# Rare B decay theory

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Higgs Tasting Workshop
Benasque, 19 May 2016

# Contents

1. Rare exclusive B decays: generalities
   (generalised) form factors an factorisation

2. Bs->mu mu: the gold-plated mode

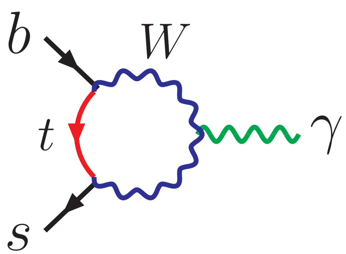
3. B -> K(*) l l: selected topics

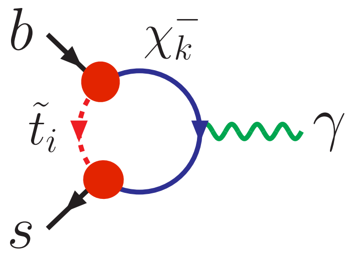
# (You know the) motivation

- After the Higgs discovery, the naturalness problem is a reality. But even natural new physics may lie beyond the LHC energy reach. ATLAS & CMS may point to that.

- This puts precision Higgs and flavour at the centre of the quest for physics beyond the Standard Model

- Natural BSM models tend to have a flavour problem
  eg SUSY




new sources of flavour violation

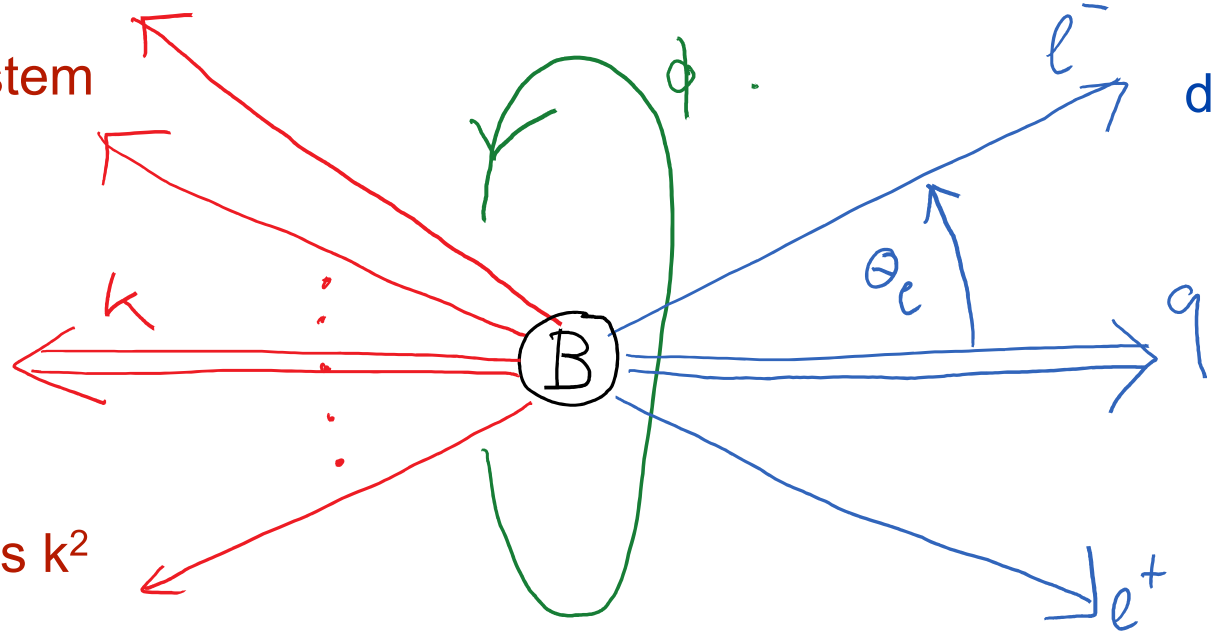
$B \to K^* \gamma$, $K^* ll$
$B \to X_s \gamma$

- Unprecedented statistics & interesting results from LHCb, with Belle2 rapidly approaching

# Semileptonic decays

hadronic system

dilepton




hadronic mass $k^2$

dilepton mass $q^2$

hadronic angles & energies
equivalently:
  angular momentum L'
  helicity λ'
  (+ more if >2 hadrons)

one hadronic/leptonic
relative angle Φ
if >1 hadron

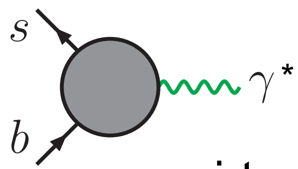
leptonic angle
equivalently:
  angular momentum L
  helicity λ

B has spin zero => $\lambda = \lambda'$

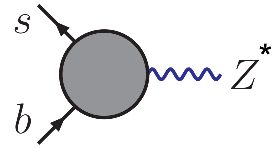
Observing Φ requires interference  $A(\lambda_1)\, A(\lambda_2)^*$  $\exp(i\,(\lambda_1 - \lambda_2)\Phi\,)$

# semileptonic ΔB=ΔS=1 Hamiltonian

in SM mainly




virtuality $\sim M_W^2$




$C_9$ : dilepton from vector current (L=1)

$$Q_{9V} = \frac{\alpha_{\rm em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l)$$

$C_{10}$ : dilepton from axial current (L=1 or 0)

$$Q_{10A} = \frac{\alpha_{\rm em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma^5 l)_A$$

- both can be obtained from Z' exchanges
- or leptoquarks

Descotes-Genon et al; Altmannshofer et a
Crivellin et al; Gauld et al; ...

Alonso-Grinstein-Martin Camalich; Hiller-Schmaltz; Allanach et al; Gripajos et al; ...

$C_7$ : dilepton produced through photon (virtuality $q^2$, pole at $q^2$=0)

$$Q_{7\gamma} = \frac{e}{16\pi^2} m_b\,(\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}$$

- strongly constrained from inclusive b->s decay

BSM: also parity-transformed operators ($C_9'$, $C_{10}'$, $C_7'$)

**$C_9$, $C_{10}$ can depend on the lepton flavour.**

**Universal BSM effects in $C_9$ mimicked by a range of SM effects**

# hadronic ΔB=ΔS=1 Hamiltonian

Four-quark operators with net ΔB=ΔS=1
in SM mainly (from tree-level W exchange):

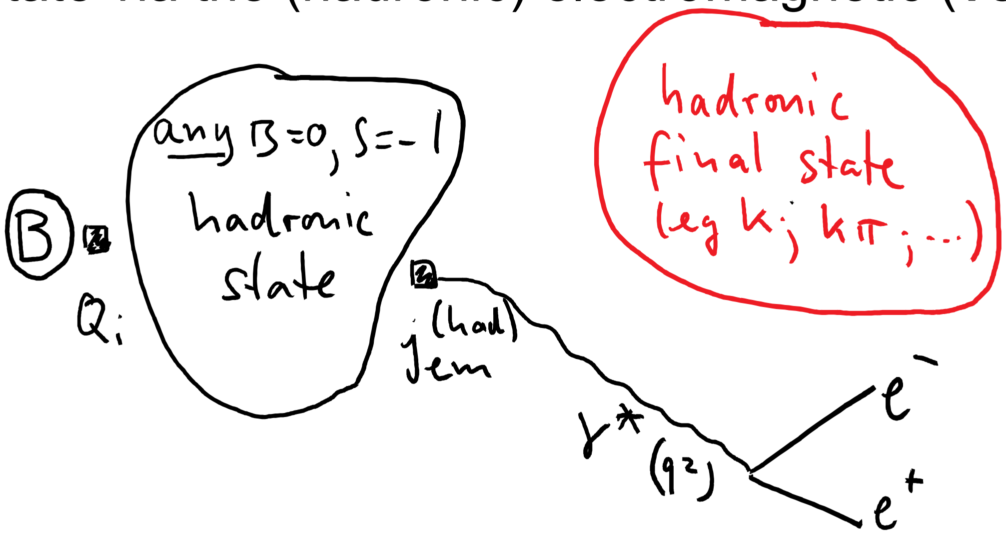
$$Q_1 = (\bar{s}\gamma_\mu P_L b)(\bar{u}\gamma^\mu P_L u)$$

$$Q_2 = (\bar{s}^i\gamma_\mu P_L b^j)(\bar{u}^j\gamma^\mu P_L u^i)$$

to lesser extent also (hadronic) QCD penguin operators

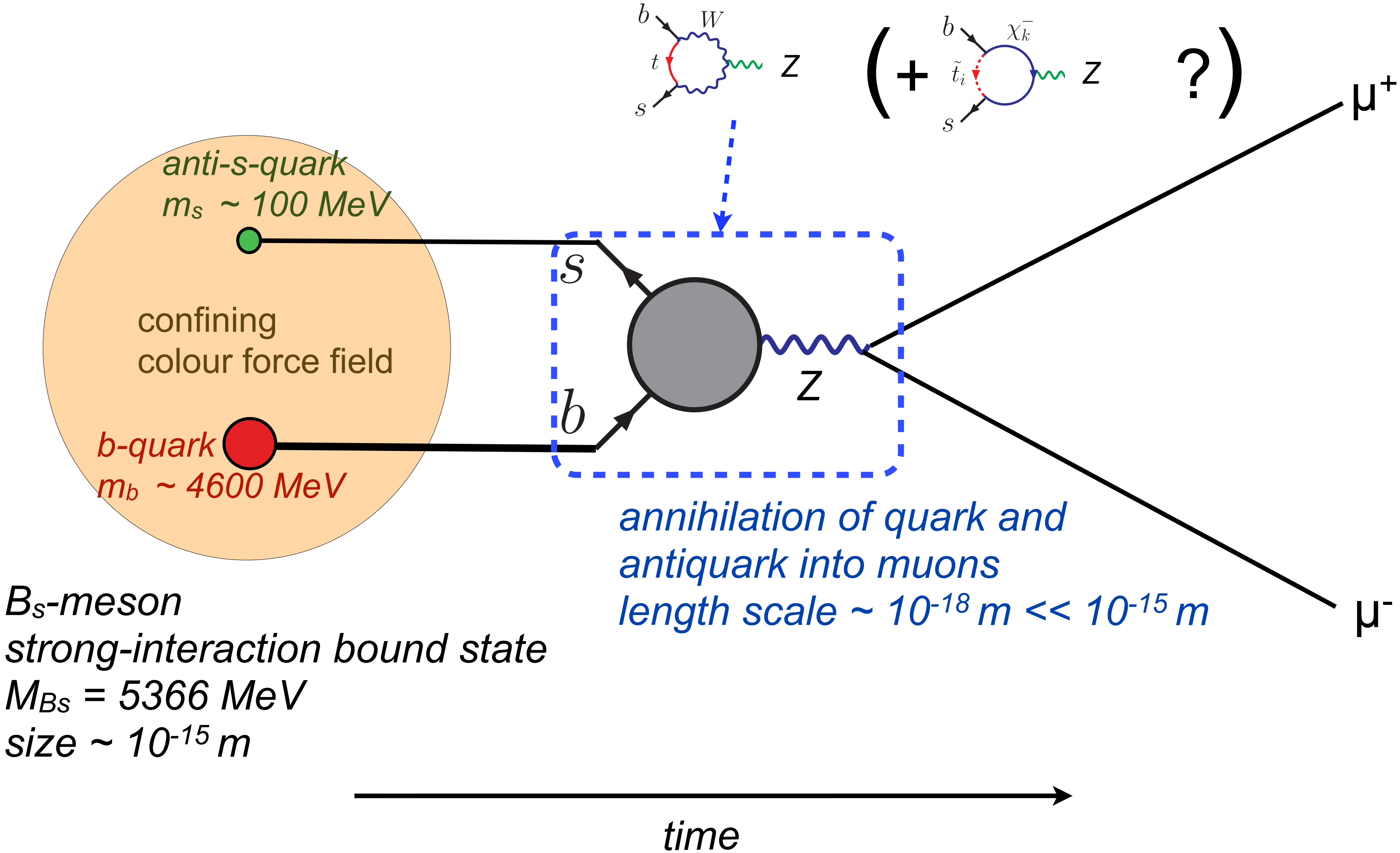
*

dilepton is produced by conversion of a hadronic intermediate state via the (hadronic) electromagnetic (**vector**) current



