Modified Gravity: f(R) and Scalar tensor theories

Oral Presentation in ASP2016

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Modified Gravities:

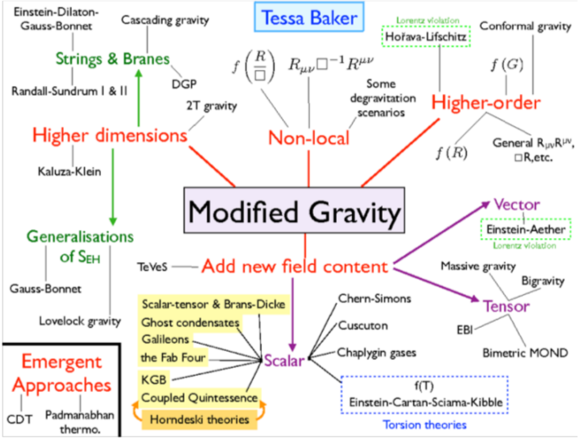


Figure : A picture showing a summary of modified gravities, Credit: Javier Chagoya Swansea University

Introduction: Two ways to get Einstein Field Equations

They are two ways to get Einstein Field Equations:

- From line element, get the metric, Bianchi identities, Riemann tensor, then Ricci scalar and arrive at the EFEs.
- The second: from Hilbert-Einstein action make variation and get the field equations.

$$I_{HE} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}[R + \mathcal{L}_m]. \tag{1}$$

- The Einstein Field Equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy momentum tensor.

Action for Scalar-tensor theory

The scalar tensor theory has the action:

$$I_{ST} = \int d^4x\sqrt{-g}\left[\phi R - \frac{W(\phi)}{\phi}\nabla_\mu\phi\nabla^\mu\phi - 2\Lambda(\phi) + \mathcal{L}_m(\Psi, g_{\mu\nu})\right], \quad (3)$$

where $W(\phi)$ is coupling parameter, $\mathcal{L}_m$ is Lagrangian density of matter field Ψ and $\Lambda(\phi)$ is arbitrary function (cosmological constant).

The action in Brans-Dicke theory is for $W = const$ and $\Lambda(\phi) = 0$:

$$I_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{W}{\phi}\nabla_\mu\phi\nabla^\mu\phi + \mathcal{L}_m(\Psi, g_{\mu\nu})\right]. \quad (4)$$

f(R) as sub-class of scalar-tensor theories

The action that represent f(R) gravity is

$$I_{f(R)} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}[f(R) + \mathcal{L}_m]. \tag{5}$$

The action in scalar-tensor theory has the form:

$$I_{f(\phi)} = \frac{1}{2\kappa} \int d^4 \sqrt{-g} \left[f(\phi(R)) + \mathcal{L}_m \right], \tag{6}$$

where $f(\phi(R))$ is the function of $\phi(R)$ and we consider the scalar field ϕ to be

$$\phi = f' - 1. \tag{7}$$

Here the prime indicate differentiation with respect to R.

f(R) Theory in Scalar-Tensor Language(Field Equations)

The field equations from the above action are given as:

$$G_{ab} = \frac{\kappa}{\phi+1} T^m_{ab} + \frac{1}{(\phi+1)}\Big[\frac{1}{2} g_{ab}\big(f - (\phi+1)R\big) + \nabla_a \nabla_b \phi - g_{ab} \Box \phi\Big]. \tag{8}$$

The scalar filed ϕ obeys the Klein-Gordon equation:

$$\Box\phi - \frac{1}{3}\big(2f - (\phi+1)R + T^m\big) = 0, \tag{9}$$

where T^m is the trace of the matter energy momentum tensor. The effective potential $V(\phi)$ is defined in the way that

$$V'(\phi) = \frac{dV}{d\phi} = \frac{1}{3}\big(2f - (\phi+1)R\big). \tag{10}$$

Some results: $f(R) = \beta R^n$

The model is

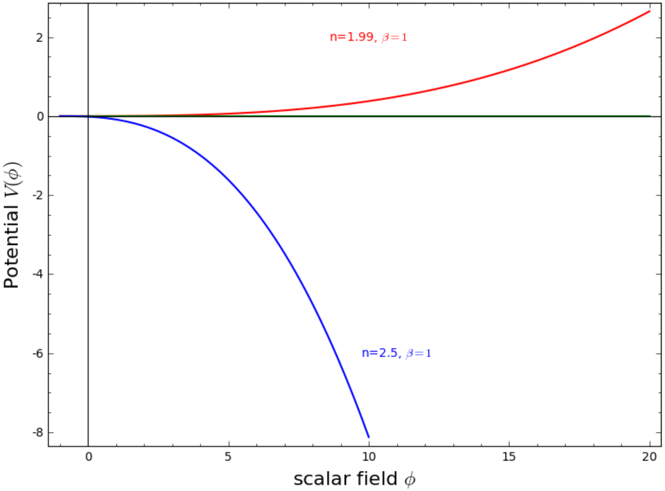
$$f(\phi) = \beta\left(\frac{\phi + 1}{n\beta}\right)^{\frac{n}{n-1}}. \tag{11}$$

The potential is

$$V(\phi) = \frac{\beta(n-1)(2-n)}{3(2n-1)(n\beta)^{n/(n-1)}}(\phi+1)^{(2n-1)/(n-1)}, \ \text{n} \neq 0, \frac{1}{2} \text{ and } 1. \tag{12}$$

The potential $V(\phi)$ dependence on the scalar field ϕ for the case of the βR^n, is presented in figure below.

