Measuring a_μ^{HLO} at per mille precision via a muon beam

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SUMMARY: We propose an experiment to measure the running of the fine structure constant in the space-like region by scattering high energy muons on atomic electrons in a low-Z target, through the process $\mu e \to \mu e$. The differential cross section of the process, measured as a function of $t = q^2 < 0$ (the squared difference of the initial and final momenta of the muon or the electron), provides direct sensitivity to the leading hadronic contribution to the muon g-2, a_μ^{HLO}. By using a muon beam of 150 GeV, with a 2×10^7 muons/s intensity, currently available at the CERN North Area, a statistical uncertainty of $\sim 0.3\%$ can be achieved on a_μ^{HLO} after two years of data taking.

The q^2 of the muon-electron interaction is determined by the electron (or muon) scattering angle: the experiment is primarily based on a precise measurement of the scattering angles of the two outgoing particles, by means of state-of-the-art silicon detectors. A low-Z solid target is preferred (Be, C) in order to provide the required event rate, limiting at the same time the effect of multiple scattering as well as of other types of muon interactions (pair production, bremsstrahlung and nuclear interactions). An advantage of the muon beam is the possibility of employing a modular apparatus, with the target subdivided in subsequent layers. The normalization of the cross section is provided by the very same $\mu e \to \mu e$ process in the low-q^2 region, where the effect of the hadronic corrections on the fine structure constant is negligible. Such a simple and robust technique has the potential to keep systematic effects under control, having the goal of reaching a systematic uncertainty of the same order as the statistical one.

This direct measurement of a_μ^{HLO} could be competitive with the present determination obtained with the dispersive approach via time-like data and would allow a stringent test of the Standard Model with the experimental measurements of the future g-2 experiments at Fermilab and J-PARC.

I. INTRODUCTION

In searching for new physics, low-energy high-precision measurements are complementary to the LHC high energy frontier. The long standing discrepancy of about 3σ between the experimental value of the muon anomalous magnetic moment $a_\mu = (g-2)/2$ and the Standard Model (SM) prediction, $\Delta a_\mu(\text{Exp} - \text{SM}) \sim (28 \pm 8) \times 10^{-10}$ [1], has been considered during these years as one of the most compelling indicators of physics beyond the SM. It strongly constrains speculative new theories such as supersymmetry, dark gauge bosons or extra dimensions. The accuracy of the theoretical prediction (5×10^{-10}) is limited by the strong interaction effects, which cannot be computed perturbatively at low energies. Using analyticity and unitarity, it was shown long ago that the leading-order (LO) hadronic contribution to the muon anomalous moment, a_μ^{HLO}, can be computed via a dispersion integral on the cross section for low-energy hadronic e^+e^- annihilation [2]. a_μ^{HLO} gives the main uncertainty $(\sim 4 \times 10^{-10})$, with a fractional accuracy of 0.6%. An alternative evaluation of a_μ^{HLO} can be obtained by lattice QCD calculations [3]; even if the current results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [4].

The $\mathcal{O}(\alpha^3)$ hadronic light-by-light contribution, a_μ^{HLbL}, which has the second largest error in the theoretical evaluation, contributing with an uncertainty of $(2.5 - 4) \times 10^{-10}$ [5], cannot at present be determined from data and its calculation relies on the use of specific models [6].

From the experimental side, the error achieved by the BNL E821 experiment $\delta a_\mu^{\text{Exp}} = 6.3 \times 10^{-10}$ (corresponding to 0.54 ppm) [7] is dominated by the available statistics. New experiments at Fermilab and J-PARC, aiming at measuring the muon anomaly to a precision of 1.6×10^{-10} (0.14 ppm), are under way [8, 9].

