Evaporation of the de Sitter horizon

Tommi Markkanen
King's College London

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Outline

1. Introduction
2. Open quantum systems and decoherence
3. Loss of information from a Horizon
4. Backreaction
5. Conclusions

de Sitter space (FLRW)

- Einstein's equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \underbrace{M_{\rm pl}^{-2} T_{\mu\nu}}_{=0}$$

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The cosmologists' line element

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

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- The de Sitter solution:

$$\dot{H} = 0 \quad \Leftrightarrow \quad a(t) = e^{Ht}\,, \qquad H = \sqrt{\Lambda/3}$$

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- Inflation of the early Universe $\approx$ de Sitter
- Observations consistent with $\rho_\Lambda / M_{\rm pl}^4 = \Lambda / M_{\rm pl}^2 \sim 10^{-120}$

Stability

- Stability in a quantized theory has been studied by many:

Stable

- Gibbons & Hawking (77)
- ...

Not stable

- Mottola (85) & (86)
- Tsamis & Woodard, many papers, e.g. (93)
- Abramo, Brandenberger & Mukhanov (97)
- Goheer, Kleban & Susskind (03)
- Polyakov (07)
- Anderson & Mottola (14)
- Dvali, Gomez & Zell (17)
- and many more, see TM (16a,b) & (17)

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An instability could be important for inflation, the cosmological constant problem and the fate of the Universe !

Friedmann equations

- A system with vacuum energy and matter:

  $$\rho_\Lambda \quad \text{and} \quad \rho_m$$

- Spacetime homogeneous and isotropic

$$\begin{cases} 3H^2 M_{\rm pl}^2 & = \rho_m + \rho_\Lambda \\ -(3H^2 + 2\dot{H})M_{\rm pl}^2 & = p_m + p_\Lambda \end{cases},$$

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A thermal system:
$\rho_m + p_m > 0$
?!

Schrödinger's Cat

image: http://braungardt.trialectics.com/sciences/physics/quantum-mechanics/schrodingers-cat/

$$|\mathrm{cat}\rangle = \frac{1}{\sqrt{2}}|\mathrm{alive}\rangle + \frac{1}{\sqrt{2}}|\mathrm{dead}\rangle$$

Open systems

- An observable system, $\mathcal{S}$ and a hidden environment, $\mathcal{E}$
- *Entanglement*

$$|\Psi\rangle_{t=t_0} = |s_0\rangle \otimes |\varepsilon_0\rangle \longrightarrow \sum_i c_i |s_i\rangle \otimes |\varepsilon_i\rangle$$

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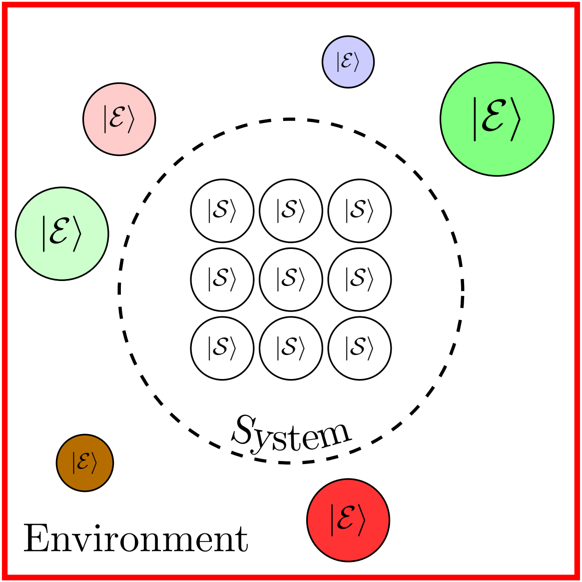
- $\mathcal{E}$ is unobservable $\Rightarrow$ sum over $|\varepsilon_i\rangle$ with equal weights:

