Matrix elements and Monte Carlo calculations with k_T-factorization

Andreas van Hameren
Institute of Nuclear Physics
Polish Academy of Sciences
Kraków

presented at

REF 2016, *Resummation, Evolution, Factorization*

9-11-2016, University of Antwerp, Antwerp

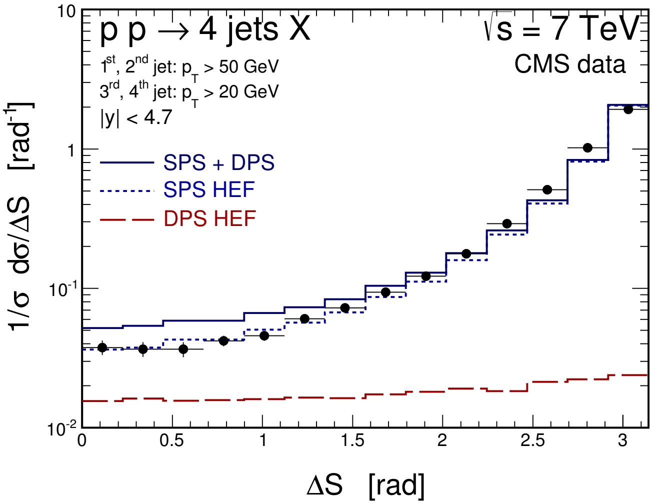
This work was supported by grant of National Science Center, Poland, No. 2015/17/B/ST2/01838.

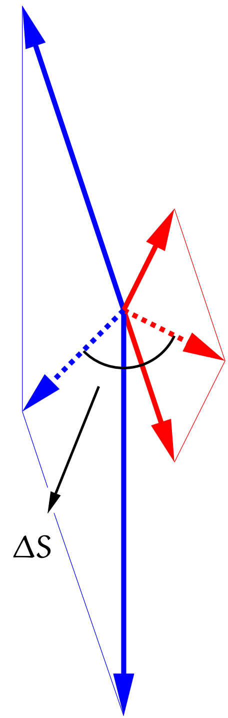
Outline

- Factorized cross section calculation
- Off-shell amplitudes
- BCFW recursion for amplitudes with off-shell partons
- KaTie: for parton-level event generation with k_T-dependent initial states

Four jets with k_T-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016





- ΔS is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- k_T-factorization allows for the necessary momentum inbalance.

Factorization for hadron scattering

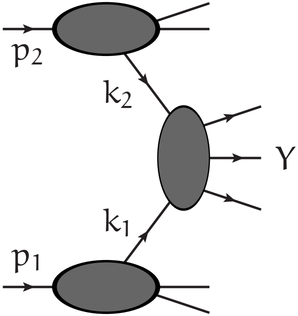
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x}\, f_{i,a}(x,\mu)\, \delta^4(k - x\,p_i)$

k_T-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x}\, \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\,p_i - k_T)$

- The *parton level* cross section $d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$ can be calculated within perturbative QCD.
- The *parton distribution functions* $f_{i,a}$ and $\mathcal{F}_{i,a}$ must be modelled and fit against data.
- Unphysical scale μ is a price to pay, but its dependence is calculable within perturbative QCD via *evolution equations*.



Factorization for hadron scattering

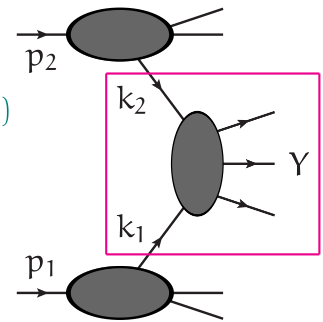
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x} f_{i,a}(x,\mu)\, \delta^4(k - x\, p_i)$

k_T-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x} \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\, p_i - k_T)$

$$\hat{\sigma} = \int d\Phi(1,2 \to 3,4,\ldots,n)\, \big|\mathcal{M}(1,2,\ldots,n)\big|^2\, \mathcal{O}(p_3,p_4,\ldots,p_n)$$

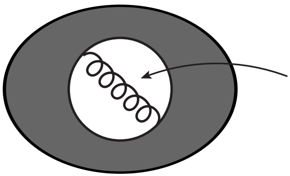


phase space includes summation over color and spin
squared amplitude calculated perturbatively
observable includes phase space cuts, or jet algorithm

Gauge invariance

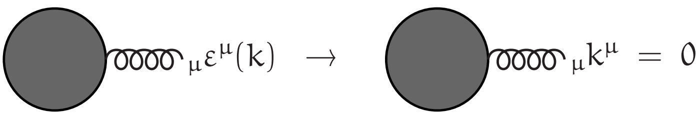
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:



$$\left\{ \begin{array}{l} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2 + \xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{array} \right.$$

Ward identity:



- Only holds if all external particles are on-shell.
- k_T-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- How to define amplitudes with off-shell intial-state momenta?

