One-loop effective actions and higher spins

L. Bonora[a], M. Cvitan[b], P. Dominis Prester[c], S.Giaccari[b], B. Lima de Souza[a], T. Štemberga[b]

[a] International School for Advanced Studies and INFN, [b] University of Zagreb, [c] University of Rijeka

Abstract

We explore the idea that the one-loop effective action of a free massive field theory coupled to external sources (via conserved currents) contains complete information about the classical dynamics of such sources. We show several examples of this fact for (scalar and fermion) free field theories in various dimensions coupled to (bosonic) sources with a large number of spins. We concentrate on two-point correlators, so the one-loop effective action we construct contains only the quadratic terms and the relevant equations of motion for the sources we obtain are the linearized ones.

Motivation

- Better understanding of higher spin theories - we do not know the interaction nor the full form of the symmetry transformation for spin $s > 2$.

Introduction

We consider the Lagrangian for the free fermion model

$$L_0 = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

We can construct on-shell conserved currents for any spin s

$$J^{(s)}_{\mu_1\ldots\mu_n} = i^{s-1}\bar{\psi}\gamma_\mu \sum_{j=0}^{\left\lfloor\frac{s-1}{2}\right\rfloor} \frac{(u_\mu - v_\mu)^{s-2j-1}(2w\eta_{\mu\mu} - 4u_\mu v_\mu)^j}{(2j+1)!(s-2j-1)!}\psi \qquad (1)$$

where $u_\mu = \overrightarrow{\partial}_\mu, v_\mu = \overleftarrow{\partial}_\mu, \langle uv\rangle = u^\mu v_\mu$ and $w = \langle uv\rangle - m^2$. The explicit form of currents (1) is:

$$\begin{aligned}
&\text{Spin } s=1 \quad J^{(1)}{}_{\mu_1\ldots\mu_1} = \bar{\psi}\gamma_\mu\psi\\
&\text{Spin } s=2 \quad J^{(2)}{}_{\mu_1\ldots\mu_2} = i\bar{\psi}\gamma_\mu\left(u_\mu - v_\mu\right)\psi\\
&\text{Spin } s=3 \quad J^{(3)}{}_{\mu_1\ldots\mu_3} = \bar{\psi}\gamma_\mu\left(-\frac{u_\mu{}^2}{2} + \frac{5u_\mu v_\mu}{3} - \frac{v_\mu{}^2}{2} - \frac{w\eta_{\mu\mu}}{3}\right)\psi
\end{aligned}$$

The currents (1) couple minimally to external spin s fields $\varphi^{\mu_1\cdots\mu_s}$

$$S_{int} \sim \int d^dx\varphi^{\mu_1\cdots\mu_s}(x)J_{\mu_1\ldots\mu_s}(x) \qquad (2)$$

In particular

$$\begin{aligned}
&\text{Spin 1} \quad \varphi_\mu = A_\mu \quad \text{gauge field}\\
&\text{Spin 2} \quad \varphi_{\mu\nu} = h_{\mu\nu} \quad \text{graviton field}\\
&\text{Spin 3} \quad \varphi_{\mu\nu\lambda} = b_{\mu\nu\lambda} \quad \text{spin - 3 field}
\end{aligned}$$

Due to the (on-shell) current conservation, coupling (2) is invariant under (infinitesimal) gauge transformations $\delta\varphi_{\mu_1\ldots\mu_s}(x) = s\partial_{(\mu_1}\Lambda_{\mu_2\ldots\mu_s)}$.
Starting from the generating function

$$Z[\varphi] = e^{iW[\varphi]} = \int D\bar{\psi}D\psi e^{i(S_0 + S_{int}[\varphi])}$$

