The Grimus-Neufeld model with todays measurements

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We present the constraints between the neutrino sector and the two-Higgs-doublet sector of the model introduced by W. Grimus and H. Neufeld [1] coming from the prediction of the mass of the second lightest neutrino by loop corrections that are dominated by the Higgs bosons in the loop. Solving for the mass squared differences of the neutrino we determine the Yukawa couplings of the second Higgs doublet to the gauge singlet that drives the seesaw mechanism using the neutrino mixing matrix as input. The predictive power of the model then lies in pocesses that use this second Yukawa coupling.

Lagrangians of the GN model

The Higgs sector: a Two Higgs Doublet Model.

The Gauge sector: from the Standard Model (SM) gauge group $\mathcal{G}_{\rm SM} = U(1)_{\rm Y} \otimes SU(2)_{\rm L} \otimes SU(3)_{\rm color}$.
In the following, $SU(3)_{\rm color}$ will be ignored.

The Gauge-Higgs sector puts the Higgs fields into the fundamental representation of the gauge groups $U(1)_{\rm Y}$ and $SU(2)_{\rm L}$.

$$\mathcal{L}_{\rm GH} = -\tfrac14 B_{\mu\nu}B^{\mu\nu} - \tfrac14 W^j_{\mu\nu}W^{j\,\mu\nu} + (D^\mu\phi_a)^\dagger(D_\mu\phi_a) - V \ , \quad (1)$$

with the Higgs potential

$$V = Y_{a\bar b}\phi^\dagger_{\bar a}\phi_b + \tfrac12 Z_{a\bar b c\bar d}(\phi^\dagger_{\bar a}\phi_b)(\phi^\dagger_{\bar c}\phi_d) \ , \quad (2)$$

is invariant under the gauge symmetries of $U(1)_{\rm Y}$ and $SU(2)_{\rm L}$, and the Higgs doublet symmetry [2]

$$\phi_a \to \phi'_a = U_{a\bar b}\phi_b \quad \text{with} \quad U_{a\bar b} \in U(2) \ , \quad (3)$$

and triggers spontaneous symmetry breaking.

The Fermion-Gauge sector puts the fermion fields into the fundamental representation of $\mathcal{G}_{\rm SM}$.

$$\mathcal{L}_{\rm GF} = \sum_\psi \bar\psi\, i\not{D}\, \psi \quad (4)$$

The gauge couplings define the **flavour states**:

$$q^0_L = \begin{pmatrix} u^0_L \\ d^0_L \end{pmatrix} , \ u^0_R , \ d^0_R , \ \ell^0_L = \begin{pmatrix} \nu^0_L \\ e^0_L \end{pmatrix} , \ e^0_R , \text{ and } N^0_R , \quad (5)$$

where N^0_R is a gauge singlet Majorana fermion.

The Fermion-Higgs sector defines Yukawa couplings, provided the Higgs doublet symmetry, eq. (3), is fixed.

With the definition of the adjoint doublet

$$\tilde\phi_{\bar a} = \epsilon\phi^*_a =: \begin{pmatrix} \phi^{0*}_{\bar a} & -\phi^-_{\bar a} \end{pmatrix}^\top \quad (6)$$

we define (ignoring the quarks)

$$\mathcal{L}_{\rm FH} = -\bar\ell^0_{Lj}\phi_a Y^{\bar a}_{Ljk} e^0_{Rk} - \bar\ell^0_{Lj}\tilde\phi_{\bar a}\tilde Y^a_{Lj} N^0_R + h.c. \ . \quad (7)$$

In the **Higgs basis** [2]

$$\phi_1 = \frac{v}{\sqrt2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + H_1 = \frac{1}{\sqrt2}\begin{pmatrix} G^+_r + iG^+_i \\ v + H_{1r} + iG^0 \end{pmatrix} \quad (8)$$

$$\phi_2 = H_2 = \frac{1}{\sqrt2}\begin{pmatrix} H^+_r + iH^+_i \\ H_{2r} + iH_{2i} \end{pmatrix} , \quad (9)$$

we get the Yukawa couplings

$$(Y^{(a)}_E)_{jk} := Y^{\bar a}_{Ljk} \quad \text{and} \quad (Y^{(a)}_N)_k := \tilde Y^a_{Lk} \ . \quad (10)$$

Diagonalizing the charged leptons

$$\tfrac{v}{\sqrt2}(Y^{(1)}_E)_{jk} \to {\rm diag}\{m_e, m_\mu, m_\tau\} \quad (11)$$

defines the flavour states of the neutrinos $\{\nu_e, \nu_\mu, \nu_\tau\}$.

