Braneworld cosmology and holography

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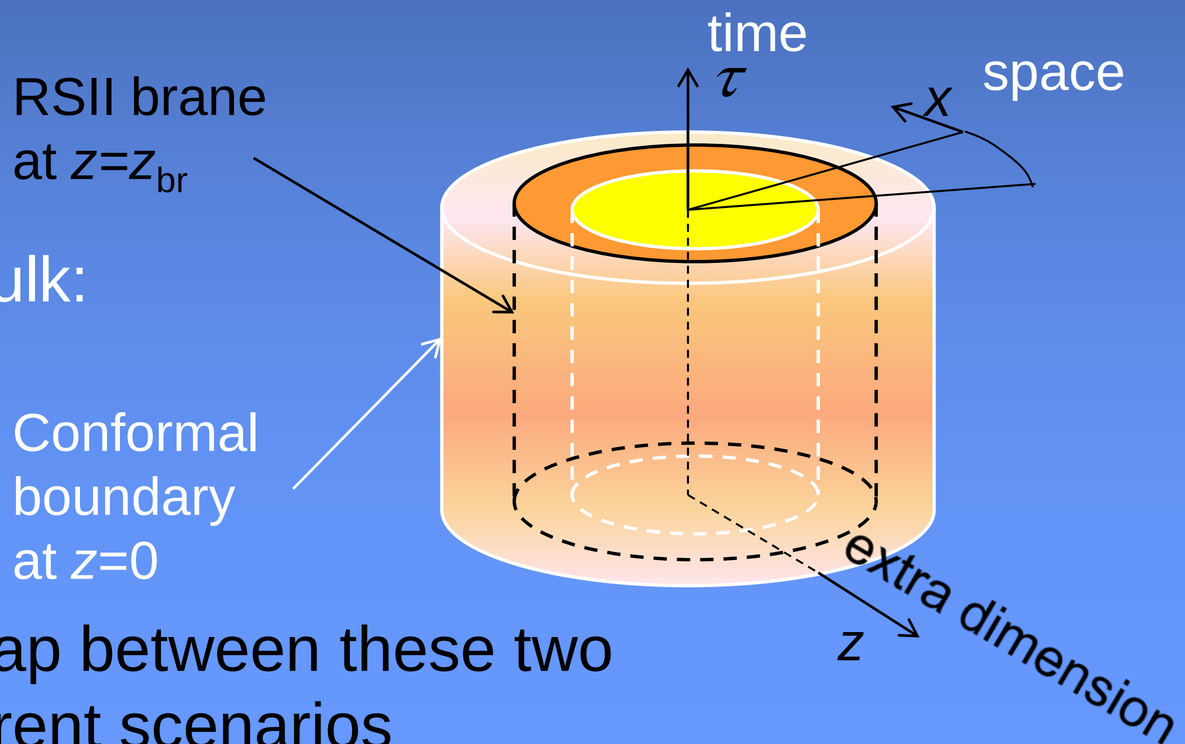
Basic idea

Braneworld cosmology is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.

We will consider two types of braneworlds in a 5-dim asymptotically Anti de Sitter space (AdS_5)

1. In a holographic braneworld universe a 3-brane is located at the boundary of the asymptotic AdS_5. The cosmology is governed by matter on the brane in addition to the boundary CFT
2. In the second Randall-Sundrum (RS II) model a 3-brane is located at a finite distance from the boundary of AdS_5. The model was proposed as an alternative to compactification of extra dimensions.

Foliation of the bulk:



. There exists a map between these two substantially different scenarios

This talk is based on:

N.B., Phys Rev D 93 (2016) [arXiv:1511.07323]

and related works

E. Kiritsis, JCAP **0510** (2005) [hep-th/0504219].
P. Brax and R. Peschanski, Acta Phys. Polon. **B 41** (2010) [arXiv:1006.3054]

P.S. Apostolopoulos, G. Siopsis and N. Tetradis, Phys. Rev. Lett. **102** (2009) [arXiv:0809.3505]

Second Randall-Sundrum model (RS II)

RS II was proposed as an alternative to compactification of extra dimensions. If extra dimensions were large that would yield unobserved modification of Newton's gravitational law. Eperimental bound on the volume of *n* extra dimensions $V^{1/n} \leq 0.1\,\mathrm{mm}$

Long *et al*, Nature **421** (2003).

