Tutorial on beam dynamics

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ALBA - CELLS

Transverse emittance

- **Linacs and accelerators not dominated by synchrotron radiation**
- **(Liouville theorem applies)**
- ***The geometrical emittance decreases with the acceleration (proportional to 1/$\beta\gamma$). The normalized emittance is constant if only linear fields and conservative forces are influencing beam dynamics. Space charge, proportional to current, and other collective effects, usually increase the emittance.***
- ***Usually horizontal and vertical emittance are equal***

$$\varepsilon_x = \varepsilon_y = \frac{\varepsilon_{norm}}{\beta\gamma}$$

Transverse emittance

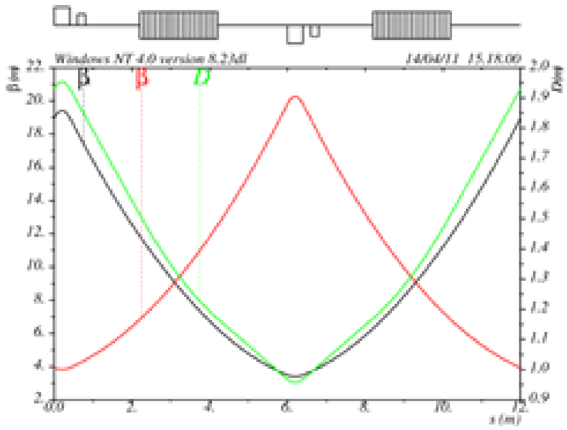
- **Synchrotrons for electrons and positrons, dominated by synchrotron radiation**
- **(Liouville theorem does not apply)**
- **The horizontal emittance is determined by the equilibrium between SR emission and radiation damping. It is proportional to γ^2**
- ***The vertical emittance shrinks due to radiation damping to almost zero, but it is dominated by the transfer of horizontal oscillations to vertical plane due to coupling, produced by tilted magnetic fields***