Only the diagonalization of the non-charged fermions will define the correct neutrino mixing matrix.

The Fermion-Majorana sector gives the gauge singlet N^0_R a Majorana mass and allows the Seesaw mechanism.

Majorana and chiral fermions

The GN model couples Majorana fermions only with chiral fermions: both can be written as Weyl spinors; for a recent writeup see [3].

Since the GN model incorporates only a scalar coupling between the left-chiral fermions and the Majorana gauge singlet, it makes sense to specify the Majorana fermion over its **right-chiral** component.

Then we get the mass terms for the Majorana fermion, written in Dirac notation,

$$\begin{aligned}\mathcal{L}_{\rm M} &= -\tfrac12 M^*_R \bar N^0 P_L \widehat N^0 - \tfrac12 M_R \overline{\widehat N^0} P_R N^0 \\ &= -\tfrac12 |M_R| (\bar N^0 P_L N^0 + \bar N^0 P_R N^0) \ , \quad (12)\end{aligned}$$

with the Majorana spinor N^0 obeying the Majorana condition

$$N^0 = \eta_N \widehat N^0 \ , \quad (13)$$

with the Majorana phase $\eta_N = M^*_R/|M_R|$, and

$$\widehat N^0 = \gamma^0 \mathbf{C} N^{0*} \ , \quad (14)$$

denotes the Lorentz covariant conjugate of N^0.

PMNS matrix

The neutrino mixing matrix, or PMNS matrix, is defined by the coupling to the $W^\pm$, the charged current coupling:

$$\mathcal{L}_{\rm cc} = -\frac{g}{\sqrt2} W^-_\mu V^{\rm PMNS}_{ka} \bar\psi_{ek}\gamma^\mu P_L \zeta_a + h.c. \ , \quad (15)$$

where ζ_a describe the physical neutrino mass eigenstates.

The Seesaw @ tree-level

The Seesaw mechanism mixes left-chiral neutral leptons with the Majorana gauge singlet. Therefore it is **convenient** to formulate it using **Weyl spinors** [3].
We get the mass matrix for the right chiral Weyl spinors

$$\mathcal{M}_{3+1} = \begin{pmatrix} 0 & \frac{v}{\sqrt2}(Y^{(1)}_N) \\ \frac{v}{\sqrt2}(Y^{(1)}_N)^\top & M_R \end{pmatrix} \quad (16)$$

which is diagonalized by a unitary matrix $\mathcal{U}$:

$$\mathcal{U}^\top \mathcal{M}_{3+1}\mathcal{U} = {\rm diag}\{m_1, m_2, m_3, m_4\} \ . \quad (17)$$

Since $\mathcal{M}_{3+1}$ has rank 2, only two masses are bigger than zero:

$$m_4 > m_3 > m_2 = m_1 = 0 \ . \quad (18)$$

Rewriting eq. (17) linear in $\mathcal{U}$, we get $[\mathcal{M}_{3+1}\mathcal{U}]_{ka}$

$$= (\mathcal{M}_{3+1})_{kj}\mathcal{U}_{ja} = \begin{pmatrix} 0 & 0 & \mathcal{U}^*_{e3}m_3 & \mathcal{U}^*_{e4}m_4 \\ 0 & 0 & \mathcal{U}^*_{\mu3}m_3 & \mathcal{U}^*_{\mu4}m_4 \\ 0 & 0 & \mathcal{U}^*_{\tau3}m_3 & \mathcal{U}^*_{\tau4}m_4 \\ 0 & 0 & \mathcal{U}^*_{s3}m_3 & \mathcal{U}^*_{s4}m_4 \end{pmatrix} , \quad (19)$$