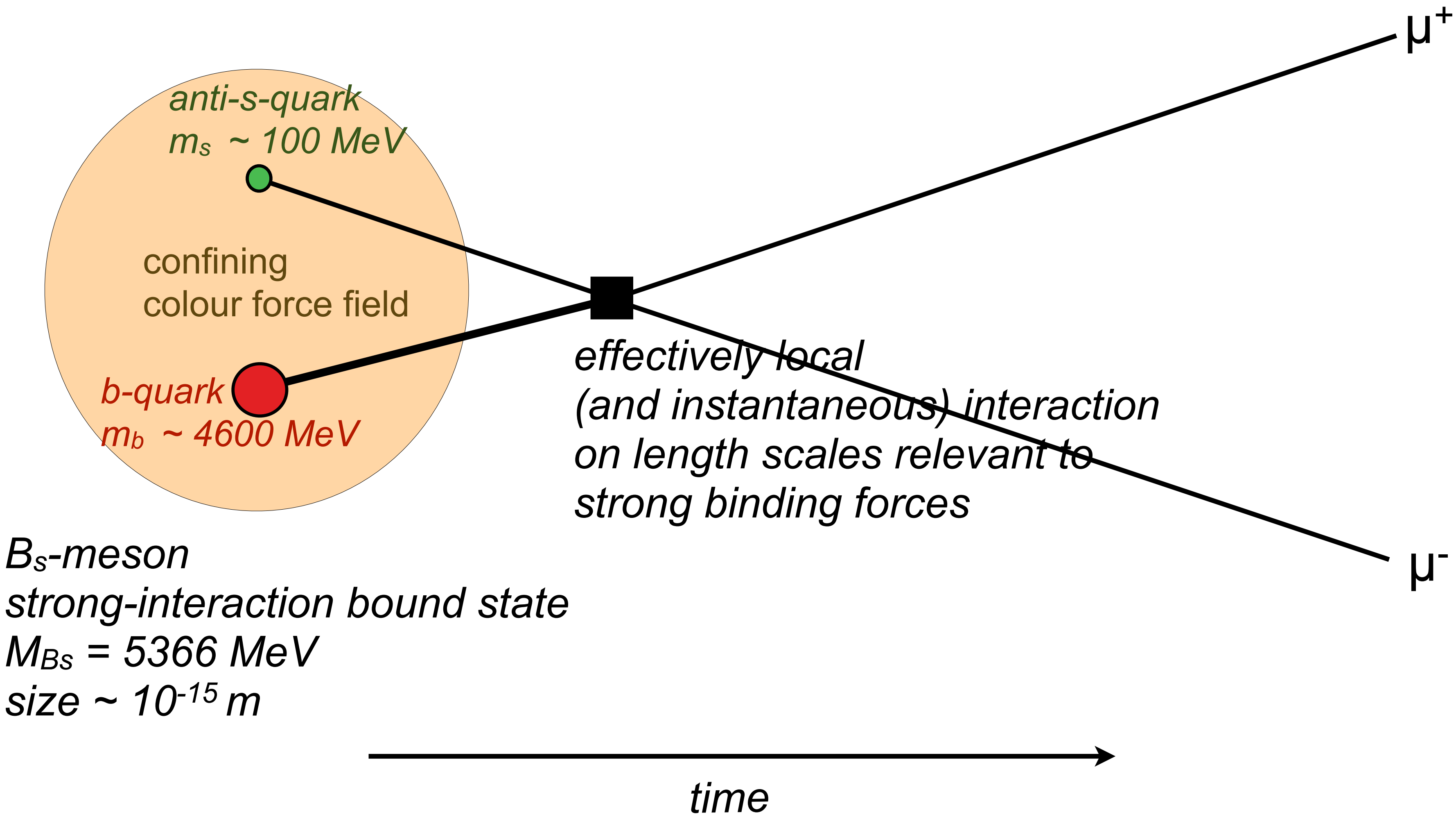
# Zero hadrons: $B_s \to \mu\mu$

**C & P forbid creation through vector current!**
**No hadronic intermediate states, no $C_9$**



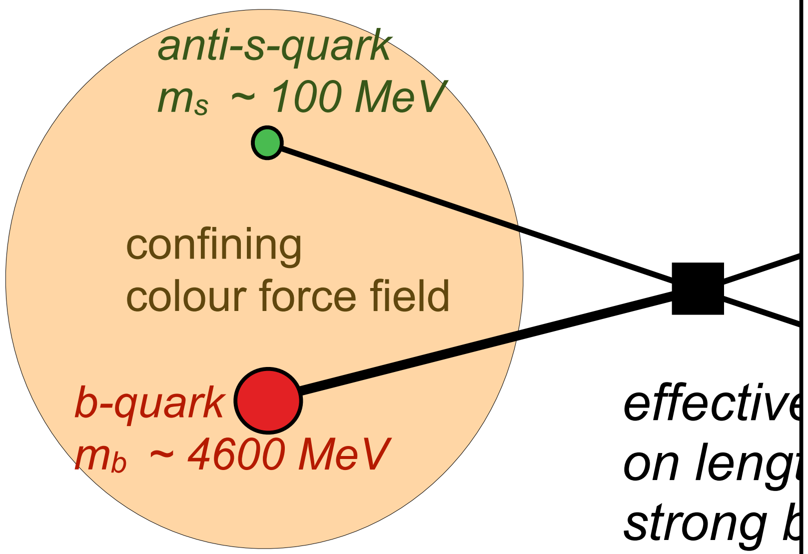
# Factorisation (Wilsonian)

more accurately drawn to scale



# weak Hamiltonian

more accurately drawn to scale



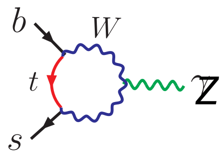

*effective*
*on leng*
*strong b*

*$B_s$-meson*
*strong-interaction bound state*
*$M_{Bs}$ = 5366 MeV*
*size ~ $10^{-15}$ m*

technically: effective local four-fermion interaction (local operator)

$$Q_{10A} = \frac{\alpha_{\rm em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma^5 l)$$

coupling constant (Wilson coefficient) $C_{10}$
**calculable in perturbation theory, including BSM effects**

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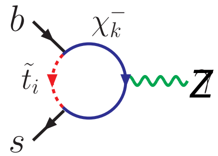
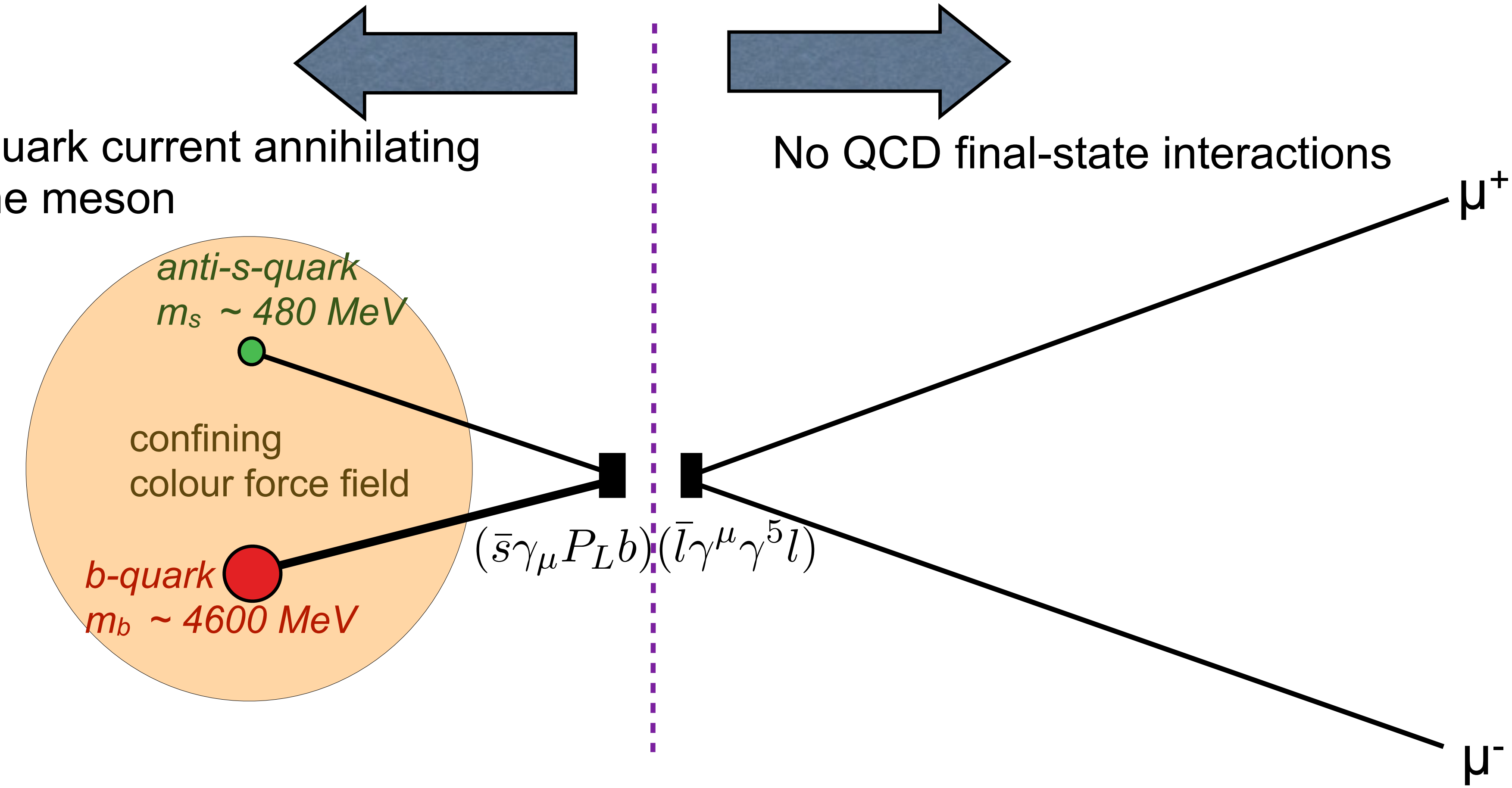



?)

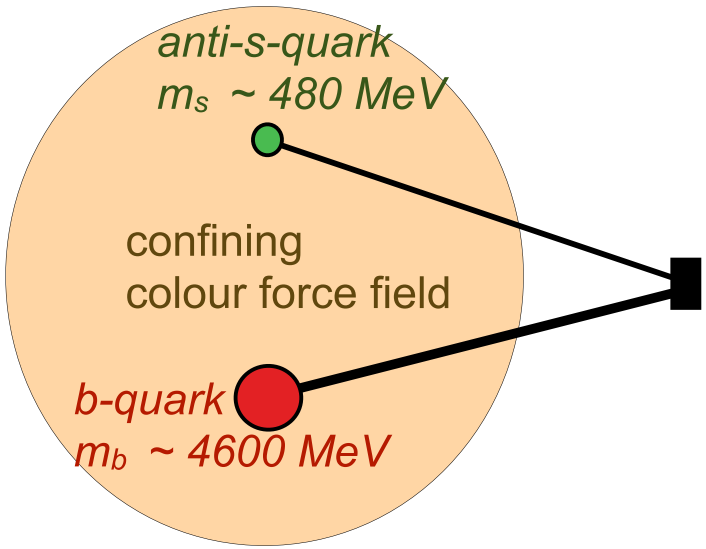
# Implies "naive" factorisation




parameterised by a decay constant
not calculable in perturbation theory

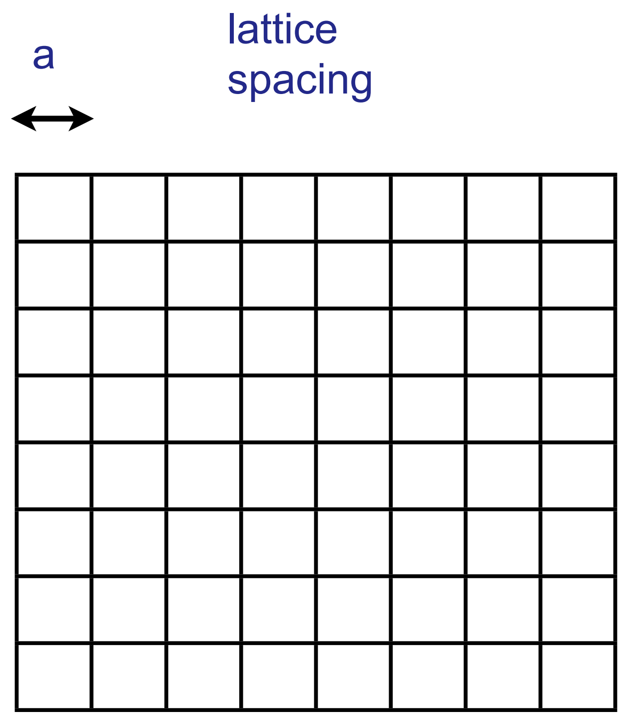
quantum electrodynamics only
very well controlled theoretically

# Calculating the decay constant




$$\langle 0|\bar{s}\gamma_\mu P_L b|B_s\rangle = -\frac{1}{2}p_\mu f_{B_s}$$

$B_s$ (leptonic) decay constant



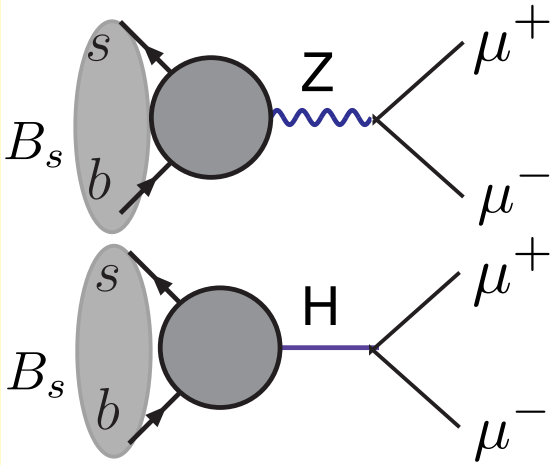

numerical first-principles calculation possible with lattice-regularised path integral (lattice QCD) (expansion in $1/m_b$ needed)

$$f_{B_s} = (224 \pm 5)\,\mathrm{MeV}$$

Flavour Lattice Averaging Group 2013
Eur.Phys.J.C74 (2014) 2890

# $C_{10}$ and $B_s$ -> mu mu

[slide based on talk by M Steinhauser, BEACH 2014]




very NP sensitive (Z penguin $C_{10}$, heavy Higgses)

SM helicity suppression

- NLO QCD corrections [Buchalla,Buras'93'99; Misiak,Urban'99]
- leading-$m_t$ NLO electroweak corrections [Buchalla,Buras'98]
- uncertainty (from higher orders): $\approx 7\%$

exp uncertainty will reach this during HL run

- NNLO QCD
- NLO EW

[Bobeth,Gorbahn,Hermann,Misiak,Stamou,Steinhauser'13]

**missing** $\mathcal{O}(\alpha_{em})$

- no enhancement factor (like $\frac{1}{\sin^2\theta_W}$, $\frac{m_t^2}{M_W^2}$ or $\ln^2\frac{M_W^2}{\mu_b^2}$)
- soft Bremsstrahlung: $B_s \to \mu^+\mu^- + (n\gamma)$ $(n = 0, 1, 2, \ldots)$
- Can QED corrections ($\alpha_{em}/\pi \approx 2\times 10^{-3}$) remove helicity suppression factor ($m_\mu^2/M_{B_s}^2 \approx 10^{-4}$)?

helicity suppression remains

New prediction

$$\overline{\mathcal{B}}_{s\mu} = (3.65 \pm 0.06)\, R_{t\alpha}\, R_s \times 10^{-9} = 3.65 \pm 0.23 \times 10^{-9}$$