$$\varepsilon_x \propto \gamma^2 \qquad \varepsilon_y = \kappa \varepsilon_x$$

$$\kappa < 1\% \; usually$$

Lattice choice for low emittance synchrotrons

FODO lattice

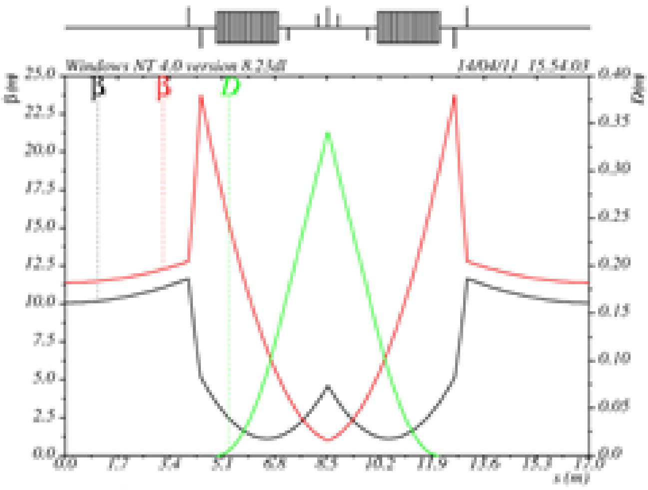


Minimum emittance

$$\epsilon_{min} \approx 1.2 C_q \gamma^2 \theta^3$$

$$C_q \approx 3.83\ 10^{-13} m$$

Double bend achromat

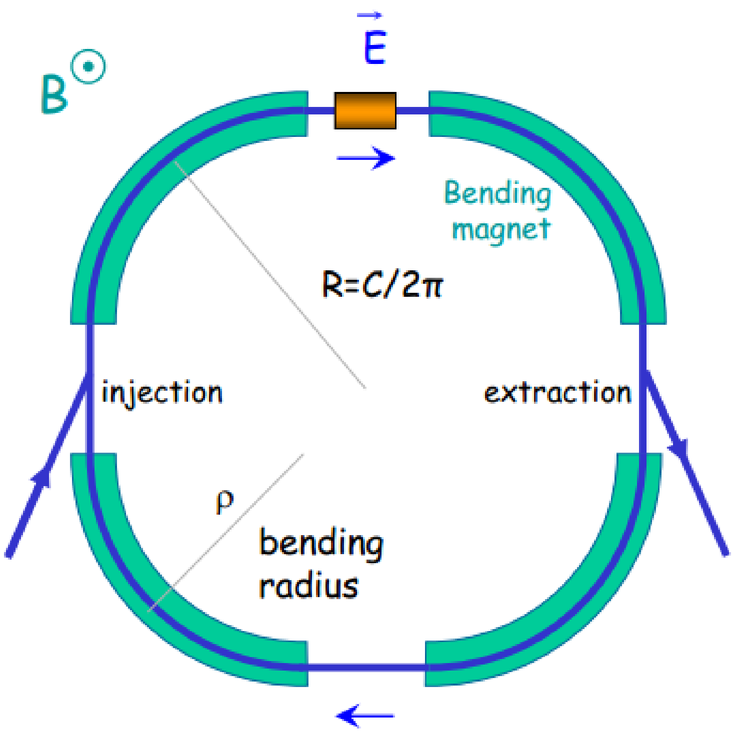


Minimum emittance

$$\epsilon_{min} \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$$

Synchrotrons – storage rings

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:



$e\hat{V}\sin\phi$ ⟶ Energy gain per turn

$\phi = \phi_s = cte$ ⟶ Synchronous particle

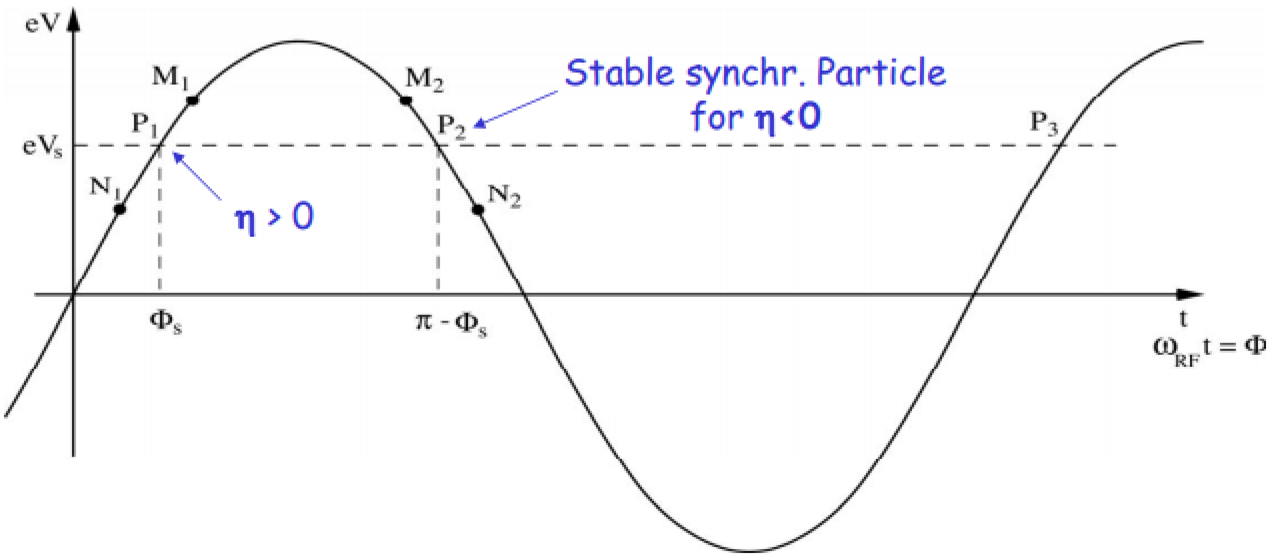
$\omega_{RF} = h\omega_r$ ⟶ RF synchronism (h - harmonic number)

$\rho = cte \quad R = cte$ ⟶ Constant orbit

$B\rho = P/e \Rightarrow B$ ⟶ Variable magnetic field

Phase Stability in a Synchrotron

From the definition of η it is clear that below transition an increase in energy is followed by a higher revolution frequency (increase in velocity dominates) while the reverse occurs above transition ($v \approx c$ and longer path) where the momentum compaction (generally > 0) dominates.