giving

$$\mathcal{U}_{s1} = \mathcal{U}_{s2} = Y^{(1)}_{Nk}\mathcal{U}_{k1} = Y^{(1)}_{Nk}\mathcal{U}_{k2} = 0 \ , \quad (20)$$

indicating, that we can parametrize the mixing matrix as

$$\mathcal{U}_{ka} = \begin{pmatrix} \vec u_1 & \vec u_2 & \vec u'_3 & \vec u'_4 \\ 0 & 0 & \mathcal{U}_{s3} & \mathcal{U}_{s4} \end{pmatrix}_{ka} , \quad (21)$$

with $\vec u_k = \vec u'_k/|\vec u'_k|$ normalized. Since $\mathcal{U}$ is unitary, we have

$$\vec u^\dagger_1\vec u_3 = \vec u^\dagger_2\vec u_3 = \vec u^\dagger_1\vec u_4 = \vec u^\dagger_2\vec u_4 = 0 \ , \quad (22)$$

and

$$\tfrac{v}{\sqrt2}\vec Y^{(1)}_N\mathcal{U}_{s3} = \vec u'^*_3 m_3 \quad \text{and} \quad \tfrac{v}{\sqrt2}\vec Y^{(1)}_N\mathcal{U}_{s4} = \vec u'^*_4 m_4 \ , \quad (23)$$

giving $\vec u'_4 = \frac{m_3}{m_4}\frac{\mathcal{U}_{s4}}{\mathcal{U}_{s3}}\vec u'_3 \ \Rightarrow \ \vec u_4 = \vec u_3$. The normalization

$$\begin{aligned} s^2 &:= |\mathcal{U}_{s3}|^2 = 1 - |\mathcal{U}_{s4}|^2 = |\vec u'_4|^2 = \tfrac{m_3^2}{m_4^2}\tfrac{1-s^2}{s^2}|\vec u'_3|^2 \\ &= \tfrac{m_3^2}{m_4^2}\tfrac{1-s^2}{s^2}(1 - |\mathcal{U}_{s3}|^2) = \tfrac{m_3^2}{m_4^2}\tfrac{(1-s^2)^2}{s^2} \quad (24)\end{aligned}$$

gives the seesaw angle

$$\tfrac{s^2}{1-s^2} := \tfrac{s^2}{c^2} = \tfrac{m_3}{m_4} \ , \quad (25)$$

and with the definition

$$\tfrac{v^2}{2}(Y^{(1)}_N)^\top(Y^{(1)}_N)^* = \tfrac{v^2}{2}(Y^{(1)}_N)^\top(Y^{(1)}_N)^* := m_D^2 \ , \quad (26)$$

and the identification, eq. (23), the seesaw relation:

$$m_D^2 = \left|\tfrac{v}{\sqrt2}\vec Y^{(1)}_N\right|^2 = \tfrac{|\vec u'_3|^2}{|\mathcal{U}_{s3}|^2}m_3^2 = \tfrac{c^2}{s^2}m_3^2 = m_4 m_3 \ . \quad (27)$$

The 34-element of eq. (19)

$$\tfrac{v}{\sqrt2}(Y^{(1)}_N)^\top\vec u_3 + M_R\mathcal{U}_{s3} = \tfrac{m_3}{\mathcal{U}_{s3}}|\vec u_3|^2 + M_R\mathcal{U}_{s3} = \mathcal{U}^*_{s3}m_3 \ , \quad (28)$$

gives us the phase of $\mathcal{U}_{s3} =: \eta_3|\mathcal{U}_{s3}| = \eta_3 s$ from

$$0 = \tfrac{m_3}{\eta_3 s}c^2 + M_R\eta_3 s - \tfrac{s}{\eta_3}m_3 = \tfrac{s}{\eta_3}\left[m_3(\tfrac{m_4}{m_3} - 1) + M_R\eta_3^2\right] \ , \quad (29)$$

as $\eta_3 = \pm i$, since $m_4 > m_3$.