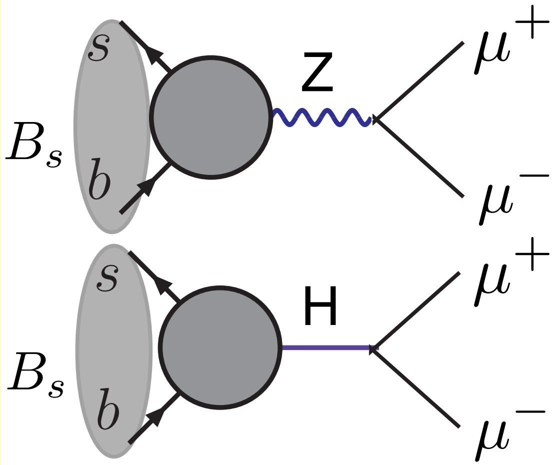
$$\overline{R}_{ql} = \frac{\overline{\mathcal{B}}_{ql}}{\ldots} = \frac{1 + \mathcal{A}^{ll}_{\Delta\Gamma}\, y_q}{\ldots}\left(|S|^2 + |P\right.$$

parametric uncertainties dominate

$$R_s = \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{1.615}$$

# $C_{10}$ and $B_s$ -> mu mu

[slide based on talk by M Steinhauser, BEACH 2014

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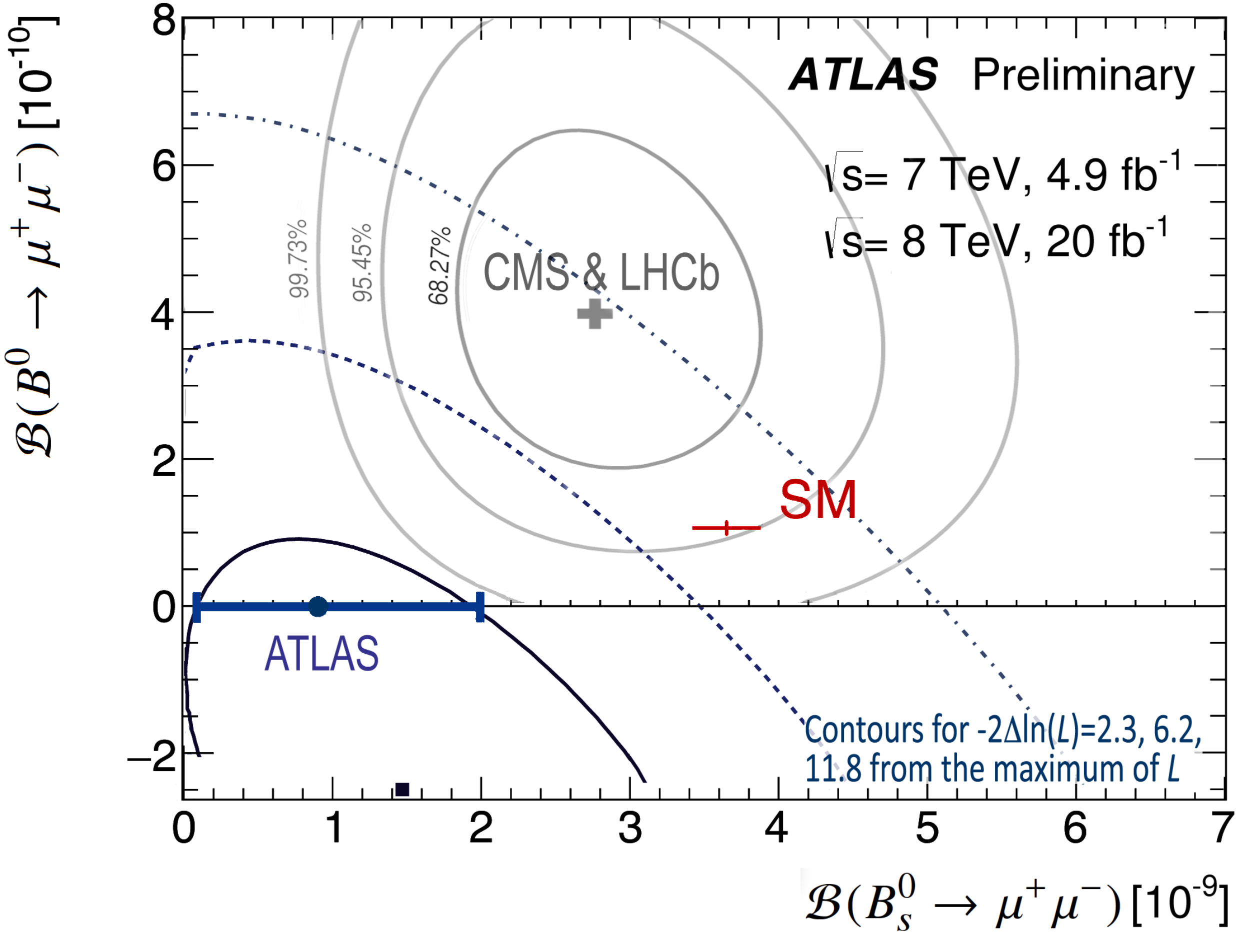
$R = \left(\frac{f_{B_s}[\mathrm{MeV}]}{\ldots}\right)^2 \left(\frac{|V_{cb}|}{\ldots}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{\ldots}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{\ldots}$

No contamination from long-distance charm. Precision ready for HL-LHC

# $B_s \to \mu\mu$: experiment




S Palestini (ATLAS), Moriond 2016

Some indication of a suppression w.r.t. SM: $C_{10} < C_{10}^{SM}$ ?

good prospects from LHCb, (increasingly) CMS; ATLAS
eventually HL-LHC (completely dominated by experimental error)

# Two hadrons: $B \to K^*(K\pi)l^+l^-$

Resonant production: hadronic angular momentum L'=1

leptonic angular momentum L=1 (L=0 helicity-suppressed)

classify decay amplitudes according to leptonic mechanism and helicity $\lambda$

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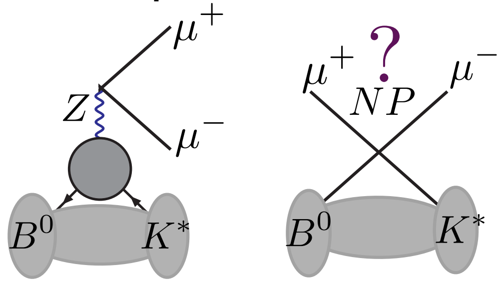
- axial leptonic current



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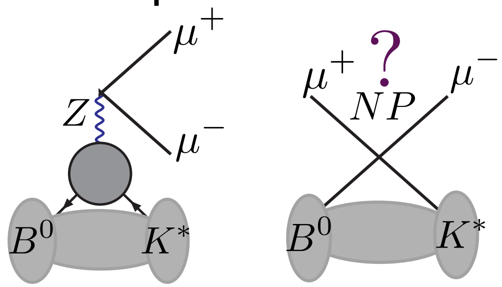


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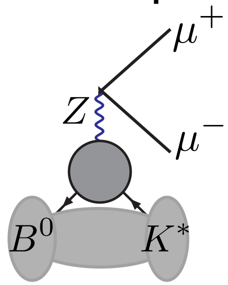


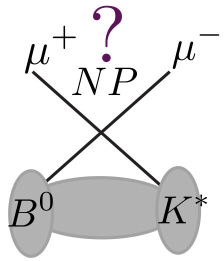


K* helicity

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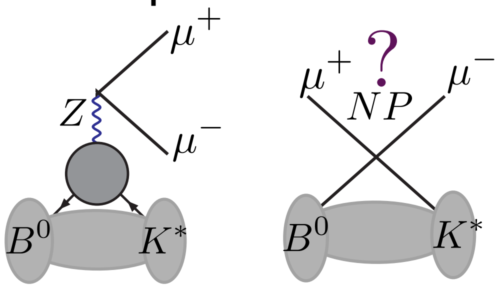


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one form factor (nonperturbative) per helicity
amplitudes factorize naively
[one more amplitude if not neglecting lepton mass]

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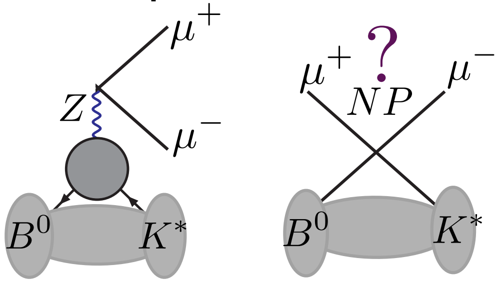


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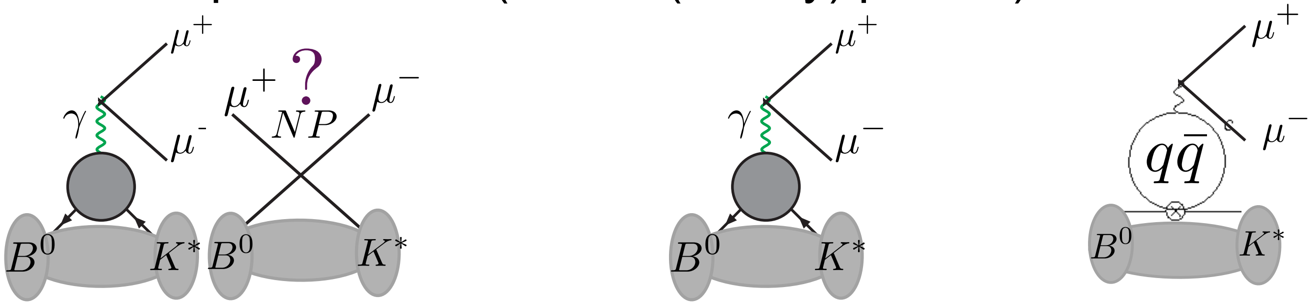
- vector lepton current (in SM: (mainly) photon)

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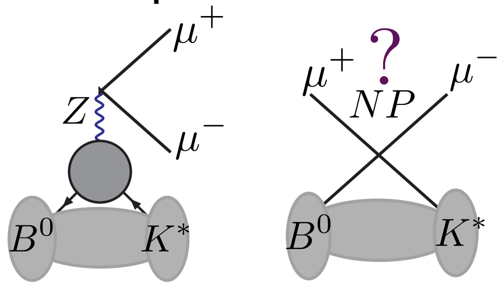


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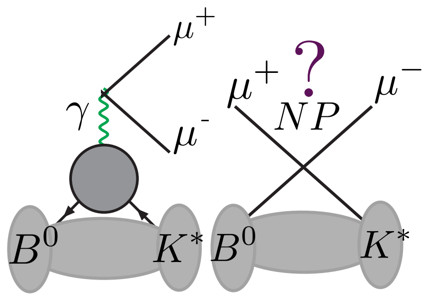


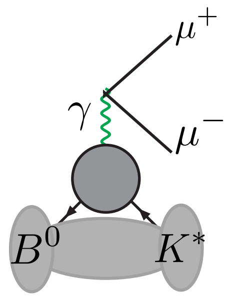


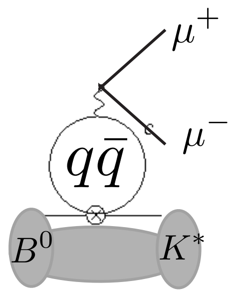


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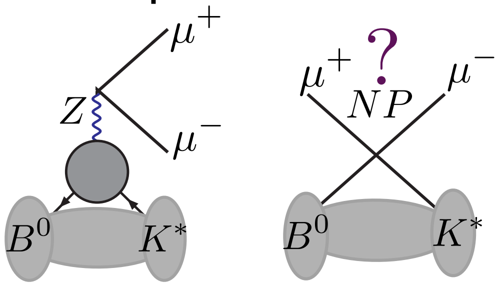


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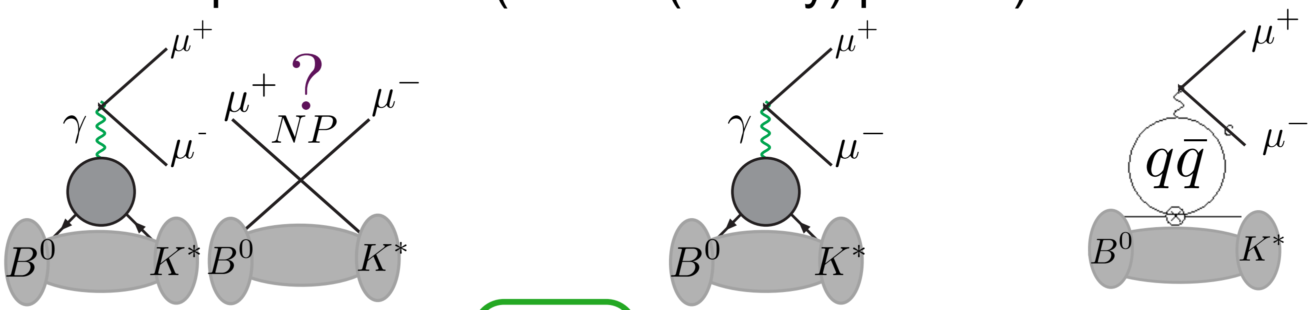


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photon pole at $q^2=0$

intermediate hadronic states
do **not** factorize naively

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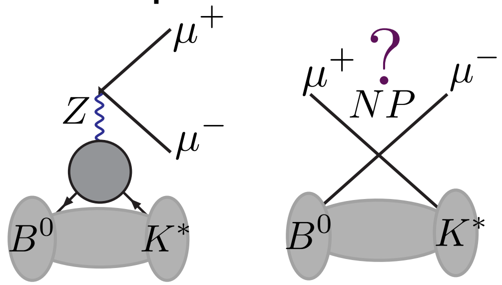


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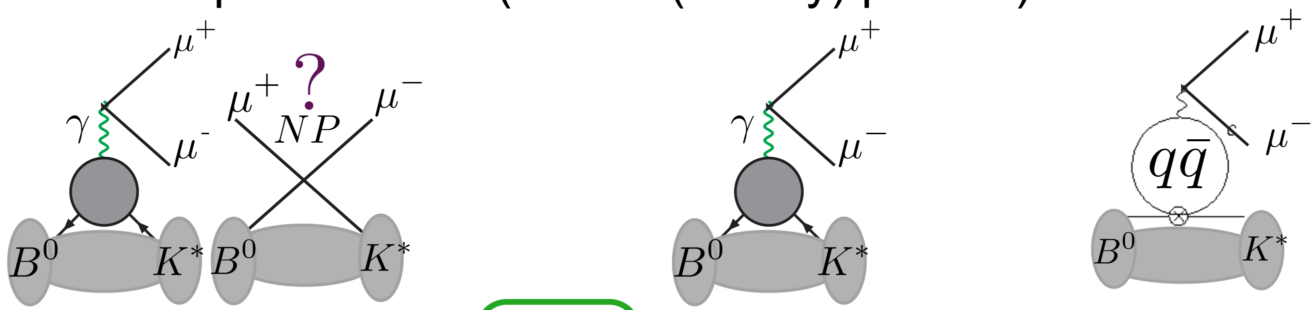