RSII brane-world does not rely on compactification to localize gravity at the brane, but on the curvature of the bulk ("warped compactification"). The negative cosmological constant $\Lambda^{(5)}$ acts to "squeeze" the gravitational field closer to the brane. One can see this in Gaussian normal coordinates on the brane at $y = 0$, for which the AdS_5 metric takes the form

$$ds^2_{(5)} = e^{-2y/\ell}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

warp factor

L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)

"Sidedness"

In the original RSII model one assumes the Z_2 symmetry

$$z \leftrightarrow z_{\mathrm{br}}/z \qquad \text{or} \qquad y - y_{\mathrm{br}} \leftrightarrow y_{\mathrm{br}} - y$$

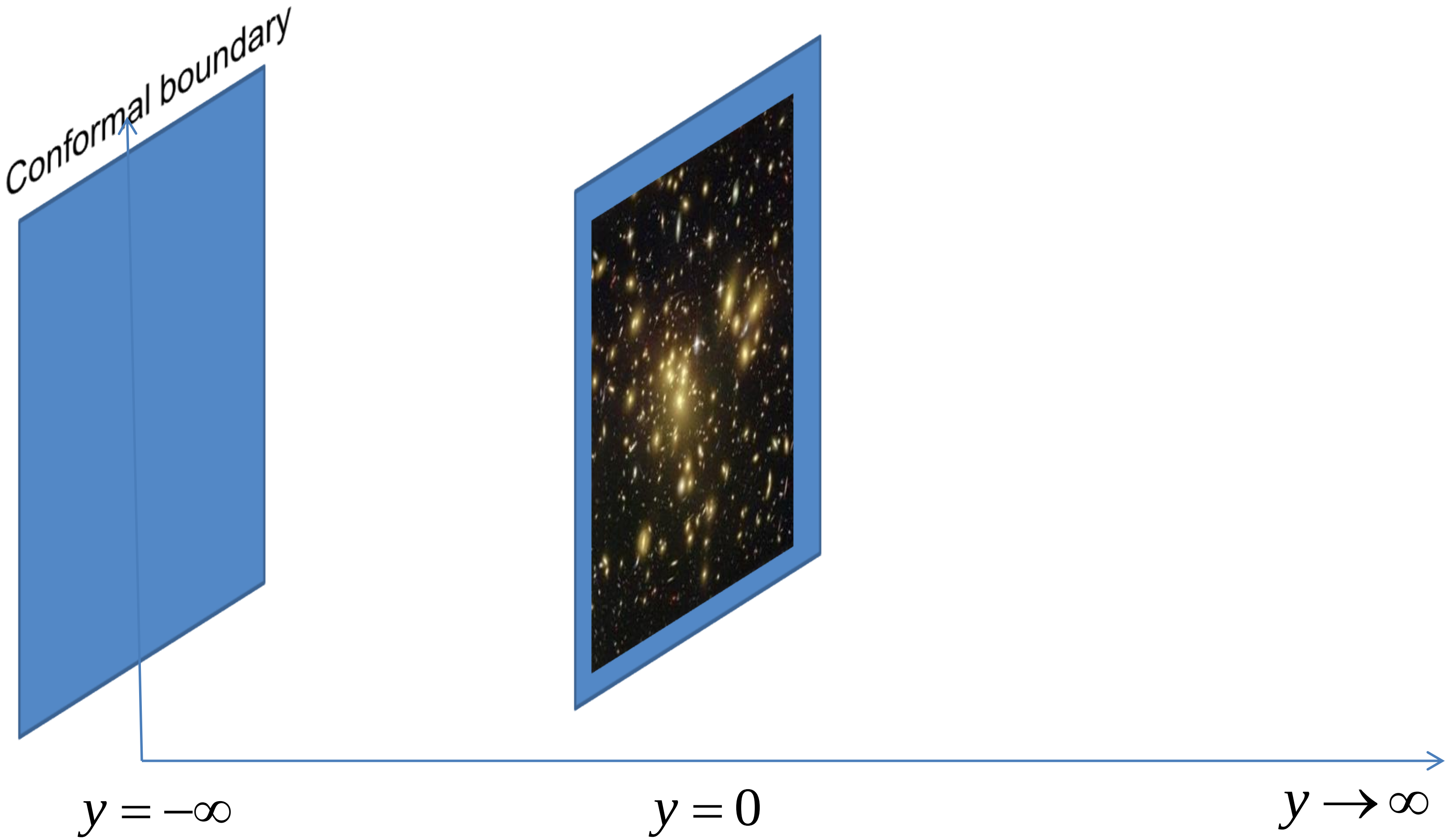
so the region $0 < z \leq z_{\mathrm{br}}$ is identied with $z_{\mathrm{br}} \leq z < \infty$ with observer's brane at the fixed point $z = z_{\mathrm{br}}$. The braneworld is sitting between two patches of AdS_5, one on either side, and is therefore dubbed "two-sided". In contrast, in the "one-sided" RSII model the region $0 < z \leq z_{\mathrm{br}}$ is simply cut off.