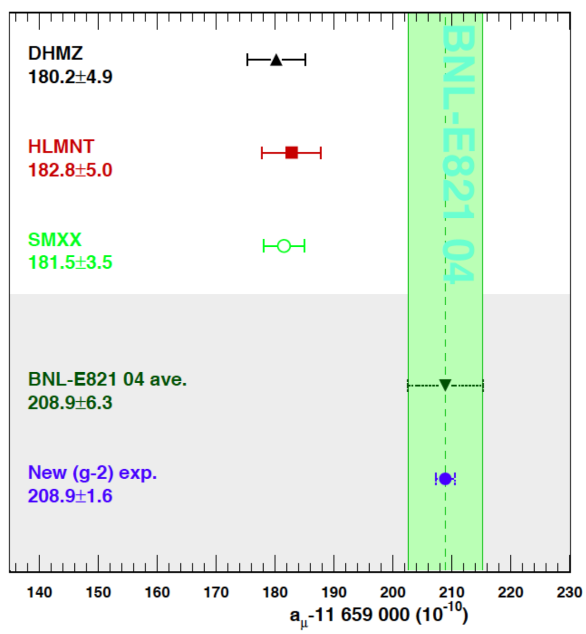

FIG. 1. Comparison between a_μ^{SM} and a_μ^{Exp}. DHMZ is Ref. [10], HLMNT is Ref. [11]; "SMXX" is the same central value with a reduced error as expected by the improvement on the hadronic cross section measurement; "BNL-E821 04 ave." is the current experimental value of a_μ; "New (g-2) exp." is the same central value with a fourfold improved precision as planned by the future g-2 experiments at Fermilab and J-PARC.

Figure 1 from Ref. [1] shows the status of the g-2 discrepancy compared with what could be expected after the new g-2 measurements at Fermilab and JPARC, in case the central value would remain the same. A more recent evalutation leads to a larger 4σ discrepancy [12]. Together with a fourfold improved precision on the experimental side, an improvement on the LO hadronic contribution is highly desirable. Differently from the dispersive approach, which uses *time-like* data from annihilation cross sections highly fluctuating at low energy due to resonances and threshold effects, our proposal is to determine a_μ^{HLO} from a measurement of the effective electromagnetic coupling in the space-like region [13], where the vacuum polarization is a smooth function of the squared momentum transfer. A method to determine the running of the electromagnetic coupling in small-angle Bhabha scattering was proposed in [14] and applied to LEP data in [15]. Application to the measurement of a_μ^{HLO} from Bhabha $(e^+e^- \to e^+e^-)$ scattering data was discussed in [13].

The hadronic contribution to the runnning of α can also be determined unambiguously by using the t-channel μe scattering process, from which $a_\mu^{\rm HLO}$ can be obtained.

II. THEORETICAL FRAMEWORK

With the help of dispersion relations and the optical theorem, the leading-order hadronic contribution to the muon g-2 is given by the well-known formula [2, 16]

$$a_\mu^{\rm HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{\hat{K}(s) R_{\rm had}(s)}{s^2}, \tag{1}$$

where $R_{\rm had}(s)$ is the ratio of the total $e^+e^- \to hadrons$ and $e^+e^- \to \mu^+\mu^-$ cross sections, $\hat{K}(s)$ is a smooth function and m_μ (m_π) is the muon (pion) mass. We remark that $R_{\rm had}(s)$ in the integrand function of eq.(1) is highly fluctuating at low energy due to resonances and threshold effects.

The dispersive integral in eq. (1) is usually calculated by using the experimental value of $R_{\rm had}(s)$ up to a certain value of s [17–19] and by using perturbative QCD (pQCD) [20] in the high-energy tail.

For the calculation of $a_\mu^{\rm HLO}$, an alternative formula can also be exploited [13, 21], namely

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi} \int_0^1 dx\, (1-x)\, \Delta\alpha_{\rm had}[t(x)]\,, \tag{2}$$

where

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0 \tag{3}$$

is a space-like (negative) squared four-momentum and $\Delta\alpha_{\rm had}(t)$ is the hadronic contribution to the running of the fine structure constant. In contrast with the integrand function of eq.(1), the integrand in eq.(2) is smooth and free of resonances.

$\Delta\alpha_{\rm had}(t)$, where $t = q^2 < 0$, is related to the effective fine structure constant $\alpha(q^2)$ by:

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \tag{4}$$

where

$$\Delta\alpha(q^2) = \Delta\alpha_{\rm lep}(q^2) + \Delta\alpha_{\rm had}(q^2)$$

and $\Delta\alpha_{\rm lep}(q^2)$ is the leptonic (+ top quark) contribution, which is known with high precision. By measuring the running of $\alpha(q^2)$, the hadronic contribution $\Delta\alpha_{\rm had}(t)$ can be extracted.