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photon pole at $q^2$=0

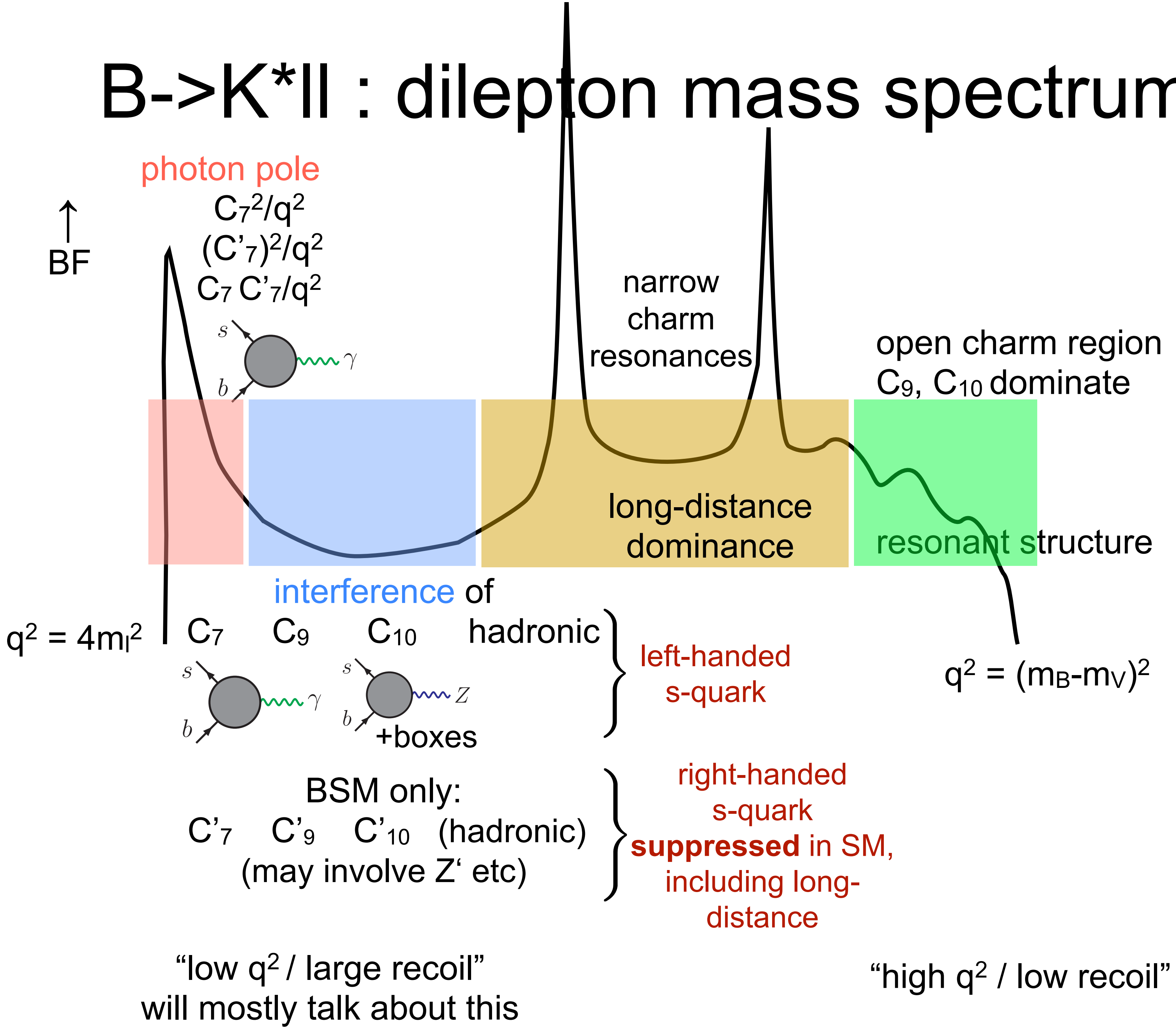
two form factors interfere for each helicity

intermediate hadronic states
do **not** factorize naively

natural and **transparent** discussion in terms of 6 (7 if $m_l$ != 0) helicity amplitudes

SJ, Martin Camalich 2012

# B->K*ll : dilepton mass spectrum



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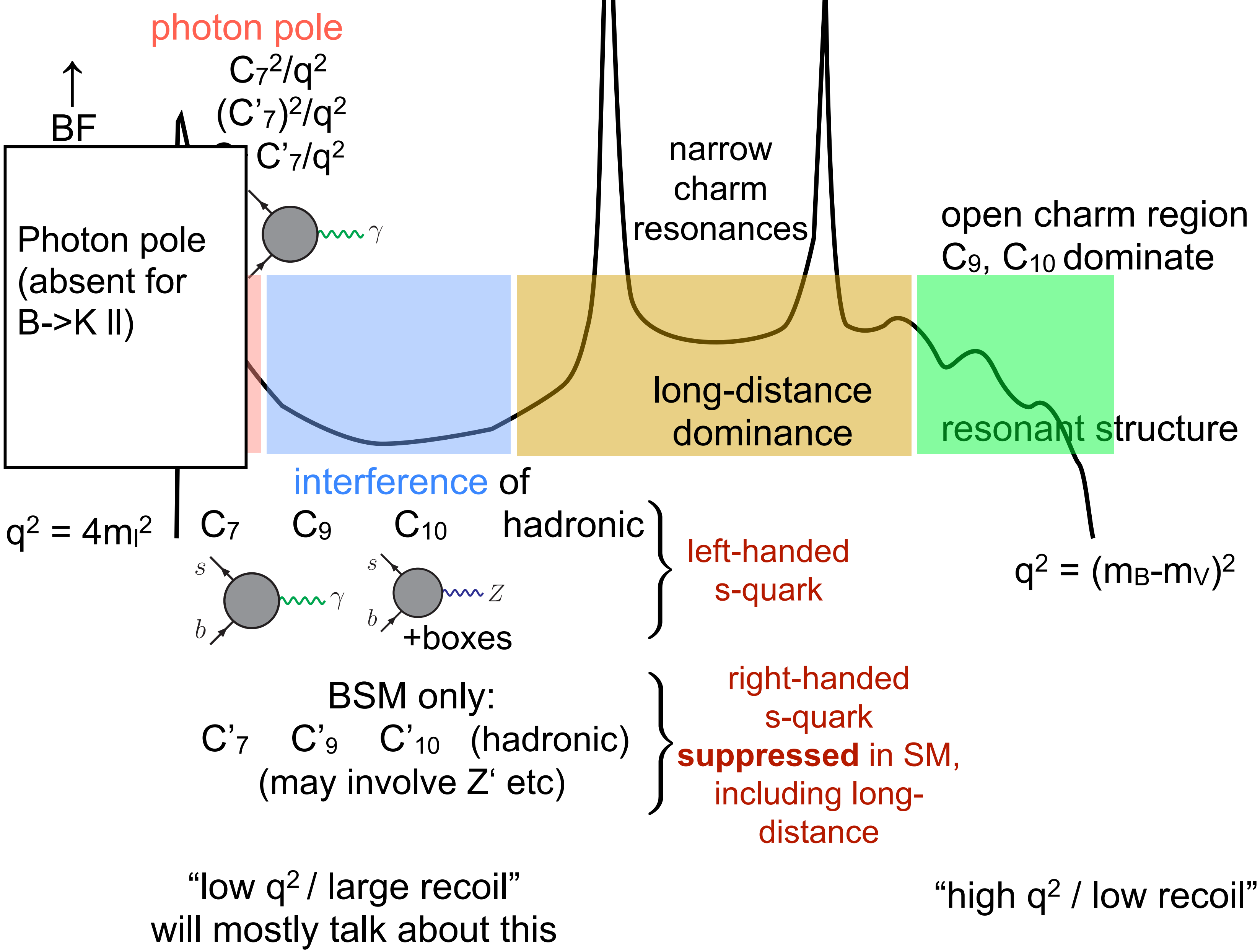
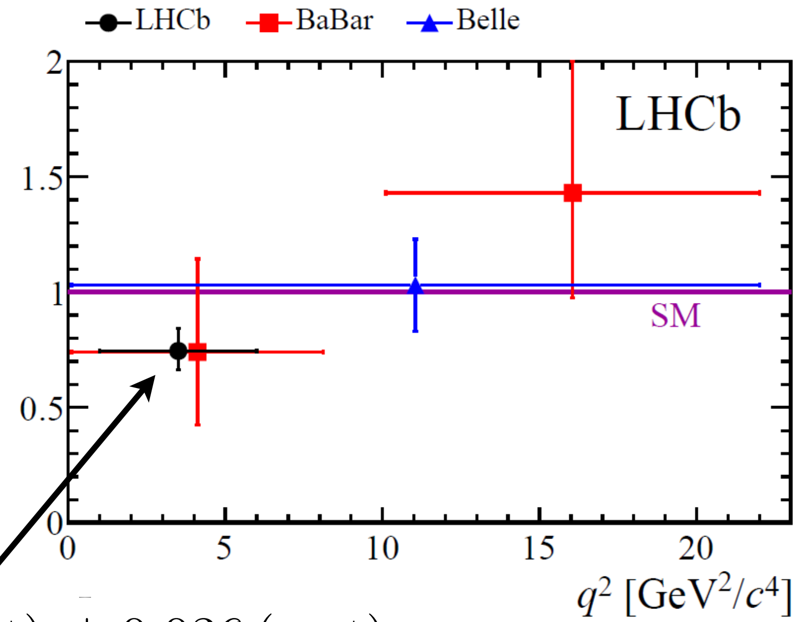


# Lepton universality violation

$$R_K = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{{\rm d}\Gamma[B^+\to K^+\mu^+\mu^-]}{{\rm d}q^2}{\rm d}q^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{{\rm d}\Gamma[B^+\to K^+e^+e^-]}{{\rm d}q^2}{\rm d}q^2}$$




$$0.745^{+0.090}_{-0.074}\,(\text{stat})\ \pm 0.036\,(\text{syst})$$

LHCb, PRL 113 (2014) 151601

All form-factor and non-local hadronic uncertainties cancel (lepton-universal) if lepton masses negligible (as is the case for 1 GeV$^2$ lower cutoff) Hiller, Krueger 2003
$R_K^{(th)} \approx 1$

a large effect ! (Would be consistent with reduced $C_{10}^{(\mu)}$ or $C_9^{(\mu)}$ )

**Main theory concern is role of soft photon radiation. No published theoretical study.**
**Informal consensus that effect is at percent level at most.**

# Further lepton universality tests

SM predicts lepton universality to great accuracy. In particular, apart from lepton mass effects all helicity amplitudes coincide and hence, to our accuracy, the theory error on any LUV ratio or difference is zero. Altmannshofer, Straub; Hiller, Schmaltz; SJ, Martin Camalich

Two particular classes of observables:

(1)

$$R_{K^*_X} = \frac{\mathcal{B}(B \to K^*_X \mu^+ \mu^-)}{\mathcal{B}(B \to K^*_X e^+ e^-)}. \qquad X = L,\, T$$

$$R_i = \frac{\langle \Sigma_i^\mu \rangle}{\langle \Sigma_i^e \rangle} \qquad \Sigma_i = \frac{I_i + \bar{I}_i}{2}$$

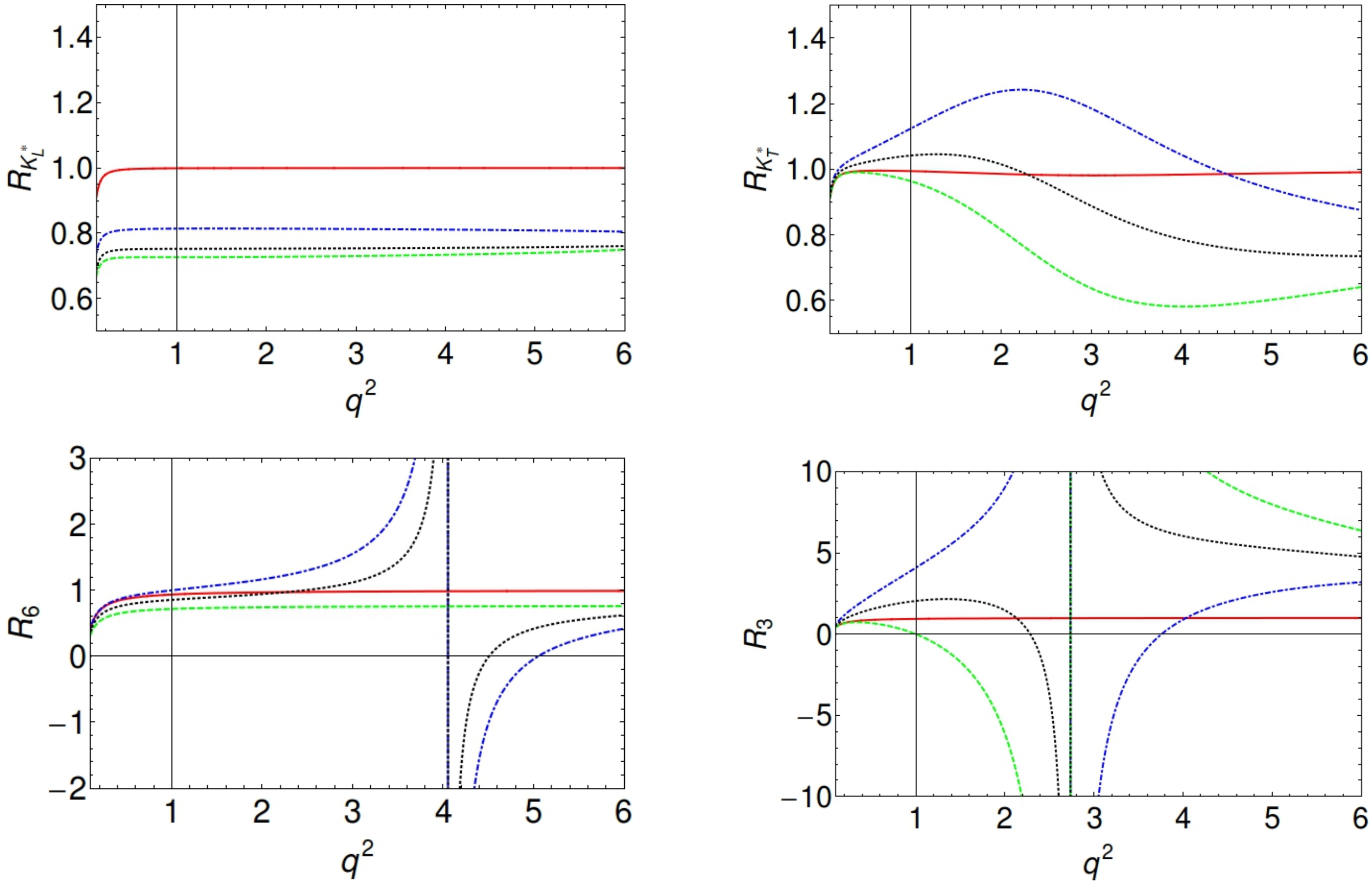
(2) lepton-flavour-dependence of position of zero-crossings

$$\Delta_0^i \equiv (q_0^2)_{I_i}^{(\mu)} - (q_0^2)_{I_i}^{(e)}$$

SJ, Martin Camalich 1412.3183

# What would a signal look like?

SJ, Martin Camalich
1412.3183




Any observed deviation from one ($R_i$) or zero ( $\Delta_0^i$) would be a clear BSM signal

Different BSM explanations of $R_K$ discriminated

# Optimised angular observables

=functions of the angular coefficients for which form factors drop out in the heavy quark limit if perturbative QCD corrections neglected.