1-sided and 2-sided versions are equivalent from the point of view of an observer at the brane. However, in the 1-sided RSII model, by shifting the boundary in the bulk from $z = 0$ to $z = z_{\mathrm{br}}$, the model is conjectured to be dual to a cutoff CFT coupled to gravity, with $z = z_{\mathrm{br}}$ providing the cutoff. This connection involves a single CFT at the boundary of a single patch of AdS_5. In the 2-sided RSII model one would instead require two copies of the CFT, one for each of the AdS_5 patches.

M. J. Duff and J. T. Liu, Phys. Rev. Lett. 85, (2000); Class. Quant. Grav. 18 (2001)

RSII Cosmology – Dynamical Brane

Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at y=0 while making the metric in the bulk time dependent.



The Friedmann equations on the brane are modified.

Assuming $$ds^2_{(4)} = n^2(t)dt^2 - a^2(t)d\Omega^2_\kappa$$

$$d\Omega^2_\kappa = d\chi^2 + \frac{\sin^2\sqrt{\kappa}\chi}{\kappa}\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

and a fine tuning of the brane tension $\sigma = \sigma_0 \equiv 3/(8\pi G_N \ell^2)$

one finds

$$\mathcal{H}^2 = \frac{8\pi G_N}{3}\rho + \boxed{\left(\frac{4\pi G_N \ell}{3}\right)^2 \rho^2 + \frac{\mu\ell}{a^4}}$$

$$\mathcal{H}^2 = H^2 + \frac{\kappa}{a^2}$$

$$= \frac{\left(\partial_t a\right)^2}{a^2 n^2} + \frac{\kappa}{a^2}$$

Quadratic deviation from the standard FRW.
Decays rapidly as $\sim a^{-8}$ in the radiation epoch

dark radiation
due to a black hole in the bulk – should not exceed 10% of the total radiation content in the epoch of BB nucleosynthesis

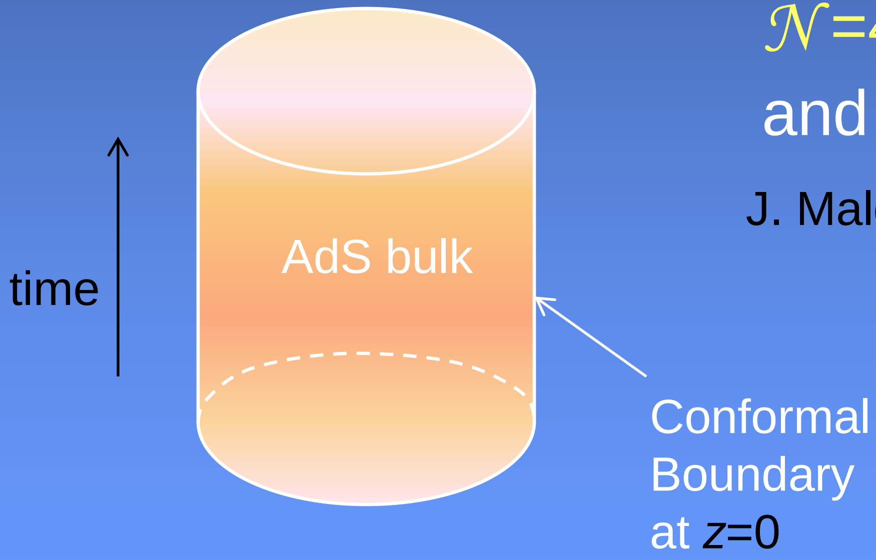
RSII cosmology is thus subject to astrophysical tests

