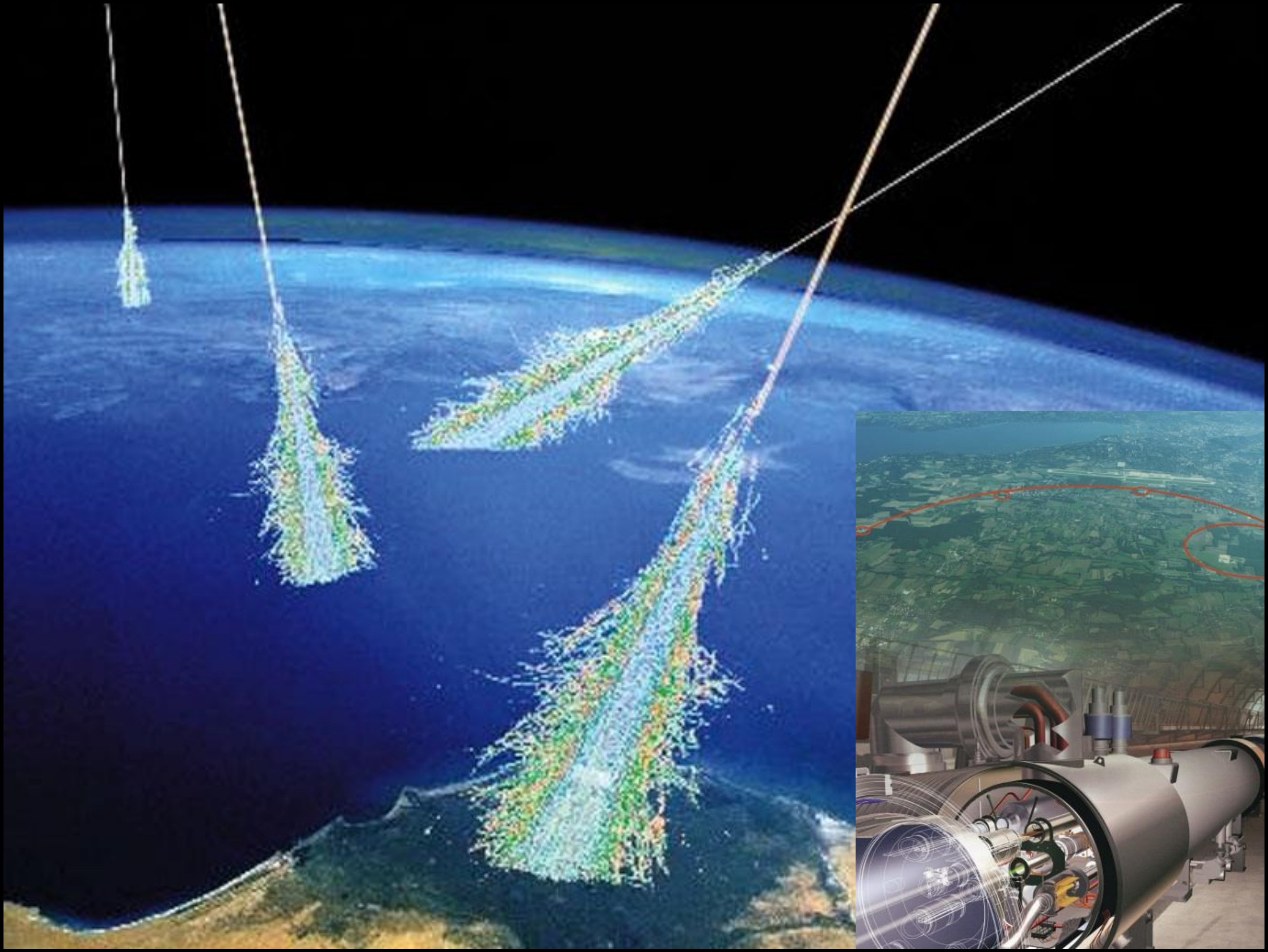
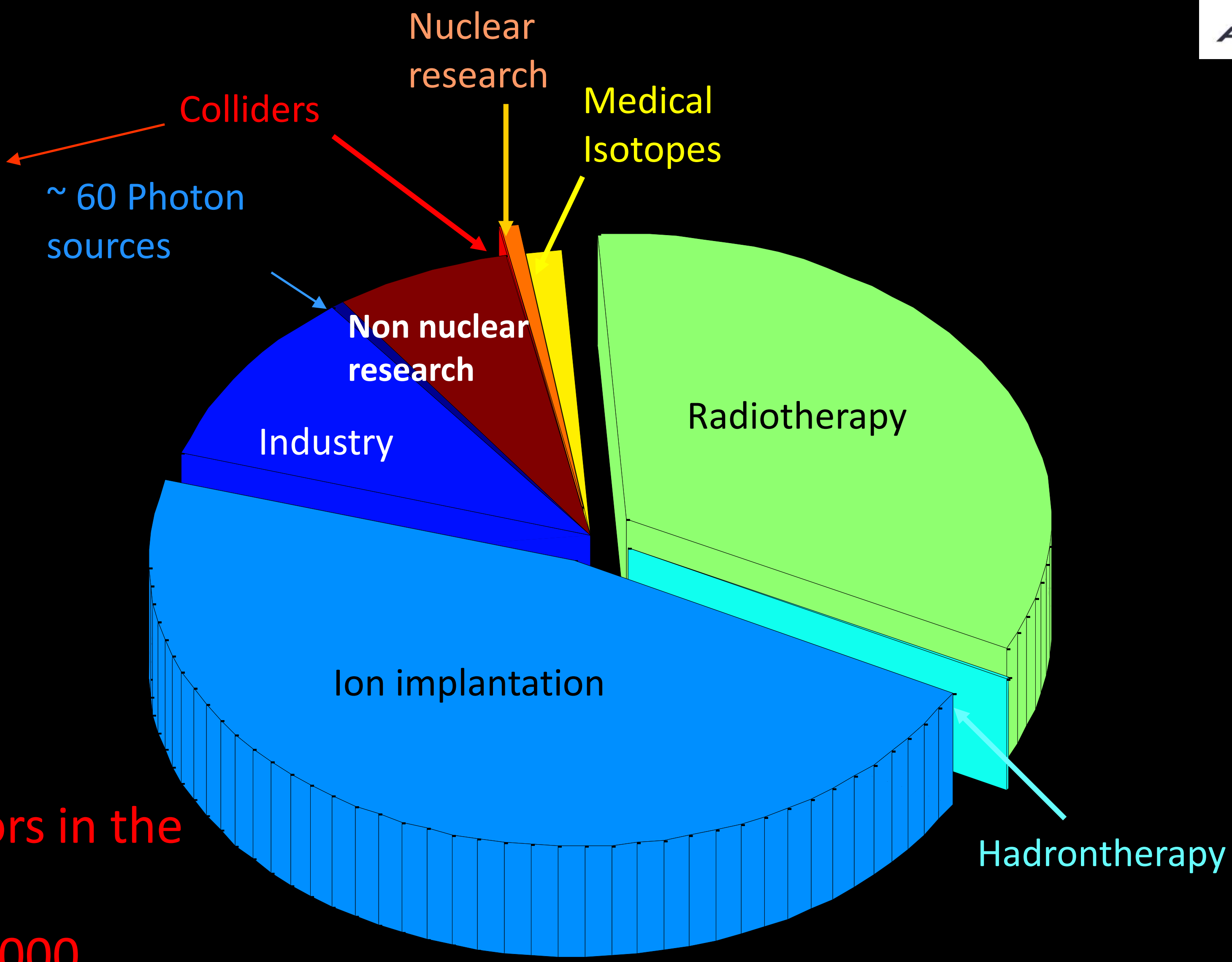
Introduction to Accelerators



1 CERN
1 Italy
2 Russia
1 China
1 USA
1 Japan



Accelerators in the world: around 30000 (15000 in 2000)

Livingston's chart of accelerators – ~ One century of history

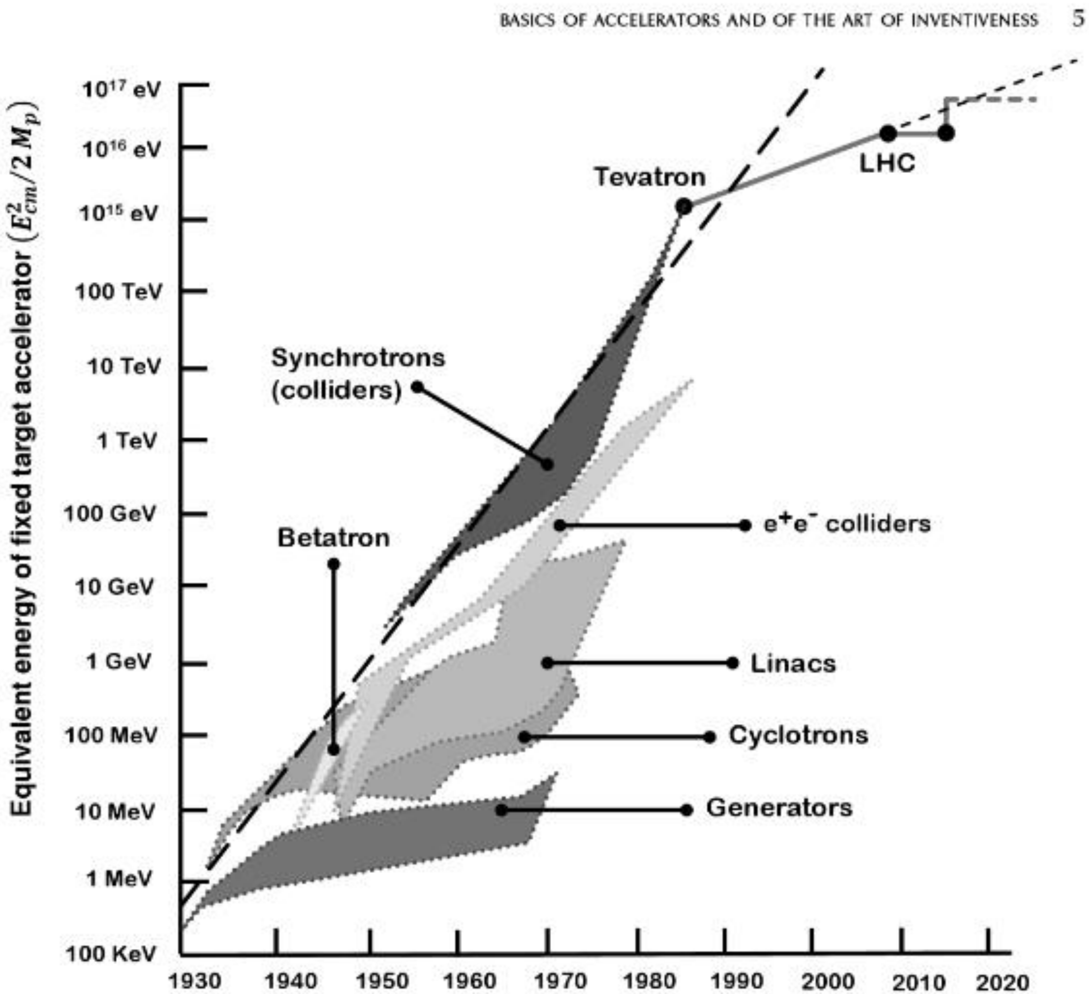
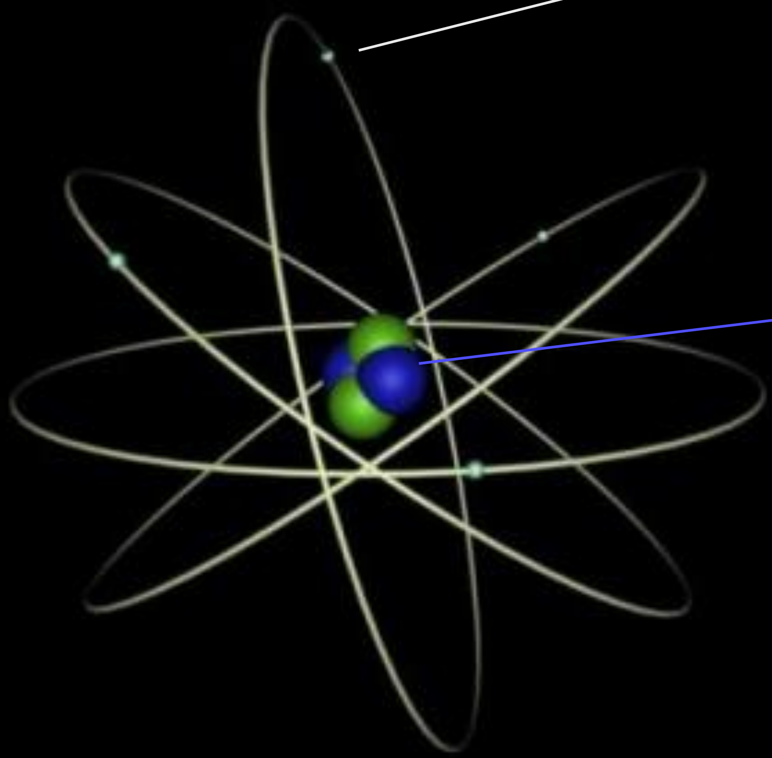


FIGURE 1.6
Livingston plot of evolution of accelerators.

