Consistency relations for large-scale structures: applications to observables.

Luca Alberto Rizzo
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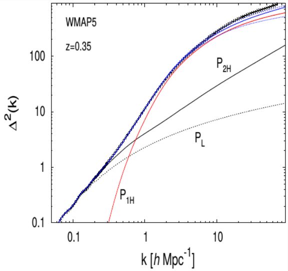
26/04/2017

Large-scale structure formation

Perturbation theory

Analytical results limited to large scales



Valageas,Nishimichi & Taruya,PhysRevD.87.083522

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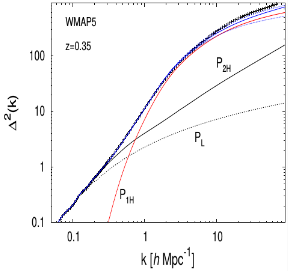

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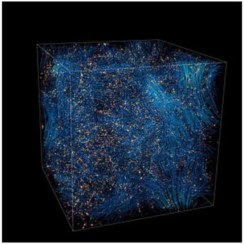
Small-scale structure formation

Numerical simulations and/or phenomenological models

Time consuming and based on parametrization



Adamek et al, Nature Physics 12, 346349 (2016)

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CONSISTENCY RELATIONS FOR LARGE-SCALE STRUCTURE

Factorization of $\left\langle \tilde{\delta}_{\mathrm{L0}}(\mathbf{k}_1) \cdots \tilde{\delta}_{\mathrm{L0}}(\mathbf{k}_\ell) \;\; \tilde{\rho}_1(\mathbf{k}_1) \cdots \tilde{\rho}_n(\mathbf{k}_n) \right\rangle$

in terms of $\langle \tilde{\rho}_1(\mathbf{k}_1) \cdots \tilde{\rho}_n(\mathbf{k}_n) \rangle$ and $P_L(k)$

$\tilde{\delta}_{\mathrm{L0}}(\mathbf{k}_i)$ linear density contrast

$\tilde{\rho}_i(\tilde{\delta}_{\mathrm{L0}}(\mathbf{k}_i))$ cosmological field ($\mathbf{k}_i$ can be non-linear)

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CRs are valid at non-linear level and only rely on the weak equivalence principle and Gaussian initial conditions

Assuming Gaussian initial condition and $\ell = 1$

$$C^{1,n}(\vec{x}) = \langle \delta_{L0}(\vec{x}) \rho_1 \ldots \rho_n \rangle = \int \mathcal{D}\delta_{L0} \; e^{-\delta_{L0} \cdot C_{L0}^{-1} \cdot \delta_{L0}/2} \; \delta_{L0}(\vec{x}) \rho_1 \ldots \rho_n$$

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integrating by parts over δ_{L0} and expanding in Fourier space

$$\tilde{C}^{1,n}(\mathbf{k}') = P_{L0}(k')\, \tilde{R}^{1,n}(-\mathbf{k}'),$$

where we defined the Fourier-space correlation and response function as

$$\tilde{C}^{1,n}(\mathbf{k}') = \langle \tilde{\delta}_{L0}(\mathbf{k}')\tilde{\rho}_1 \ldots \tilde{\rho}_n \rangle, \quad \tilde{R}^{1,n}(\mathbf{k}') = \langle \frac{\mathcal{D}[\tilde{\rho}_1 \ldots \tilde{\rho}_n]}{\mathcal{D}\tilde{\delta}_{L0}(\mathbf{k}')} \rangle.$$

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$$\tilde{\delta}(\mathbf{k}_i, \tau) = \int \frac{d\mathbf{x}}{(2\pi)^3} e^{-\imath \mathbf{k}_i \cdot \mathbf{x}} \delta(\mathbf{x}, \tau) = \int \frac{d\mathbf{q}}{(2\pi)^3} e^{-\imath \mathbf{k}_i \cdot \mathbf{x}}$$

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$$R^{1,n} = \left\langle \int \frac{d\mathbf{q}_1 \cdots \mathbf{q}_n}{(2\pi)^{3n}} \sum_{n}^{i=1} \mathbf{k}_i \cdot \frac{\mathcal{D}\mathbf{\Psi}}{\mathcal{D}\tilde{\delta}_{\mathrm{L0}}(\mathbf{k}', \tau)} e^{-\mathbf{k}_1 \cdot (\mathbf{q}_1 + \mathbf{\Psi}_1) - \cdots - \mathbf{k}_n \cdot (\mathbf{q}_n + \mathbf{\Psi}_n)} \right\rangle,$$

where we introduced the displacement field $\mathbf{\Psi}(\mathbf{q}, t) = \mathbf{x}(\mathbf{q}, t) - \mathbf{q}$.