Trace over the environment

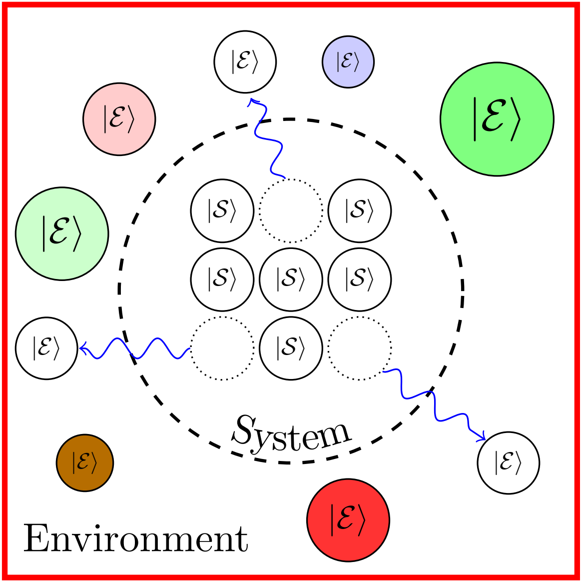
$$\hat{\rho} = |\Psi\rangle\langle\Psi| \longrightarrow \hat{\rho} = \mathrm{Tr}_{\mathcal{E}}\{|\Psi\rangle\langle\Psi|\} = \sum_i c_i |s_i\rangle\langle s_i| \,; \quad \langle\varepsilon_i|\varepsilon_j\rangle = \delta_{ij}$$

- $\hat{\rho}$ diagonal in the 'pointer basis' *chosen* by $\mathcal{E}$
  - 'Decoherence', Zurek (03); Schlosshauer (04) (reviews)

Initially the System and the Environment are separable



Interactions lead to entanglement



Features of spacetimes with horizons

- Black hole evaporation; Hawking (75), the Unruh effect; Unruh (76) and thermality of de Sitter; Gibbons & Hawking (77)

- A horizon divides the initial state into $\mathcal{S}$ and $\mathcal{E}$
  - Crucial for the information paradox, Hawking (76)

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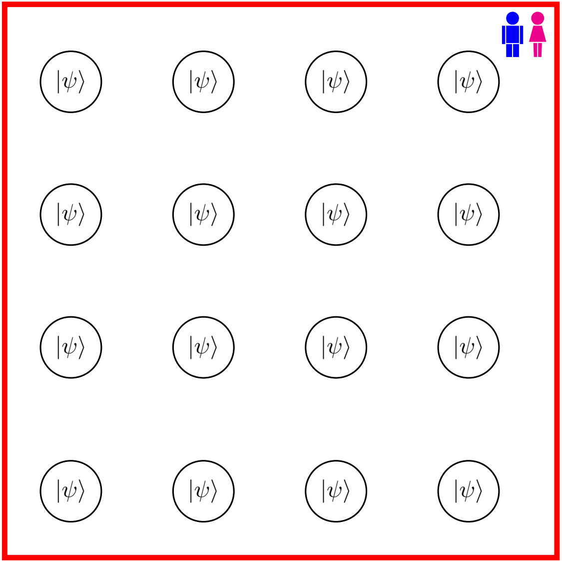
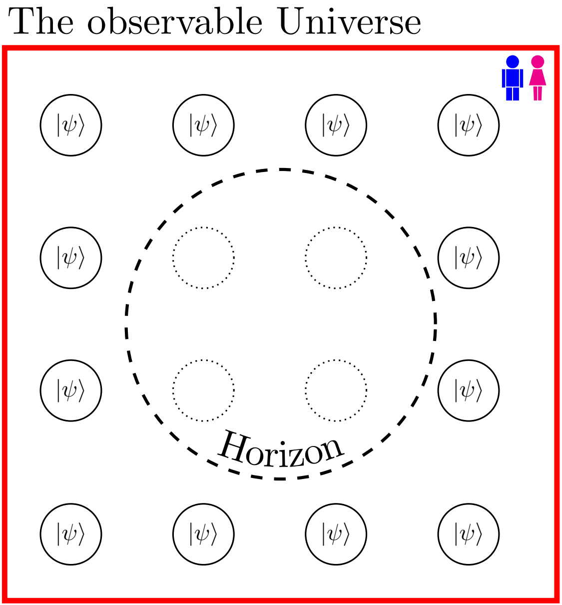
Here a completely different focus than in a calculation of primordial perturbations!