Nearly nine decades of continued growth in the energy reach of accelerators
Driven by continuous innovation in acceleration techniques
Many new acceleration techniques developed to keep pushing the energy frontier

Particle in the accelerators =
= charged particles



Electrons
Mass = 9.1 * 10^{-31} kg

Protons
Mass = 1,7 × 10^{-27} kg

and Ions

ATOM

Relativity

For the most part, we will use SI units, except

- *Energy: eV (keV, MeV, etc) [1 eV = 1.6x10^{-19} J]*
- *Mass: eV/c^2 [proton = 1.67x10^{-27} kg = 938 MeV/c^2]*
- *Momentum: eV/c [proton @ β=.9 = 1.94 GeV/c]*

$$\beta \equiv \frac{v}{c} = \frac{pc}{E}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$\text{momentum } p = \gamma m v$$

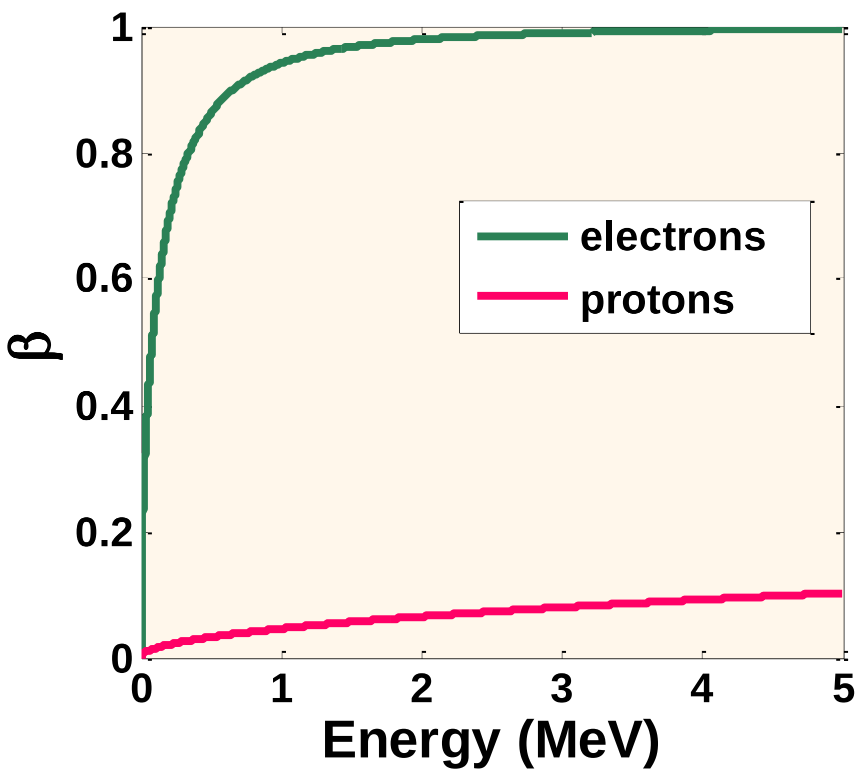
$$\text{total energy } E = \gamma m c^2$$

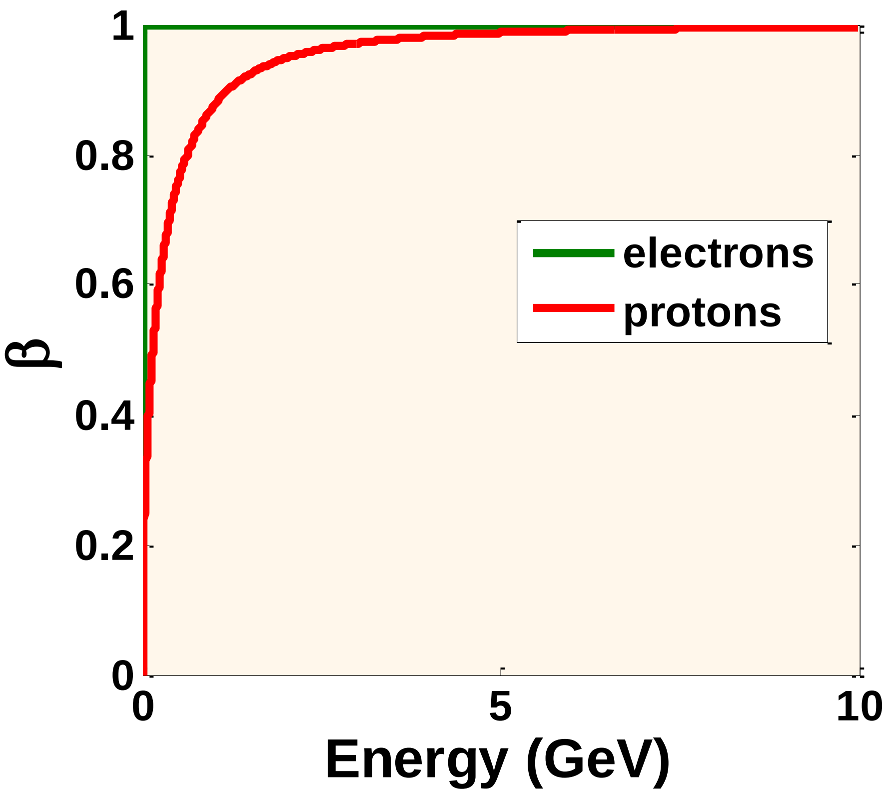
$$\text{kinetic energy } K = E - mc^2$$

$$E^2 = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2}$$

Particle velocity as a function of kinetic energy

$$\beta = \frac{v}{c} \qquad \beta = 1 \longrightarrow \text{Particle at light velocity } c$$





Electrons are ultrarelativistic at few MeV

Protons at few GeV (mass ~2000 times electron mass)

Lorentz's Force

Particle dynamics are governed by the Lorentz force law

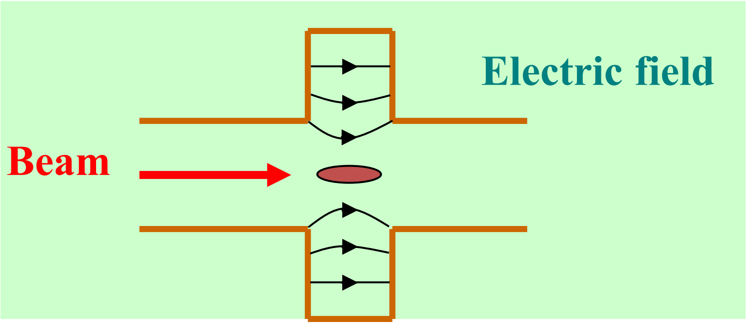
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} \quad \text{for } v << c$$

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{for any } v$$

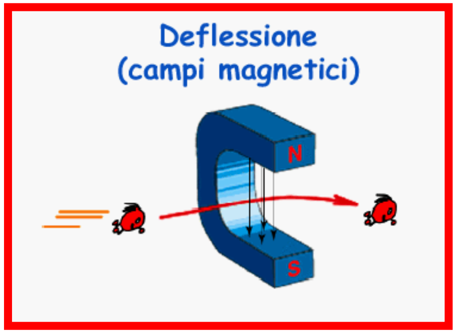
Acceleration

$$\vec{E} = \text{electric field}$$



Bending and focusing

$$\vec{B} = \text{magnetic field}$$



Main systems of an accelerator

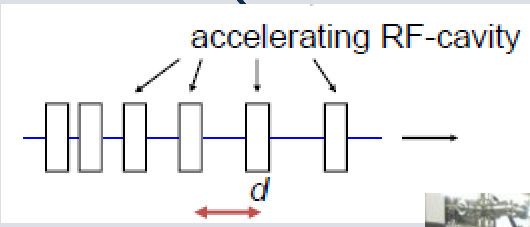
- Sources
- Rf cavities
- Magnets
- Vacuum system
- Beam diagnostics
- Control system

Main accelerators types and their major utilization

LINACs Radiotherapy, FELs, neutron sources, colliders, injectors	**Cyclotrons** Isotope production, proton therapy, nuclear physics, neutron sources
Synchrotrons colliders, photon sources, injectors	**Others: betatrons, van der Graaf, electrostatic, FFAG, ERL,…**

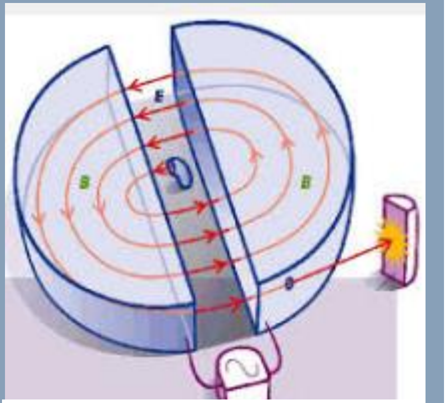
Main accelerators types and their major utilization

Linac (Linear Accelerator)



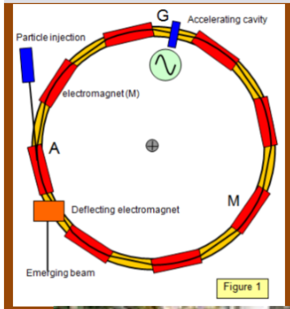
$d=\beta\lambda/2$

Cyclotron



$$\omega_o = \frac{qB}{m} = \omega_{RF} = \text{costante}$$

Synchrotron



Main accelerators applications and their typical parameters

Colliders (LHC, SuperKEKB,..) e+, e-, p $\bar{p}$, ions *Energy, Luminosity*	**Photon Sources (ESRF, LCLS, ALBA,…)** electrons *Energy, Brilliance*
HPPA (SNS, GSI, ESS,..) Protons *Energy, Power*	**Medical applications (radio and hadrontherapy)** e-, p, C ions, *Energy, Dose*

