Selected Topics in Lattice Flavour Physics

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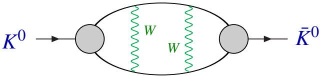
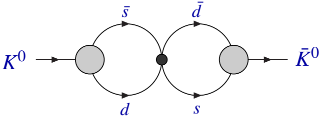
1. Introduction

- The mission of lattice calculations is to evaluate hadronic effects.
- “Standard" lattice calculations in flavour physics are of matrix elements of local operators between single hadron states $\langle h_2(p_2)|O(0)|h_1(p_1)\rangle$ (or $\langle 0|O(0)|h(p)\rangle$).
- For example, in the evaluation of ϵ_K, we need to calculate (schematically)



(gluons and quark loops not shown.)

- The process is short-distance dominated and so we can approximate the above by a perturbatively calculable (Wilson) coefficient C times



where the black dot represents the insertion of the local operator $(\bar{s}\gamma_\mu(1-\gamma^5)d)\,(\bar{s}\gamma_\mu(1-\gamma^5)d)$.

  - In the standard model only this single operator contributes.
  - In generic BSM theories there are 5 possible $\Delta S=2$ operators contributing.

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- In recent years the precision with which such standard quantities can be computed has improved immensely.
  - Computations can now be performed at with 2, 2+1, 2+1+1 dynamical quarks at physical masses.
- Some sample results from the FLAG collaboration:

See tables 1 and 2 in arXiv:1607.00299

	$N_f = 2$	$N_f = 2+1$	$N_f = 2+1+1$
f_K/f_π	1.205(6)(17)	1.192(5)	1.193(3)
$\hat{B}_K$	0.727(22)(12)	0.7625(97)	0.717(18)(16)
$f_+(0)$	0.9560(57)(62)	0.9677(27)	0.9704(24)(22)

- In this talk I want to introduce some new directions in lattice flavour physics.

Outline of talk:

1. Introduction
2. $K \to \pi\pi$ decays
3. Electromagnetic (and isospin breaking) contributions to decay amplitudes
4. Summary and conclusions

Thank you to my collaborators from the RBC-UKQCD Collaboration and from Rome (em corrections) for such stimulating collaborations on the topics of this talk.

2. Status of RBC-UKQCD calculations of $K \to \pi\pi$ decays

- In 2015 RBC-UKQCD published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large errors:

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

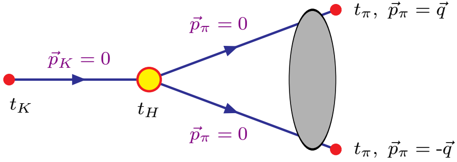
- This single result hides much important (and much more precise) information which we have determined along the way. For example using our matrix elements, but updating the treatment of perturbative short-distance effects:
  - A.Buras, M.Gorbahn, S.Jäger, M.Jamin, arXiv:1507.06345

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{BGJJ}} = (1.9 \pm 4.5) \times 10^{-4}\,.$$

  - T.Kitahari, U.Nierste, P.Tremper, arXiv:1607.06727

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{KNT}} = (0.96 \pm 4.96) \times 10^{-4}\,.$$

- This is by far the most complicated project that I have ever been involved with.



- $K \to \pi\pi$ correlation function is dominated by lightest state, e.g. for $I = 2$ with $\vec{p}_K = 0$ this is the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A_1 e^{-2m_\pi t_\pi} + A_2 e^{-2E_\pi t_\pi} + \cdots$$

(For $I = 0$ there is also a constant term A_0 on the right-hand side.)

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

(For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.)

- Requiring that the $E^0_{\pi\pi} = m_K \;\Rightarrow$ the volume must be tuned accordingly.
  non-trivial

- Moreover - since the two-pion potential is attractive in the $I = 0$ channel and repulsive in the $I = 2$ channel, on a given volume

$$E^{0,\,I=0}_{\pi\pi} < E^{0,\,I=2}_{\pi\pi}$$

  and the tuning has to be done separately in each channel.

- For the evaluation of A_2, it is sufficient to impose antiperiodic boundary conditions for the d-quark.
  - Isospin breaking by the boundary conditions is harmless here.