Connection with AdS/CFT

AdS/CFT correspondence is a holographic duality between gravity in d+1-dim space-time and quantum CFT on the d-dim boundary. Original formulation stems from string theory:

Equivalence of 3+1-dim
$\mathcal{N}$=4 Supersymmetric YM Theory
and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)



Examples of CFT:
quantum electrodynamics,
Yang Mills gauge theory,
massless scalar field theory,
massless spin ½ field theory

In the RSII model by introducing the boundary in AdS_5 at $z = z_{br}$ instead of $z = 0$, the model is conjectured to be dual to a cutoff CFT coupled to gravity, with $z = z_{br}$ providing the IR cutoff (corresponding to the UV catoff of the boundary CFT) The on-shell bulk action is IR divergent because physical distances diverge at $z=0$. Asymptotically AdS metric

$$ds^2_{(5)} = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

near $z=0$ can be expanded as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + \cdots$$

Explicit expressions for $g^{(2n)}_{\mu\nu},\ n = 2,4$, in terms of arbitrary $g^{(0)}_{\mu\nu}$ may be found in

de Haro, Solodukhin, Skenderis, Comm. Math. Phys. **217** (2001)

The variation of the action yields Einstein's equations on the boundary

$$R_{\mu\nu}-\frac{1}{2}Rg^{(0)}_{\mu\nu}=8\pi G_{\mathrm{N}}\left(\gamma\left\langle T^{\mathrm{CFT}}_{\mu\nu}\right\rangle+T^{\mathrm{matt}}_{\mu\nu}\right)$$

where

$$\langle T^{\mathrm{CFT}}_{\mu\nu}\rangle=-\frac{\ell^3}{4\pi G_5}\left\{g^{(4)}_{\mu\nu}-\frac{1}{8}\left[(\mathrm{Tr}g^{(2)})^2-\mathrm{Tr}(g^{(2)})^2\right]g^{(0)}_{\mu\nu}-\frac{1}{2}(g^{(2)})^2_{\mu\nu}+\frac{1}{4}\mathrm{Tr}g^{(2)}g^{(2)}_{\mu\nu}\right\}$$

de Haro, Solodukhin, Skenderis, Class. Quant. Grav. **18** (2001)

We start from AdS-Schwarzschild static coordinates and make the coordinate transformation $t=t(\tau,z),\ r=r(\tau,z)$.
The line element will take a general form

$$ds^2_{(5)}=\frac{\ell^2}{z^2}(g_{\mu\nu}dx^{\mu}dx^{\nu}-dz^2)=\frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2-a^2(\tau,z)d\Omega^2_k-dz^2\right]$$

Holographic cosmology

Assuming the induced metric at the boundary to be an FRW type

$$ds^2_{(0)} = g^{(0)}_{\mu\nu} dx_\mu dx_\nu = d\tau^2 - a_{\rm h}^2(\tau) d\Omega_k^2$$

from Einstein's equations on the boundary we obtain the holographic Friedmann equation

$$\mathcal{H}_{\rm h}^2 = \frac{8\pi G_{\rm N}}{3}\rho_{\rm h} + \boxed{\frac{\ell^2}{4}\left(\mathcal{H}_{\rm h}^4 + \frac{4\mu\ell}{a_{\rm h}^4}\right)} \qquad \mathcal{H}_{\rm h}^2 = H_{\rm h}^2 + \frac{\kappa}{a_{\rm h}^2}$$

quadratic deviation

dark radiation

The second Friedmann equation can be derived from energy-momentum conservation

$$\frac{\ddot{a}_{\rm h}}{a_{\rm h}}\left(1 - \frac{\ell^2}{2}\mathcal{H}_{\rm h}^4\right) + \mathcal{H}_{\rm h}^2 = \frac{4\pi G_{\rm N}}{3}(\rho_{\rm h} - 3p_{\rm h})$$

quadratic deviation

where $\rho_{\rm h} = T_{00}^{\rm matt}, \;\; p_{\rm h} = -T^{{\rm matt}\,i}_{\;\;\;\;\;\;\;i}$