Colliders

Colliders have made the history of particle physics

Beam beam collision *Fixed target collision*

$$\boxed{W \cong 2\sqrt{E_1 E_2}}$$

$$\boxed{W \cong \sqrt{2E\, m_t}}$$

W = 1 GeV in the center of mass

$E_1 = E_2 = 0.5$ GeV *E = 1000 GeV (e-)*

W = 100 GeV in the center of mass

$E_1 = E_2 = 50$ GeV *$E = 10^7$ GeV (e-)*

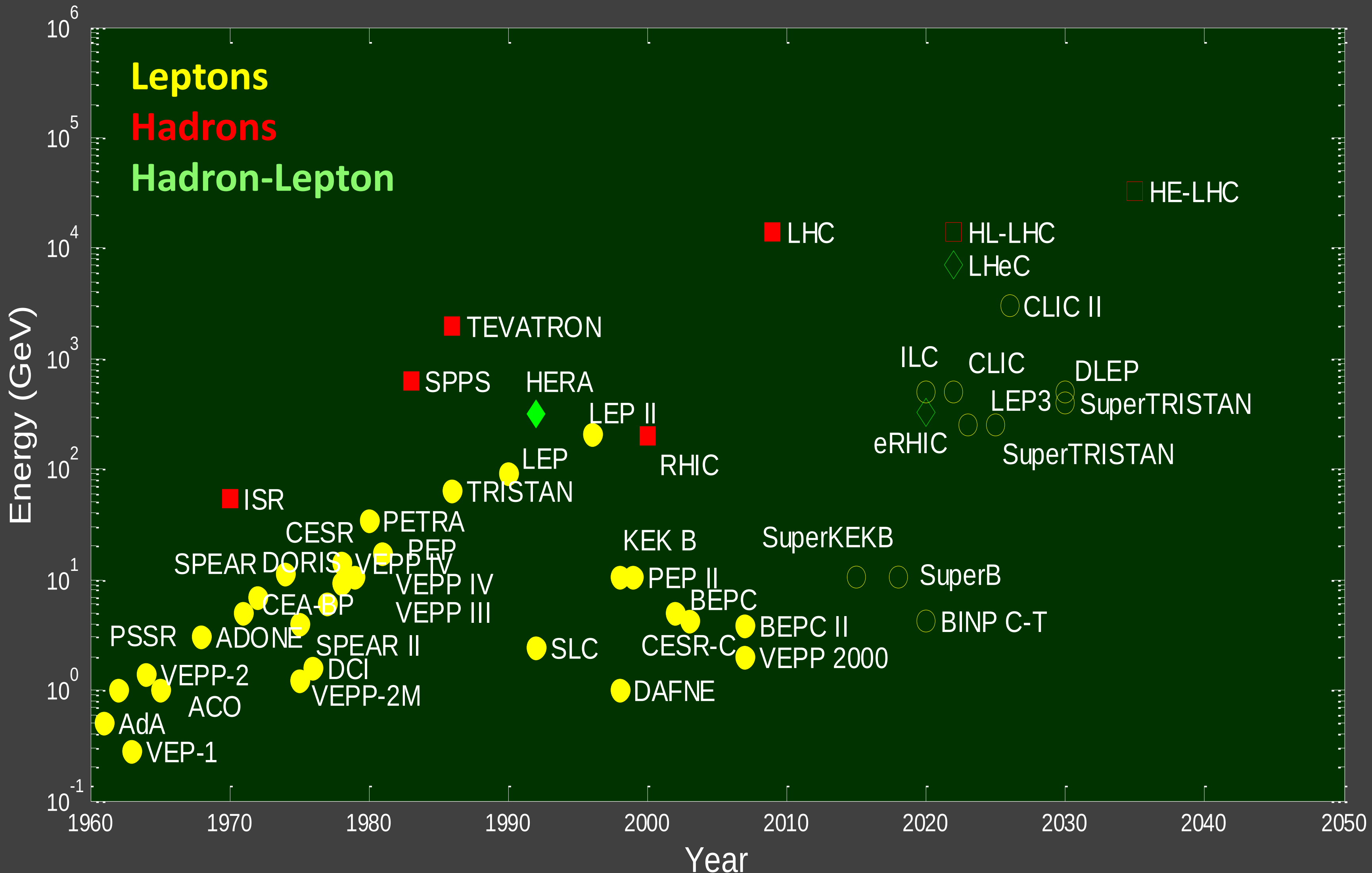
Colliders

Hadron colliders -> Discovery experiments

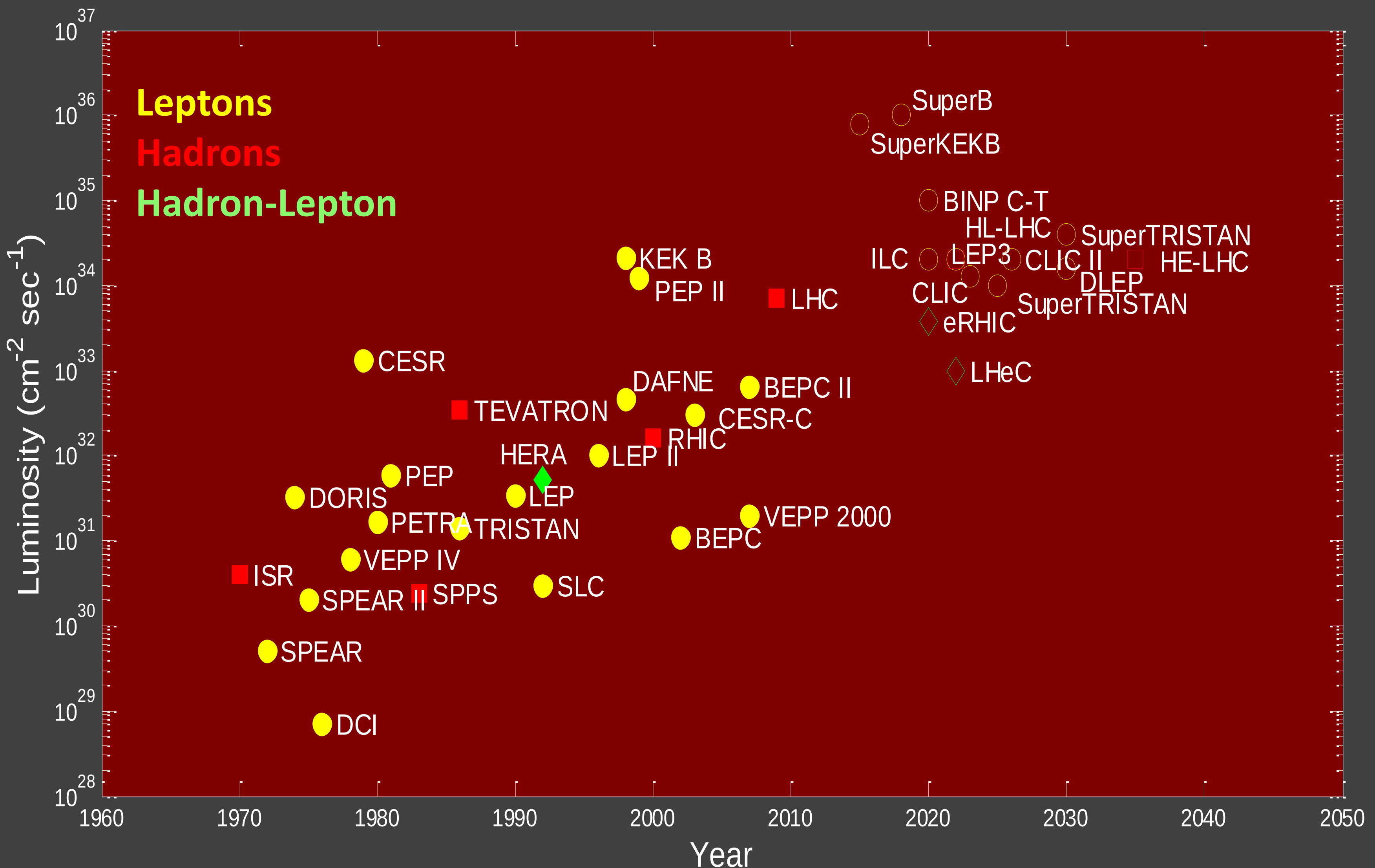
Lepton colliders - > Precision experiments

Hadron-Lepton Colliders -> Leptons used to probe hadron structures

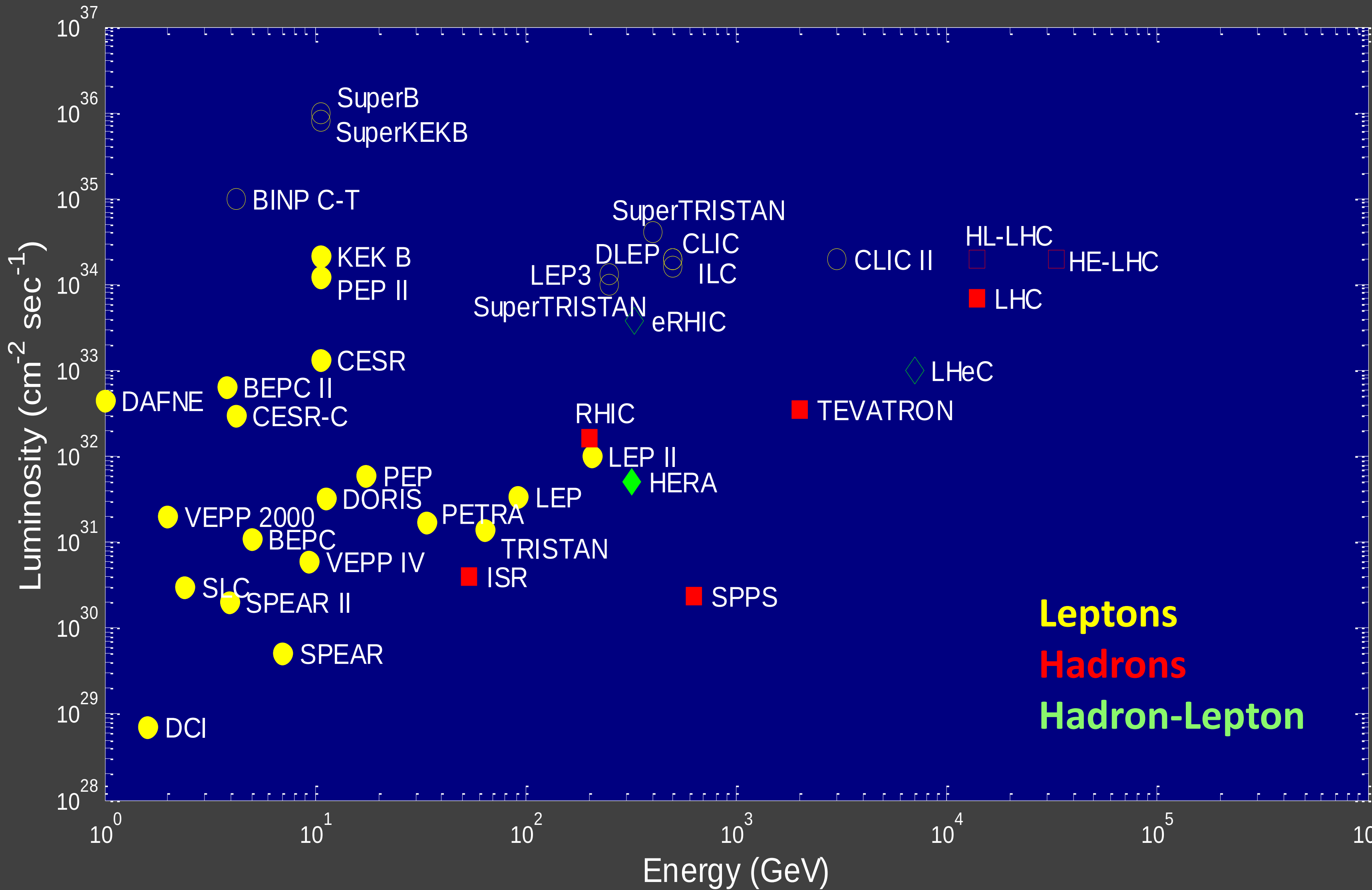
Collider energies



Collider luminosities



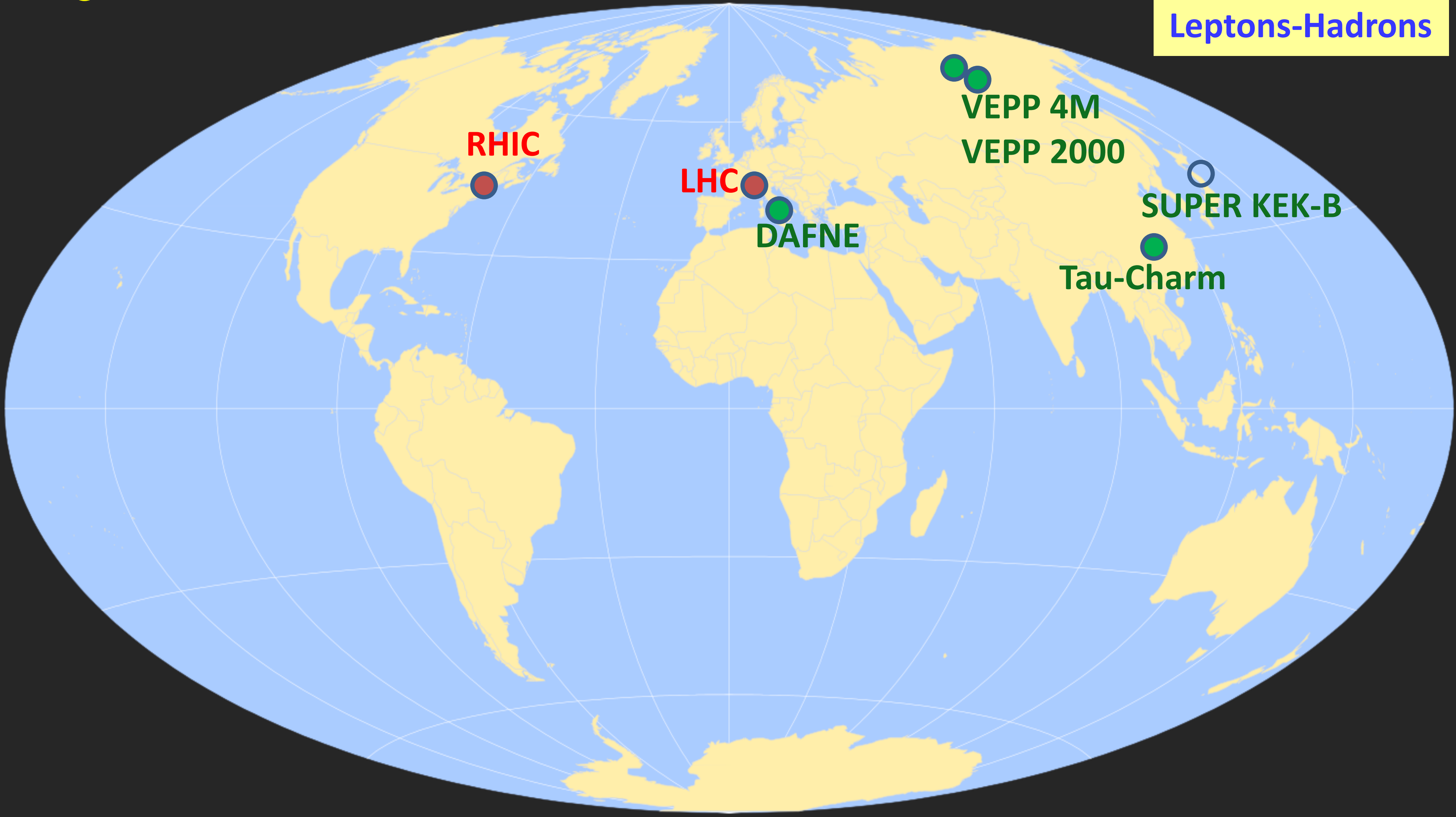
Luminosity versus Energy



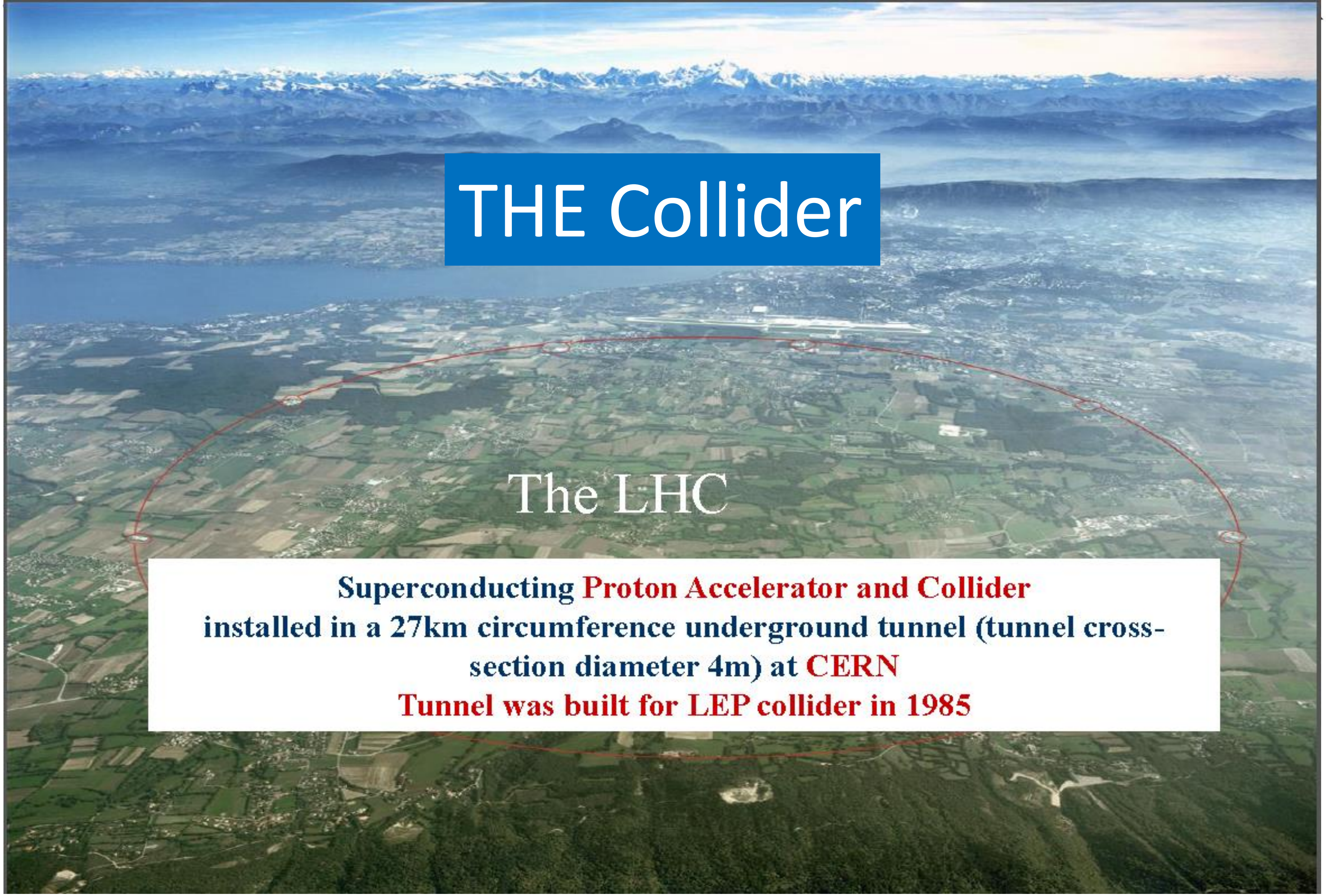
Particle Colliders - 2016

- In operation
- In commissioning

Hadrons
Leptons
Leptons-Hadrons



THE Collider



Superconducting Proton Accelerator and Collider installed in a 27km circumference underground tunnel (tunnel cross-section diameter 4m) at CERN