The maximum number of bunches in a synchrotron is

h = revolution frequency/rf frequency

h = harmonic number

The velocity of a 1 GeV proton is with respect to the velocity of a 1 MeV electron

- A – larger
- B – the same
- X – smaller

$\gamma = (E_o + E_{kin})/E_o$ $\beta = \text{sqrt}(1 - 1/\gamma^2)$

β(e- 1MeV) = 0.94 , β(p 1GeV) = 0.86

A linac for protons accelerates the beam from 200 keV to 2 MeV. The horizontal emittance, disregarding space charge,

A – increases

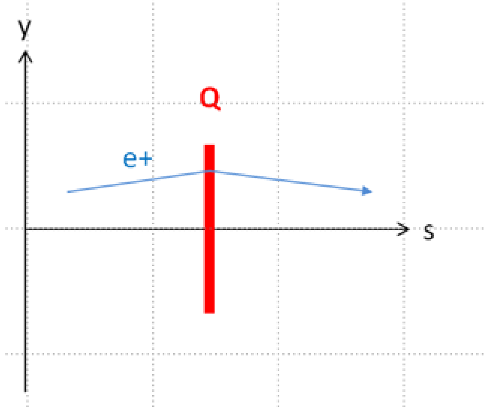
X – decreases

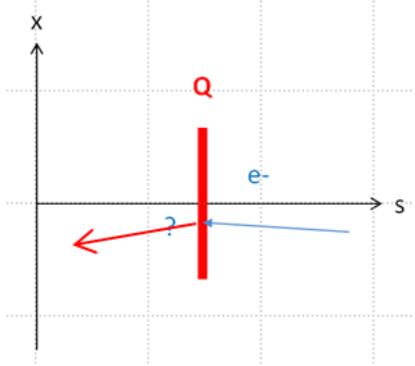
C - remains constant

A positron is focused in the vertical plane by the quadrupole of the figure. An electron with the same energy enter the quadrupole in the opposite direction with coordinates (x, x', y, y') = (-0.1 mm, 1.2 mrad, 0 mm, 0.0mrad)

- A – is horizontally focused
- X - is horizontally defocused

A quadrupole acts on a positive charged particle with the same effect of a negative charged particle travelling in the opposite direction





A synchrotron for electrons at 3 GeV has a magnetic structure with 36 dipoles. The magnetic field in the dipoles is 1.5 T. The bending radius in the dipoles is

- X – less than 10 m
- B – equal to 10 m
- C – larger than 10 m

For e- at 3 GeV, $B\rho$ ~ 10 Tm, => ρ = 10/1.5 = 6.7 m

Sesame is a synchrotron where the electron beam is injected at 800 MeV and then ramped up to 2.5 GeV. If at the maximum energy the emittance is 10 nm rad, which is the equilibrium emittance at the injection energy?

- ε proportional to γ^2
- $\varepsilon(0.8) = \varepsilon(2.5) * (0.8/2.5)^2 = 1.02$ nm rad

Luminosity

intuitively: higher L if there are more particles and more tightly packed

Peak luminosity

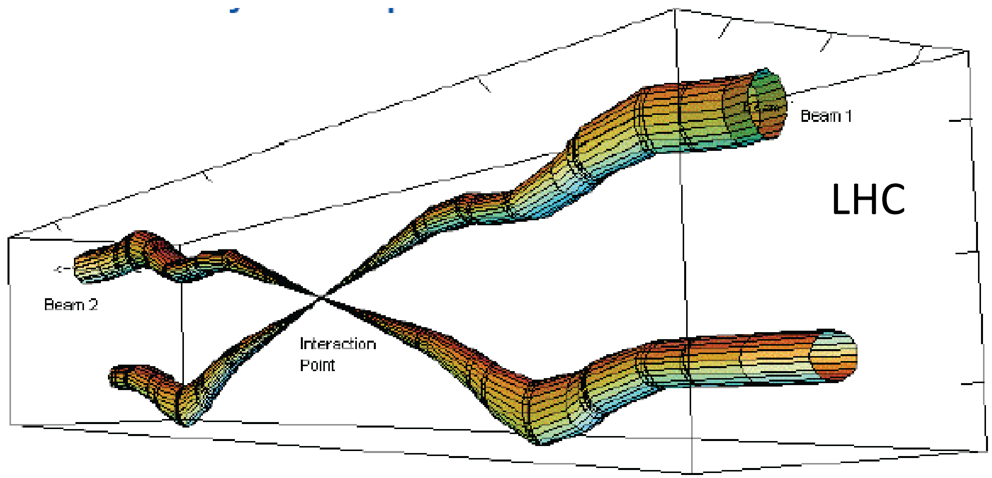
$$L = f_{rev} \frac{kN_1N_2}{4\pi\sigma_x\sigma_y}$$