Amplitudes with off-shell gluons

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$ and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$

$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$
$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$
$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q)p^\mu \quad \text{with} \quad x(q) \equiv \frac{q\cdot k}{q\cdot p}$$

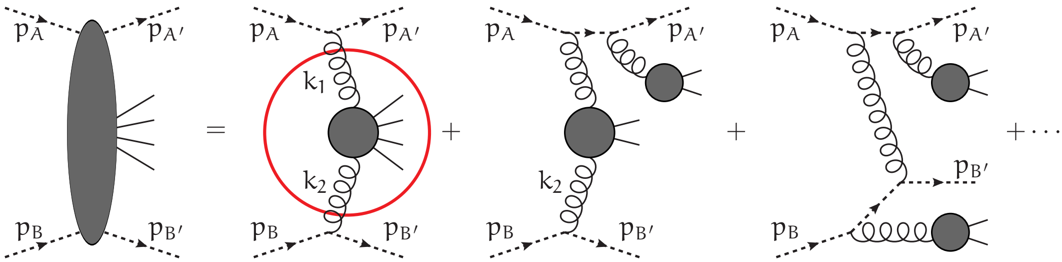
Construct k_T^μ explicitly in terms of p^μ and q^μ:

$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\[2ex] \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:
Embed the process in an on-shell process with auxiliary partons

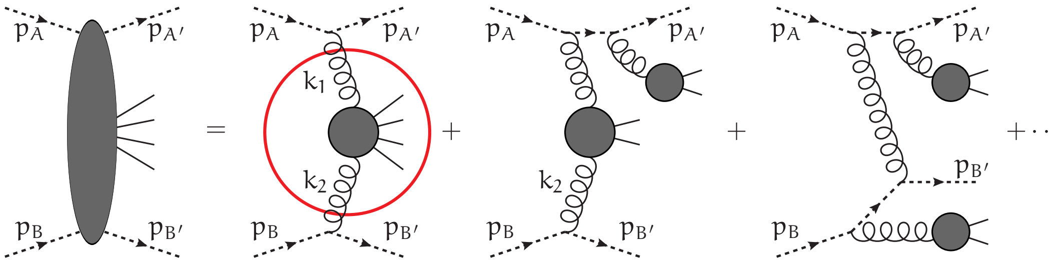


$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$

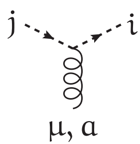
Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

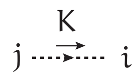


$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$
$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$
$$\Lambda \to \infty$$