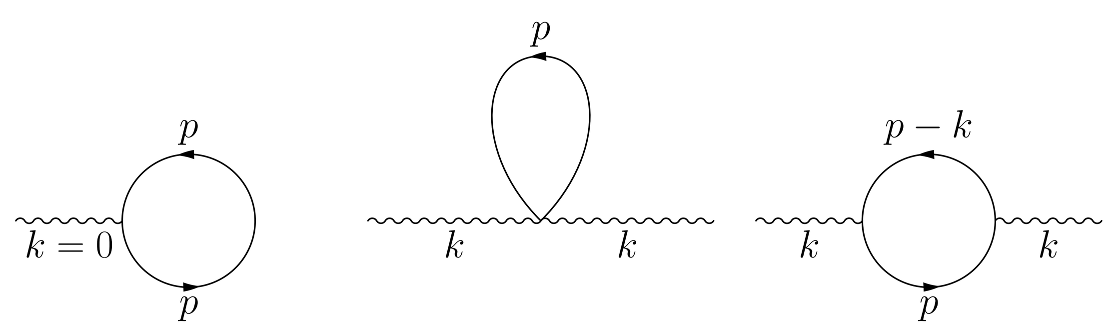
we will compute the effective action for the external source fields up to the quadratic order:

$$\begin{aligned}
W[\varphi] &= W[0] + \int \mathrm{d}^dx\varphi^{\mu_1\cdots\mu_s}(x)J_{\mu_1\ldots\mu_s}(x) \qquad (3)\\
&+\frac{1}{2!}\int \mathrm{d}^dx\mathrm{d}^dy\varphi^{\mu_1\cdots\mu_s}(x)\varphi^{\nu_1\cdots\nu_s}(y)J_{\mu_1\ldots\mu_s\nu_1\ldots\nu_s}(x,y) + \ldots
\end{aligned}$$

where $J_{\mu_{11}\ldots\mu_{1s}\ldots\mu_{n1}\ldots\mu_{ns}}(x_1,\ldots,x_n) = \frac{\delta^n(W[\varphi])}{\delta\varphi^{\mu_{11}\cdots\mu_{1s}}(x_1)\ldots\delta\varphi^{\mu_{n1}\cdots\mu_{ns}}(x_n)}\Big|_{\varphi=0}$.
The one-loop 1-pt correlator for the external field is (up to the linear order):

$$\begin{aligned}
\langle\langle J_{\mu_1\ldots\mu_s}(x)\rangle\rangle &= \frac{\delta(W[a])}{\delta a^{\mu_1\cdots\mu_s}(x)} \qquad (4)\\
&= J_{\mu_1\ldots\mu_s}(x) + \int \mathrm{d}^dy\varphi^{\nu_1\cdots\nu_s}(y)J_{\mu_1\ldots\mu_s\nu_1\ldots\nu_s}(x,y) + \ldots
\end{aligned}$$

To obtain the effective action and the one-loop 1-pt correlator, in general, we compute the following Feynman diagrams:



We can derive general expressions for 1-pt and 2-pt correlators expressed in terms of hypergeometric functions for any dimension d, but these results are not easily "readable". For this reason we expand in UV ($m/k \to 0$) and IR ($m/k \to \infty$).

Spin 1

The 1-point function (even part) reads

$$\langle\langle J_\mu(x)\rangle\rangle = 2^{-d+\lfloor\frac{d}{2}\rfloor}m^{d-2}\pi^{-\frac{d}{2}}\sum_{n=1}^{\infty}\frac{(-1)^n n\, m^{-2n}\Gamma\left(2n-\frac{d}{2}\right)}{2^n(2n+1)!!}\Box^{n-1}\partial^\nu F_{\mu\nu}$$

The effective action in the even parity sector is

$$W[A] = 2^{-1-d+\lfloor\frac{d}{2}\rfloor}m^{d-2}\pi^{-\frac{d}{2}}\int \mathrm{d}^dx\sum_{n=1}^{\infty}\frac{(-1)^n n\, m^{-2n}\Gamma\left(2n-\frac{d}{2}\right)}{2^n(2n+1)!!}F_{\mu\nu}\Box^{n-1}F^{\mu\nu} \overset{\text{IR}}{\sim} m^{d-4}\int d^dxF_{\mu\nu}F^{\mu\nu}$$

