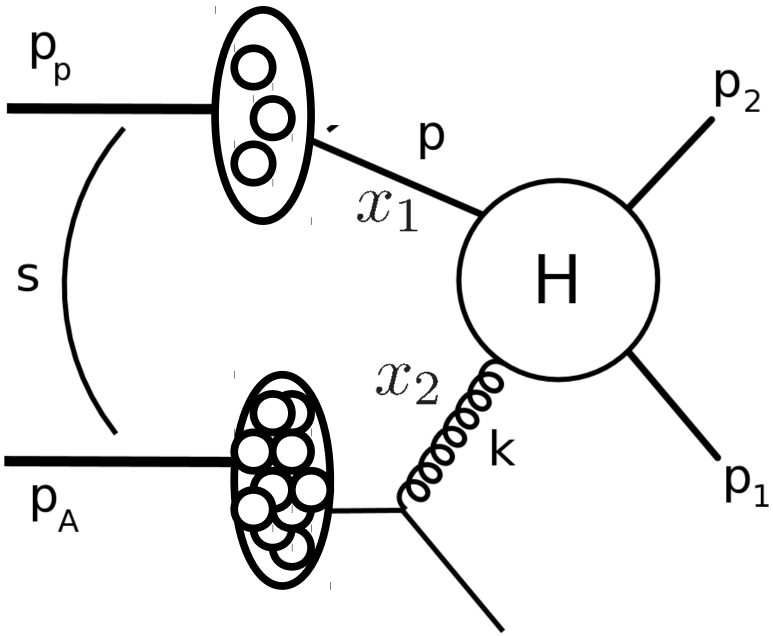
$$\frac{\langle A|O(\xi',\xi)|A\rangle}{\langle A|A\rangle} = \langle O(\xi',\xi)\rangle_{x_2}$$



$a^{-2}\,\mathcal{F}^{(2)}_{gg}\;\times(2\pi^3\,g^2\,L^{-2})$
$a^{-2}\,\mathcal{F}^{(1)}_{gg}\;\times(2\pi^3\,g^2\,L^{-2})$
$a^{-2}\,|\mathrm{Re}\,\mathcal{F}^{(2)}_{gg}|\;\times(2\pi^3\,g^2\,L^{-2})$
$a^{-2}\,\mathcal{F}^{(3)}_{gg}\;\times(2\pi^3\,g^2\,L^{-2})$
$a^{-2}\,\mathcal{F}^{(5)}_{gg}\;\times(2\pi^3\,g^2\,L^{-2})$
$a^{-2}\,\mathcal{F}^{(6)}_{gg}\;\times(2\pi^3\,g^2\,L^{-2})$
0.10
0.15
0.20
$p_\perp/Q_s(y)$

Forward di-jet production in dilute-dense collisions

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



$$x_1 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \sim 1$$

➢ Dilute

$$x_2 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \ll 1$$

➢ Dense

$$|k_t|^2 = |p_{1t} + p_{2t}|^2$$

➔ Momentum imbalance

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization $k_t \sim Q_s \ll P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

F. Dominguez, C. Marquet, B. Xiao and F. Yuan (2011)

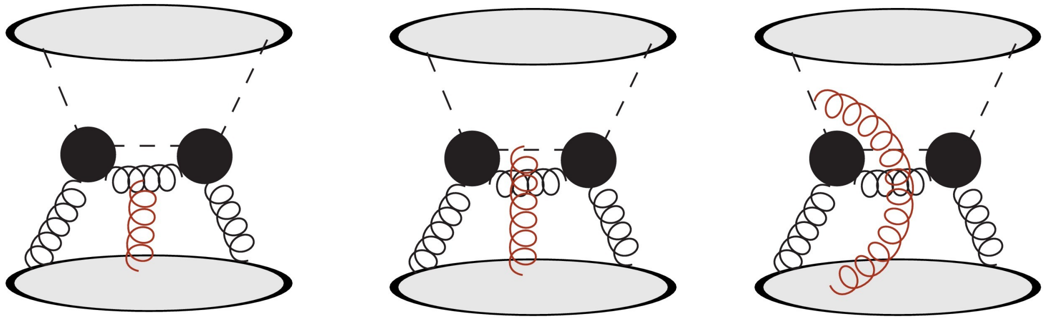
- Valid in the large-N_c limit;
  - Parton distributions of collinear factorization for the large-x projectile;
    - Five k_t dependent unintegrated gluon distributions for the small-x target;
      - On-shell hard factors.
        - Equivalent to CGC at large N_c and in the collinear limit.