  CTS & G.Villadoro, hep-lat/0411033

- For A_0 this is not possible and we have had to develop the implementation of G-parity boundary conditions in which $(u, d) \to (\bar{d}, -\bar{u})$ at the boundary .

  U. Wiese, Nucl.Phys. B375 (1992) 45 , RBC-UKQCD, C.h.Kim hep-lat/0311003
  - This has been the key development making the calculation of A_0 possible.

- Computations were performed on a $32^3 \times 64$ lattice with the Iwasaki and DSDR gauge action and $N_f = 2+1$ flavours of Möbius Domain Wall Fermions.

$$a^{-1} = 1.379(7)\,\text{GeV},\ m_\pi = 143.2(2.0)\,\text{MeV}, (E_\pi = 274.8(1.4)\,\text{MeV})$$

- The $\pi\pi$ energies are

$$E_{\pi\pi}^{I=0} = (498 \pm 11)\,\text{MeV} \quad E_{\pi\pi}^{I=2} = (565.7 \pm 1.0)\,\text{MeV}$$

to be compared with $m_K = (490.6 \pm 2.4)$ MeV.

- Lüscher's quantisation condition $\Rightarrow E_{\pi\pi}^{I=0}$ corresponds to

$$\delta_0(m_K) = (23.8 \pm 4.9 \pm 1.2)^\circ\,,$$

which is somewhat smaller than phenomenological expectations.

  - For $I=2$ are results are in line with expectations. arXiv:1502.00263

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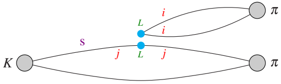
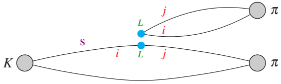
- Re A_2 is dominated by a simple operator:

$$O_{(27,1)}^{3/2} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L \, (\bar{u}^j d^j)_L$$

and two diagrams:

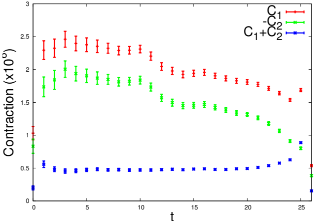


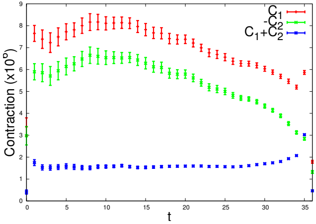
C_1



C_2

- Re A_2 is proportional to $C_1 + C_2$.
- The two dominant contributions to A_2 have opposite signs $\Rightarrow$ significant cancellation $\Rightarrow$ major contribution to the $\Delta I = 1/2$ rule. arXiv:1212.1474
This is confirmed on our latest computation of A_2 arXiv:1502.00263





1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

   "K to $\pi\pi$ decay amplitudes from lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

   "Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"

   Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

   "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm

   Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

   "Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2"

   Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

   "Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"

   P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D. Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3. A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
   "$K \to \pi\pi\ \Delta I = 3/2$ decay amplitude in the continuum limit,"
   T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
   Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].

4. A_0 at physical kinematics and a single coarse lattice spacing.
   "Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
   Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
   Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

3. Isospin breaking effects

- "Standard" QCD calculations have been performed in the isospin limit, i.e. with $m_u = m_d$, so to improve the precision still further isospin breaking effects (including electromagnetism) need to be included.
- These are

$$O\left(\frac{m_u - m_d}{\Lambda_{\rm QCD}}\right) \quad \text{and} \quad O(\alpha)\,,$$

  i.e. $O(1\%)$ or so.
- Such calculations for the spectrum have been performed for a few years now, with perhaps the most noteworthy result being BMW Collaboration, arXiv:1406.4088

$$m_n - m_p = 1.51(16)(23)\,\text{MeV}$$

  to be compared to the experimental value of 1.2933322(4) MeV.
- I stress that including electromagnetic effects, where the photon is massless of course, required considerable theoretical progress, e.g.

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\cdots \Rightarrow \frac{1}{L^3T}\sum_k \frac{1}{k^2}\cdots$$

  and we have to control the contribution of the zero mode in the sum.