Equation (2) involves $\Delta\alpha_{\rm had}(q^2)$ evaluated at negative space-like momenta $t < 0$. In fig. 2 (left) we plot the integrand $(1-x)\Delta\alpha_{\rm had}[t(x)]$ of eq. (2) using the output of the routine `hadr5n12` [23] (which uses time-like hadroproduction data and perturbative QCD). The range $x \in (0,1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The peak of the integrand occurs at $x_{\rm peak} \simeq 0.914$ ($t_{\rm peak} \simeq -0.108$ GeV^2) and $\Delta\alpha_{\rm had}(t_{\rm peak}) \simeq 7.86 \times 10^{-4}$ (see fig. 2 (right)).

III. EXPERIMENTAL PROPOSAL

We propose to use eq. (2) to compute $a_\mu^{\rm HLO}$ exploiting the measurement of $\Delta\alpha_{\rm had}(t)$ in the space-like region.

Measuring the running of $\alpha(t)$ in the space-like region, using a muon beam with $E_\mu^i \simeq 150$ GeV on a fixed electron target, with a technique similar to the one described in [24] for a measurement of the pion form factor, is very appealing for the following reasons:

- it is a pure t-channel process (at least at LO in perturbation theory), making the dependence on t of the LO differential cross section proportional to $[\alpha(t)]^2$:

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left[\frac{\alpha(t)}{\alpha_0}\right]^2 \tag{5}$$

  where $\alpha(t)$ is defined in eq. (4);