Tree level eigenstates

$$\mathcal{U} = \begin{pmatrix} (\vec u_1, \vec u_2, \vec u_3) & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & \begin{matrix} ic & s \\ -is & c \end{matrix} \end{pmatrix} \quad (30)$$

from eq. (21), containing the three independent and orthonormal 3-vectors $\vec u_k$, diagonalizes the seesaw mass matrix $\mathcal{M}_{3+1}$, eq. (16), and relates the flavour states $\{\nu^0_{Lk}, N^0_R\}$ with the tree level mass eigenstates ζ^0_a:

$$\begin{aligned} \nu^0_{Le} &= (\vec u_1)_e\zeta^0_1 + (\vec u_2)_e\zeta^0_2 + (\vec u_3)_e(ic\zeta^0_3 + s\zeta^0_4) \quad (31)\\ \nu^0_{L\mu} &= (\vec u_1)_\mu\zeta^0_1 + (\vec u_2)_\mu\zeta^0_2 + (\vec u_3)_\mu(ic\zeta^0_3 + s\zeta^0_4) \quad (32)\\ \nu^0_{L\tau} &= (\vec u_1)_\tau\zeta^0_1 + (\vec u_2)_\tau\zeta^0_2 + (\vec u_3)_\tau(ic\zeta^0_3 + s\zeta^0_4) \quad (33)\\ N^0_R &= -is\zeta^0_3 + c\zeta^0_4 \ . \quad (34)\end{aligned}$$

Tree-level Yukawas

The same three independent and orthonormal 3-vectors $\vec u_k$, appearing in eq. (21), can be used to define the Yukawa couplings of the GN model. We already defined the first Yukawa in eq. (23):

$$\vec Y^{(1)}_N = \frac{\sqrt2}{v}\frac{|\vec u'_3|m_3}{\mathcal{U}_{s3}}\vec u^*_3 = \frac{\sqrt2 m_D}{v}\vec u^*_3 \ . \quad (35)$$

Requiring, that the lightest neutrino should not couple to any Higgs we can parametrize the second Yukawa coupling

$$\vec Y^{(2)}_N = d\vec u^*_2 + d'\vec u^*_3 \ . \quad (36)$$

Dominant one loop corrections

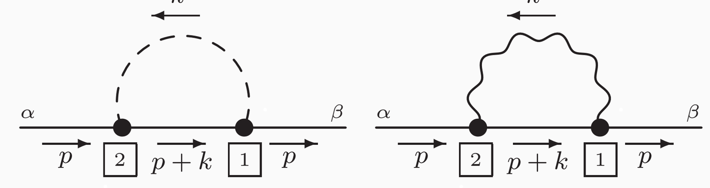
Abbreviating $M_D := \frac{v}{\sqrt2}Y^{(1)}_N$, Grimus and Lavoura (G–L) [4] construct the effective neutrino mass matrix

$$\mathcal{M}^{\rm tree}_\nu = -M_D\, M_R^{-1}\, M_D^\top \ , \quad (37)$$

from the mass matrix, eq. (16), and calculate the correction

$$\begin{aligned}\mathcal{M}_\nu = \ &\mathcal{M}^{\rm tree}_\nu + \delta M_L - \delta M_D\, M_R^{-1}\, M_D^\top - M_D\, M_R^{-1}\, \delta M_D^\top \\ &+ M_D\, M_R^{-1}\, \delta M_R\, M_R^{-1}\, M_D^\top \ . \quad (38)\end{aligned}$$