3. Isospin breaking effects (cont).

- Calculating electromagnetic corrections to decay amplitudes has an added major complication, not present in computations of the spectrum,

the presence of infrared divergences

- This implies that when studying weak decays, such as e.g. $K^+ \to \ell^+\nu$ the physical observable must include soft photons in the final state

$$\Gamma(K^+ \to \ell^+\nu_\ell(\gamma)) = \Gamma(K^+ \to \ell^+\nu_\ell) + \Gamma(K^+ \to \ell^+\nu_\ell\gamma)\,.$$

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- Last year we proposed a method for including electromagnetic corrections in decay amplitudes and are developing it further as well as testing it numerically.

N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino & M.Testa, arXiv:1502.00257

- I stress (and will explain) that in order to implement this method successfully, it will be be necessary to work with the experimental community to ensure that we are calculating quantities which correspond to the experimental measurements.

- For illustration, I consider consider leptonic decays of the pion but the discussion is general.
  - The discussion applies to the leptonic and semileptonic decays of all pseudoscalar mesons and can be readily adapted to other processes.
  - We do not use ChPT. For a ChPT based discussion of f_π, see

J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479

- At $O(\alpha^0)$

$$\Gamma(\pi^+ \to \ell^+\nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2 .$$

  - All the hadronic effects are contained in the leptonic decay constant f_π.

$$\langle 0\,|\,\bar d\gamma^\mu\gamma^5\,u\,|\,\pi^+(p)\rangle = ip^\mu f_\pi\,.$$

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- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+\nu_\ell) + \Gamma(\pi^+ \to \ell^+\nu_\ell\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

  where the suffix denotes the number of photons in the final state.

- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

  F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for our community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+\nu_\ell) + \Gamma(\pi^+ \to \ell^+\nu_\ell\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\end{aligned}$$

- In principle, particularly as techniques and resources improve in the future, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.
- At present we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
    - For pions and kaons at least, a cut-off ΔE of $O(10\text{ - }20\,\text{MeV})$ appears to be appropriate both experimentally and theoretically.

      F.Ambrosino et al., KLOE collaboration, hep-ex/0509045. arXiv:0907.3594
    - Question: What is the best way to translate the photon energy and angular resolutions at LHC, Belle II etc. into the rest frame of the decaying mesons?

- We now write

$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))\,.$$

  - pt stands for *point-like*.
  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated non-perturbatively and $\Gamma_0^{\rm pt}$ in perturbation theory.

- Finite-volume effects take the form:

$$\Gamma_0^{\rm pt}(L) = C_0(r_\ell) + \tilde{C}_0(r_\ell)\log(m_\pi L) + \frac{C_1(r_\ell)}{m_\pi L} + \dots\,,$$

  where $r_\ell = m_\ell/m_\pi$ and m_ℓ is the mass of the final-state charged lepton.

  The exhibited terms are *universal*, i.e. independent of the structure of the meson!

  - We have calculated the coefficients (using the $\mathrm{QED_L}$ regulator of the zero mode).

- The leading structure-dependent FV effects in $\Gamma_0 - \Gamma_0^{\rm pt}$ are of $O(1/L^2)$.

V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo, arXiv:1611.08497

- Writing

$$\Gamma_0^{\rm pt}(L) = \Gamma_0^{\rm tree}\left\{1 + 2\frac{\alpha}{4\pi}\,Y(L)\right\}\,,$$

we find

$$\begin{aligned} Y(L) \;&=\; \left(1+r_\ell^2\right)\left[2(K_{31}+K_{32}) + \frac{\left(\gamma_E + \log\left[\frac{L^2 m_\pi^2}{4\pi}\right]\right)\log\left[r_\ell^2\right]}{(1-r_\ell^2)} + \frac{\log^2\left[r_\ell^2\right]}{2\,(1-r_\ell^2)}\right] + \\ &\quad + \frac{(1-3\,r_\ell^2)\,\log\left[r_\ell^2\right]}{(1-r_\ell^2)} - \log\left[\frac{M_W^2}{m_\pi^2}\right] + \log[m_\pi^2 L^2] - \frac{1}{2}K_P + \frac{1}{12} + \\ &\quad + \frac{1}{m_\pi L}\left(\frac{2r_\ell^2}{1-r_\ell^2}\left(K_{21}+K_{22}-2\pi\left(\frac{1}{1+r_\ell^2}+\frac{1}{r_\ell}\right)\right) - \frac{\pi(1+r_\ell^2)}{(1-r_\ell^2)}\,(K_{11}+K_{12}-3)\right)\,, \end{aligned}$$

where $r_\ell = m_\ell/m_\pi$ and the $K's$ are constants ($K_{31}+K_{32}$ depends on the direction of $\vec{p}_\ell$).