E.g.

neglecting strong phase differences [tiny; take into account in numerics]

close to $q^2 = 0$ (photon pole dominance)

Krueger,Matias 2005; Egede et al 2008
Becirevic, Schneider 2011
Matias, Mescia, Ramon, Virto 2012
Descotes-Genon et al 2012

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 2\frac{\mathrm{Re}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx \frac{\mathrm{Im}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

$\Big\} \approx 0$ (in SM)

(Melikhov 1998)
Krueger, Matias 2002
Lunghi, Matias 2006
Becirevic, Schneider 2011
Becirevic, Kou, et al 2012

$$P_5' = \frac{\mathrm{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2)}} = \frac{C_{10}\left(C_{9,\perp} + C_{9,\parallel}\right)}{\sqrt{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp}^2 + C_{10}^2)}}$$

**in SM**, neglecting power corrections and pert. QCD corrections

where

$$C_{9,\perp} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, m_B}{q^2} C_7^{\mathrm{eff}}$$

$$C_{9,\parallel} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, E}{q^2} C_7^{\mathrm{eff}}$$

$C_7$ and $C_9$ opposite sign
destructive interference enhances vulnerability to anything that violates the large-energy form factor relations (or more generally underestimated errors on form facors

much less of an issue in than to $P_1$ or $P_3^{CP}$ than eg in $P_5'$ (and others)

# Optimised angular observables

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Two approximate null tests of the SM

What are the leading corrections?

where

$$C_{9,\perp} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, m_B}{q^2}\, C_7^{\mathrm{eff}}$$

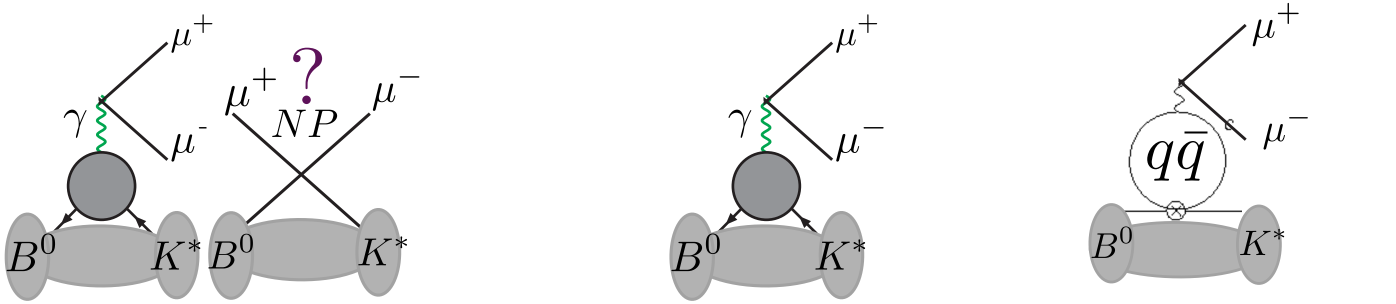
$$C_{9,\parallel} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, E}{q^2}\, C_7^{\mathrm{eff}}$$

$C_7$ and $C_9$ opposite sign
destructive interference enhances vulnerability to anything that violates the large-energy form factor relations (or more generally underestimated errors on form facors

much less of an issue in than to $P_1$ or $P_3^{CP}$ than eg in $P_5'$ (and others)

# B->Vll vector amplitudes

**Only helicity +1 and -1 contribute to $P_1$ and $P_3^{CP}$ : sin,cos(2Φ) dependence**




$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2)C_9 - V_{-\lambda}(q^2)C_9' + \frac{2\,m_b m_B}{q^2}\left(\tilde{T}_\lambda(q^2)C_7 - \tilde{T}_{-\lambda}(q^2)C_7'\right) - \frac{16\,\pi^2 m_B^2}{q^2}\,h_\lambda(q^2)$$

no photon pole: vanishing relative contribution as $q^2$->0

photon pole at $q^2$=0

Only one form factor, drops out up to interference

photon pole at $q^2$=0

complicated nonlocal correction

Helicity +1 power suppressed in the heavy-quark limit — Burdman, Hiller 2000

form factor $T_+$ doubly suppressed (further $q^2/m_B^2$ factor) — SJ, Martin Camalich 2012

nonlocal term known to be singly suppressed ($\Lambda/m_b$) — Beneke, Feldmann, Seidel 2001

could be the dominant uncertainty for null tests — Grinstein et al 2004; Khodjamirian et al 2010; (Ball, Jones, Zwicky 2006)

however, extra suppression ~ $\Lambda/m_b$ — SJ, Martin Camalich 2012

# Form factor relations

The heavy-quark limit is highly predictive both for form factor ratios and for virtual-charm effects, for instance:

Charles et al 1999
Beneke, Feldmann 2000
Beneke, Feldmann, Seidel 2001-4

$$\frac{T_-(q^2)}{V_-(q^2)} = 1 + \frac{\alpha_s}{4\pi} C_F \left[ \ln \frac{m_b^2}{\mu^2} - L \right] + \frac{\alpha_s}{4\pi} C_F \frac{1}{2} \frac{\Delta F_\perp}{V_-} \quad \text{where} \quad L = -\frac{2E}{m_B - 2E} \ln \frac{2E}{m_B}$$

"vertex" correction:
**parameter-free**

"spectator scattering":
mainly dependent on B meson LCDA
**but $a_s$ suppressed**

- Eliminates form factor dependence from some observables (eg $P_2$' and zero of $A_{FB}$) almost completely, up to $\Lambda/m_b$ power corrections

Descotes-Genon, Hofer, Matias, Virto

- pure HQ limit: $T_-(0)/V_-(0) \sim 1.05 > 1$ Beneke,Feldmann 2000

- compare to: $T_-(0)/V_-(0) = 0.94 +/- 0.04$ [D Straub, priv comm based on Bharucha, Straub, Zwicky 1503.05534]
light-cone sum rule computation with correlated parameter variations.
Difference consistent with $\Lambda/m_b$ power correction;
remarkable 5% error

# General parameterisation of power corrections

SJ, Martin Camalich 2012

At most 1-2% over entire 0..6 GeV^2 range -> ignore

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

$a_F$, $b_F$ are $O(\Lambda/m_b)$
- varied at +/-10% of generic leading-power analogue (+/-0.03 and +/-0.1 respectively) for error bars on previous slides

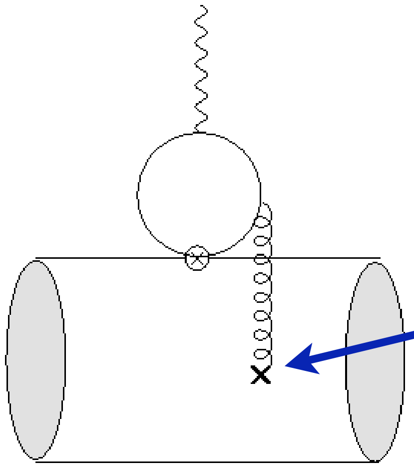
One can eliminate two $a_F$ and $b_F$ by choice of two reference ("soft") form factors. **However,** unambiguous heavy-quark limit for form factor ratios (eg $T_-/V_-$): These are **invariant** under change of form factor scheme, as are **any observables**

Any calculation (eg LCSR) can be expressed in terms of the general parameterisation - but then one is using dynamical/model input beyond the heavy-quark expansion

Proposal ( Descotes-Genon et al 2014 ) to center ranges for $a_F$, $b_F$ around LCSR predictions (but replace the corresponding errors by ad hoc 10% ranges).

No theoretical justification given for this. **Practical** effect is to obtain predictions similar to LCSR - this is so by construction, and is not an independent check.

# Charm loop estimate



$$h_\lambda|_{c\bar{c}} = \frac{1}{m_B^2}\frac{2}{3}\epsilon^{\mu*}(\lambda)\int d^4y\, e^{iq\cdot y}\langle M|T[(\bar{c}\gamma^\mu c)(y)(C_1^c Q_1^c + C_2^c Q_2^c)(0)]|\bar{B}\rangle$$

consider soft gluon (in B rest frame)

From collinear factorisation viewpoint this represents the endpoint region, which is known to give a power-suppressed contribution

perform a "light-cone OPE"
(This is equivalent to expanding the charm loop, treating $\Lambda^2/(4\, m_c^2) \sim \Lambda/m_b$ )

Buchalla, Isidori

Khodjamirian et al 2010

obtain

$$h_\lambda|_{c\bar{c},\mathrm{LD}} = \epsilon^{\mu*}(\lambda)\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$$

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta\Big(\omega - \frac{in_+\cdot D}{2}\Big)\tilde{G}^{\alpha\beta}b_L$$

(a nonlocal, light-cone operator)

need estimate of $\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$ (which goes into $H_V^\lambda$)

light-cone SR based on Khodjamirian et al 2010 for K* helicity amplitudes SJ, Martin Camalich 2012
**outcome: helicity hierarchy remains for the endpoint region**
same conclusion for (anyway CKM-suppressed) light-quark LD effects at low $q^2$ (estimated via VMD)

# Predictions at very low $q^2$

SJ, Martin Camalich
1412.3183

| Bin [$\mathrm{GeV}^2$] | $Br$ [$10^{-8}$] | $P_1$ | $P_2$ | $P_3^{CP}$ [$10^{-4}$] |
|---|---|---|---|---|
| [0.1, 0.98] | $9.5^{+5.2}_{-3.5}$ | $0.024^{+0.053}_{-0.055}$ | $-0.16^{+0.05}_{-0.04}$ | $0.1^{+0.7}_{-0.8}$ |
| Electron | $26^{+12}_{-9}$ | $0.030^{+0.047}_{-0.044}$ | $-0.073^{+0.020}_{-0.016}$ | $0.1^{+0.6}_{-0.6}$ |

[0.0004,1.12+/-0.06]

- Very clean, very insensitive to form factor input
- Boost in BR: nearly 3x more electrons, most of the extra ones in the relevant $q^2$ region -> partly offsets lower efficiency in LHCb

| | Result | QCDF | Fact. p.c.'s | Non-fact. p.c.'s |
|---|---|---|---|---|
| $P_1$ | $0.030^{+0.047}_{-0.044}$ | $^{+0.008}_{-0.003}$ | $\pm 0.012$ | $^{+0.028}_{-0.026}$ |
| $P_3^{CP}$ [$10^{-4}$] | $0.1^{+0.7}_{-0.6}$ | $\pm 0.3$ | $\pm 0.2$ | $\pm 0.3$ |

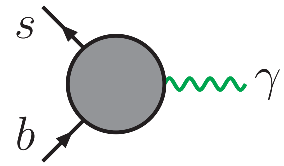
Experiment (electrons)

$$A_{\mathrm{T}}^{(2)} = -0.23 \pm 0.23 \pm 0.05$$
$$A_{\mathrm{T}}^{\mathrm{Im}} = +0.14 \pm 0.22 \pm 0.05$$
$$A_{\mathrm{T}}^{\mathrm{Re}} = +0.10 \pm 0.18 \pm 0.05$$

LHCb, 1501.03028, JHEP 1504 (2015) 064

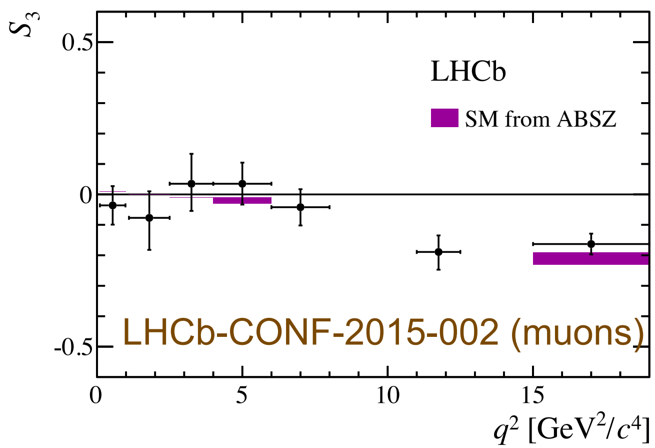
# Constraint on dipoles

$C_7$ : electromagnetic dipole coupling
(strongly constrained by inclusive B->$X_s$ gamma)

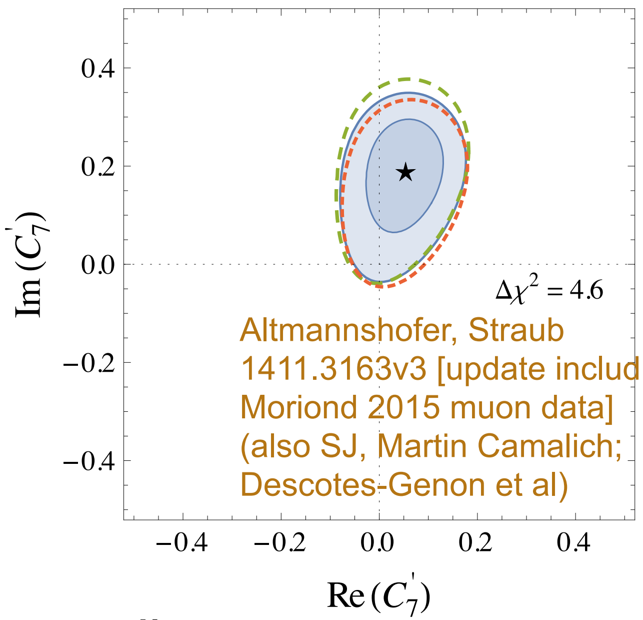



operators with right-handed strange quarks
(constrained by other angular observables)