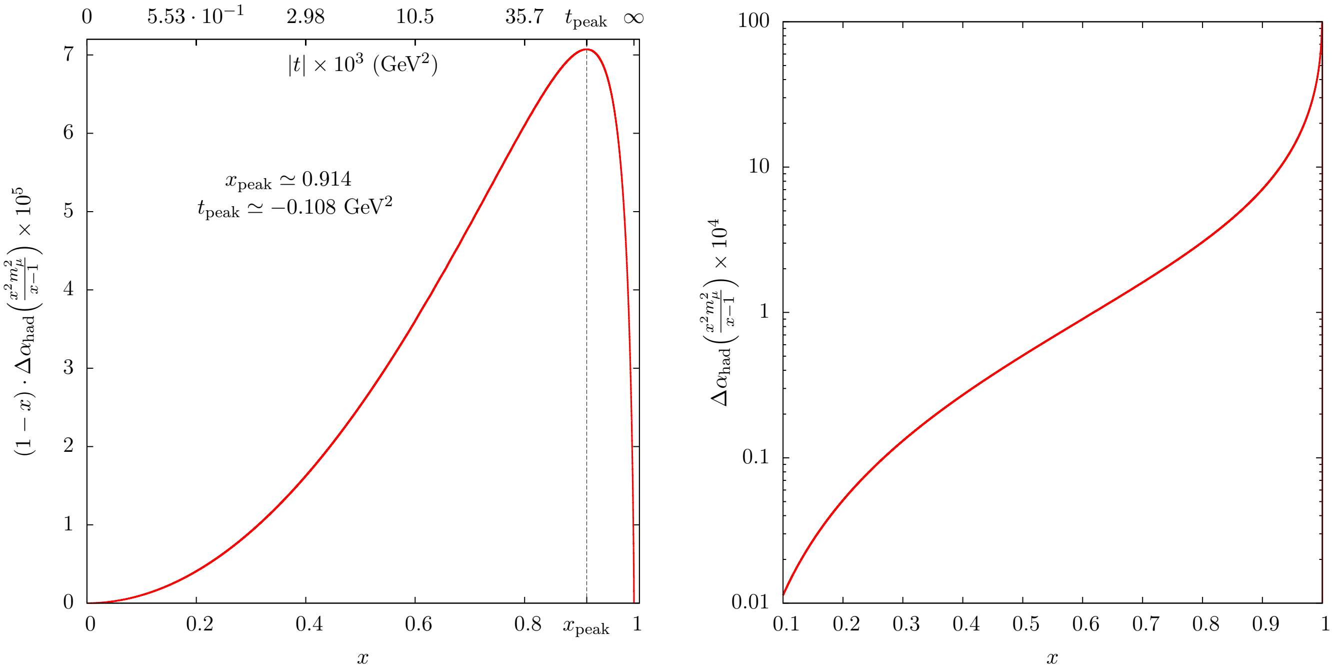

FIG. 2. Left: The integrand $(1-x)\Delta\alpha_{\text{had}}[t(x)] \times 10^5$ as a function of x and t. Right: $\Delta\alpha_{\text{had}}[t(x)] \times 10^4$.

- with fixed target the t variable is related to the energy E_e^f or the angle θ_e^f of the scattered electron, given the incoming muon energy E_μ^i:

$$t = (p_\mu^i - p_\mu^f)^2 = (p_e^i - p_e^f)^2 = 2m_e^2 - 2m_e E_e^f$$
$$E_e^f = m_e \frac{1+r^2 c_e^2}{1-r^2 c_e^2}$$
$$\theta_e^f = \arccos\left(\frac{1}{r}\sqrt{\frac{E_e^f/m_e - 1}{E_e^f/m_e + 1}}\right), \tag{6}$$

where

$$r \equiv \frac{\sqrt{(E_\mu^i)^2 - m_\mu^2}}{E_\mu^i + m_e}; \quad c_e \equiv \cos\theta_e^f.$$

- the boosted kinematics of the collision allows all the scattering angles to be accessed by a single detector element in the laboratory system. By requiring a minimum energy threshold for the electron $E_e^f > 1$ GeV, the electron scattering angle θ_e^f spans the range $0-31.85$ mrad while the electron energy E_e^f is in the range $1-139.8$ GeV. By using the same detector for all the phase-space, many systematic errors, *e.g.* on the efficiency, will cancel out (at least at first order) in the relative ratios of event counts in the high and low q^2 regions (signal and normalization regions);

- the 150 GeV muon momentum gives $s \simeq 0.161$ GeV^2, which implies that t ($-s < t < 0$) is located in the peak region of the integrand function of the formula (2), as visible in fig. 2 (left);

- the angle of the scattered electron and muon are correlated as shown in fig. 3 (for muons of 150 GeV). This constraint is extremely important to select elastic scattering events, rejecting background events from radiative or inelastic processes and to minimize systematic effects in the determination of t. Note that for scattering angles of 2-3 mrad there can be an ambiguity between the outgoing electron and muon, as their angles and momenta are similar. To associate them correctly it is necessary to identify the two particles by means of downstream dedicated detectors (calorimeter and muon detectors).

We propose to measure the running of $\alpha(t)$ and in turn $\Delta\alpha_{\text{had}}(t)$, from $\mathcal{O}(30)$ experimental points, for instance evenly-spaced in t or x in the kinematic range of the proposed experiment. The values of $\Delta\alpha_{\text{had}}(t)$ for large $|t|$ (*e.g.*

