One-loop spectrum of the minimal calculable unified model [1]

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Neuron FUND FOR SUPPORT OF SCIENCE

Motivation

Experiment :

future ***proton decay*** searches (DUNE, Hyper-K, ...) $\longrightarrow$ $\mathcal{O}(10)$ increase in $\tau_p(p^+ \to \pi^0 e^+, K^+\bar{\nu})$ sensitivity

Theory :

proton lifetime estimates plagued by large ***uncertainties***

Matching experimental efforts requires improving the theoretical predictions for τ_p by *NLO* calculation

Theoretical uncertainties

Main sources of uncertainties in τ_p estimates in GUTs

- ***hadronic matrix elements*** $\langle \pi^{+,0}, K^{+,0}, \eta | \dots | p^+ \rangle$
  lattice/χPT $\longrightarrow$ $\Delta \sim \mathcal{O}(20-40\%)$
- ***flavour structure*** *of the BLV currents*
  - ∗ partial widths $\Gamma(p^+ \to \text{final state})$ in general depend on unknown *rotation matrices*
  - ∗ *Planck scale effects* :
    $\Delta \sim \mathcal{O}(1\%)$ inaccuracy in Yukawa fits due to *shifts in* $d=4$ *matching* $\longrightarrow$ potentially large changes in the *mixing angles* and *BLV currents*
- ***effective mediator mass*** *"M_{GUT}" determination*:
  in non-SUSY predominantly ***heavy gauge bosons*** :
  $(3,2,-\frac{5}{6})$ or $(3,2,+\frac{1}{6})$ $\longrightarrow$ $\tau_p \propto \alpha_{GUT}^{-1} \frac{M_{GUT}^4}{m_p^5}$

  @ NLO:
  - ∗ running of α_i ***@ 2-loop*** $(\Delta\alpha_i^{exp}(m_Z))$
  - ∗ *threshold effects* ***@ 1-loop*** (scalar spectrum)
  - ∗ *Planck scale effects*: $d=5$ gauge kinetic form operators cause uncontrolled & inhomogeneous *shifts in matching* $\longrightarrow$ $\Delta M_{GUT} \sim \mathcal{O}(10^3)$

The minimal calculable model

SO(10) GUT *spontaneously* broken by scalar **45** :

leading Planck-scale operators *absent* (group theory)

$$\mathcal{L} \propto \frac{\langle 45 \rangle}{M_P} F^{\mu\nu} F_{\mu\nu} = 0$$

$\omega_\mathbf{b}, \omega_\mathbf{r} \subset \langle 45 \rangle$ breaks ***SO(10)*** , $\chi \subset \langle 16 \rangle$ or $\sigma \subset \langle 126 \rangle$ reduces ***rank***

- renormalizable SUSY $\mathbf{45} \oplus \mathbf{16}$:
  $\langle 45 \rangle \propto \langle 16 \rangle = \mathbf{1}_{SU(5)}$ $\longrightarrow$ no breaking to SM ✗
- renormalizable non-SUSY $\mathbf{45} \oplus \mathbf{16}$ [2]:
  $m_{\nu_R} \sim 10^4$ GeV, non-renormalizable Yukawas ✗
- renormalizable non-SUSY $\mathbf{45} \oplus \mathbf{126}$ [3]:
  @ tree level : **RULED OUT** ✗
  tachyonic spectrum or ***flipped SU(5)*** *vacuum* [4, 5, 6]

$$\left.\begin{array}{l} m^2_{(8,1,0)_{\mathbf{45}}} \propto (\omega_\mathbf{b} - \omega_\mathbf{r})(2\omega_\mathbf{b} + \omega_\mathbf{r}) > \mathbf{0} \\ m^2_{(1,3,0)_{\mathbf{45}}} \propto (\omega_\mathbf{r} - \omega_\mathbf{b})(\omega_\mathbf{b} + 2\omega_\mathbf{r}) > \mathbf{0} \end{array}\right\} \to -2 < \frac{\omega_\mathbf{b}}{\omega_\mathbf{r}} < -\frac{1}{2}$$

@ 1-loop : **QUANTUM SALVATION** ??

potentially ***realistic*** *spectrum* (non-tachyonic, perturbative, seesaw scale σ) in the region where ***1-loop radiative corrections*** to masses *dominate*

Effective potential approach

Expansion of ***Coleman-Weinberg effective potential*** [7] and VEVs in number of loops ($\hbar$):

$$V = V_0 + \frac{\hbar}{64\pi^2}\mathrm{Tr}\left[\mathbf{M_S}^4(\phi)\left(\log\frac{\mathbf{M_S}^2(\phi)}{\mu_r^2} - \frac{3}{2}\right) + 3\mathbf{M_G}^4(\phi)\left(\log\frac{\mathbf{M_G}^2(\phi)}{\mu_r^2} - \frac{5}{6}\right)\right] + \mathcal{O}(\hbar^2)$$