SJ, Martin Camalich 2012, 2014;
various global fits 2014-2015




LHCb-CONF-2015-002 (muons)




Altmannshofer, Straub
1411.3163v3 [update including
Moriond 2015 muon data]
(also SJ, Martin Camalich;
Descotes-Genon et al)

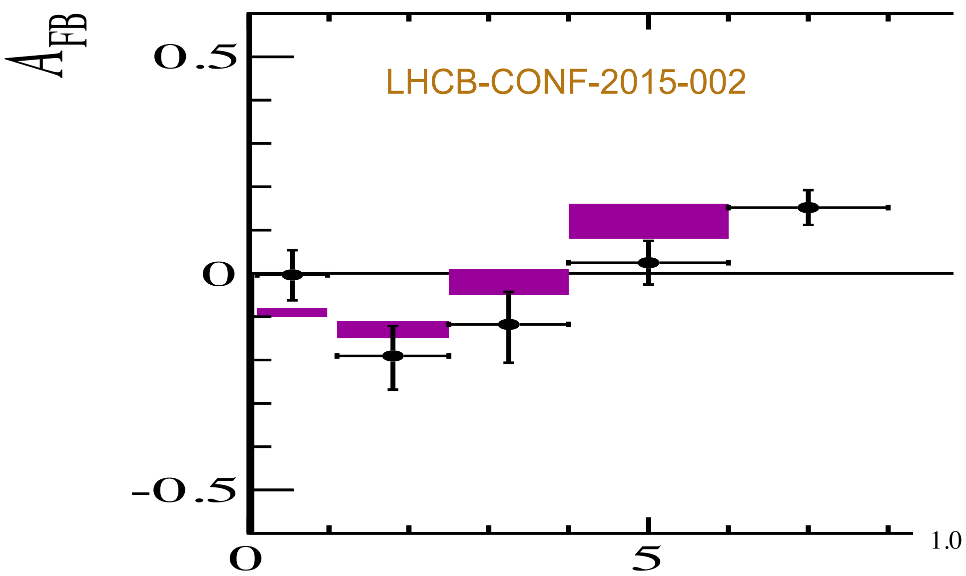
+ results on B->K* $e^+e^-$

JHEP 1504 (2015) 064

operators with scalar or pseudoscalar couplings
(gigantic effects in $B_s$ -> mu mu due to SU(2)xU(1) symmetry)

Grinstein, Martin Camalich 2014

# Forward-backward asymmetry




LHCb Moriond 2015 (3 $fb^{-1}$)
downward shift of $A_{FB}$ relative to LCSR-based prediction
(Bharucha, Straub, Zwicky 2015)

Such a shift is largely equivalent to a **rightward shift** of the zero crossing.

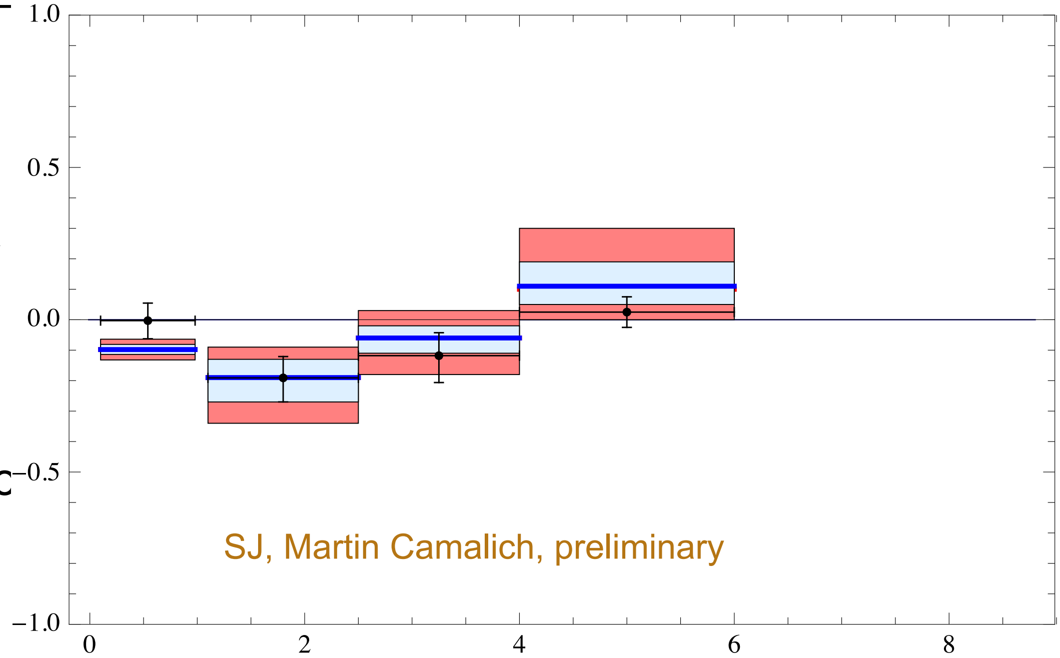
Zero crossing in LCSR has been significantly lower than heavy-quark limit for many years (as low as <3 $GeV^2$)

blue line: pure heavy-quark limit, **no power corrections**

light blue: "68% Gaussian" theory error (including power corrections)

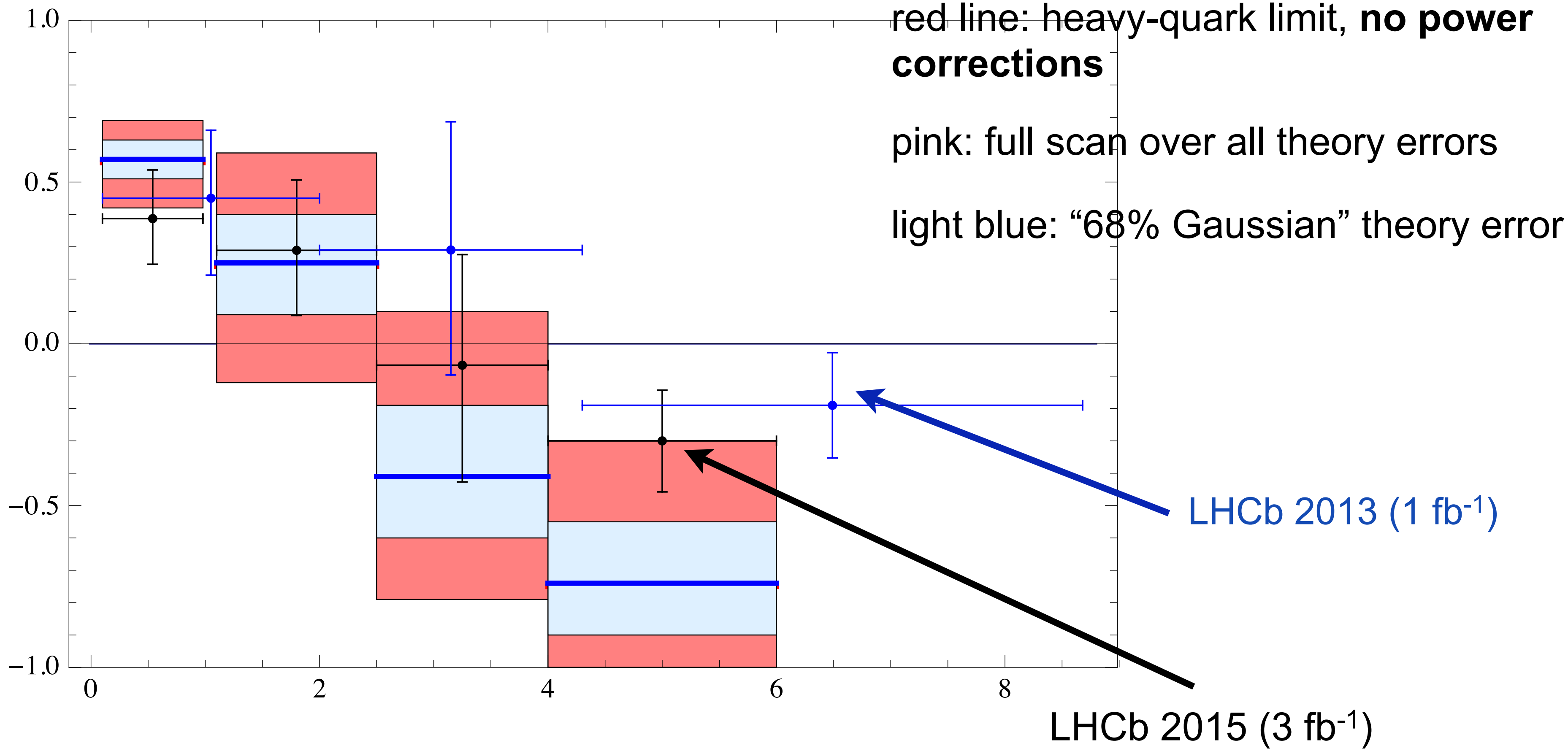
pink: full scan over all theory errors

Surprising that pure HQ limit appears to agree reasonably well with data !




"Clean" observables at present precision have noticeable form factor dependence

# Angular observable $P_5'$ SJ, Martin Camalich




(Ignore 6..8 GeV bin, above perturbative charm threshold and very close to resonances.)

For Gaussian errors [corresponding to what most authors employ], there is a noticeable deviation in a single bin; but also here less drastic than with LCSR-based theory

# Power corrections: analytical

SJ, Martin Camalich 1412.3183

Compare

$$P_5' = P_5'|_\infty \left(1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} + \frac{a_{V_0} - a_{T_0}}{\xi_\parallel} 2\, C_7^{\rm eff} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} + 8\pi^2 \frac{\tilde{h}_-}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{C_{9,\perp} + C_{9,\parallel}} + \text{further terms}\right) + \mathcal{O}(\Lambda^2/m_B^2)$$

(truncated after 3 out of 11 independent power-correction terms!)
also, dependence on soft form factors reappears at PC level

and

$$P_1 = \frac{1}{C_{9,\perp}^2 + C_{10}^2} \frac{m_B}{|\vec{k}|} \left( -\frac{a_{T_+}}{\xi_\perp} \frac{2\, m_B^2}{q^2} C_7^{\rm eff} C_{9,\perp} - \frac{a_{V_+}}{\xi_\perp} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) - \frac{b_{T_+}}{\xi_\perp} 2 C_7^{\rm eff} C_{9,\perp} - \frac{b_{V_+}}{\xi_\perp} \frac{q^2}{m_B^2} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) + 16\pi^2 \frac{h_+}{\xi_\perp} \frac{m_B^2}{q^2} C_{9,\perp} \right) + \mathcal{O}(\Lambda^2/m_B^2).$$

(complete expression)

Further notice that $a_{T+}$ vanishes as $q^2$->0, $h_+$ helicity suppressed [will show], and the other three terms lacks the photon pole.

Hence $P_5$' **much** less clean than $P_1$ (especially the latter at very low $q^2$)

# Power corrections, scheme independence

SJ, Martin Camalich 1412.3183

Example

manifestly form-factor-scheme-independent

$$P_5' = P_5'|_\infty \Bigg( 1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})}$$

$$+ \frac{a_{V_0} - a_{T_0}}{\xi_\|} 2\, C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\|}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})}$$

$$+ 8\pi^2 \frac{\tilde{h}_-}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{C_{9,\perp} + C_{9,\|}} + \text{further terms} \Bigg) + \mathcal{O}(\Lambda^2/m_B^2)$$

heavy-quark-limit result

(“charm loop” power correction)

(**truncated** after 3 out of 11 independent power-correction terms!)

Many independent power-correction parameters appear.

They appear only in form-factor-scheme-independent combinations.

Example: choose either $V_-$ as “soft” (reference) form factor, then $a_{V-}$=0,
or can choose $T_-$, then $a_{T-}$=0.
Because $V_-/T_-$ is fixed in QCD, the difference ($a_{V-}$ - $a_{T-}$) agrees in both schemes, up to $O(\Lambda^2/m_b^2)$.

Numerical differences between different schemes are estimators of higher powers (beyond the truncated parameterisation).

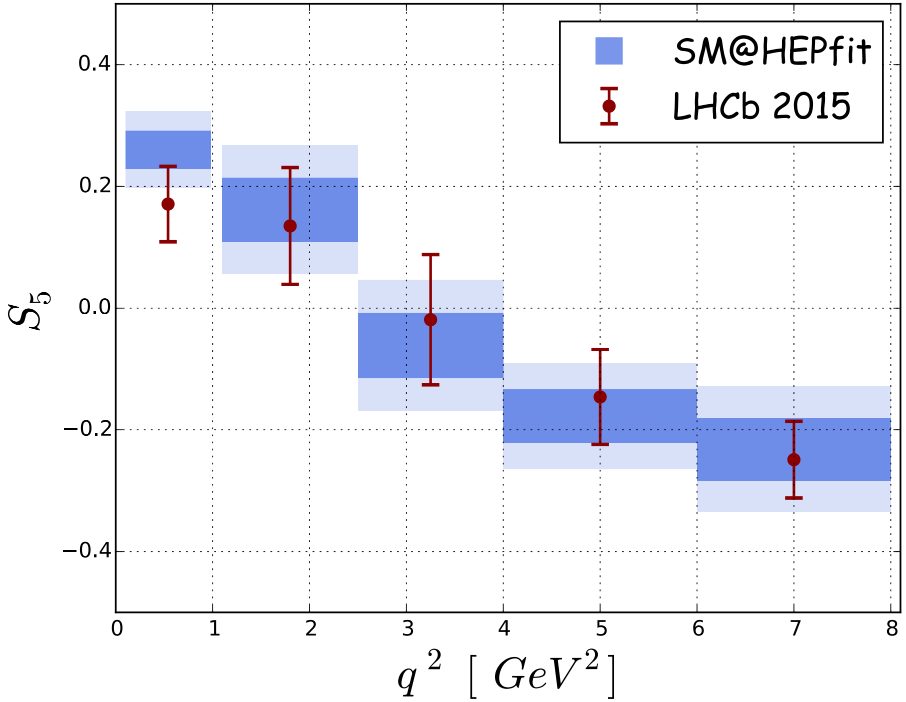
# Charming penguin?