We will compute $\mathcal{D}\mathbf{\Psi}/\mathcal{D}\tilde{\delta}_{\mathrm{L0}}(\mathbf{k}', \tau)$ in linear and non-linear case.

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For $\boldsymbol{\Psi} = \boldsymbol{\Psi}_L$, using continuity equation we have

$$k' \to 0 \; : \; \frac{\mathcal{D}\boldsymbol{\Psi}_L}{\mathcal{D}\tilde{\delta}_{L0}(\mathbf{k}',\tau)} \to \imath \frac{\mathbf{k}'}{k'^2} D(\tau),$$

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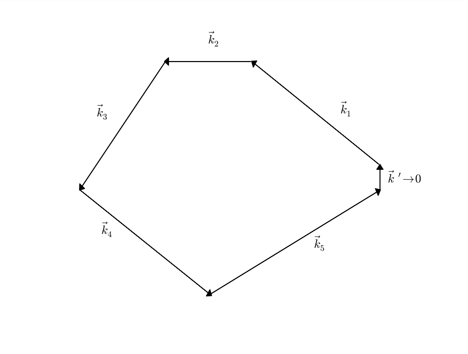
Same happens for $\mathbf{\Psi}$ in the squeezed limit



The response function looks like

$$R^{1,n}_{k'\to 0} = \left\langle \tilde{\delta}(\mathbf{k}_1, \tau_1) \cdots \tilde{\delta}(\mathbf{k}_n, \tau_n) \right\rangle \frac{\mathbf{k}'}{k'^2} \cdot \sum_{i=1}^{n} \mathbf{k}_i \; D(\tau_i),$$

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and so the consistency relations for density fields

$$\left\langle \tilde{\delta}_{\mathrm{L0}}(\mathbf{k}') \tilde{\delta}(\mathbf{k}_1, \tau_1) \cdots \tilde{\delta}(\mathbf{k}_n, \tau_n) \right\rangle'_{k'\to 0} =$$

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Different-time correlators needed!

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$$\langle \tilde{\delta}(\mathbf{k}') \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{j=n+1}^{n+m} \tilde{\lambda}(\mathbf{k}_j) \rangle'_{k'\to 0} = P_L(k') \frac{d\ln D}{d\tau}$$

$$\times \frac{\mathbf{k}'}{k'^2} \cdot \sum_{i=n+1}^{n+m} \mathbf{k}_i \ \langle \tilde{\delta}(\mathbf{k}_i) \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{\substack{j=n+1 \\ j\neq i}}^{n+m} \tilde{\lambda}(\mathbf{k}_j) \rangle'$$

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At the lowest order

$$b_g \langle \tilde{\delta}_g(\mathbf{k}') \tilde{\delta}_g(\mathbf{k}) \tilde{\lambda}_g(-\mathbf{k}) \rangle'_{k' \to 0} = -\frac{\mathbf{k} \cdot \mathbf{k}'}{k'^2} \frac{d \ln D}{d\tau} P_L^{\delta_g \delta_g}(k') \times P^{\delta_g \delta_g}(k),$$

Large-scale limit bias b_g for galaxy bias.

A connection to observables via 2D smoothed galaxy density contrast

$$\delta_g^s(\vec{\theta}) = \int d\tau \, I_g(\tau) \int d\mathbf{k} \, \tilde{W}_\Theta(k_\perp r) e^{\imath k_\parallel r + \mathrm{i}\mathbf{k}_\perp \cdot r\vec{\theta}} \tilde{\delta}_g(\mathbf{k}, \tau)$$

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and the smoothed CMB temperature anisotropies due to ISW effect

$$\Delta_{\mathrm{ISW}}^s(\vec{\theta}) = \int d\tau\, I_{\mathrm{ISW}}(\tau) \int d\mathbf{k}\, \tilde{W}_\Theta(k_\perp r) e^{\mathrm{i}k_\parallel r + \mathrm{i}\mathbf{k}_\perp \cdot r\vec{\theta}} \times \frac{\tilde{\lambda} + \mathcal{H}\tilde{\delta}}{k^2}$$