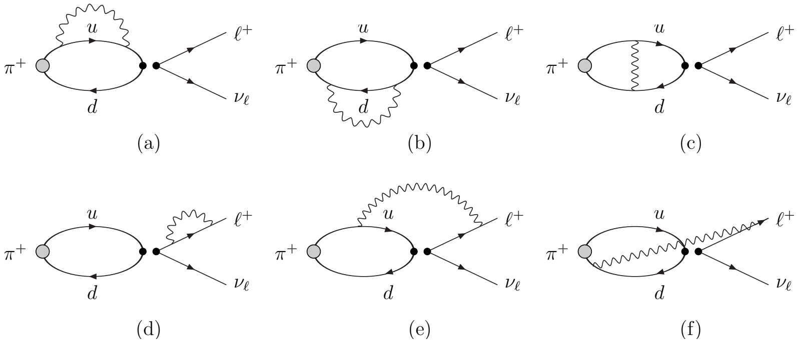
- For the spectrum

$$m_\pi(L) = m_\pi\left[1 - q^2\alpha\left(\frac{\kappa}{m_\pi L}\left(1+\frac{2}{m_\pi L}\right)\right) + O\left(\frac{1}{(m_\pi L)^3}\right)\right]$$

where $\kappa = 1.41865$ is a universal constant and the structure dependent terms start at $O(1/L^3)$.

S.Borsanyi et al., arXiv:1406.4088

- I don't have time to discuss many other theoretical and technical issues:
  - What is G_F at $O(\alpha)$?
  - The calculation of $\Gamma_0^{\rm pt} + \Gamma_1(\Delta E)$
  - Estimates of structure-dependent contributions to $\Gamma_1(\Delta E)$.
  - The efficient evaluation of many diagrams including



- Results from an exploratory study were presented at Lattice2016 in July.

  V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino, arXiv:1610.09668

4. Conclusions

- For "standard" quantities such as f_K/f_π, B_K or $f^+(0)$, the precision of lattice calculations is now $O(1\%)$ or better. FLAG collaboration, arXiv:1607.00299
- In $K\to\pi\pi$ decays
  - as a result of our work, the computation of A_2 is now fully controlled (and becoming “standard");
  - the $\Delta I = 1/2$ rule has a number of components, of which the significant cancelation between the two dominant contributions to ReA_2 is a major one.
  - We have completed the first calculation of ϵ'/ϵ with controlled errors $\Rightarrow$ motivation for further refinement (systematic improvement by collecting more statistics, working on larger volumes, $\geq$2 lattice spacings etc.)
  - ϵ'/ϵ is now a quantity which is amenable to lattice computations.
- To push *precision flavour physics* still further requires control of IB effects, including electromagnetic corrections.
  - A number of groups are studying the IB effects in the spectrum.
  - We have proposed a strategy for computing electromagnetic corrections to decay amplitudes and calculated the universal FV corrections up to and including $O(1/L)$. (At $O(1/L^2)$ the corrections are structure dependent.)

4. Conclusions (cont.)

I briefly also mention another important aspect of RBC-UKQCD's research program:

- Long-distance contributions to flavour changing processes

$$\iint d^4x\, d^4y\, \langle f \,|\, T[Q_1(x)\, Q_2(y)] \,|\, i \rangle\,.$$

Applications include:

(a) Δm_K and ϵ_K. See:

N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931
Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916
Z.Bai (RBC-UKQCD), arXiv:1411.3210, arXiv:1611.06601

(b) The rare kaon decays $K \to \pi \ell^+ \ell^-$, $K^+ \to \pi^+ \nu\bar{\nu}$. See:

N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1507.03094, arXiv:1605.04442
N.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS, arXiv:1608.07585

Lattice QCD is an indispensable element of precision flavour physics!