Case of $f(R) = \beta R^n$: Potential



Case of $f(R) = \beta R^n$: K-G Equation

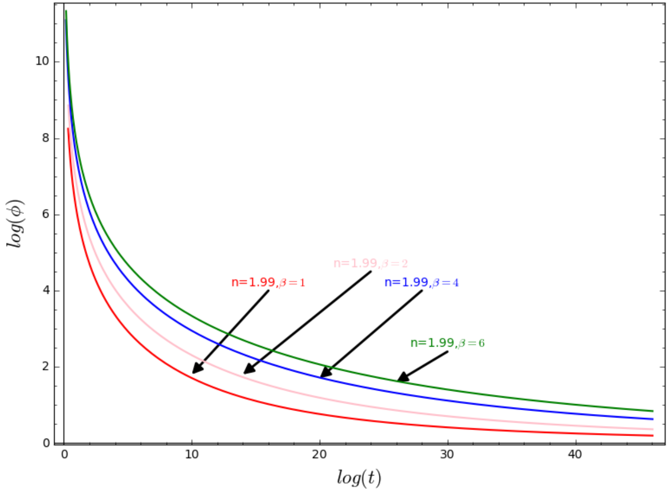
The Klein-Gordon Equation is

$$\Box\phi - \frac{1}{3}\Big[\frac{2\beta - n\beta}{(n\beta)^{n/(n-1)}}(\phi+1)^{n/(n-1)} + \frac{3\omega - 1}{\phi + 1}\tilde{\mu}^m\Big] = 0. \qquad (13)$$

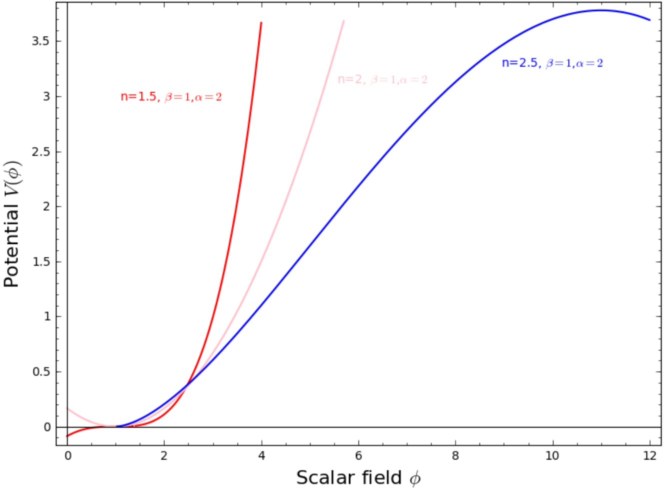
Assumptions:

- ϕ depends on time only.
- spatial dependence in the Covariant d'Alembert operator is dropped out.
- μ_m is negligible in the early universe.

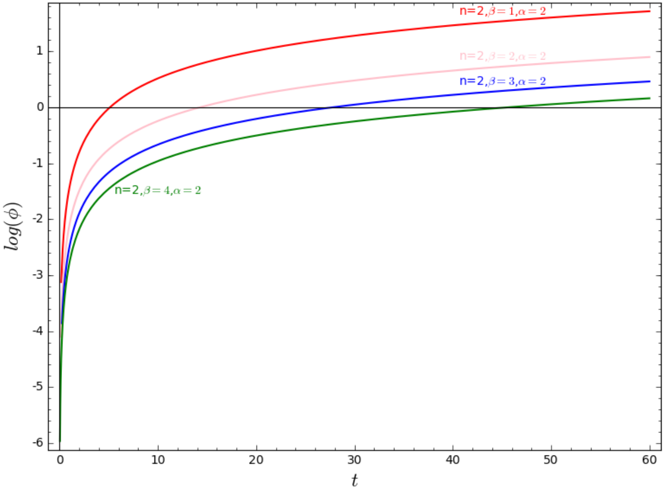
Case of $f(R) = \beta R^n$: K-G Solution



Case of $f(R) = \alpha R + \beta R^n$: Potential $V(\phi)$



Case of $f(R) = \alpha R + \beta R^n$: K-G Solution



Conclusion

- $f(R)$ gravity is a subclass of Brane-Dicke scalar tensor theory with the coupling constant $W = 0$.
- Two $f(R)$ models have been taken as special cases namely $f(R) = \beta R^n$ and $f(R) = \alpha R + \beta R^n$.
- The dependence of $V(\phi)$ on scalar field ϕ is parabolic. This is in agreement with the existing literature about the inflation potential.
- We have obtained solution to Klein-Gordon equation for both cases.
- As time grows the scalar field decreases and approaches zero asymptotically. This inspection is also in agreement with the literature on the scalar field behavior.

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Thanks for your
attention!