$$I_g(\tau) = \left| \frac{dz}{d\tau} \right| n_g(z), \;\; I_{\mathrm{ISW}}(\tau) = 8\pi\mathcal{G}\bar{\rho}_0 \frac{e^{-\tau_{\mathrm{opth}}}}{a}$$

$$\mathbf{k} = \left(k_\parallel, \mathbf{k}_\perp\right)$$

$\tilde{W}_\Theta(k_\perp r)$ Fourier transform of 2D symmetric window function

Let us consider

$$\xi_3(\delta_g^s, \delta_{g_1}^s, \Delta_{\mathrm{ISW}_2}^s) = \langle \delta_g^s(\vec{\theta})\, \delta_{g_1}^s(\vec{\theta}_1)\, \Delta_{\mathrm{ISW}_2}^s(\vec{\theta}_2) \rangle,$$

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$$\times P_L(k_\perp, \tau)P_{g_1,m}(k_{1\perp}, \tau)J_1(k_\perp r|\vec{\theta} - \vec{\theta}_2|) \times J_1(k_{1\perp} r|\vec{\theta}_1 - \vec{\theta}_2|)$$

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The geometric factor may not require the computation of the right-hand side.

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THANK YOU FOR THE ATTENTION

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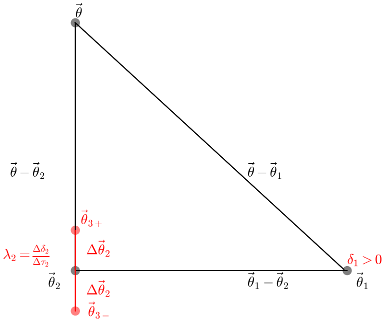

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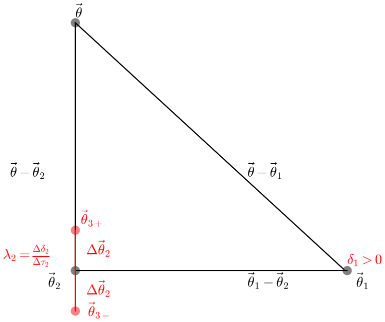

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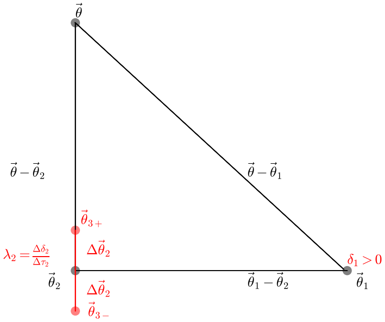
$\lambda_{2+} = -\frac{\delta_2 - \delta_{3+}}{\Delta\tau} > 0$ and $\lambda_{2-} = -\frac{\delta_2 - \delta_{3-}}{\Delta\tau} > 0$, regardless of $\delta(\vec{\theta}) > 0$ or $\delta(\vec{\theta}) < 0$;



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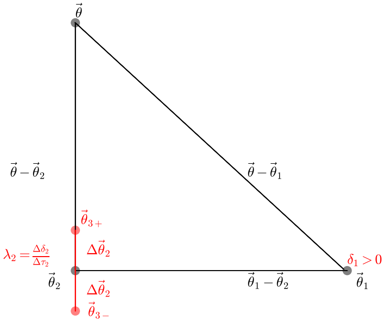
$\lambda_{2+} = -\frac{\delta_2 - \delta_{3+}}{\Delta\tau} > 0$ and $\lambda_{2-} = -\frac{\delta_2 - \delta_{3-}}{\Delta\tau} > 0$, regardless of $\delta(\vec{\theta}) > 0$ or $\delta(\vec{\theta}) < 0$;

dependency of $\langle\delta_1\lambda_2\rangle$ on $\delta(\vec{\theta})$ is quadratic $\Rightarrow$ first-order response vanishes $\Rightarrow$ ξ_3 vanishes



Let us consider $\Delta\delta_{\rm L0}(\mathbf{k}')$, from which we have

$$\Delta\mathbf{\Psi}(\mathbf{q}) = \int d\mathbf{k}' \frac{\mathcal{D}\mathbf{\Psi}_L}{\mathcal{D}\tilde{\delta}(\mathbf{k}',\tau)}\Delta\tilde{\delta}_{\rm L0}(\mathbf{k}'),$$

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$$k' \to 0 \,:\, \frac{\mathcal{D}\boldsymbol{\Psi}}{\mathcal{D}\tilde{\delta}(\mathbf{k}',\tau)} \to \imath \frac{\mathbf{k}'}{k'^2} D(\tau),$$