Holographic map

The time dependent bulk spacetime with metric

$$ds^2_{(5)} == \frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2 - a^2(\tau,z)d\Omega_k^2 - dz^2\right]$$

may be regarded as a z-foliation of the bulk with FRW cosmology on each z-slice. In particular:
at $z=z_{br}$: RSII cosmology, at $z=0$: holographic cosmology.

A map between z-cosmology and $z=0$-cosmology can be constructed using

$$a^2 = a_h^2\left[\left(1-\frac{\mathcal{H}_h^2 z^2}{4}\right)^2 + \frac{1}{4}\frac{\mu z^4}{a_h^4}\right], \qquad n = \frac{\dot{a}}{\dot{a}_h},$$

and the inverse relation

$$a_h^2 = \frac{a}{2}\left(1+\frac{\mathcal{H}^2 z^2}{2} + \mathcal{E}\sqrt{1+\mathcal{H}^2 z^2 - \frac{\mu z^4}{a^4}}\right) \qquad \mathcal{E} = \begin{cases} \pm 1 & \text{one-sided} \\ -1 & \text{two-sided} \end{cases}$$

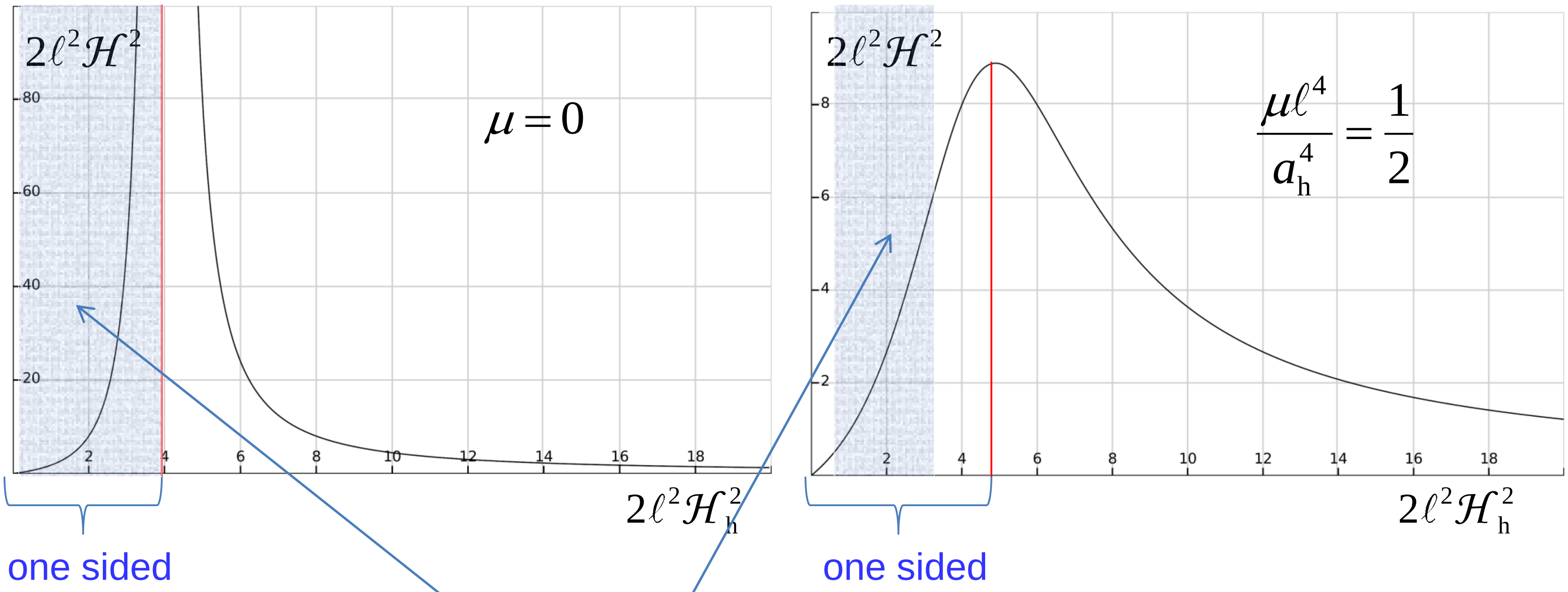
Holographic map

holographic
cosmology

$$\begin{tikzcd}
z=0 & \boxed{ds_{\mathrm{h}}^2 = d\tau^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] \arrow[d, "z"'] & \boxed{ds_{\mathrm{h}}^2 = \frac{1}{n^2} d\tilde{\tau}^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[d, "z"] \\
z = z_{\mathrm{br}} & \boxed{ds^2 = n^2 d\tau^2 - a^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] & \boxed{ds^2 = d\tilde{\tau}^2 - a^2 d\Omega_k^2}
\end{tikzcd}$$

RSII
cosmology

Hubble rate at $z=\sqrt{2}\ell$ as a function of the Hubble rate at $z=0$



physical region ($\rho_{\mathrm{h}} \geq 0$)

$$2-2\sqrt{1-\mu\ell^4/a_{\mathrm{h}}^4} \leq \mathcal{H}_{\mathrm{h}}^2\ell^2 \leq 2+2\sqrt{1-\mu\ell^4/a_{\mathrm{h}}^4}$$

The regime of large $\mathcal{H}_{\mathrm{h}}$ violates the weak energy condition $\rho \geq 0$

Effective energy density

We analyze two cosmologic scenarios:

Holographic scenario: Primary cosmology is on the AdS boundary at $z = 0$. Observers on the RSII brane on an arbitrary z-slice experience an emergent cosmology which is a reflection of the boundary cosmology.

RSII scenario: Primary cosmology is on the RSII brane at $z = z_{br}$ The cosmology on the $z = 0$ brane emerges as a reflection of the RSII cosmology.