G–L argue that the dominant contribution comes from δM_L, which does not have a counterterm, as it vanishes at tree level. As a result from calculating the loops



they get the finite and simplified contribution

$$\delta M_L = \tfrac{3m_Z^2}{32\pi^2 m_4}\vec Y^{(1)}_N \ln\tfrac{m_4^2}{m_Z^2}\vec Y^{(1)\top}_N + \sum_{b=1}^{3}\tfrac{m_b^2}{32\pi^2 m_4}\vec\Delta_b \ln\tfrac{m_4^2}{m_b^2}\vec\Delta^\top_b \ , \quad (39)$$

with the coupling

$$\begin{aligned}\vec\Delta_b &= q_{b2}\vec Y^{(2)}_N - iq_{b1}\vec Y^{(1)}_N \\ &= q_{b2}d\vec u^*_2 + (q_{b2}d' - iq_{b1}\tfrac{\sqrt2 m_D}{v})\vec u^*_3 \ , \quad (40)\end{aligned}$$

where q_{bk} describes the mixing of the neutral Higgses [2]:

$$h_b = q_{b1}H_{1r} + \mathcal{R}{\rm e}[q_{b2}]H_{2r} + \mathcal{I}{\rm m}[q_{b2}]H_{2i} \ . \quad (41)$$

This gives a loop level mass matrix

$$\begin{aligned}\mathcal{M}^{\rm loop}_\nu &= \delta M_L - M_D\, M_R^{-1}\, M_D^\top \\ &= \vec u^*_2\vec u^\dagger_2 A + (\vec u^*_2\vec u^\dagger_3 + \vec u^*_3\vec u^\dagger_2)B + \vec u^*_3\vec u^\dagger_3 C \ , \quad (42)\end{aligned}$$

with

$$\begin{aligned} A &= d^2 f_1 \ , \quad B = d\,d' f_1 + id\tfrac{m_D}{v}f_2 \ , \text{ and} \\ C &= d'^2 f_1 + 2id'\tfrac{m_D}{v}f_2 + \tfrac{m_D^2}{v^2}f_3 \ . \quad (43)\end{aligned}$$

The functions f_i are

$$\begin{aligned} f_1 &= \tfrac{1}{32\pi^2}\left[s^2_{12}\tfrac{m_h^2}{m_4}\ln\tfrac{m_4^2}{m_h^2} + c^2_{12}\tfrac{m_H^2}{m_4}\ln\tfrac{m_4^2}{m_H^2} - \tfrac{m_A^2}{m_4}\ln\tfrac{m_4^2}{m_A^2}\right] \quad (44)\\ f_2 &= \tfrac{s_{12}c_{12}}{32\pi^2}\left[\tfrac{m_h^2}{m_4}\ln\tfrac{m_4^2}{m_h^2} - \tfrac{m_H^2}{m_4}\ln\tfrac{m_4^2}{m_H^2}\right] \quad (45)\\ f_3 &= \tfrac{1}{16\pi^2}\left[\tfrac{3m_Z^2}{m_4}\ln\tfrac{m_4^2}{m_Z^2} - \tfrac{c^2_{12}m_h^2}{m_4}\ln\tfrac{m_4^2}{m_h^2} - \tfrac{s^2_{12}m_H^2}{m_4}\ln\tfrac{m_4^2}{m_H^2} - \tfrac{v^2}{m_4}\right] \quad (46)\end{aligned}$$

Mixing of the tree level states

The loop level mass matrix, eq. (42) introduces a mixing between the states ζ^0_2 and ζ^0_3. Using the tree level defined $\vec u_k$ from eq. (21), reduces the singular value problem

$$U^\top\mathcal{M}^{\rm loop}_\nu U = {\rm diag}\{0, \tilde m_2, \tilde m_3\} \ , \quad (47)$$

($\tilde m_k$ being 1-loop masses), with $U = \begin{pmatrix} \vec u_1 & \vec u_2 & \vec u_3 \end{pmatrix}\cdot R$ to

$$R^\top\begin{pmatrix} A & B \\ B & C \end{pmatrix}R = \begin{pmatrix} \tilde m_2 & 0 \\ 0 & \tilde m_3 \end{pmatrix} \ . \quad (48)$$

We parametrize

$$R = e^{\frac{i}{2}\alpha}\begin{pmatrix} e^{\frac{i}{2}(\gamma+\delta)}\cos\vartheta & e^{\frac{i}{2}(\gamma-\delta)}\sin\vartheta \\ -e^{-\frac{i}{2}(\gamma-\delta)}\sin\vartheta & e^{-\frac{i}{2}(\gamma+\delta)}\cos\vartheta \end{pmatrix} \ . \quad (49)$$