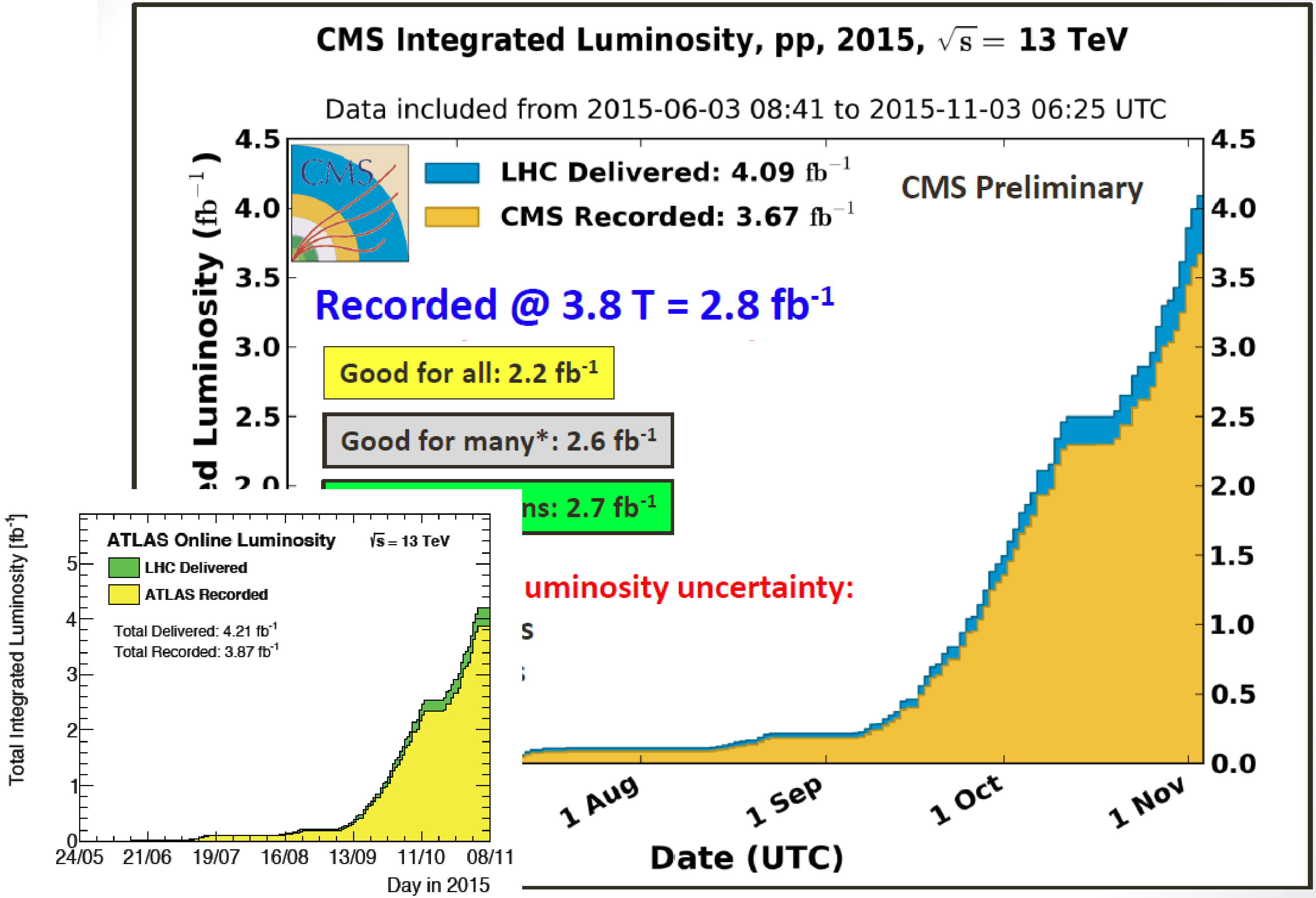
Tunnel was built for LEP collider in 1985

LHC arc view

27 km cooled down to 2K
The coolest place in the world

13 TeV dataset

Run 2 – LHC integrated luminosity (CMS)



CMS Collaboration - 13 TeV Results 15/12/2015

Medical applications

Isotope production

Cancer therapy

Radio therapy

Hadron therapy

Cancer therapy: X-rays

- 1937: first X-rays irradiation from van de Graaff electrostatic machine
- 1956 : first patient treated with radiotherapy at Stanford – eye tumor
- Since then 40 millions patients have been treated
- Today 50% of cancer patients are X-ray treated (70% in industrialized countries), either alone or in combination with other techniques.
- Electron Linacs (4 – 22 MeV) produce e- to be shot on a metallic target and produce X-rays

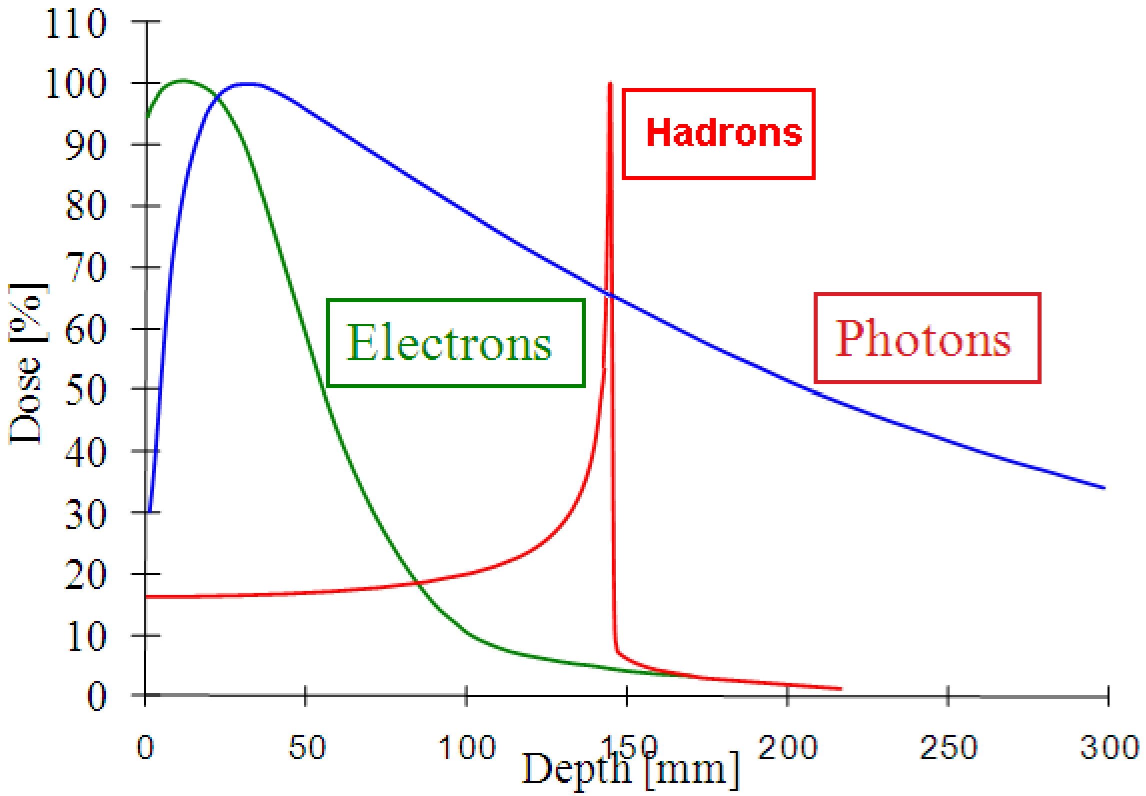
Courtesy of Department of Radiation Oncology

The first patient to receive radiation therapy from the medical linear accelerator at Stanford was a 2-year-old boy.

- Commercially produced
  - Thousands of compact and fully reliable Linacs are daily treating patients
  - Control of radiation dose and radiation fields decrease collateral effects
  - IMRT = Intensity Modulated Radiation Therapy

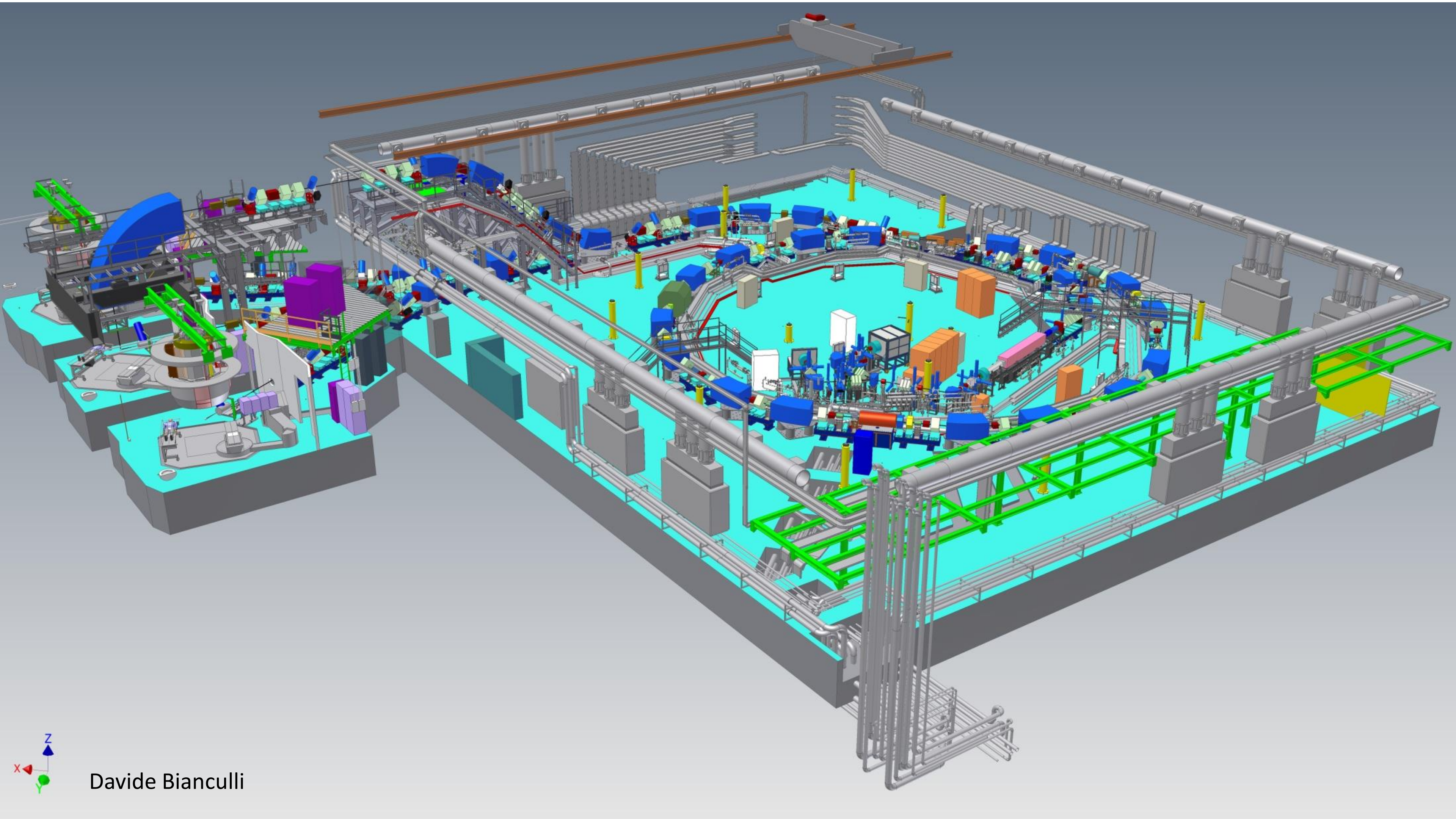
X-Rays and hadrontherapy

Hadrontherapy uses hadrons (protons and ions)
Particles at high energy deposit relatively little energy as they enter an absorbing material but tend to deposit extremely large amounts of energy in a very narrow peak, the Bragg peak, as they reach the end of their range: Very localized depth-dose deposition



The depth and magnitude of the Bragg peak is determined by the mass and charge, as well as the particle initial energy