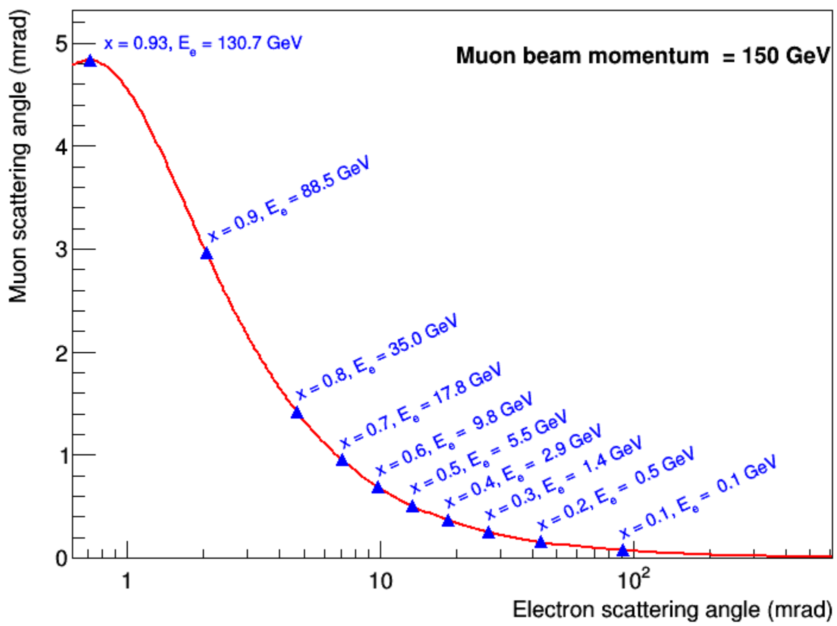

FIG. 3. The relation between the muon and electron scattering angles for 150 GeV incident muon momentum.

$|t| > 10$ GeV^2) can be calculated by means of pQCD. The data and the pQCD regime can then be fitted together with the help of Padé approximants and the fitted curve can be integrated to give a value for $a_\mu^{\rm HLO}$ according to eq. (2).

We estimate the statistical sensitivity of this experiment to be $\sim 0.3\%$ on the value of $a_\mu^{\rm HLO}$ after 2 years of data taking (1 year $= 2 \times 10^7$ sec), by assuming a muon beam of 150 GeV with an intensity of 2×10^7 muons/sec and a detector layout as described in the following.

IV. PRELIMINARY CONSIDERATIONS ON THE DETECTOR

In order to perform the planned measurement to the required precision, a dedicated detector is necessary. We describe here a possible setup to measure the observables:

- the direction and momentum of the incident muon;
- the directions of the outgoing electron and muon.

The CERN muon beam M2, used at 150 GeV, has the characteristics needed for such a measurement. The beam intensity appears to be adequate to provide the required event yield. The beam time structure allows to tag the incident muon while keeping low the background related to incoming particles (*e.g.* electrons). The electrons contamination is very small. The beam provides both positive and negative muons, which we plan to use.

The target consists of atomic electrons. To reach the required statistics, given the M2 beam intensity, the target must provide an adequate amount of material to get enough electron scattering centres. The target has to be made of low-Z materials to minimize the impact of the multiple scattering and the background due to bremsstrahlung and pair production processes. A promising idea for the detector, still under investigation, is to use 20 layers of Be coupled to Si planes, spaced by intermediate air gaps, located at a relative distance of one meter from each other. Figure 4 shows the basic layout on which we are working at the moment.

The arrangement provides both a distributed target with low-Z and the tracking system. As downstream particle identifiers we plan to use a calorimeter for the electrons and a muon system for the muons (a filter plus the active planes). This particle identifier system is required to solve the muon-electron ambiguity for electron scattering angles around (2-3) mrad (fig. 4). The preliminary studies of such an apparatus, performed by using GEANT4, indicate an angular resolution for the outgoing particles of ~ 0.02 mrad.

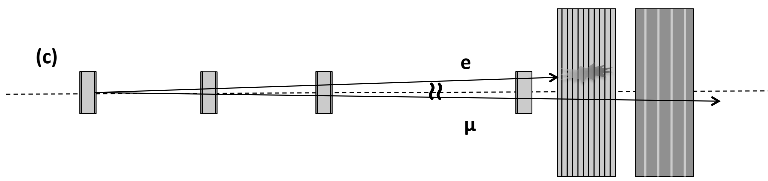

FIG. 4. Scheme of a possible layout for the detector.

The detector acceptance must cover the region of the signal, with the electron emitted at extremely forward angles and high energies, as well as the normalization region, where the electron has much lower energy (around 1 GeV) and an emission angle of some tens of mrad. The forward kinematics of the process allow the detector to cover almost 100% of the acceptance.

To tag each incoming muon, we estimate that a detector of the kind of those in use by COMPASS or NA62 can be used. A detailed simulation for optimizing and validating the detector technology and geometry is in progress. Beam tests in the North Area could be planned once the proposed project will be at a more advanced stage.