Initially, complete information

The observable Universe



Information loss from a black hole



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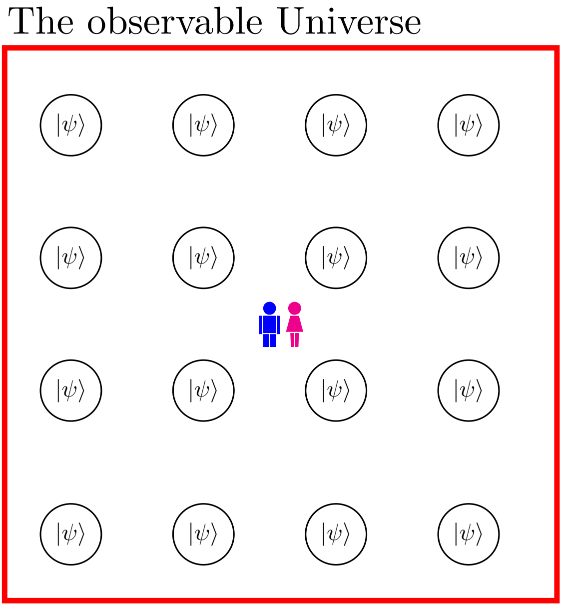

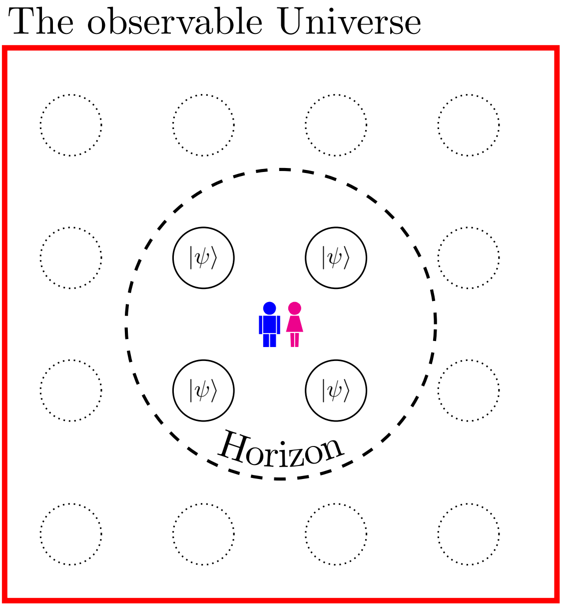
The observable Universe
$|\psi\rangle$
$|\psi\rangle$
$|\psi\rangle$
$|\psi\rangle$
Horizon

Backreaction

The energy-momentum tensor

- The Bunch-Davies vacuum contains unobservable states

$$|0^{\mathrm{BD}}\rangle = \sum_i c_i |n_i,\, \mathrm{IN}\rangle \otimes |n_i,\, \mathrm{OUT}\rangle$$

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$$\mathrm{Tr}_{\rm OUT}\{|0^{\rm BD}\rangle\langle 0^{\rm BD}|\} \propto \prod_{\mathbf{k}} \sum_{n_{\mathbf{k}}=0}^{\infty} \exp\Big\{-\frac{k}{T_H} n_{\mathbf{k}}\Big\} |n_{\mathbf{k}}\rangle\langle n_{\mathbf{k}}| \, ; \quad T_H = \frac{H}{2\pi}$$

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The energy-momentum (far from the horizon)

$$\rho_m \equiv T_{00} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{k}{e^{2\pi k/H} - 1}\,,$$