ϑ has to be determined from the hermitian equation

$$U^\dagger XU := U^\dagger\mathcal{M}^{\rm loop\dagger}_\nu\mathcal{M}^{\rm loop}_\nu U = {\rm diag}\{0, \tilde m_2^2, \tilde m_3^2\} \ , \quad (50)$$

giving the masses

$$\tilde m^2_{2,3} = \tfrac12{\rm Tr}[X] \pm \sqrt{\tfrac14{\rm Tr}[X]^2 - \det[X]} \ . \quad (51)$$

We find

$$t^2_\vartheta := \tan^2\vartheta = \frac{\tilde m_3^2 - |B|^2 - |C|^2}{|B|^2 + |C|^2 - \tilde m_2^2} \ , \quad (52)$$

which we can use to determine

$$\begin{aligned} e^{2i\alpha} &= \tfrac{\tilde m_2\tilde m_3}{AC - B^2} \quad (53)\\ (e^{i\gamma})_{1,2} &= \tfrac{(t^2_\vartheta-1)B \pm\sqrt{4ACt^2_\vartheta + (t^2_\vartheta-1)^2B^2}}{2At_\vartheta} \quad (54)\\ (e^{i\delta})_{1,2} &= e^{i\alpha}\tfrac{(t^2_\vartheta+1)B \pm\sqrt{4ACt^2_\vartheta + (t^2_\vartheta-1)^2B^2}}{2\tilde m_3 t_\vartheta} \quad (55)\\ &= -e^{-i\alpha}\tfrac{2\tilde m_2 t_\vartheta}{(t^2_\vartheta+1)B \pm\sqrt{4ACt^2_\vartheta + (t^2_\vartheta-1)^2B^2}} \ . \quad (56)\end{aligned}$$

Loop level eigenstates

$\mathcal{U}$ from eq. (21) has to be replaced by

$$U = \begin{pmatrix} (\vec u_1, \vec u_2, \vec u_3) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \begin{matrix} ic & s \\ -is & c \end{matrix} \end{pmatrix} \quad (57)$$

from eq. (47). Redefining the vectors $\vec u_2$ and $\vec u_3$ by their mixing matrix R, eq. (49):

$$\tilde u_2 = R_{22}\vec u_2 + R_{32}\vec u_3 \qquad \tilde u_3 = R_{23}\vec u_2 + R_{33}\vec u_3 \ , \quad (58)$$

we get the loop-level eigenstates

$$\begin{aligned} \nu_{Le} &= (\vec u_1)_e\zeta_1 + (\tilde u_2)_e\zeta_2 + (\tilde u_3)_e(ic\zeta_3 + s\zeta_4) \quad (59)\\ \nu_{L\mu} &= (\vec u_1)_\mu\zeta_1 + (\tilde u_2)_\mu\zeta_2 + (\tilde u_3)_\mu(ic\zeta_3 + s\zeta_4) \quad (60)\\ \nu_{L\tau} &= (\vec u_1)_\tau\zeta_1 + (\tilde u_2)_\tau\zeta_2 + (\tilde u_3)_\tau(ic\zeta_3 + s\zeta_4) \quad (61)\\ N_R &= -is\zeta_3 + c\zeta_4 \ . \quad (62)\end{aligned}$$

Connecting parameters

We express parameters in the Lagrangian by the physical observables:

The CP conserving Higgs potential needs the masses of the Higgs bosons $m_b = \{m_h, m_H, m_A, m_{H^\pm}\}$, the mixing angle s_{12}, and the coefficients Z_2, Z_3, and Z_7, that do not contribute to the mass matrix [2].

The Gauge-Higgs sector fixes the gauge couplings and the vacuum expectations value through the masses of the gauge bosons.

The Fermion-Higgs sector defines the Yukawa couplings, and gives as physical observables the masses of the charged fermions and the mixing matrices $V_{\rm CKM}$ and $V_{\rm PMNS}$.

The Fermion-Majorana sector has the Majorana mass M_R.