In the IR (large m) we get Maxwell action. For dimension $d = 3$ we also get a odd parity part which in the IR reads

$$W[A] \overset{\text{IR}}{\sim} \frac{1}{8\pi}\int \mathrm{d}^3x\epsilon^{\mu\nu\lambda}\left(A_\mu\partial_\nu A_\lambda\right)$$

which is the lowest order term of the Chern - Simons action $S_{CS} = \frac{\kappa}{4\pi}\int \mathrm{d}^3xTr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$

Spin 2

Let us consider a free fermion theory coupled to gravity

$$S[h] = \int \mathrm{d}^dx\, e\left[i\bar{\psi}E^\mu_a\gamma^a\nabla_\mu\psi - m\bar{\psi}\psi\right], \quad \nabla_\mu = \partial_\mu + \frac{1}{8}\omega_{\mu bc}\left[\gamma^b,\gamma^c\right]$$

where E^m_a is the inverse vierbein. If we expand the metric around the flat spacetime, $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, then, contrary to spin-1 case, interaction is not linear in the field $h_{\mu\nu}$.
The one-loop 1-pt function (energy-momentum tensor) defined as $\langle\langle T_{\mu\nu}(x)\rangle\rangle = \frac{2}{\sqrt{g}}\frac{\delta W}{\delta h^{\mu\nu}(x)}$ becomes

$$\langle\langle T_{\mu\nu}(x)\rangle\rangle = -2^{-1-d+\lfloor\frac{d}{2}\rfloor}m^d\pi^{-\frac{d}{2}}\left[\Gamma\left(-\frac{d}{2}\right)g_{\mu\nu} + \sum_{n=1}^{\infty}\frac{(-1)^n m^{-2n}\Gamma\left(n-\frac{d}{2}\right)}{2^{n+1}(2n+1)!!}\left((2n-1)\Box^{n-1}G_{\mu\nu} + (n-1)\Box^{n-2}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)R\right)\right]$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R$ is Einstein tensor. The energy-momentum tensor is divergence free:

$$\nabla^\mu\langle\langle T_{\mu\nu}(x)\rangle\rangle = 0$$

For the effective action in the IR we obtain (in the even parity sector)

$$W[h] \overset{\text{IR}}{\sim} -2^{-1-d+\lfloor\frac{d}{2}\rfloor}m^d\pi^{-\frac{d}{2}}\int \mathrm{d}^dx\sqrt{g}\left[\Gamma\left(-\frac{d}{2}\right) - \frac{1}{24}m^{-2}\Gamma\left(1-\frac{d}{2}\right)R - \frac{1}{80}m^{-4}\Gamma\left(2-\frac{d}{2}\right)\left(R^2_{\mu\nu\lambda\rho} - 2R^2_{\mu\nu} + \frac{1}{3}R^2\right)\right] + \cdots$$

The divergent part of the effective action for $d = 4$ is

$$W[h] \sim \frac{1}{4\pi^2\varepsilon}\int \mathrm{d}^4x\left(m^4\sqrt{g} - \frac{1}{24}m^2R - \frac{1}{80}\mathcal{W}^2\right) + \cdots$$

First term is a cosmological constant term and the second is the linearized Einstein-Hilbert action. The third term (m^0 term) is Weyl density $\mathcal{W}^2 = R^2_{\mu\nu\lambda\rho} - 2R^2_{\mu\nu} + \frac{1}{3}R^2$ (conformal structure in 4d). For $d = 3$ we also have a odd parity part

$$W[h] \overset{\text{IR}}{\sim} \frac{1}{192\pi}\int \mathrm{d}^3x\,\epsilon_{\sigma\nu\rho}h^{\mu\nu}\partial^\sigma\left(\partial_\mu\partial_\lambda - \eta_{\mu\lambda}\Box\right)h^{\lambda\rho}$$

which corresponds to the quadratic order of gravitational Chern-Simons term $S_{CS} = -\frac{\kappa}{96\pi}\int \mathrm{d}^3x\,\epsilon^{\mu\nu\lambda}\left(\partial_\mu\omega_\nu{}^{ab}\omega_{\lambda ab} + \frac{2}{3}\omega_{\mu a}{}^b\omega_{\nu b}{}^c\omega_{\lambda c}{}^a\right)$.