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2) \otimes H_{ag\to cd} \otimes \mathcal{F}_{ag}(x_2,k_t)$$

$$\mathcal{F}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \langle A|\text{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}\right)F^{i-}(0)\right]|A\rangle$$



Igor's talk on Wednesday

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;\mathbf{0})U(\pm\infty,\xi^+;\boldsymbol{\xi})$$

$$\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

$$U(a,b;\mathbf{x}) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

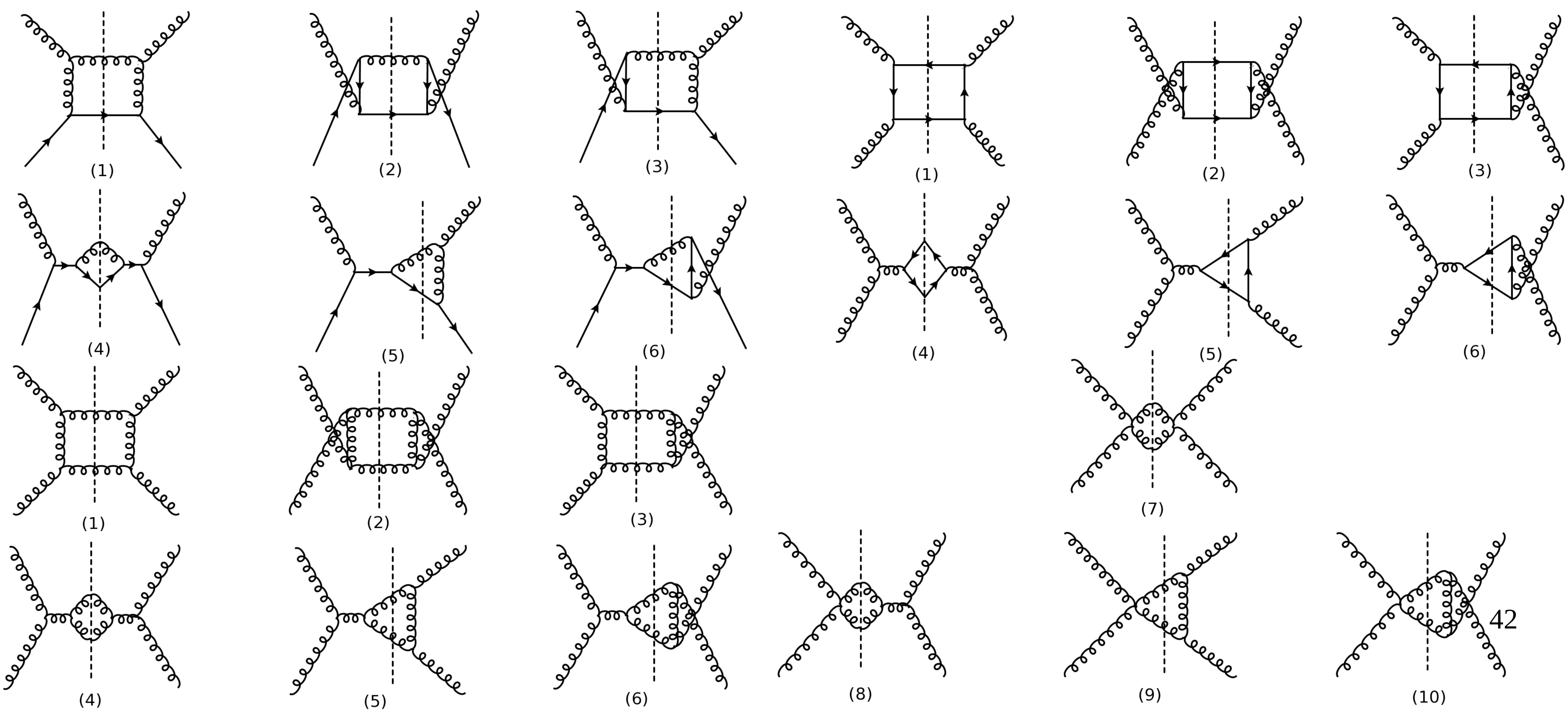
- TMD gluon distributions are gauge invariant, but process dependent.

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2) \otimes H_{ag\to cd} \otimes \mathcal{F}_{ag}(x_2,k_t)$$

C. J. Bomhof, P. J. Mulders and F. Pijlman (2006)



Forward di-jet production in dilute-dense collisions

- Improved Transverse Momentum Dependent factorization

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

- Includes all finite N_c corrections;
  - Eight unintegrated gluon distributions, two independent per channel;

- Outlook: Show equivalence between CGC and HEF at finite N_c

Forward di-jet production in dilute-dense collisions

- High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

- Parton distributions of collinear factorization for the large-x projectile;
  - One k_t dependent unintegrated gluon distribution for the small-x target;
    - Off-shell hard factors.