Neutrino oscillations tell us $\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm atm}$, and the mixing matrix $V_{\rm PMNS}$, eq. (15). Since $\tilde m_1 = 0$, the masses $\tilde m_2$ and $\tilde m_3$ are directly given by the differences.

Using the loop corrected neutrino mass matrix with eqs. (47) and (50) we can use the masses $\tilde m_2$ and $\tilde m_3$, obtained by the differences, to replace them by two of the parameters $\{m_D, M_R, d, |d'|, \varphi = \arg[d']\}$:

- Being the heavy scale, we keep M_R as an input parameter
- We can relate m_D to the tree-level seesaw mass, eq. (27), $m^0_3 = m^2_D/M_R$.
- Using the determinant of eq. (47) we get

$$\tilde m_2\tilde m_3 = |AC - B^2| = \frac{d^2m_D^2(f_1f_3 + f_2^2)}{(32\pi^2 v)^2} \ , \quad (63)$$

  which gives uniquely

$$d = \frac{32\pi^2 v}{m_D}\sqrt{\frac{\tilde m_2\tilde m_3}{(f_1f_3 + f_2^2)}} \ . \quad (64)$$

- for $|d'|$ we get a fourth order equation from the trace of eq. (50):

$$\begin{aligned} 0 &= -\tilde m_2^2 - \tilde m_3^2 + |A|^2 + 2|B|^2 + |C|^2 \quad (65)\\ &= a_4|d'|^4 + a_3|d'|^3 + a_2|d'|^2 + a_1|d'| + a_0 \ , \end{aligned}$$

  with

$$\begin{aligned} a_0 &= d^4f_1^2 + 2d^2\tfrac{m_D^2}{v^2}f_2^2 + \tfrac{m_D^4}{v^4}f_3^2 - m_2^2 - m_3^2 \quad (66)\\ a_1 &= 4\sin\varphi\tfrac{m_D}{v}f_2[d^2f_1 - \tfrac{m_D^2}{v^2}f_3] \quad (67)\\ a_2 &= 2[d^2f_1^2 + (2f_2^2 + \cos2\varphi f_1f_3)\tfrac{m_D^2}{v^2}] \quad (68)\\ a_3 &= 4\sin\varphi\tfrac{m_D}{v}f_1f_2 \quad (69)\\ a_4 &= f_1^2 \ . \quad (70)\end{aligned}$$

  We can have up to four solutions, depending on input values, but most generally $|d'| \approx \mathcal{O}(d)$.

- $\varphi \in (-\pi, \pi]$ stays a free parameter

"Loop" Yukawas

Assuming $\tilde m_4 > m_W$ we can approximate $V_{\rm PMNS}$ in the charged current, eq. (15), by the left upper 3×3 block of the loop level mixing U_{ka}, eq. (57):

$$(\vec u_1, \tilde u_2, \tilde u_3)_{ka} = V^{\rm PMNS}_{ka} \ . \quad (71)$$

Using the unitarity of R, we can determine

$$\vec u^*_2 = R_{22}\tilde u^*_2 + R_{23}\tilde u^*_3 \qquad \vec u^*_3 = R_{32}\tilde u^*_2 + R_{33}\tilde u^*_3 \ , \quad (72)$$

getting the Yukawa couplings as:

$$\begin{aligned} \vec Y^{(1)}_N &= \tfrac{\sqrt2 m_D}{v}(R_{32}\tilde u^*_2 + R_{33}\tilde u^*_3) \quad (73)\\ \vec Y^{(2)}_N &= (dR_{22} + d'R_{32})\tilde u^*_2 \\ &\quad + (dR_{23} + d'R_{33})\tilde u^*_3 \ . \quad (74)\end{aligned}$$

Predictions

With the limit $\frac12 < \frac{m_3^{\rm tree}}{\tilde m_3} < 2$ and the seesaw scale $M_R \approx \tilde m_4$ we have only the phase φ as a free parameter for the neutrino Yukawa couplings.

$\Rightarrow$ any process governd by $\vec Y^{(i)}_N$ is tightly constrained by the GN model [5, 6, 7, 8, 9, 10, 11].

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