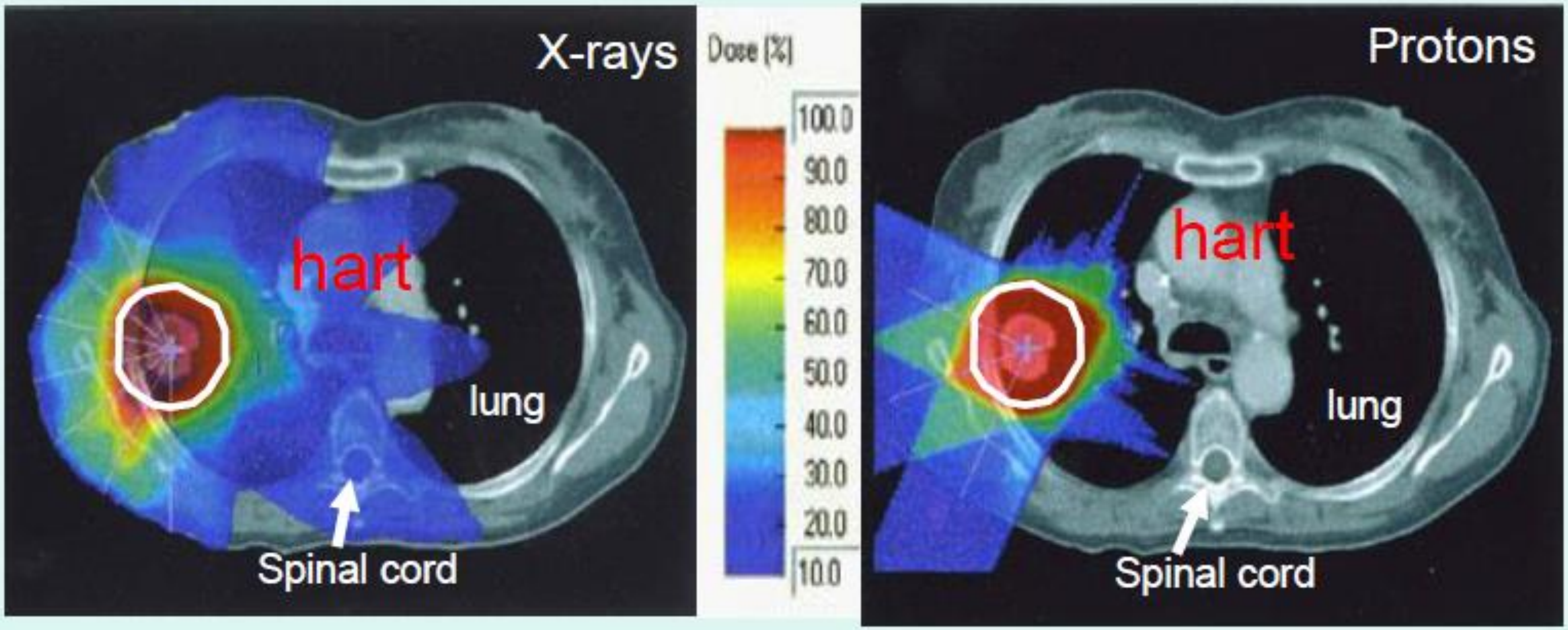
CNAO (Pavia): Accelerator and Treatment Rooms



X-rays vs. Protons

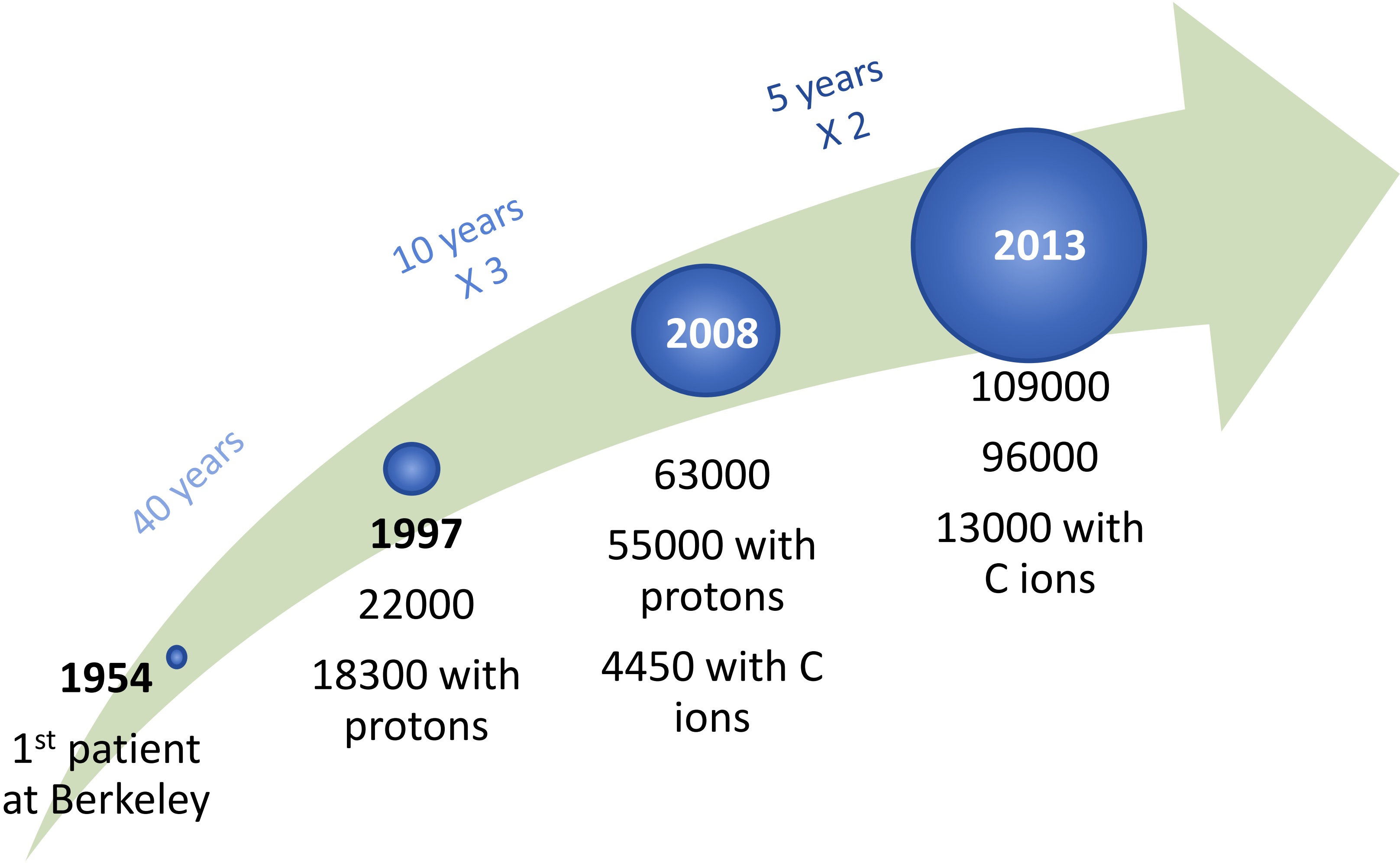
X-ray beams (IMRT) from 7 directions

Proton beams from 3 directions



pictures: Medaustron

Growth of Hadrontherapy



Basics on beam dynamics

Rigidity: Relation between radius and momentum

$$B\rho = \frac{p}{q}$$

How hard (or easy) is a particle to deflect?

- Often expressed in [Tm] (easy to calculate B)
- Be careful when q≠e!!

$$B\rho[Tm] \approx 3.33 \frac{p[\frac{GeV}{c}]}{q[e]}$$

Electrons and protons (now in MeV)

$$E_o = 0.511\ MeV \quad or\ 938.27\ MeV$$
$$E_{tot} = E_{kin} + E_o$$
$$\gamma = \frac{E_{tot}}{E_o}, \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$
$$p = \beta E_{tot}$$

$$B\rho = 3.33\ 10^{-3} p$$

Energia [MeV]	*Rigidità Bρ [Tm]*	
	p	*e⁻*
1	0,14	0,005
10	0,44	0,035
100	1,45	0,34
1.000	5,66	3,34
10.000	36,35	33,36
100.000	336	336
1.000.000	3335	3335

Ions

Kinetic energy per nucleon: E_{kin}

Total charge = Qe

$$E_e = 0.511\ MeV \qquad E_p = 938.27\ MeV \qquad E_n = 939.57 MeV$$

$$N_e, Z, N \Rightarrow A = Atomic\ mass\ (Z + N)$$

$$E_o = ZE_p + NE_p + N_e E_e - A * 0.8$$

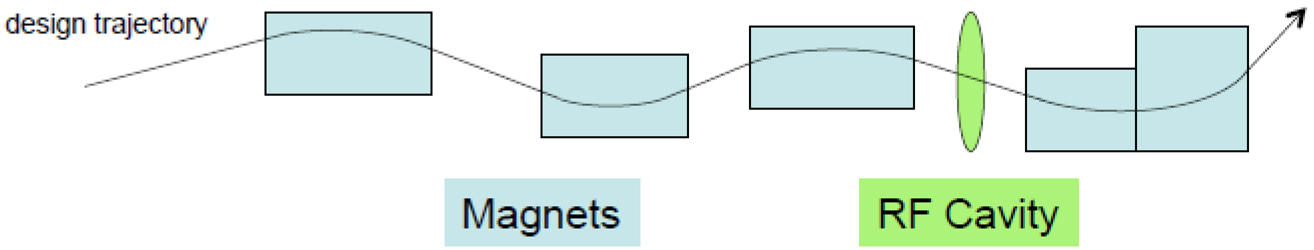
$$E_{tot} = A\ E_{kin} + E_o$$

$$\gamma = \frac{E_{tot}}{E_o}, \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$p = \beta E_{tot}$$

$$B\rho = 3.33\ 10^{-3} p/\mathrm{Q}$$

Simplified Particle Motion



Design trajectory

- Particle motion will be expanded around a **design trajectory** or **orbit**
- This orbit can be over linacs, transfer lines, rings

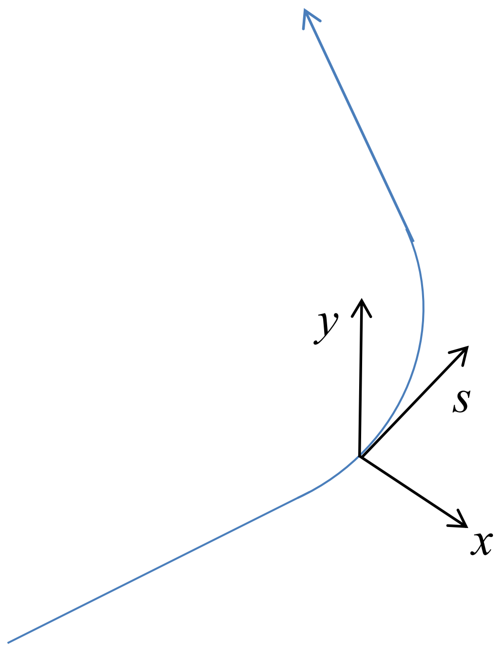
Separation of fields: Lorentz force

- Magnetic fields from static or slowly-changing magnets transverse to design trajectory
- Electric fields from high-frequency RF cavities in direction of design trajectory
- Relativistic charged particle velocities

The accelerator from the particle point of view is a sequence of

- Drifts – No external fields – Particles go straight
- Magnetic fields – Particles are bent according to the magnetic rigidity
- Electric fields – Particles go straight, gain or loose energy

Single particle dynamics in magnetic fields



Reference system

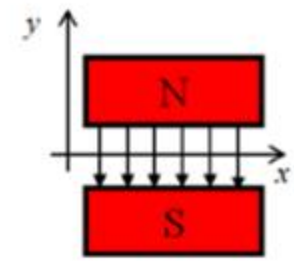
x : horizontal

y : vertical

s : longitudinal along the trajectory

Properties of Typical Magnets

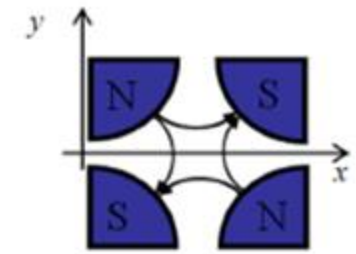
Dipoles: used for guiding the particle trajectories



$B_x = 0$

$B_y = B_o = proportional\ to\ B\rho \qquad B_o/B\rho = 1/\rho\ [m^{-1}]$

Quadrupoles: used to focus the particle trajectories

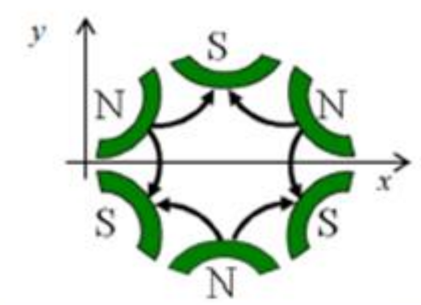


$B_x = G\,y$

$B_y = -G\,x \qquad k = G/B\rho = (1/B\rho)\, dB_x/dx\ [m^{-2}]$

$G = proportional\ to\ B\rho$

Sextupoles: used to correct chromatism and non linear terms



$B_x = 2\,S\,x\,y$

$B_y = S\,(x^2 - y^2) \qquad k^2 = S/B\rho = (1/B\rho)\, d^2B/dx^2\ [m^{-3}]$

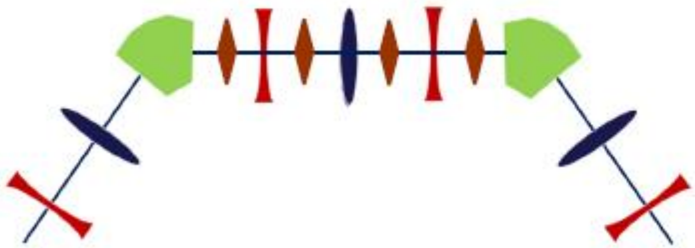
$S = proportional\ to\ B\rho$

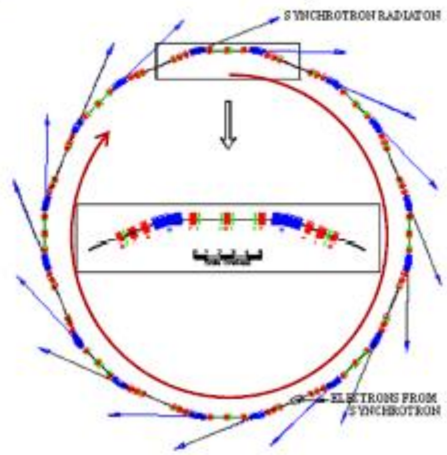
Fields

Normalized fields (with magnetic rigidity)

Lattice in an accelerator

Lattice: Sequence of magnets interleaved with drifts (used for diagnostics, vacuum pumping, Injection, extraction, etc)





The first step in calculating a lattice is to consider only the linear components of it (quadrupoles and dipoles). Non linear effects and chromatic aberration corrections will be evaluated later.

The trajectory of the reference particle (the particle with nominal energy and initial position and divergence set to zero) along the optics is calculated.

All the other beam particles are represented in a frame moving along the reference trajectory, and where the reference particle is always in the center.