$\Rightarrow$



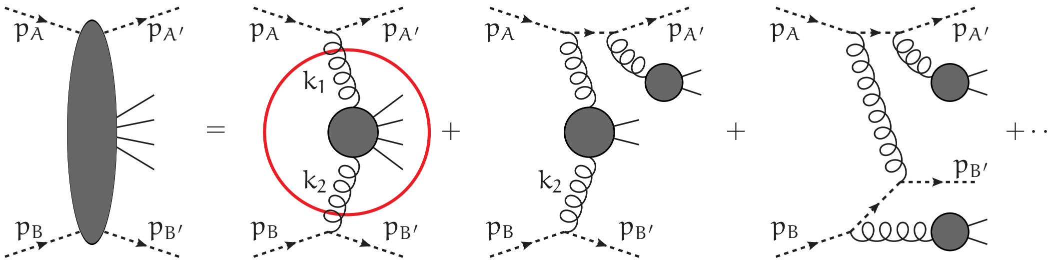
$= -i\,T^a_{i,j}\,p_1^\mu$

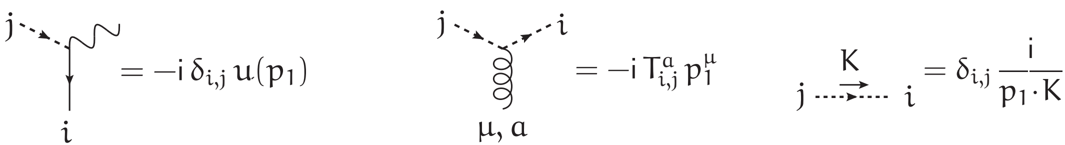


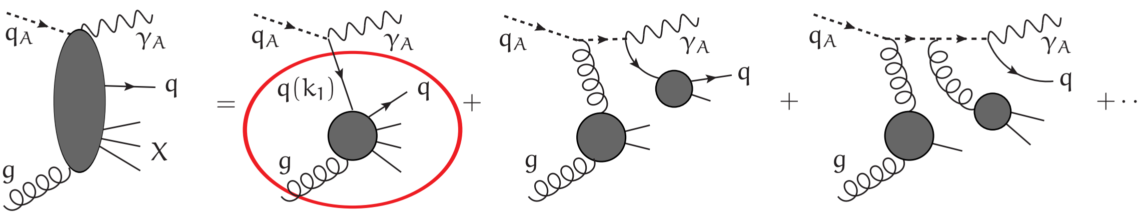
$= \delta_{i,j}\,\frac{i}{p_1 \cdot K}$

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

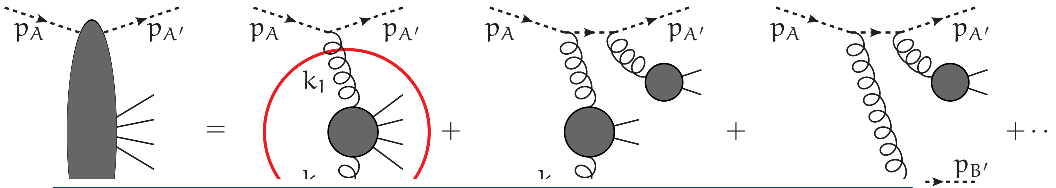


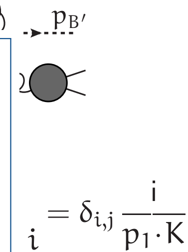




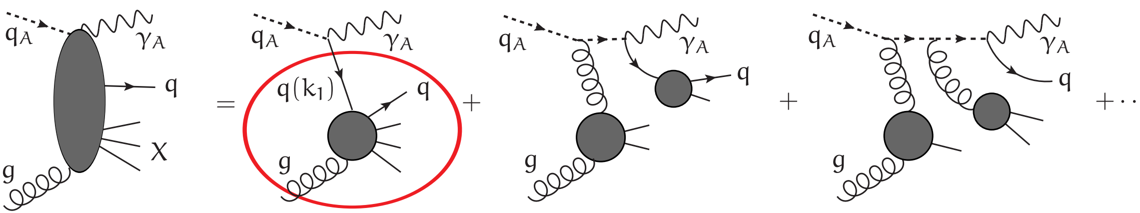
Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

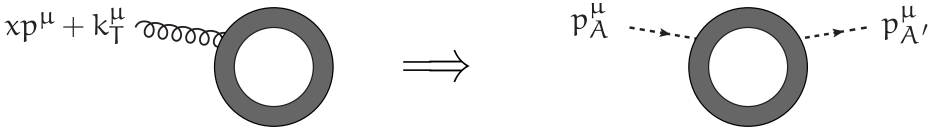




In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014



Off-shell one-loop amplitudes



$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \quad , \quad p_{A'}^\mu = -(\Lambda - x)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu \ ,$$

where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \ .$$

With this choice, the momenta $p_A, p_{A'}$ satisfy the relations

$$p_A^2 = p_{A'}^2 = 0 \quad , \quad p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu$$

for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$.

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}\big(\Lambda^{-1}\big)$$

Taking this limit after loop integration will lead to singularities $\log\Lambda$.

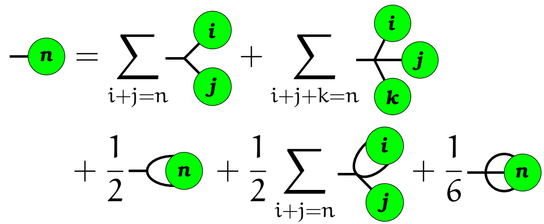
Tree-level amplitudes with off-shell recursion

The scattering amplitude is the residue of the connected Green function obtained for the kinematical situation in which all external particles are on-shell, and satisfy momentum conservation.