Spin 3

The 1-point function (even parity) is given by

$$\langle\langle J_{\mu_1\mu_2\mu_3}(x)\rangle\rangle = 2^{-d+\lfloor\frac{d}{2}\rfloor}m^d\pi^{-\frac{d}{2}}\sum_{n=1}^{\infty}(-1)^n m^{-2n}\Gamma\left(n-\frac{d}{2}\right)\left(a_n\Box^{n-1}\partial^\nu F_{\mu_1\mu_2\mu_3\nu} + b_n\Box^{n-2}(\eta_{\mu_1\mu_2}\Box - \partial_{\mu_1}\partial_{\mu_2})\partial^\nu F'_{\nu\mu_3}\right)$$

where a_n and b_n are numerical coefficients and the symmetrisation in μ_i indices is understood. The tensor $F_{\alpha\mu\nu\rho}$ is obtained from the spin-3 Ricci tensors by taking traces of spin-3 Riemann tensor defined with $R_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3} = \partial_{\nu1}\partial_{\nu2}\partial_{\nu3}\phi_{\mu_1\mu_2\mu_3}|_{[\mu_i,\nu_j]}$

$$F_{\alpha\mu\nu\rho} \equiv R'_{\alpha(\mu\nu\rho)} - \frac{1}{2}R''_{\alpha(\mu}\eta_{\nu\rho)} = \partial_{[\alpha}\left(\mathcal{F}_{(\mu]\nu\rho)} - \frac{1}{2}\mathcal{F}_{(\mu]}\eta_{\nu\rho)}\right)$$

where Fronsdal tensor $\mathcal{F}_{\mu\nu\rho} = \eta^{\alpha\beta}\Gamma_{\alpha\beta;\mu\nu\rho}$ is given in terms of a second-order Christoffel-like connection.
A covariant conservation should be written also for the HS currents, but for $s > 2$ we will content ourselves with the lowest non-trivial order in which the conservation law reduces to

$$\partial^{\mu_1}\langle\langle J^{(s)}_{\mu_1\ldots\mu_s}(x)\rangle\rangle = 0$$

The effective action in the IR coming from the lowest local term in the even sector is

$$W[b] \overset{\text{IR}}{\sim} \frac{2^{3-d+\lfloor\frac{d}{2}\rfloor}m^{d-2}\pi^{-\frac{d}{2}}}{135}\Gamma\left(1-\frac{d}{2}\right)\int \mathrm{d}^dx\left((\mathcal{F}_{\mu\nu\lambda})^2 - (\mathcal{F}'_\mu)^2\right)$$

For the UV for $d = 3$ in the odd parity sector we find

$$W[b] \sim \int \mathrm{d}^3x\,\epsilon^{\mu\nu\sigma}\left(5\,\Gamma^{\alpha\beta}{}_{;\mu\rho\lambda}\partial_\sigma\Gamma^{\rho\lambda}{}_{;\nu\alpha\beta} - 3\,\mathcal{F}_{\mu\rho\lambda}\partial_\sigma\mathcal{F}_\nu{}^{\rho\lambda}\right)$$

which corresponds to Pope-Townsend action for traceless spin-3 field $b_{\mu\nu\lambda}$.

Conclusion

- The same analysis can be done for scalar free field theory coupled to external sources.
- We considered 1-pt and 2-pt correlators → the one-loop effective action contains only the *quadratic* terms and the relevant equations of motion for the sources we obtain are the *linearized* ones.
- Forthcoming research - higher order-point correlators → information about non-linear structure of higher-spins.

References

[1] L. Bonora, M. Cvitan, P. Dominis Prester, B. Lima de Souza and I. Smolić, JHEP **1605** (2016) 072 doi:10.1007/JHEP05(2016)072 [arXiv:1602.07178 [hep-th]].

[2] L. Bonora, M. Cvitan, P. Dominis Prester, S. Giaccari, B. Lima de Souza and T. Štemberga, arXiv:1609.02088 [hep-th].

LaTeX TikZposter