Coordinate systems used to describe the motion are usually locally Cartesian or cylindrical (typically the one that allows the easiest field representation)

Linear transverse motion
(in presence of dipoles and quadrupoles)

Lorentz force +
Linear magnetic fields +
Derivative along s instead of t

$$x'' + \left(\frac{1}{\rho^2} - k\right)x = 0$$ H

$$y'' + ky = 0$$ V

$$k = \frac{g}{p/e} = \frac{1}{B\rho}\frac{dB_y}{dx}$$

Solution of equations of motion:

$a_1 = x_0 \quad a_2 = \frac{x'_0}{\sqrt{K}}$

Horizontal : $K = \frac{1}{\rho^2} - k$

Vertical : $K = k$

Here x represents both x or y

$$x(s) = x_0 \cos\left(\sqrt{K}s\right) + x'_0 \frac{1}{\sqrt{K}} \sin\left(\sqrt{K}s\right)$$
$$x'(s) = -x_0 \sqrt{K} \sin\left(\sqrt{K}s\right) + x'_0 \cos\left(\sqrt{K}s\right)$$

Matrix formalism

We can write the equations in matrix formalism: coordinates at point s_1 can be obtained knowing the coordinates at s_0

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

$$M = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}$$

Example: Drift
Length: L
$K = 0$

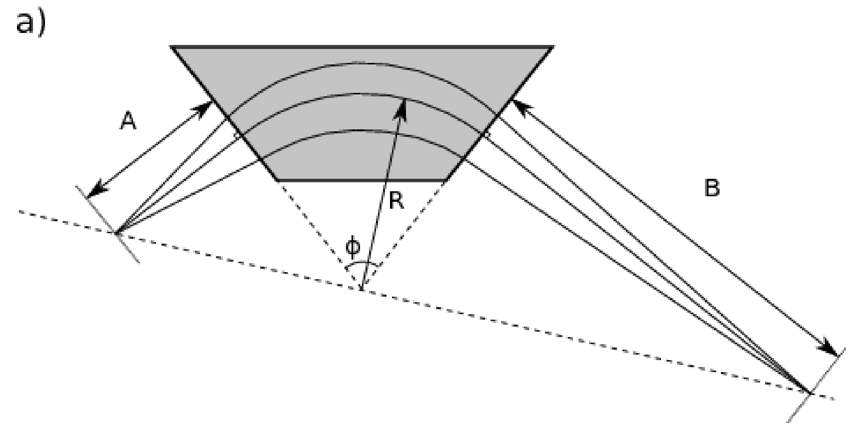
$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$x_1 = x_0 + Lx'_0$$
$$x'_1 = x'_0$$

Focusing quadrupole:
Length L, $K > 0$

$$M_Q = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}$$

Dipole



Sector magnet:
Nominal particle trajectory is perpendicular to dipole entrance
Horizontal plane: $K = {}^{1}/_{\rho^2} - k$
Vertical plane: $K = k$

If $k = 0, L = \rho\theta$

$$M_H = \begin{pmatrix} cos\theta & \rho sin\vartheta \\ -\frac{1}{\rho} sin\theta & cos\theta \end{pmatrix}$$

ρ = bending radius
θ = bending angle

$$M_V = \begin{pmatrix} 1 & \rho\vartheta \\ 0 & 1 \end{pmatrix}$$

Matrix of a lattice

System of lattice elements: Drifts (M_D), quads (M_Q), bendings (or dipoles) (M_B)

Starting with $\binom{x_0}{x'_0}$ The final position and divergence of the particle will be $\binom{x_1}{x'_1}$

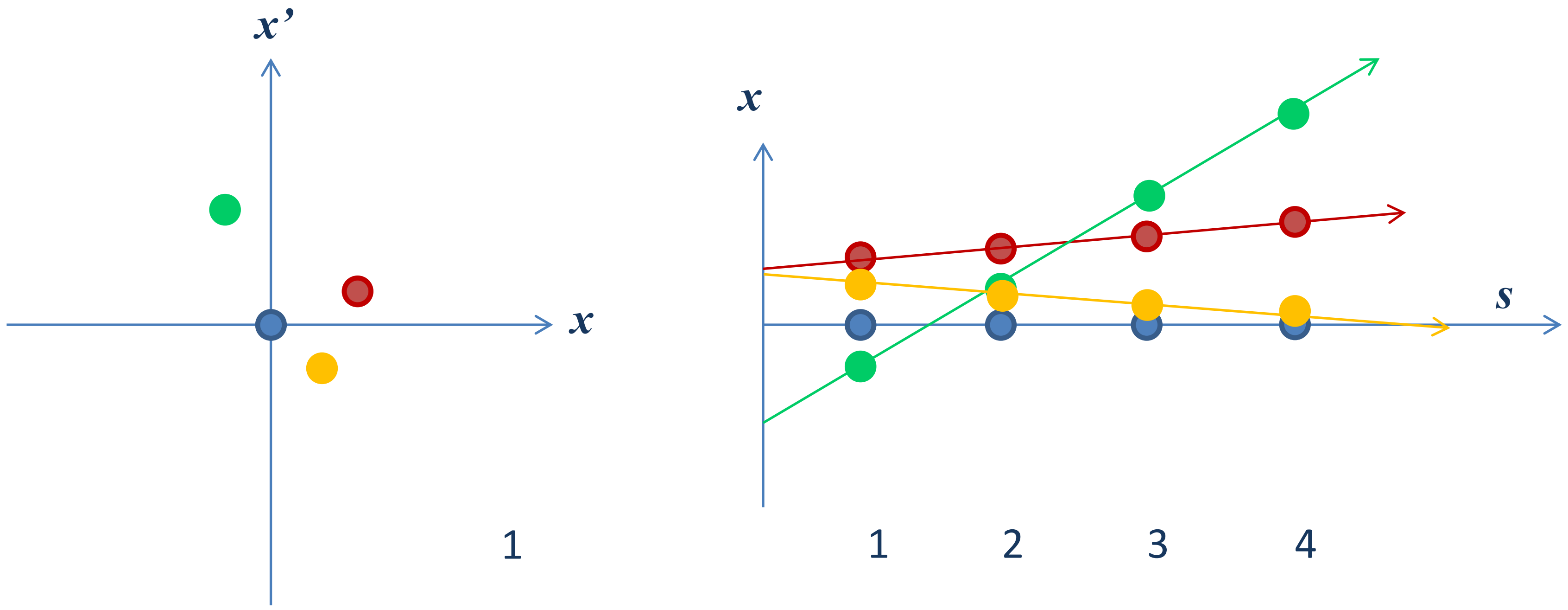
$$\binom{x_1}{x'_1} = M_{Dn} \cdot M_{Qn} \cdot M_{Dn-1} \cdots M_{B1} \cdot M_{D2} \cdot M_{Q1} \cdot M_{D1} \cdot \binom{x_0}{x'_0}$$

Or simpler

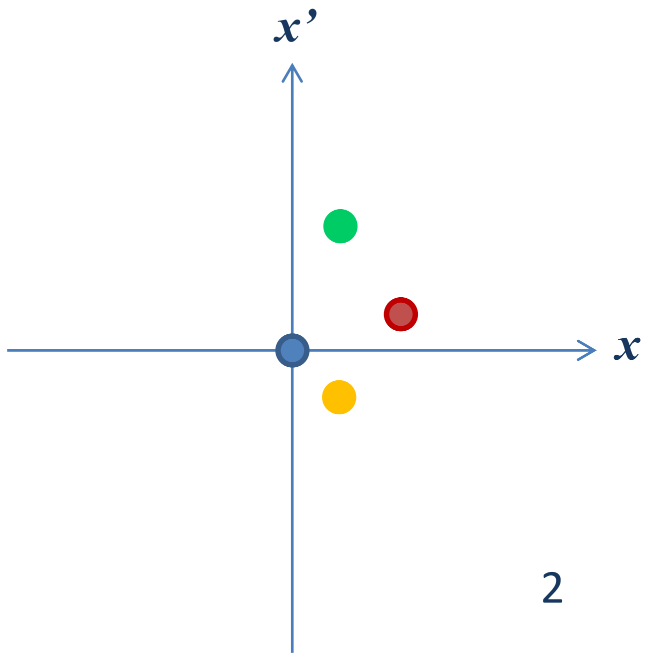
$$\binom{x_1}{x'_1} = M(s_1, s_0) \cdot \binom{x_0}{x'_0}$$

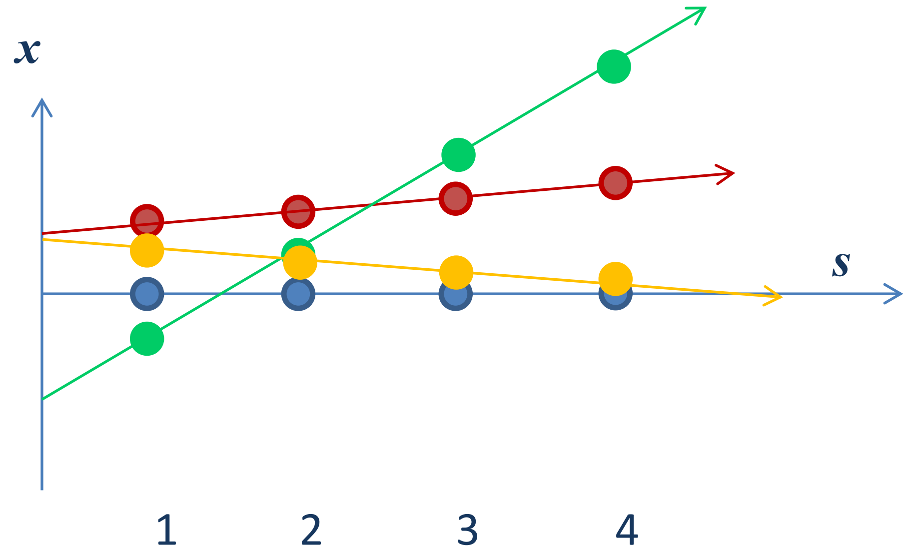
The mathematical representation of an accelerator lattice is a sequence of matrices

From single particle to emittance



From single particle to emittance



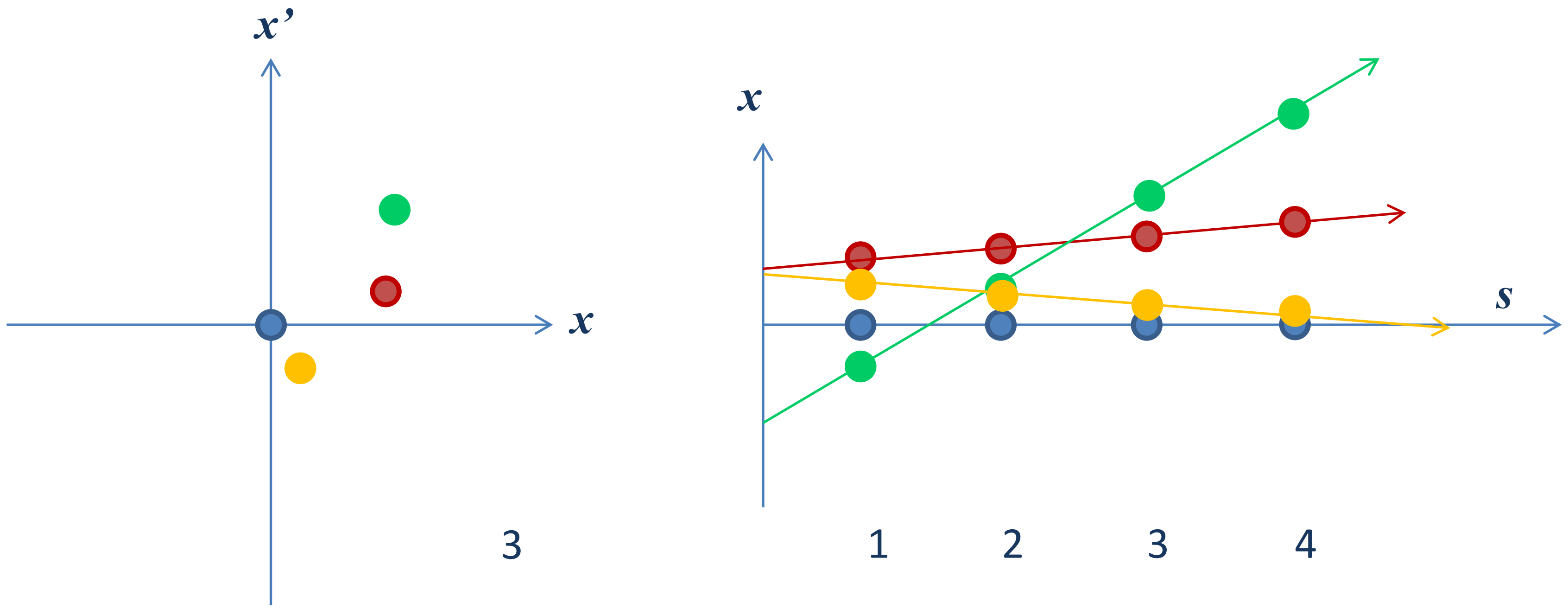


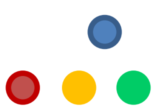
Reference particle

Other particles

(No magnetic field from 1 to 4)