Stationary conditions for the scalar potential determine the ***vacuum state***:

$$0 = \partial_a V|_v \quad \to \quad v = v_0 + \hbar\, v_1 + \mathcal{O}(\hbar^2)$$

1-loop scalar masses correspond to the *eigenvalues* of the matrix (in the vacuum state):

$$\mathbf{M}^2_{ab} \equiv \partial_a\partial_b V|_v = \partial_a\partial_b V_0|_{v_0+\hbar v_1} + \hbar\,\partial_a\partial_b V_1|_{v_0} + \mathcal{O}(\hbar^2)$$

producing ***scalar*** & ***gauge*** loop contributions of the ***polynomial*** & ***logarithmic*** kind:

$$m^2_{x,\text{1-loop}} := m^2_{x,\text{tree}} + \Delta_x^{G[\text{poly}]} + \Delta_x^{G[\log]} + \Delta_x^{S_{FIN}[\text{poly}]} + \Delta_x^{S_{INF}[\text{poly}]} + \Delta_x^{S[\log]}$$

Pseudo-Goldstone boson masses: various CHECKS

- **exact Goldstones** $(3,2,-\frac{5}{6})_{\mathbf{45}} \oplus (\bar{3},2,+\frac{5}{6})_{\mathbf{45}}$ remain massless even @ 1-loop ✓
- various **limits** (*spontaneous symmetry breaking patterns*):

	$\omega_\mathbf{b} \neq 0, \omega_\mathbf{r} \neq 0$	$\omega_\mathbf{b} = 0, \omega_\mathbf{r} \neq 0$	$\omega_\mathbf{b} \neq 0, \omega_\mathbf{r} = 0$	$\omega_\mathbf{b} = \omega_\mathbf{r} \neq 0$	$\omega_\mathbf{b} = -\omega_\mathbf{r} \neq 0$
$\sigma = 0$	$3_c\, 2_L\, 1_R\, 1_{B-L}$	$4_c\, 2_L\, 1_R$	$3_c\, 2_L\, 2_R\, 1_{B-L}$	$5\, 1_Z$	$5'\, 1_{Z'}$
$\sigma \neq 0$	$3_c\, 2_L\, 1_Y$	$3_c\, 2_L\, 1_Y$	$3_c\, 2_L\, 1_Y$	5	$3_c\, 2_L\, 1_Y$

$\omega_\mathbf{b} = \omega_\mathbf{r}$: $\mathbf{SU(5)}$ $\to$ $m^2_{(8,1,0)_{\mathbf{45}}} = m^2_{(1,3,0)_{\mathbf{45}}} = 0$ ($\mathbf{24}$-plet with $(3,2,-\frac{5}{6})_{\mathbf{45}}$) ✓

$\omega_\mathbf{b} = -\omega_\mathbf{r}$: **flipped** $\mathbf{SU(5)}$ $\to$ $m^2_{(8,1,0)_{\mathbf{45}}} = m^2_{(1,3,0)_{\mathbf{45}}}$ ($\mathbf{24}$-plet with $(3,2,+\frac{1}{6})_{\mathbf{45}}$) ✓

highly non-trivial check (**poly** $\leftrightarrow$ **log** in the infinite series of nested commutators)

- **diagramatic** computation of **gauge polynomial** and leading **scalar polynomial** (τ^2) contributions: $\Delta^{\tau^2[\text{poly}]}_{(8,1,0)_{\mathbf{45}}} = \Delta^{\tau^2[\text{poly}]}_{(1,3,0)_{\mathbf{45}}} = \frac{35\tau^2}{8\pi^2}$ ✓

Viable spectrum sample

- **PGBs @ 1-loop**: $\frac{m_{(8,1,0)_{\mathbf{45}}}}{\omega_b} = 0.35$, $\frac{m_{(1,3,0)_{\mathbf{45}}}}{\omega_b} = 0.27$
- **rest of the spectrum @ tree-level**: $1.4 \leq \frac{m_i}{\omega_b} \leq 3.7$ (5 Goldstones + 4 PGBs)

non-tachyonic spectrum ✓ perturbativity ✓ seesaw scale ✓

TO DO:

▷▷▷ SM singlet pseudo-Goldstones @ 1-loop: non-tachyonic?

▷▷▷ gauge coupling unification (2-loop RGEs)?

▷▷▷ phenomenologically viable point?

Conclusions

Ultimate goal: improve accuracy of τ_p estimates $\longrightarrow$ ***mediator mass @ NLO***

∗∗∗ ***minimal realistic GUT @ NLO*** : ***renormalizable non-SUSY SO(10)*** **with** $\mathbf{126} \oplus \mathbf{45}$

▷▷▷ only *perturbative* unified theory, with some classes of Planck scale operators (*gauge kinetic form*) under control $\longrightarrow$ can compute *radiative corrections* to masses and τ_p

▷▷▷ genuinely *quantum* theory: no available *tree level* description (*tachyons*) $\longrightarrow$ ***1-loop***

References

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