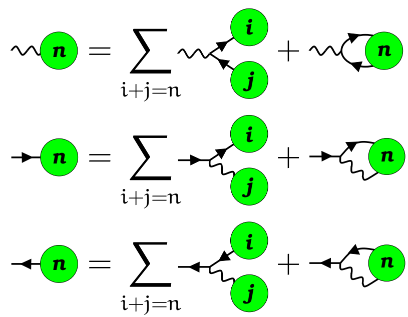
Off-shell currents, or Green functions with all external particles on-shell, satisfy the recursive *Dyson-Schwinger equations*:

- Sums are over partitions of on-shell particles over the blobs, and over possible flavors for virtual particles.
- Current with $n = \#\text{externalparticles} - 1$ is completely on-shell and gives the amplitude.
- Solution can be represented as a sum of Feynman graphs,
- but recursion can also be used to construct amplitude directly.

Theories with four-point vertices:

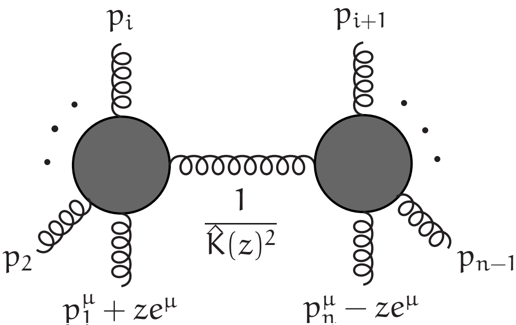


Theories with more types of currents:



BCFW recursion for on-shell amplitudes

Gives compact expression through recursion of *on-shell* amplitudes.



$$\hat{K}^\mu(z) = p_1^\mu + \cdots + p_i^\mu + ze^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_n^\mu + ze^\mu$$

$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

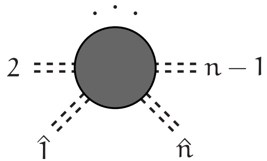
$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

$$\mathcal{A}(1^+, 2, \ldots, n{-}1, n^-) = \sum_{i=2}^{n-1} \sum_{h=+,-} \mathcal{A}(\hat{1}^+, 2, \ldots, i, -\hat{K}_{1,i}^h)\, \frac{1}{K_{1,i}^2}\, \mathcal{A}(\hat{K}_{1,i}^{-h}, i{+}1, \ldots, n{-}1, \hat{n}^-)$$

$$\mathcal{A}(1^+, 2^-, 3^-) = \frac{\langle 23\rangle^3}{\langle 31\rangle\langle 12\rangle} \quad , \quad \mathcal{A}(1^-, 2^+, 3^+) = \frac{[32]^3}{[21][13]}$$

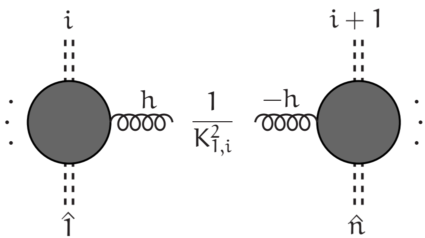
BCFW recursion for off-shell amplitudes

The BCFW recursion formula becomes

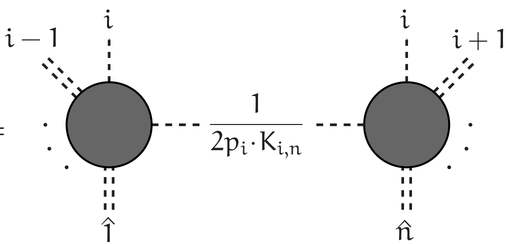


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

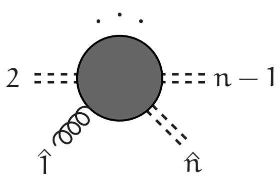
$A_{i,h} =$



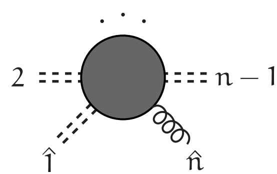
$B_i =$



$C = \frac{1}{\kappa_1}$



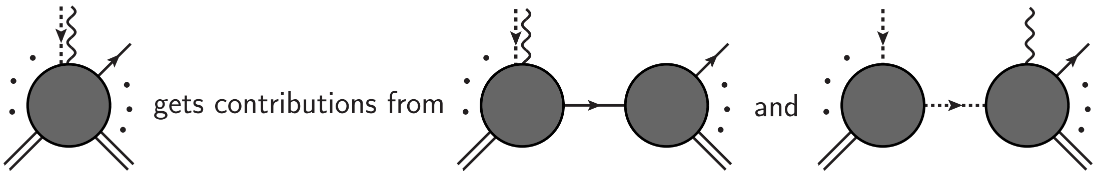
$D = \frac{1}{\kappa_1^*}$



The hatted numbers label the shifted external gluons.