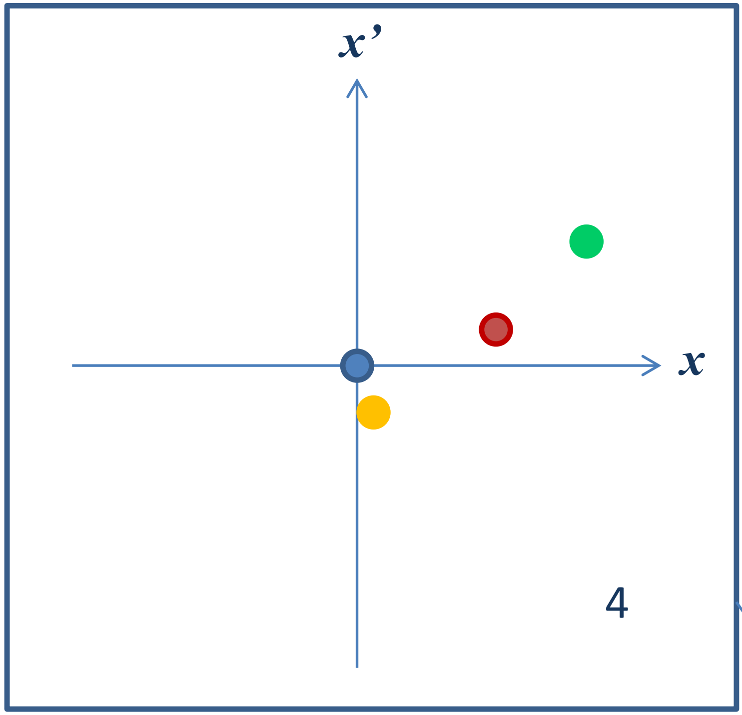
From single particle to emittance

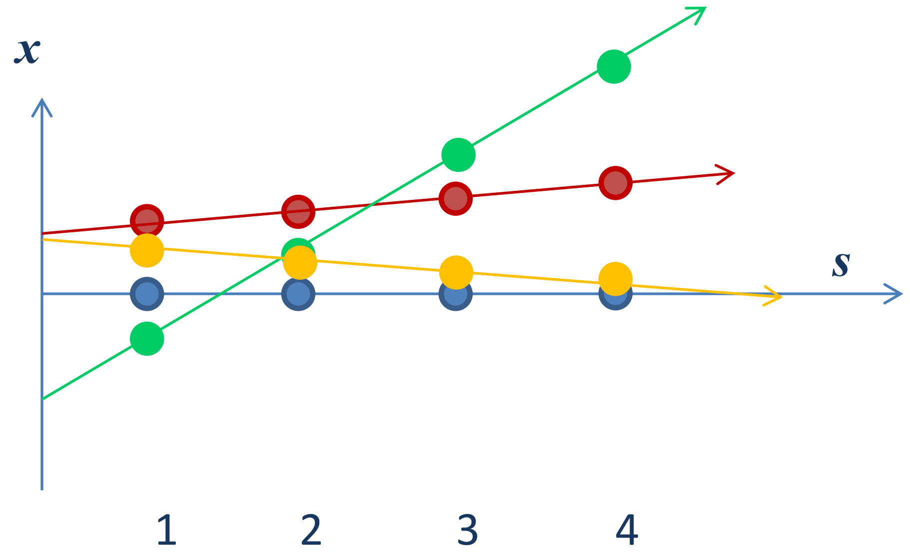




(No magnetic field from 1 to 4)

From single particle to emittance





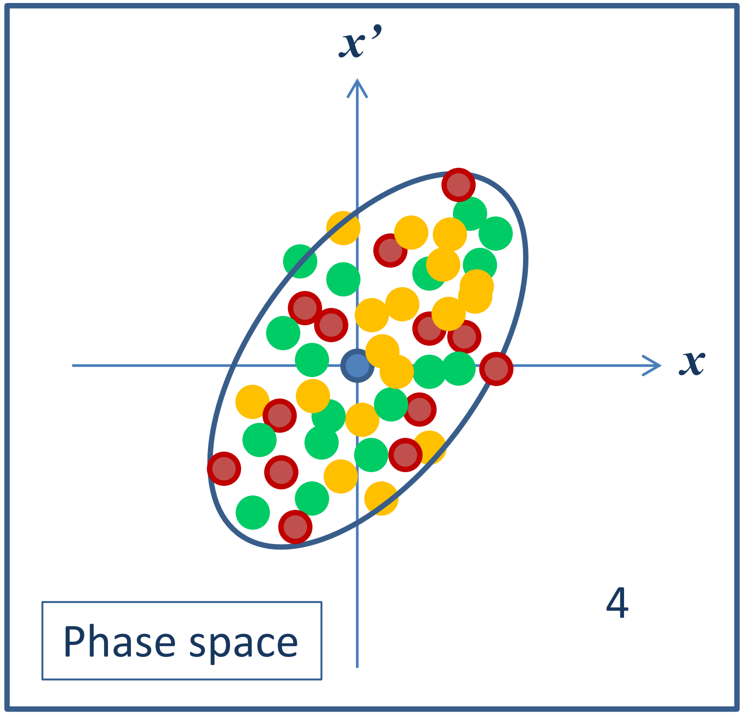
Reference particle

Other particles

Phase space

(No magnetic field from 1 to 4)

Many particles
each of them with its own position and momentum in every point of the accelerator



Reference particle

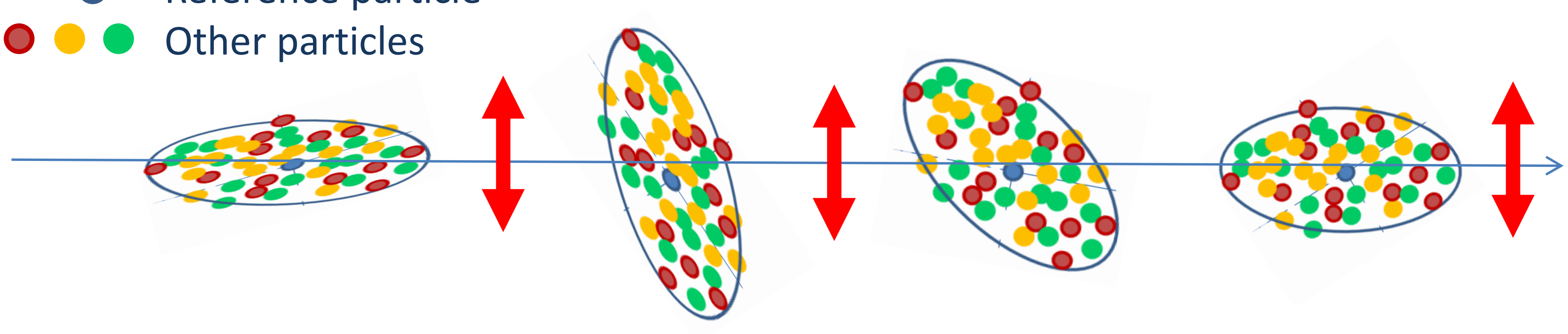
Other particles

Emittance = Area of phase space

Each beam will have emittances

- Horizontal (x, x')
- Vertical (y,y')
- Longitudinal (Time-Energy)

In the only presence of linear magnetic fields ( ) the emittance will be constant, even if the ellipse orientation and axis ratio aspect will change along s



Twiss parameters – Betatron tune

$x'' + Kx = 0$ If K = constant => motion of harmonic oscillator

$x'' + K(s)x = 0$ If K varies with s: Hill's equation

The solution of the Hill equation is given by:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}cos(\varphi(s) + \varphi_0)$$

ε and φ_0 integration constants
Inserting $x(s)$ in the equation of motion it can be shown that the phase advance is related to β by

$$\varphi(s) = \int_0^s \frac{ds}{\beta(s)}$$

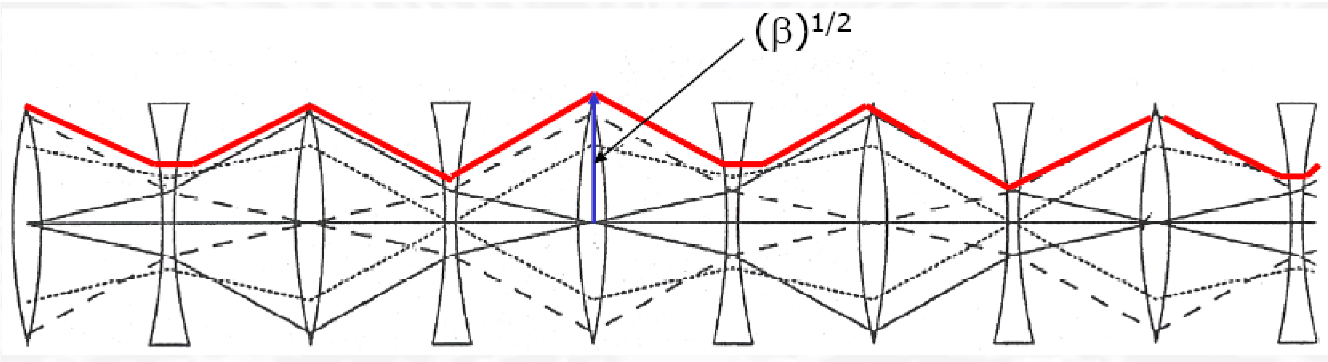
In storage rings (length of circumference = L) beta is periodic

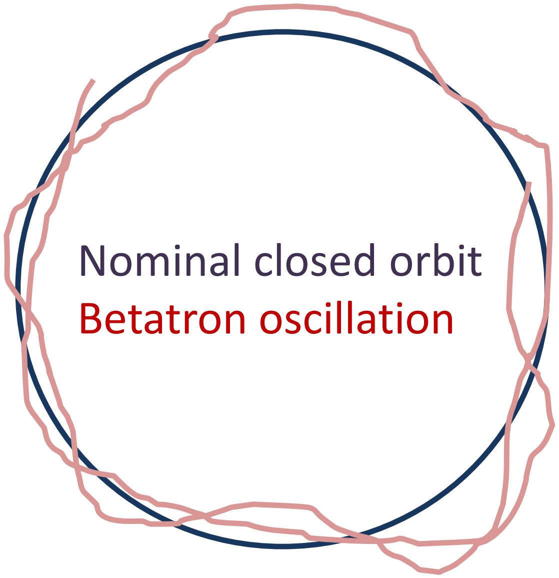
$$\beta(s + L) = \beta(s)$$

One complete turn: phase advance in one turn: ***Betatron Tune***

$$\boldsymbol{Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}(s)}}$$

Betatron oscillation





Particles oscillate around the closed orbit, a number of times which is given by the betatron tune.
The square of the β function by the emittance represents the envelope of the betatron oscillations

Twiss parameters

Amplitude of an oscillation

$$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$

$\beta(s)$represents the envelope of all particle trajectories at a given position s in a storage ring

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}cos(\varphi(s) + \varphi_0)$$

$$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\{\alpha(s)\cos(\varphi(s) + \varphi_0) + sin(\varphi(s) + \varphi_0)\}$$

$$\alpha(s) = -\frac{1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)}$$

α, β and γ are the Twiss parameters

Emittance

Inserting in x' eq.

$$\varepsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s){x'}^2(s)$$

ε is a constant of motion, not depending on *s*.

Parametric representation of an ellipse in x,x' phase space defined by alfa, beta, gamma: **Courant-Snyder invariant** emittance ε

For a single particle, different positions in the storage ring and different turns:

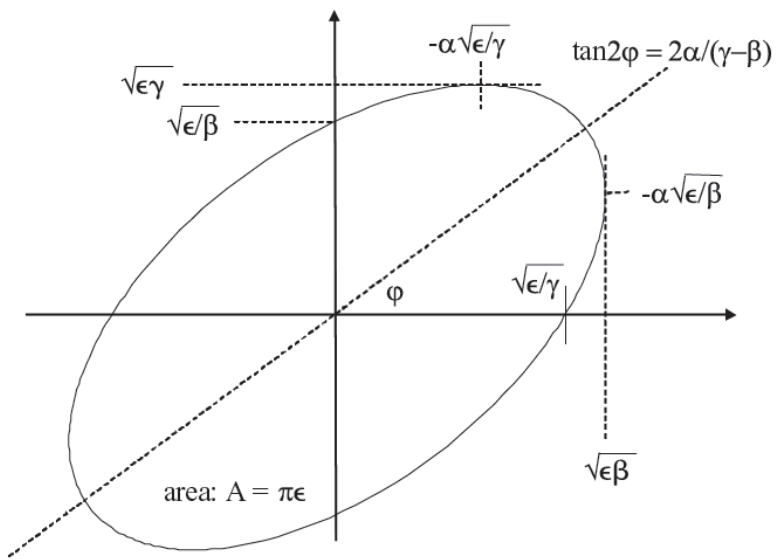
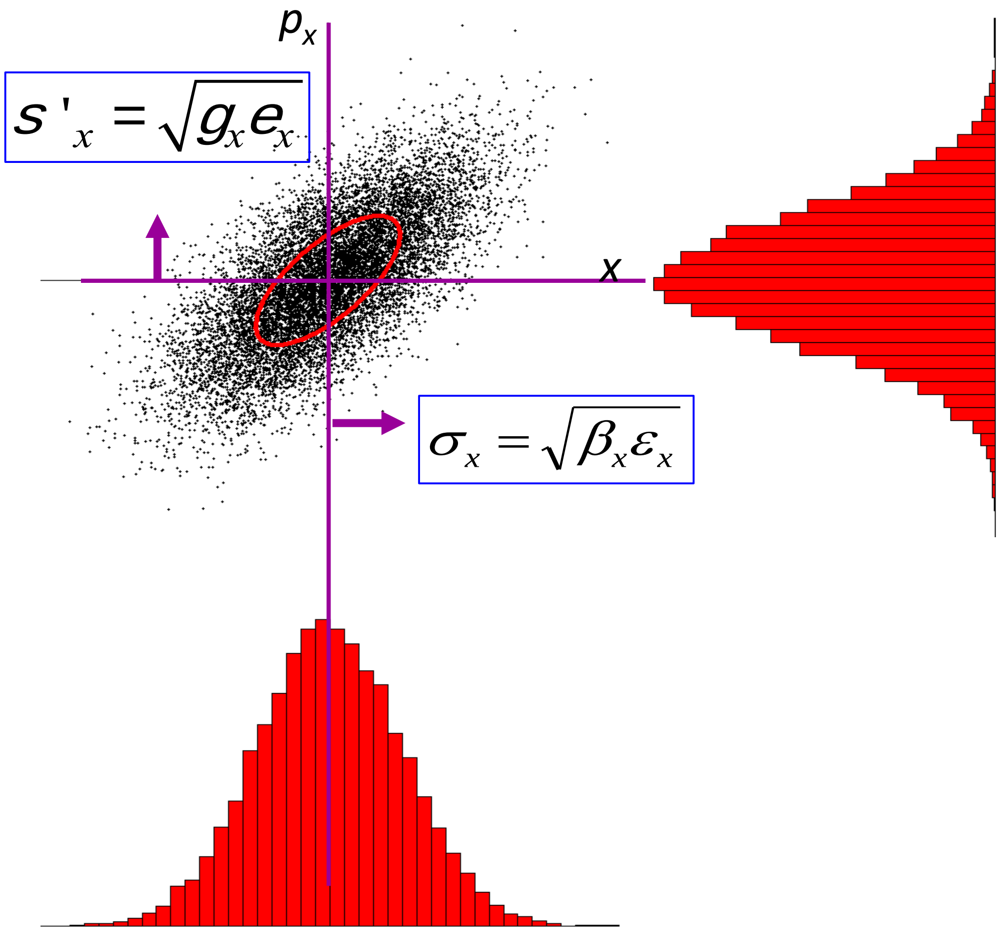