Bayesian fit based on the formalism of SJ&Martin Camalich, with conservative prior for long-distance charm

Ciuchini et al, 1512.07157

"SM" is Bayesian posterior probability

[$S_5$ closely related to $P_5'$]



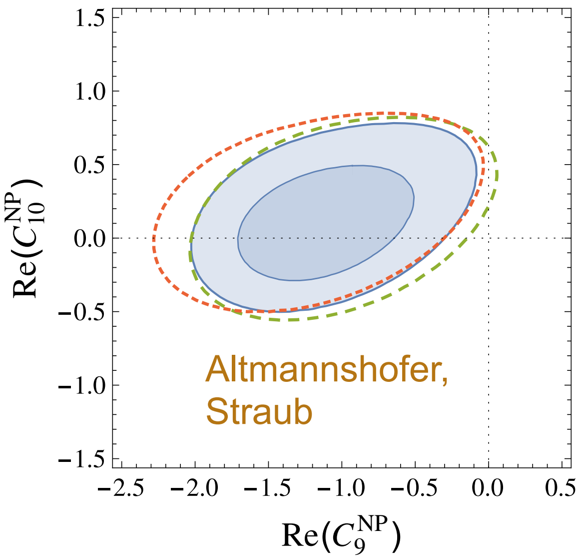

technical note: **by design** this can account for any effect depending on prior; and in particular can mimic the effect of form factor uncertainties (this work employs a LCSR prediction)

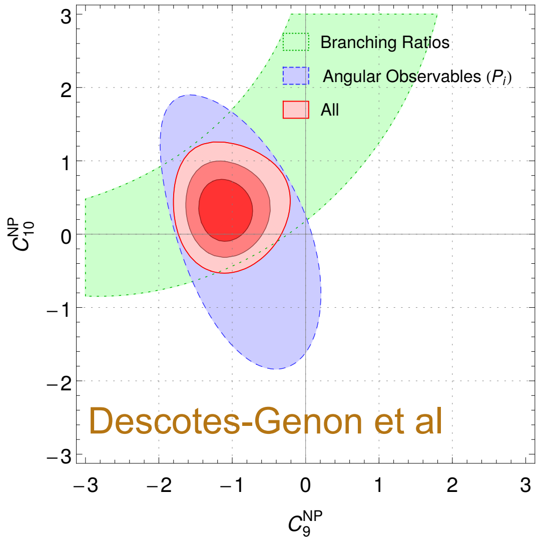
claim that interpretation in terms of shift to $C_9$ (or $C_7$) is disfavoured

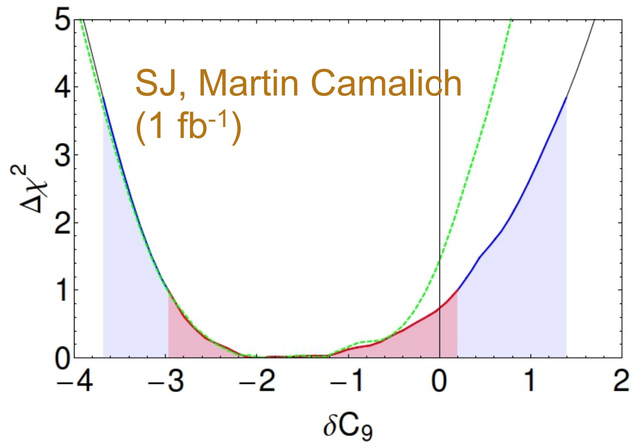
predicted suppression of long-distance contribution to $H_V^+$ confirmed by fit

# Global fits

Fits of weak Hamiltonian to data on B->K(*)ll, Bs->mu mu, B->Xs gamma, B->phi ll, B->K*gamma prefer non-SM values.




also: Bobeth et al; Hurth-Mahmoudi; Ciuchini et al; Ghosh et al,...

Most agree that best fit is for $C_9^{NP}$ ~ -1..-2 but differ on significance

Some level of degeneracy $C_9$ / $C_{10}$ (branching fractions - green band); angular observables prefer $C_9$

# Summary and outlook

Rare B decays are sensitive to BSM effects - encapsulated, under very weak assumptions, in a dimension-six weak Hamiltonian

Theoretical description generally involves nonprturbative local and nonlocal form factors which cannot at present be computed in a controlled approximation of QCD.

Some observables are not, or only weakly, sensitive to uncontrolled effects: $BR(B_s \to \mu\mu)$, $R_K$ etc , $R_{D(*)}$ ; null tests $S_3$/ $P_1$, $A_9$ / $P_3^{CP}$

Some indications of a BSM suppression of the semileptonic axial operator $C_{10}$

Eventually lattice QCD will allow to access the local form factors in a controlled manner. Prospects for nonlocal long-distance effects are less clear.

# BACKUP

# Heavy-quark limit and corrections

At most 1-2% over entire 0..6 GeV^2 range -> ignore

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

SJ, Martin Camalich 2012

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al)

(Beneke, Feldmann)

- $q^2$ dependence in heavy-quark limit not known (model by a power p, and/or a pole model)

$V_+^\infty(0) = 0$ $T_+^\infty(0)=0$ from heavy-quark/ large energy symmetry
$V_-^\infty(0) = T_-^\infty(0)$
$V_0^\infty(0) = T_0^\infty(0)$

Corrections are calculable in terms of perturbation theory, decay constants, light cone distribution amplitudes

$V_+^\infty(q^2) = 0$ $T_+^\infty(q^2)=0$

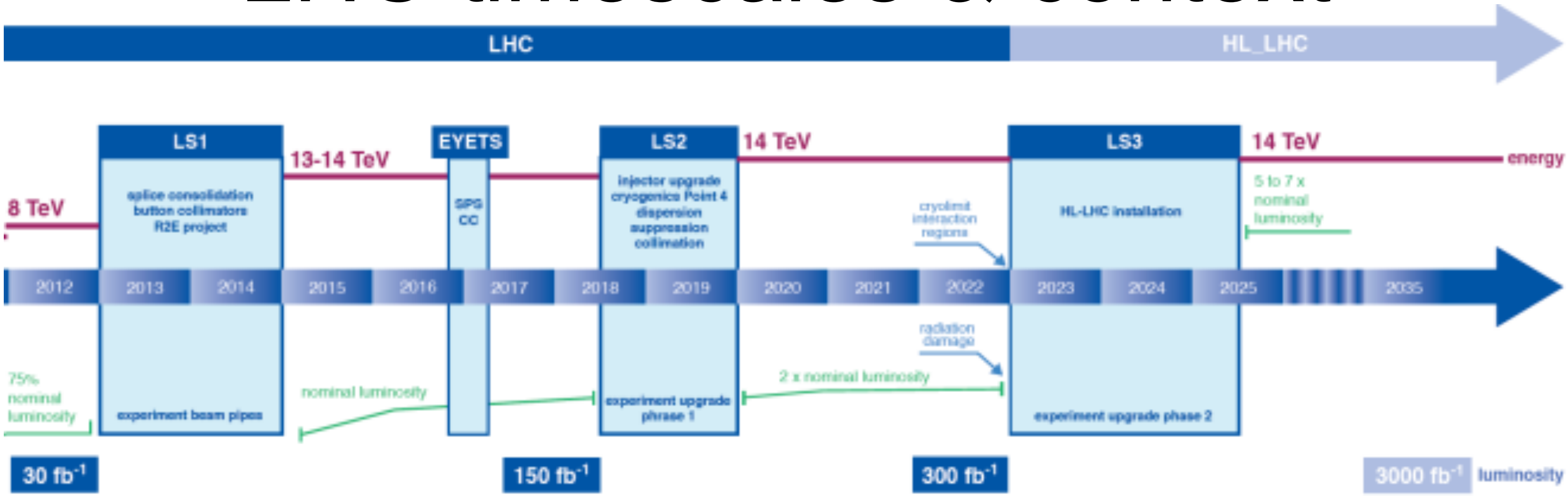
hence

$$T_+(q^2) = \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b)$$
$$V_+(q^2) = \mathcal{O}(\Lambda/m_b).$$

- **“naively factorizing” part of the helicity amplitudes $H_{V,A}^+$ strongly suppressed** as a consequence of chiral SM weak interactions
- We see the suppression is **particularly strong** near low-$q^2$ endpoint
- Form factor relations imply reduced uncertainties in suitable observables

Burdman, Hiller 1999 (quark picture)
confirmed in QCDF/SCET Beneke, Feldmann, ...

# LHC timescales & context




Belle 2 (e+e-) will report results from about 2018 and coexist with the HL-LHC

- possibility of inclusive measurements (B->$X_s$ gamma,...)
- much better acceptance & energy resolution for electrons

However, LHC will retain the statistics edge for accessible modes

- complementarity (obvious)
- interplay (eg modes for normalising $B_s$->mu mu at LHCb ?)

interplay with developments in hight pT

# Experimental prospects (LHCb)

- Some modes are no longer particularly “rare”, we have large samples of some decays already in run I.
- Extrapolating to the future:

| | channel | $1\text{fb}^{-1}$ | $3\text{fb}^{-1}$ | run II | upgrade | |
|---|---|---|---|---|---|---|
| | $B^0 \to K^{*0}\mu^+\mu^-$ | 883 | 2,400 | 10,500 | 85,000 | |
| | $B^+ \to \pi^+\mu^+\mu^-$ | 25 | 80 | 360 | 2500 | |
| | $B_s^0 \to \mu^+\mu^-$ | – | 15 | 65 | 520 | |
| | $B^0 \to K^{*0}\gamma$ | 5,300 | 17,000 | 76,000 | 500,000 | challenge to retain trigger efficiency in run II |
| [low $q^2$] | $B^0 \to K^{*0}e^+e^-$ | – | 150 | 650 | 5,200 | |

scaling naively by luminosity, assuming $\sigma_{b\bar{b}}$ scales linearly with $\sqrt{s}$

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[Tom Blake, Rare B decay workshop, Edinburgh, 12/05/15]

Huge improvements in precision
NP mass reach scales like delta$^{1/2}$ ...
... as long as theory accuracy matches experiment

# Theory needs

Form factors: very reliant on light-cone sum rules. Need independent corroboration.

- expect significant progress in lattice QCD (conceptual and numerical)

- flavour has been a driving force behind the European, and world wide, lattice programme for many years

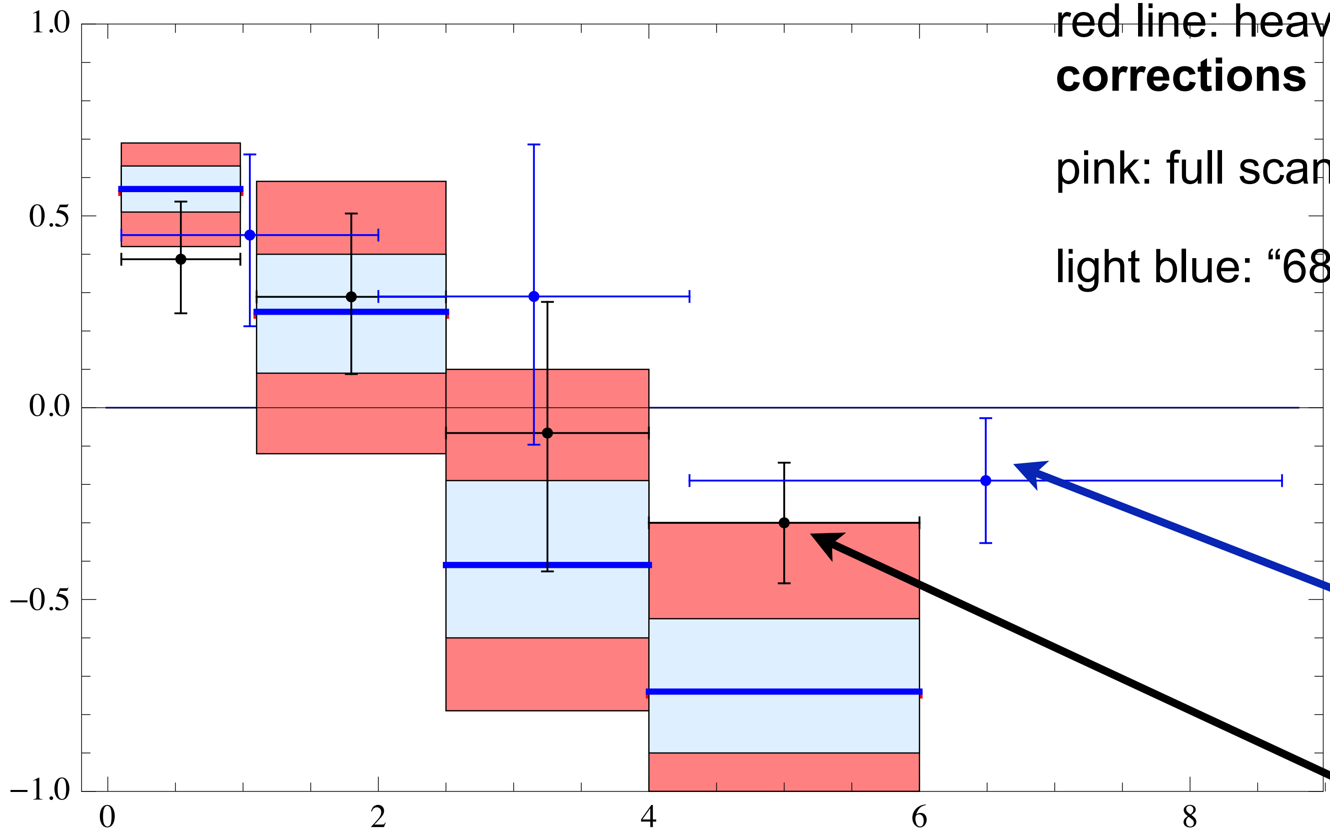
- model-independent constraints from heavy quark expansion (Beneke-Feldmann); but limited accuracy so $P_5$' anomaly significance lost. More data needed.