$$p_m \equiv T_{ii}/a^2 = \frac{1}{3}\rho_m\,,$$

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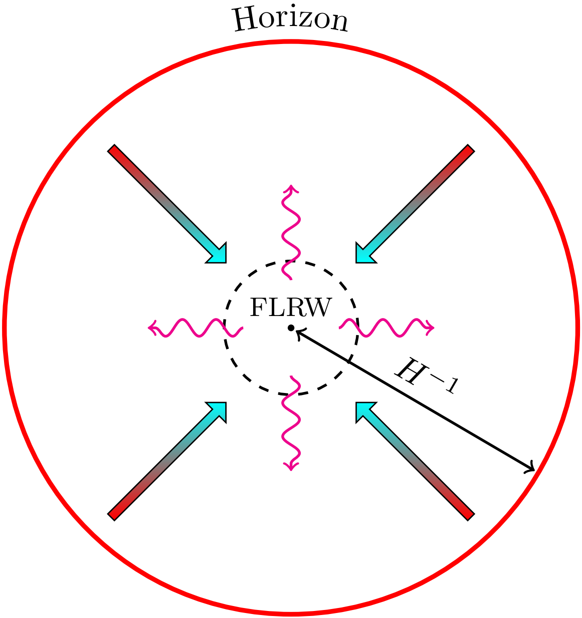
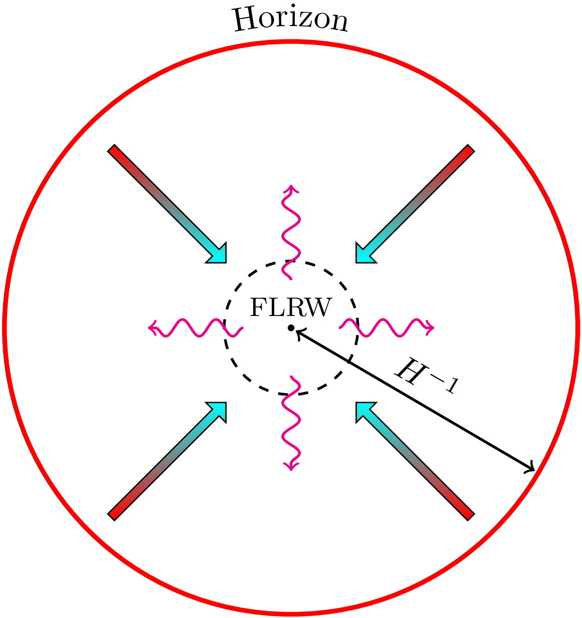
$$f_{m,\,i} \equiv T_{0i}/a = Hx^i a\big(\rho_m + p_m\big) \qquad \text{(flux)}$$



constant ρ_m

$$\dot{\rho}_m + 3H(\rho_m + p_m) + \partial^i T_{i0} = 0$$



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Horizon sources continuous particle creation !

- Backreaction is weak $\Rightarrow$ use de Sitter results to solve H

The evaporating horizon

$$-2\dot{H}M_{\rm pl}^2 = \frac{4}{3}\rho_m = \frac{H^4}{360\pi^2}\,; \quad \dot{\rho}_\Lambda = -3H(\rho_m + p_m)$$

$$\Rightarrow \quad \frac{H}{H_0} = \left(\frac{H_0^3 t}{240\pi^2 M_{\rm pl}^2} + 1\right)^{-1/3}; \quad H(0) \equiv H_0$$

- Agrees with, Padmanabhan (02) & (05)

The fate of the Universe

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- De Sitter destabilized after

$$t \sim \frac{M_{\rm pl}^2}{H_0^3} \qquad \text{(agrees with, Dvali, Gomez \& Zell (17))}$$

Thermodynamic interpretation

- ρ_Λ also contains zero-point energies of the quantum fields
  $\Rightarrow$ Particle creation balanced negative vacuum energy ('holes')
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The first law of thermodynamics in de Sitter

$$dU = TdS - PdV \qquad \Leftrightarrow \qquad 2\dot{H}M_{\rm pl}^2 = \frac{\dot{\rho}_\Lambda}{3H} \qquad \text{for}$$

$$dU = -d\left(\rho_\Lambda\frac{4\pi}{3H^3}\right), \quad TdS = \frac{H}{8\pi G}d\left(\frac{4\pi}{H^2}\right), \quad PdV = -p_\Lambda d\left(\frac{4\pi}{3H^3}\right)$$

- First principle result and thermodynamic derivation agree

Conclusions

Summary

De Sitter space for a local observer

- Entanglement across the horizon
- Trace over the hidden states leads to a thermal state
  - $\Rightarrow$ Unstable under backreaction in FLRW
- After a time $\sim M_{\rm pl}^2/H^3$ the system is no longer de Sitter
- Evaporation has a complete thermodynamic interpretation

Thank You!

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