BCFW recursion with (off-shell) quarks

- on-shell case treated in Luo, Wen 2005
- any off-shell parton can be shifted: propagators of "external" off-shell partons give the correct power of z in order to vanish at infinity
- different kinds of contributions in the recursion



- many of the MHV amplitudes come out as expected
- some more-than-MHV amplitudes do not vanish, but are sub-leading in k_T

$$\mathcal{A}(1^+, 2^+, \dots, n^+, \bar{q}^*, q^-) = \frac{-\langle \bar{q} q \rangle^3}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n \bar{q} \rangle \langle \bar{q} q \rangle \langle q 1 \rangle}$$

- off-shell quarks have helicity

$$\mathcal{A}(1, 2, \dots, n, \bar{q}^{*(+)}, q^{*(-)}) \neq \mathcal{A}(1, 2, \dots, n, \bar{q}^{*(-)}, q^{*(+)})$$

KaTie https://bitbucket.org/hameren/katie

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Hg) with several final-state particles.
- 0, 1, or 2 off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids.
- a calculation is steered by a single input file.
- employs an optimization phase in which the pre-samplers for all channels are optimized.
- during the generation phase several event files can be created in parallel.
- can generate (naively factorized) MPI events.
- event files can be processed further by parton-shower program like CASCADE (next talk by Mirko Serino).

```
Ngroup = 1
Nfinst = 3
process = g u  -> mu+ mu- u   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g u~ -> mu+ mu- u~  factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d  -> mu+ mu- d   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d~ -> mu+ mu- d~  factor = 1   groups = 1   pNonQCD = 2 0 0
lhaSet = MSTW2008nlo68cl
offshell = 1 0
tmdTableDir = /home/user0/kTfac/tables/krzysztof02/
tmdpdf = g  KMR_gluon.dat
tmdpdf = u  KMR_u.dat
tmdpdf = u~ KMR_ubar.dat
tmdpdf = d  KMR_d.dat
tmdpdf = d~ KMR_dbar.dat
tmdpdf = s  KMR_s.dat
tmdpdf = s~ KMR_sbar.dat
tmdpdf = c  KMR_c.dat
tmdpdf = c~ KMR_cbar.dat
tmdpdf = b  KMR_b.dat
tmdpdf = b~ KMR_bbar.dat
Nflavors = 5
helicity = sampling
Noptim = 1,000,000
Ecm = 7000
Esoft = 20
```

```
cut = deltaR|1,3| > 0.4
cut = deltaR|2,3| > 0.4
cut = pT|1| > 20
cut = pT|2| > 20
cut = pseudoRap|1| > 2.0
cut = pseudoRap|2| > 2.0
cut = pseudoRap|1| < 4.5
cut = pseudoRap|2| < 4.5
cut = mass|1+2| > 60
cut = mass|1+2| < 120
cut = pT|3| > 20
cut = rapidity|3| > 2.0
cut = rapidity|3| < 4.5
scale = (pT|3|+pT|1+2|+91.1882D0)/3
mass = Z 91.1882 2.4952
mass = W 80.419 2.21
mass = H 125.0 0.00429
mass = t 173.5
switch = withQCD Yes
switch = withQED Yes
switch = withWeak Yes
switch = withHiggs No
switch = withHG No
coupling = Gfermi 1.16639d-5
```

$p\,p \to Z + j$ in the forward direction

Conclusions

- k_T-factorization allows for the parton-level description of kinematical situations inaccessible with LO collinear factorization, eg. ΔS for four jets.
- Factorization prescriptions with explicit k_T dependence in the pdfs ask for hard matrix elements with off-shell initial-state partons.
- The necessary amplitudes can be defined in a manifestly gauge invariang manner that allows for Dyson-Schwinger recursion and BCFW recursion, both for off-shell gluons and off-shell quarks.
- KaTie generates parton-level events with k_T-dependent initial states.
- Next: NLO.

Backup