V. SYSTEMATIC CONSIDERATIONS

Significant contributions of the hadronic vacuum polarization to the $\mu e \to \mu e$ differential cross section are essentially restricted to electron scattering angles below 10 mrad, corresponding to electron energies above 10 GeV. The net effect of these contributions is to increase the cross section by a few per mille: a precise determination of $a_\mu^{\rm HLO}$ requires not only high statistics, but also considerable systematic accuracy, as the final goal of the experiment is equivalent to a determination of the differential cross section at ~ 10 ppm.

Such an accuracy can be achieved if the acceptance is maintained highly uniform over the entire q^2 range, including the normalization region, and over all the detector components. This motivates the choice of a purely angular measurement: an acceptance of tens of mrad can be covered with a single sensor of modern silicon detectors, positioned at a distance of about one meter from the target. It has to be stressed that particle identification (electromagnetic calorimeter and muon filter) is necessary to solve the electron-muon ambiguity at high electron momentum, where an electron-muon wrong assignment is highly unlikely. The wrong assignment probability can be measured with the data by using the rate of muon-muon and electron-electron events.

Another requirement for reaching very high accuracy is to measure all relevant contributions to systematic uncertainties from the data themselves. An important effect, which distinguishes the normalization from the signal region, is multiple scattering, as the electron energy in this region is $\mathcal{O}(1)$ GeV. Multiple scattering breaks the muon-electron two-body angular correlation, moving events out of the kinematic line in the 2D plot of Fig. 3. In addition multiple scattering, in general, causes acoplanarity, while two-body events are planar, within resolution. These facts allow multiple scattering effects to be modelled and measured using the data. An additional handle on multiple scattering could be the inclusion of a “thin target” ($\mathcal{O}(100)$ μm) in the apparatus, dedicated to the measurement of luminosity and made of the same material as the main target modules. This possibility will be studied in detail with the simulation.

The shape of the kinematic correlation between the muon and the electron (fig. 3) depends on the initial beam momentum, which defines the highest possible achievable q^2. In particular, the position of the events with equal electron and muon scattering angle in the 2D plot depends on the inverse of the square root of the beam momentum, while the maximum muon scattering angle depends only on the ratio between the electron and the muon mass. These features of the angular two-body kinematics can be used for a precise beam momentum determination with data.

In experiments dedicated to high-precision measurements, several systematic effects can be explored within the experiment itself. In this respect the proposed modularity of the apparatus will help. A test with a single module could provide a proof-of-concept of the proposed methods.

From the theoretical point of view, the control of the systematic uncertainties requires the development of high-precision Monte Carlo tools, including the relevant radiative corrections to reach the needed theoretical precision. To this aim, QED radiative corrections at leading-logarithmic level resummed at all orders of perturbation theory and matched to the exact $\mathcal{O}(\alpha)$ correction (as for instance implemented in the `BabaYaga` event generator [25] for Bhabha scattering) are mandatory, ensuring a theoretical precision at the level of some $\mathcal{O}(10^{-4})$ on the differential cross section. Moreover, by using the ratio of the cross sections in the signal and normalization regions, we expect that the theoretical systematic uncertainty is further reduced to the level of $\mathcal{O}(10^{-5})$, due to partial cancellation of common radiative corrections. Work is in progress to extend `BabaYaga` to $\mu e \to \mu e$ scattering and to quantify the actual accuracy on the computation of the ratio of cross sections by means of dedicated Monte Carlo simulations. Any further improvement in the theoretical accuracy would require the matching of QED resummation with exact two-loop corrections, which are not available at present for the $\mu e \to \mu e$ process but are within reach.

VI. CONCLUSIONS

We presented a novel approach to determine the running of $\alpha(t)$ in the space-like region and the leading hadronic contribution to the muon g-2 by scattering high energy muons on atomic electrons in a low-Z target, through the process $\mu e \to \mu e$. The cross section of the process, measured differentially as a function of $t = q^2 < 0$, provides direct sensitivity to the leading hadronic contribution to the muon g-2, $a_\mu^{\rm HLO}$. By considering a muon beam of 150 GeV with intensity of $2 \times 10^7 \mu/s$, currently available at the CERN North Area, a statistical uncertainty of $\sim 0.3\%$ can be achieved on $a_\mu^{\rm HLO}$ in two years of data taking. A preliminary detector layout has been proposed to cope with the challenge of having the systematic error at the same level.

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