- Transverse Momentum Dependent factorization $k_t \sim Q_s \ll P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd}(k_t)\, \frac{1}{1+\delta_{cd}}$$

- Parton distributions of collinear factorization for the large-x projectile;
  - Six k_t dependent unintegrated gluon distributions for the small-x target;
    - On-shell hard factors.

Forward di-jet production in dilute-dense collisions

• Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

Restore the transverse momentum dependence in the matrix elements

Off-shell matrix elements calculated with two methods:

• Feynman rules + gauge vector defined by the target four-momentum
+ longitudinal polarization vector for the off-shell gluon

S. Catani, M. Ciafaloni and F. Hautmann, 1991

• Color ordered amplitudes

Gauge invariance on the level of amplitudes;
Redundancy in hard factors removed from the start.

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

Restore the transverse momentum dependence in the matrix elements

i	1	2
$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}(\bar{s}^2+\bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\ \bar{u}\hat{u}}\right)$	$-\frac{C_F}{N_c}\frac{\bar{s}(\bar{s}^2+\bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

$$\bar{s}=(x_2p_A+p)^2 \qquad \bar{t}=(x_2p_A-p_1)^2 \qquad \bar{u}=(x_2p_A-p_2)^2$$

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\ \frac{1}{1+\delta_{cd}}$$

Unifies the regions of validity of HEF and TMD; It can be used to study forward di-jet production for any value of the momentum imbalance between the saturation scale and the moment of the jets.

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\ \frac{1}{1+\delta_{cd}}$$

Phenomenological study

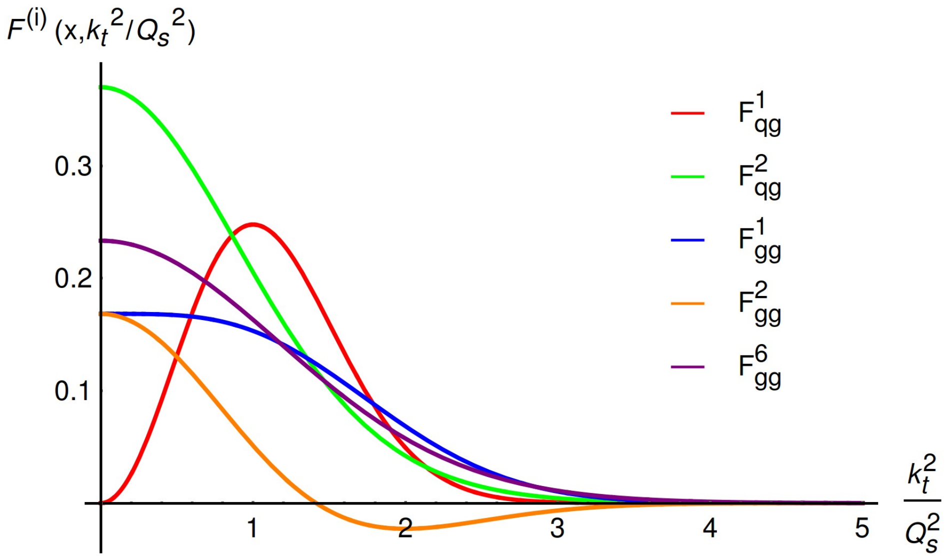
- Start with the large N_c case;
- First gluon input: Analytical model expressions for the gluon distributions;
- Improve: Numerical inputs with small-x evolution.

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

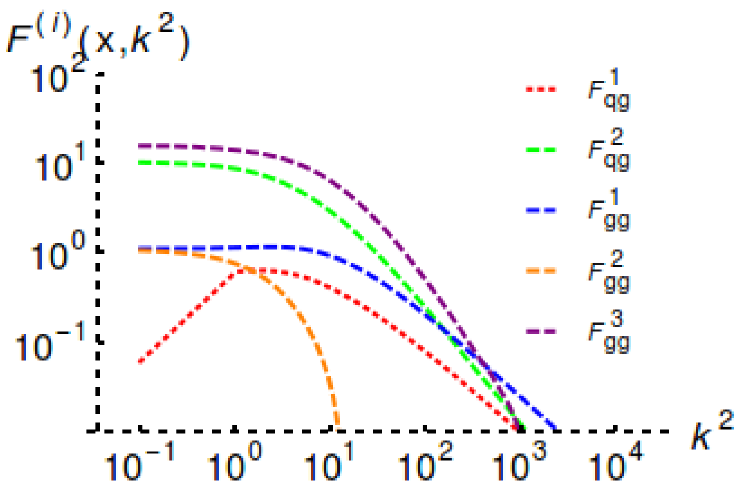
$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

- Gluon distributions in the GBW model



Kutak-Sapeta gluons with non-linear evolution:

K. Kutak and S. Sapeta, (2012)



Soon to be published