We shall assume the presence of matter on the primary brane only and no matter in the bulk

RSII scenario

In the RSII scenario the primary braneworld is the RSII brane at $z = z_{\mathrm{br}}$. Observers at the boundary brane at $z = 0$ experience the emergent cosmology. For simplicity we take $z=\ell$ and we fine tune the tension $\sigma = \sigma_0 \equiv 3/(8\pi G_{\mathrm{N}}\ell^2)$
Then, assuming the modified Friedmann equations hold on the holographic brane, the effective energy density is given by

$$\frac{\rho_{\mathrm{h}}}{\sigma_0} = \frac{4\mathcal{E}(\rho/\sigma_0 + 1 - \mathcal{E})}{(\rho/\sigma_0 + 1 + \mathcal{E})^2 + \mu\ell^4/a^4} \qquad \mathcal{E} = \begin{cases} \pm 1 & \text{one-sided} \\ -1 & \text{two-sided} \end{cases}$$

Thus, the two-sided model with positive energy density and positive μ maps into a holographic cosmology with negative effective energy density ρ_{h}. For $\mu = 0$ the density ρ_{h} diverges with ρ as $1/\rho$. The one-sided model maps into two branches: $\varepsilon = -1$ branch identical with the two-sided map and the $\varepsilon = +1$ branch with smooth positive function $\rho_{\mathrm{h}} = \rho_{\mathrm{h}}(\rho)$.

Holographic scenario

Suppose the cosmology on the z=0 brane is known, i.e., the density ρ_{h}, pressure p_{h}, and scale a_{h} are known. If there is no matter in the bulk the induced cosmology on an arbitrary z-slice is completely determined. The general expression for the effective energy density on the RSII brane is rather complicated but simplifies considerably for $z_{\mathrm{br}} = \ell$

$$\frac{\rho}{\sigma_0} = \left| \frac{1 + \rho_{\mathrm{h}}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\mathrm{h}}/\sigma_0 - \mu\ell^4/a_{\mathrm{h}}^4}}{1 - \rho_{\mathrm{h}}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\mathrm{h}}/\sigma_0 - \mu\ell^4/a_{\mathrm{h}}^4}} \right| - \frac{\sigma}{\sigma_0}$$

where $\epsilon = \pm 1$, so the mapping is not unique. The effective energy density ρ diverges in the limit $\rho_{\mathrm{h}} \to 0$.