Fig. 5.2. Phase space ellipse

Emittance and beam dimensions

- The emittance is the area of the phase space occupied by the particles
  With the emittance and the Twiss parameters in a point of the accelerator, the beam dimensions are obtained : $\sigma_{x,y}$ e $\sigma'_{x,y}$



$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

Ellipse area= $\pi\varepsilon_x$

$$\langle x^2 \rangle = \beta_x \varepsilon_x$$

$$\langle x'^2 \rangle = \gamma_x \varepsilon_x$$

$$\langle xx' \rangle = -\alpha_x \varepsilon_x$$

$$\beta_x \gamma_x - \alpha_x^2 = 1$$

Adiabatic damping

Transverse emittance while accelerating decreases proportional to increase in momentum

$$x' = \frac{dx}{dx} = \frac{dp_x}{dp}$$

Increasing the longitudinal momentum with increasing energy decreases x' and therefore the transverse emittance

Normalized emittance is defined as the invariant part:

$$\varepsilon_n = \varepsilon\beta\gamma$$

Where now β and γ are not the Twiss parameters, but:

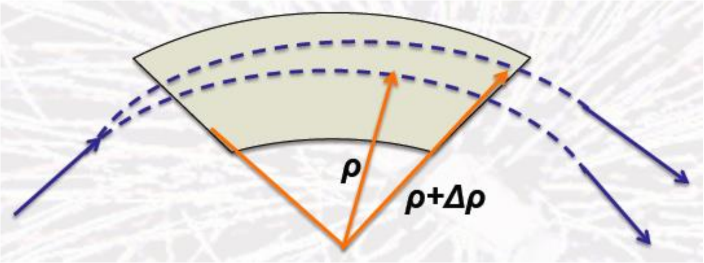
$$\beta = \frac{v}{c} \; and \; \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

Off-Momentum Particles

Magnets are chromatic elements:

Dipoles: $\rho = \frac{p}{eB} \rightarrow \frac{\Delta\rho}{\rho_0} = \frac{\Delta p}{p_0} = \frac{\Delta\theta}{\theta_0}$

Quads: $K = \frac{G}{B\rho} = \frac{Ge}{p}$



$$\Delta\theta = -\theta_0 \frac{\Delta p}{p_0}$$

$$\Delta K = -K_0 \frac{\Delta p}{p_0}$$

$\Delta p/p \ll 1$

Off-momentum particles are not oscillating around design orbit, but around a different closed orbit (chromatic closed orbit). The displacement between the design and chromatic orbits is regulated by the dispersion function *D*(*s*), which is periodic in a synchrotron.

For particles with energy deviation the Hill's equation has an extra term and is not homogeneous:

$$x'' + K(s)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

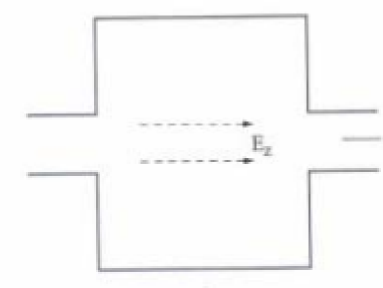
The solution is the sum of the solution of the homogenous equation + a term of dispersion:

$$x = x_{Hom} + D(s)\frac{\Delta p}{p_0}$$

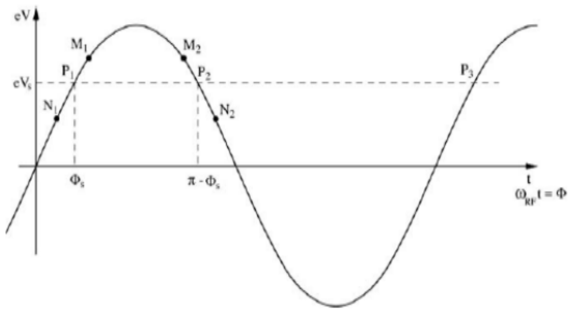
Longitudinal beam dynamics

Kinetic energy gain for a particle with charge e passing in a cavity with an electric field :

$$\Delta W = e\int_0^L \vec{E}\,(\vec{r},t)d\vec{s} = eV$$



V is the voltage gain for the particle, depends on the particle trajectory and includes the contribution of all electric fields (rf fields, space charge, interaction with vacuum chamber,..)



Assuming a sinusoidal electric field:

$$V = \hat{V}\sin(\omega_{RF}t + \phi_s)$$

The synchronous particle passes at the middle of the gap g, at time t = 0:
The energy gain is

$$\Delta W = e\hat{V}T\sin\phi_s$$

- Where T is the transit time, g is the gap length
- $T = \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda}$

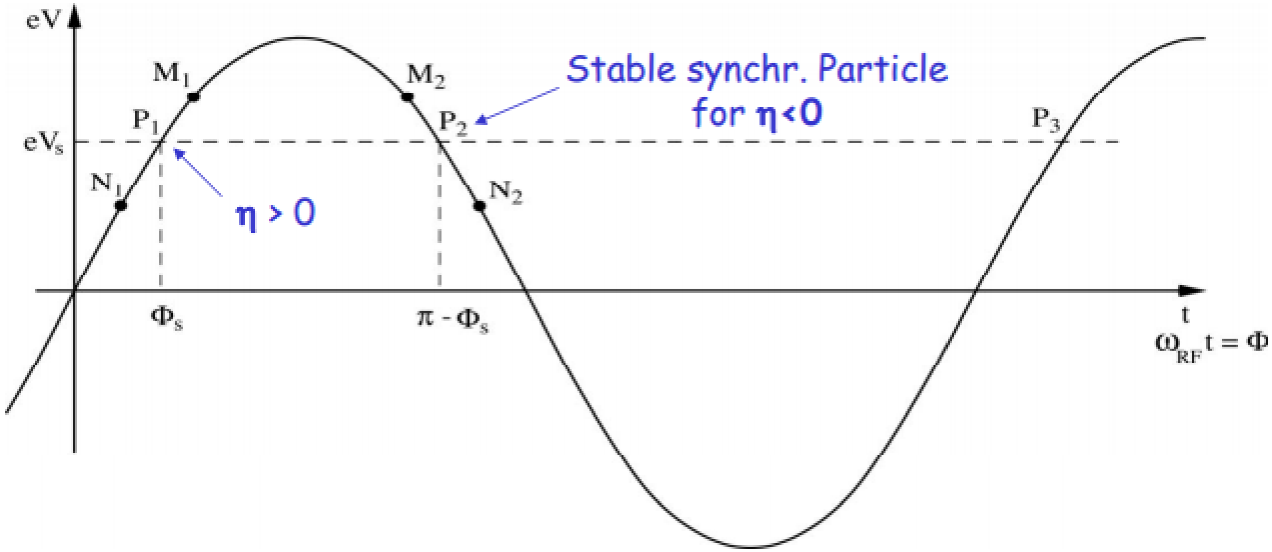
T is the transit-time factor: a factor that takes into account the time variation of the field during particle transit through the gap

In a LINAC the cavities are spaced $L = \beta_s \lambda$
β_s beta after cavity, λ wavelength
The fast particle arrives at $t_a < t_s$ and gains energy $\Delta W_a < \Delta W_s$
The slow particle arrives at $t_b > t_s$ and gains energy $\Delta W_b > \Delta W_s$
The synchronous particle arrives at the synchronous phase
a,b oscillate in phase (time) about the synchronous particle =>

synchrotron oscillation

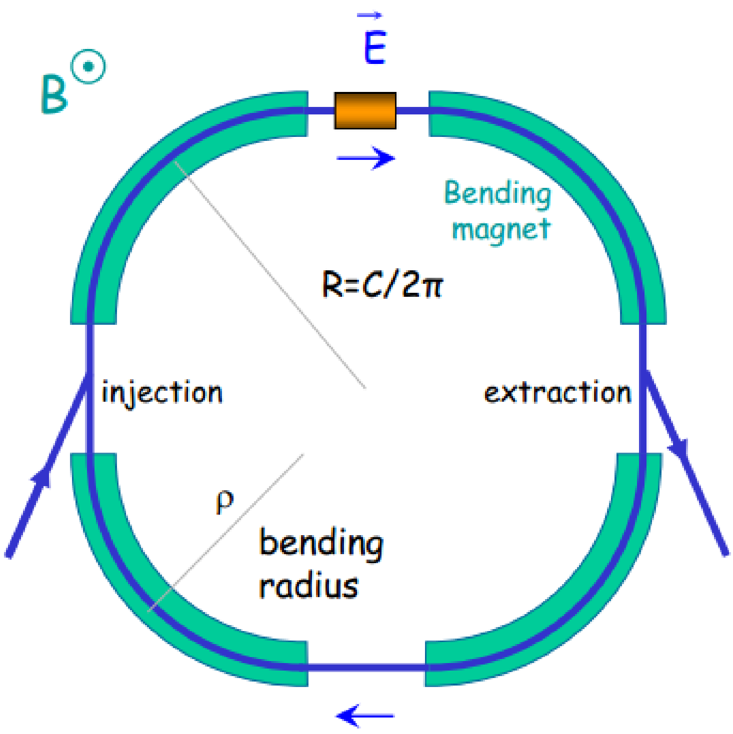
Phase Stability in a Synchrotron

From the definition of η it is clear that below transition an increase in energy is followed by a higher revolution frequency (increase in velocity dominates) while the reverse occurs above transition ($v \approx c$ and longer path) where the momentum compaction (generally > 0) dominates.



Synchrotrons – storage rings

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:



$e\hat{V}\sin\phi \longrightarrow$ Energy gain per turn

$\phi = \phi_s = cte \longrightarrow$ Synchronous particle

$\omega_{RF} = h\omega_r \longrightarrow$ RF synchronism (h - harmonic number)

$\rho = cte \quad R = cte \rightarrow$ Constant orbit

$B\rho = P/e \Rightarrow B \longrightarrow$ Variable magnetic field

Luminosity
How many particles collide in each crossing point?

Many particles per bunch

Low density

High density

Luminosity in a collider

$$R = \frac{dN}{dt} = L(t)\,\sigma_{event}$$

$$N = \sigma_{event} \int L(t)dt$$

L relates cross-section σ and event rate R = dN/dt at time t

Quantifies performance ("brilliance") of collider

Relativistic invariant and independent of physical reaction

Accelerator operation aims at maximizing the total number of events N for the experiments

σ_{event} is fixed by nature

L units : [$cm^{-2}\ s^{-1}$]

aim at maximizing $\int L(t)dt$

Integrated luminosity $\int Ldt$ is frequently expressed in $pb^{-1} = 10^{36}\ cm^{-2}$ or $fb^{-1} = 10^{39}\ cm^{-2}$

•

Luminosity

intuitively: higher L if there are more particles and more tightly packed

Peak luminosity

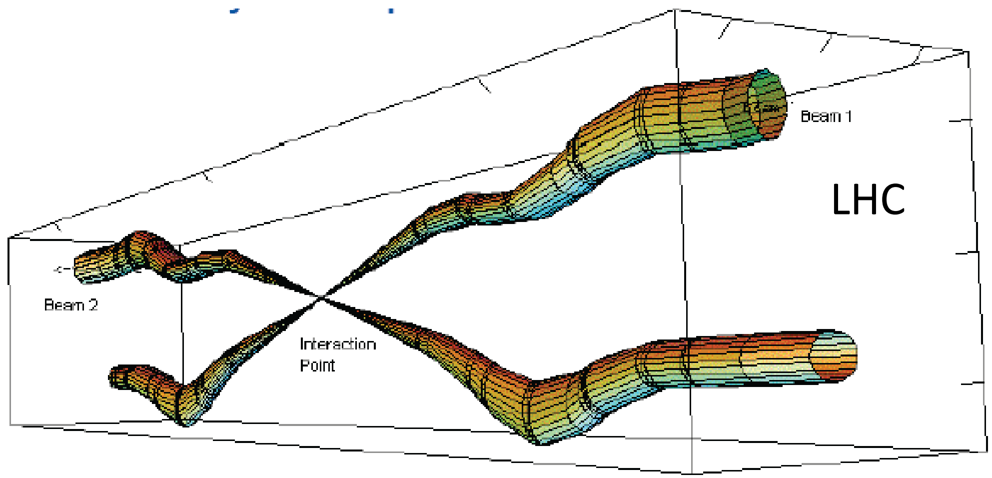
$$L = f_{rev} \frac{k N_1 N_2}{4\pi \sigma_x \sigma_y}$$

Small β*

$$\sigma^*_{x,y} = \sqrt{\varepsilon_{x,y}\,\beta^*_{x,y}}$$

$$L = \frac{kfN_1N_2}{4\pi\sqrt{\beta^*_x\,\beta^*_y\varepsilon_x\varepsilon_y}}$$

High luminosity = High intensity, high frequency, small beam dimensions



Relative beam sizes around IP1 (Atlas) in collision

Effects decreasing peak luminosity: crossing angle, hourglass effect

Beam-beam effect: action of one beam on the opposite beam particles, proportional to current

Summary

- Introduction to existing accelerators
- Electromagnetic fields
- Single particle dynamics – Bunch emittances and dynamics
- Luminosity in a circular collider
- + (not covered in this introduction) collective effects: interaction between particles and their environment and among particles themselves