New observables - to test lepton universality violation, but also to constrain hadronic inputs better from data eg Hambrock/Hiller/Zwicky 1308.4379

Systematic exploitation of LHC-Belle2 complementarity

Better (correct?) models of BSM, if anomalies accumulate

# Angular observable $P_5'$ SJ, Martin Camalich, preliminary




red line: heavy-quark limit, **no power corrections**

pink: full scan over all theory errors

light blue: “68% Gaussian” theory error

LHCb 2013 (1 $fb^{-1}$)

LHCb Moriond 2015 (3 $fb^{-1}$)

(Ignore 6..8 GeV bin, above perturbative charm threshold and very close to resonances.)

For Gaussian errors [corresponding to what most authors employ], there is a noticeable deviation in a single bin; but also here less drastic than with LCSR-based theory

# Nonlocal term / charm loop

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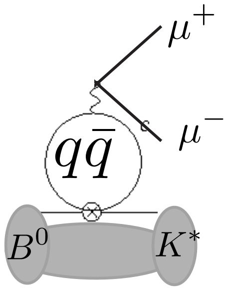
$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C_9' + \frac{2\, m_b m_B}{q^2}\left(\tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C_7'\right) - \frac{16\,\pi^2 m_B^2}{q^2}\, h_\lambda(q^2)$$



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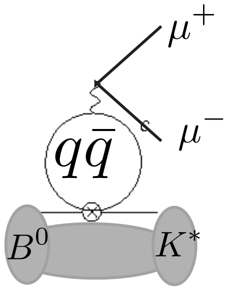


+ strong interactions!

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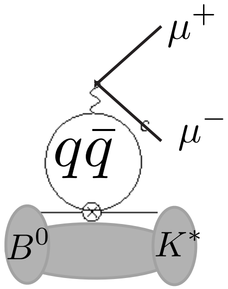


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more properly:

$$\frac{e^2}{q^2} L_V^\mu a_\mu^{\rm had} = -i\frac{e^2}{q^2}\int d^4x e^{-iq\cdot x}\langle \ell^+\ell^-|j_\mu^{\rm em,lept}(x)|0\rangle \int d^4y\, e^{iq\cdot y}\langle M|j^{\rm em,had,\mu}(y)\mathcal{H}_{\rm eff}^{\rm had}(0)|\bar{B}\rangle$$

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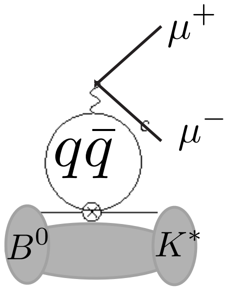


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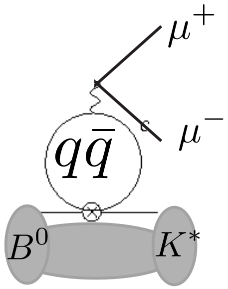


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traditional "ad hoc fix" : $C_9$ -> $C_9 + Y(q^2) = C_9^{eff}(q^2)$ ,
$C_7$ -> $C_7^{eff}$

"taking into account the charm loop"

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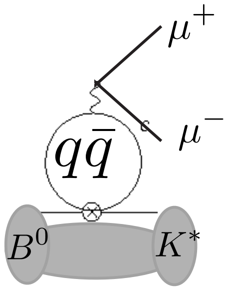


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traditional "ad hoc fix" : $C_9$ -> $C_9$ + $Y(q^2)$ = $C_9^{\rm eff}(q^2)$ , "taking into account the charm loop"
$C_7$ -> $C_7^{\rm eff}$

* for $C_7^{\rm eff}$ this seems ok at lowest order (pure UV effect; scheme independence)
* for $C_9^{\rm eff}$ amounts to factorisation of scales ~ $m_b$ (, $m_c$ ,$q^2$) and $\Lambda$ (soft QCD)
* not justified in large-N limit (broken already at leading logarithmic order)
* what about QCD corrections?
* not a priori clear whether this even gets one closer to the true result!

**only known justification** is a heavy-quark expansion in $\Lambda/m_b$ (just like inclusive decay is treated !)

Beneke, Feldmann, Seidel 2001, 2004

# Nonlocal term - another look

traditional “ad hoc fix” : $C_9 \to C_9 + Y(q^2) = C_9^{eff}(q^2)$ , $C_7 \to C_7^{eff}$

dominant effect: charm loop, proportional to ($z = 4\, m_c^2/q^2$)

$$-\frac{4}{9}\left(\ln\frac{m_q^2}{\mu^2} - \frac{2}{3} - z\right) - \frac{4}{9}(2+z)\sqrt{|z-1|}\begin{cases} \arctan\frac{1}{\sqrt{z-1}}, & z > 1, \\ \ln\frac{1+\sqrt{1-z}}{\sqrt{z}} - \frac{i\pi}{2}, & z \leqslant 1 \end{cases}$$

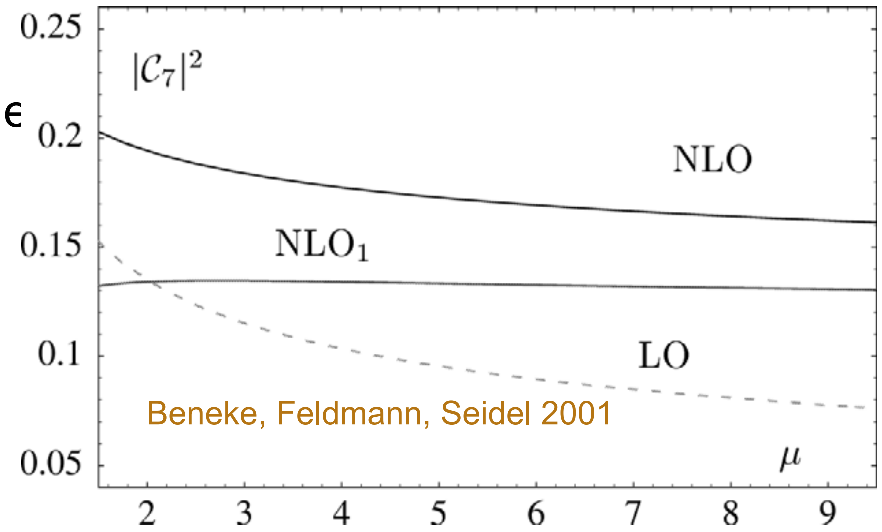
$$C_9^{\rm eff} = \begin{cases} 4.18|_{C_9} + (0.22 + 0.05i)|_Y & (m_c = m_c^{\rm pole} = 1.7\text{GeV}) \\ 4.18|_{C_9} + (0.40 + 0.05i)|_Y & (m_c = m_c^{\overline{\rm MS}} = 1.2\text{GeV}). \end{cases}$$

ie a 5% mass scheme ambiguity

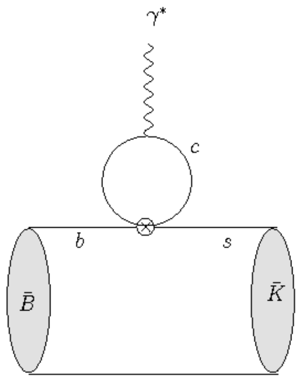
separately, one has a residual scale ambiguity of order 30% at the level of the decay amplitude

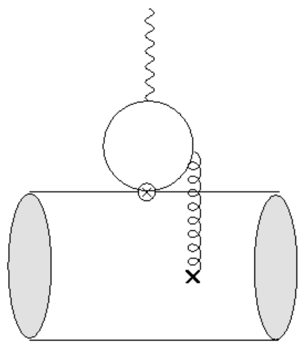
resolved in the heavy-quark expansion (to leading power)

Beneke, Feldmann, Seidel 2001, 2004



# Nonlocal terms:heavy-quark expansion



leading-power: factorises into perturbative kernels, form factors, LCDA's (including hard/hard-collinear gluon corrections to all orders)

$\alpha_s^0$ : $C_7 \rightarrow C_7^{eff}$
$C_9 \rightarrow C_9^{eff}(q^2)$
+ 1 annihilation diagram
$\alpha_s^1$ : further corrections to $C_7^{eff}(q^2)$ and $C_9^{eff}(q^2)$

(convergent) convolutions of hard-scattering kernels with meson light cone-distribution amplitudes
Beneke, Feldmann, Seidel 2001

state-of-the-art in phenomenology

unambigous (save for parametric uncertainties)

at subleading powers: breakdown of factorisation

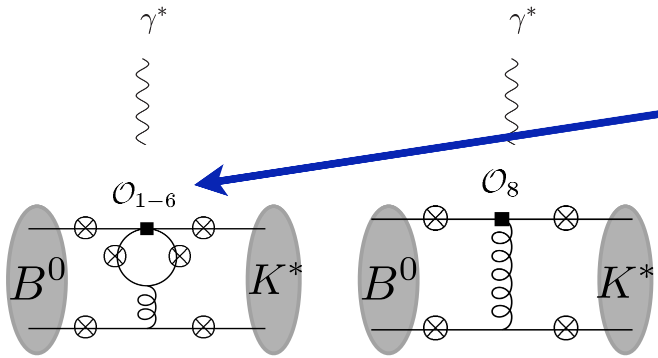
some contributions have been estimated as end-point divergent convolutions with a cut-off Kagan&Neubert 2001, Feldmann&Matias 2002

can perform light-cone OPE of charm loop & estimate resulting (nonlocal) operator matrix elements

Khodjamirian et al 2010

effective shifts of helicity amplitudes as large as ~10%

# New effect: spectator scattering




includes $Q_1^c$ , $Q_2^c$ - large Wilson coefficients

+ annihilation (+ “vertex corrections”)

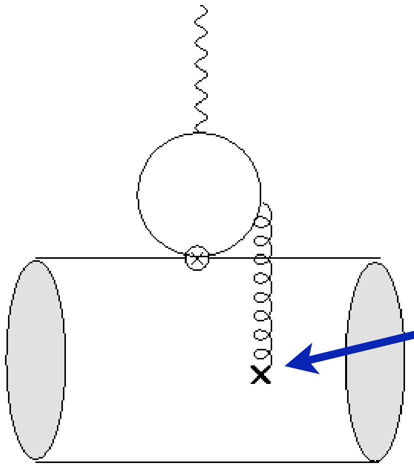
Beneke, Feldmann, Seidel 2001

leading-power: everything factorises into perturbative kernels, form factors, meson light-cone distribution amplitudes (including hard/hard-collinear gluon corrections to all orders)

$$h_\lambda = \int_0^1 du \phi_K^*(u) T(u, \alpha_s) + \mathcal{O}(\Lambda/m_b)$$

- leading power in the heavy quark limit - same as the vertex corrections going into $C_7^{eff}$, $C_9^{eff}$

# Long-distance charm loop



$$h_\lambda|_{c\bar{c}} \;=\; \frac{1}{m_B^2}\frac{2}{3}\epsilon^{\mu*}(\lambda)\int d^4y\, e^{iq\cdot y}\langle M|T[(\bar{c}\gamma^\mu c)(y)(C_1^c Q_1^c + C_2^c Q_2^c)(0)]|\bar{B}\rangle$$

consider soft gluon (in B rest frame)

From collinear factorisation viewpoint this represents the endpoint region, which is known to give a power-suppressed contribution

perform a “light-cone OPE”
(This is equivalent to expanding the charm loop, treating $\Lambda^2/(4\,m_c^2) \sim \Lambda/m_b$ ) Khodjamirian et al 2010

obtain

$$h_\lambda|_{c\bar{c},\mathrm{LD}} = \epsilon^{\mu*}(\lambda)\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$$

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta\Big(\omega - \frac{in_+\cdot D}{2}\Big)\tilde{G}^{\alpha\beta}b_L$$

(a nonlocal, light-cone operator)

need estimate of $\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$ (which goes into $H_V^\lambda$)

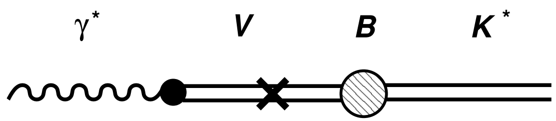
light-cone SR based on Khodjamirian et al 2010 for K* helicity amplitudes SJ, Martin Camalich 2012
**one outcome: two tests of right-handed dipol transitions remain clean**

for error estimate, introduce polynomial model in $q^2/(4m_c^2)$

# Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, presumably “duality violation”
Presumably ρ,ω,φ most important; use vector meson dominance supplemented by heavy-quark limit B➔VK* amplitudes




$$\tilde{a}_\mu^{\mathrm{had,\,lq}} = \int d^4x\, e^{-iq\cdot x} \sum_{P,P'} \langle 0| j_\mu^{\mathrm{em,lq}}(x)|P'\rangle \langle P'(x)|P(0)\rangle \langle \bar{K}^* P|\, \mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0)|\bar{B}\rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate V states.

Helicity hierarchies in **hadronic** B decays prevent large uncertainties in $H_V^+$ from this source, too.

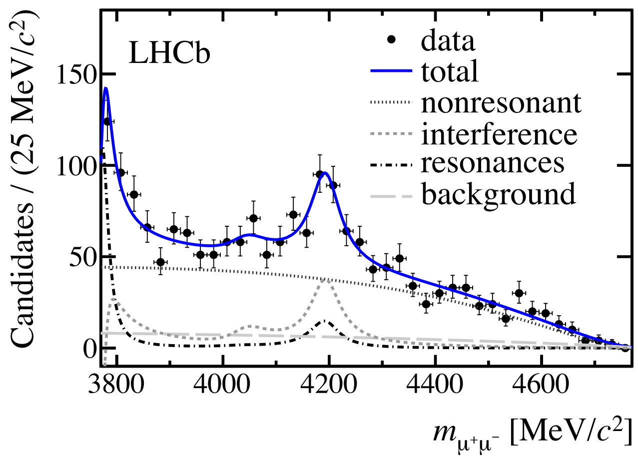
# High-$q^2$ region (sketch)

- spectator scattering mechanism power-suppressed

- above open-charm (and perturbative-charm) thresholds

- however, for $q^2 >> 4m_c^2$, OPE at amplitude level

Grinstein, Pirjol 2004; Beylich, Buchalla, Feldmann 2011




Duality violation (≡ error beyond OPE)
- expected on general grounds
for OPE above threshold

(Chibisov et al; Shifman 1990's)

- pronounced resonant
structure observed

- difficult to quantify uncertainty due to this

Beylich, Buchalla, Feldmann 2011
(Chibisov et al; Shifman 1990's)
(Lyon, Zwicky 2013)

- like in low-$q^2$, probably best to stay away from the charm threshold region in looking for new physics