For an arbitrary $z_{\mathrm{br}} \neq \ell$ in the low density regime (relevant for the one sided version only) $\rho_{\mathrm{h}} \ll \sigma_0, \quad \mu\ell^4 / a_{\mathrm{h}}^4 \ll 1$

a) for $\epsilon = -1$ the energy density at quadratic order is

$$\frac{\rho}{\sigma_0} = 1 - \frac{\sigma}{\sigma_0} + \frac{z_{\mathrm{br}}^2}{\ell^2}\frac{\rho_{\mathrm{h}}}{\sigma_0} + \frac{1}{2}\frac{z_{\mathrm{br}}^2}{\ell^2}\left(\frac{z_{\mathrm{br}}^2}{\ell^2} + 1\right)\frac{\rho_{\mathrm{h}}^2}{\sigma_0^2} + \ldots .$$

and pressure at linear order is

$$p = -(\sigma_0 - \sigma) + \frac{z_{\mathrm{br}}^2}{\ell^2} p_{\mathrm{h}} + \ldots .$$

Hence, at linear order the effective fluid on the RSII brane satisfies the same equation ofstate as the fluid on the holographic brane. The cosmological constant term will vanish on both branes if the RSII fine tuning condition is imposed.

b) for $\epsilon = +1$ we find

$$\frac{\rho}{\sigma_0} = \underbrace{\frac{z_{\rm br}^2/\ell^2+1}{z_{\rm br}^2/\ell^2-1} - \frac{\sigma}{\sigma_0}}_{\text{Cosmological constant term}} + \frac{z_{\rm br}^2/\ell^2}{(z_{\rm br}^2/\ell^2-1)^2}\frac{\rho_{\rm h}}{\sigma_0} - \frac{z_{\rm br}^2/\ell^2}{2(z_{\rm br}^2/\ell^2-1)^3}\frac{\mu\ell^4}{a_{\rm h}^4} + \dots$$

- The effective density ρ on the RSII brane differs from ρ_h on the holographic brane by a multiplicative constant.
- For $\sigma=\sigma_0$ the effective cosmological constant on the RSII brane does not vanish and is equal to

$$\Lambda_{\rm br} = \frac{6}{\ell^2}\frac{z_{\rm br}^2/\ell^2+1}{z_{\rm br}^2/\ell^2-1} - \frac{6}{\ell^2}.$$

- The effective density ρ diverges in the limit $z_{br}/\ell \to 1$

Conclusions and outlook

- We have explicitly constructed the mapping between two cosmological braneworlds: holographic and RSII .
- The cosmologies are governed by the corresponding modified Friedman equations.
- There is a clear distinction between 1-sided and 2-sided holographic map with respective 1-sided and 2-sided versions of RSII model.
- In the 2-sided map the low-density regime on the two-sided RSII brane corresponds to the high **negative** energy density on the holographic brane
- The low density regime can be made simultaneous on both branes only in the one-sided RSII

Speculations

It is conceivable that we live in a braneworld with emergent cosmology. That is, dark energy and dark matter could be emergent phenomena induced by what happens on the primary braneworld.

For example, suppose our universe is a one-sided RSII braneworld the cosmology of which is emergent in parallel with the primary holographic cosmology. If $\rho_{\rm h}$ describes matter with the equation of state satisfying $3p_{\rm h} + \rho_{\rm h} > 0$, as for, e.g., CDM, we will have an asymptotically de Sitter universe on the RSII brane. If we choose ℓ so that Λ fits the observed value, the quadratic term will be comparable with the linear term today but will strongly dominate in the past and hence will spoil the standard cosmology. However, the standard ΛCDM cosmology could be recovered by including a negative cosmological constant term in $\rho_{\rm h}$ and fine tune it to cancel Λ up to a small phenomenologically acceptable contribution.