Small β*

$$\sigma^*_{x,y} = \sqrt{\varepsilon_{x,y}\,\beta^*_{x,y}}$$

$$L = \frac{k f N_1 N_2}{4\pi\sqrt{\beta^*_x\,\beta^*_y\,\varepsilon_x\varepsilon_y}}$$

High luminosity = High intensity, high frequency, small beam dimensions



Relative beam sizes around IP1 (Atlas) in collision

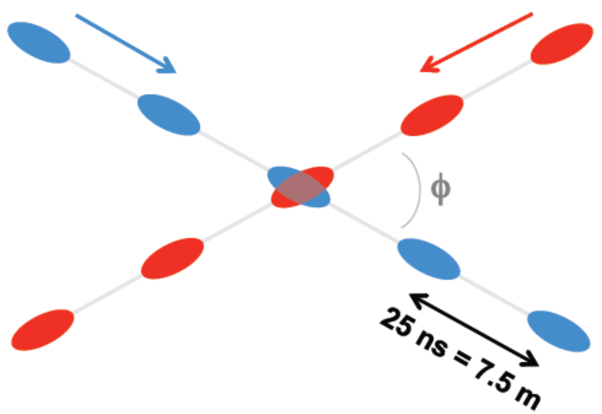
Effects decreasing peak luminosity: crossing angle, hourglass effect

Beam-beam effect: action of one beam on the opposite beam particles, proportional to current

Reduction factors 1: Crossing angle

to avoid parasitic collisions when there are many bunches

- otherwise collisions elsewhere than in interaction point only
- e.g.: CMS experiment is 21 m long, common vacuum pipe is 120 m long



luminosity is reduced as the particles no longer cross the entire length of the counter-rotating bunch

$$L = \frac{kN_1N_2f}{4\pi\sigma_x\sigma_y}\frac{1}{\sqrt{1+\left(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2}\right)^2}}$$

F

LHC
ϕ = 285 μrad
σ_s = 7.5 cm
F = 0.84

Calculate the maximum luminosity in the LHC knowing that each beam current is 200mA, the n. of bunches in collision is 1000, the circumference is 27km, $\beta_x^* = \beta_y^* = 0.5$ m, and the normalized emittances in the two planes are 1 μ rad. The energy of the two beams of protons is 6.5TeV.

Geometric emittance = normalized emittance/γ ($\beta = 1$)

$\gamma = E/E_o = 6981.7$

$\varepsilon_x = \varepsilon_y = 1\ 10^{-6}/\gamma = 1.44\ 10^{-10}$ *m rad*

Beam size $\sigma_x = \sigma_y = sqrt(\varepsilon_x \beta_x^*)$, $\sigma_x \sigma_y = \varepsilon_x \beta_x^* = 7.2\ 10^{-11} m^2$

$f_{rev} = c/C = 1.11\ 10^4$ *Hz*

Bunch current $I_b = I_{tot}/n_b = 200/1000$ *mA* $= 2\ 10^{-4}$ *A*

Number of particles per bunch $= I_b T_{rev}/e = I_b C/ec = 1.12\ 10^{11}$

$L = kNNf/4\pi\sigma_x\sigma_y = 1.54\ 10^{34}\ cm^{-2}\ sec^{-1}$

- In the collider DAFNE, where the two rings have a circumference of C = 98 m, two beams of positrons and electrons collide at one Interaction Point, with the following parameters: $\beta_x = 2$ m, $\beta_y = 5$ mm, $\varepsilon_x = 5e\text{-}07$ m rad, coupling 0.8%.
- A total number of 100 bunches with a total current of 1.5A for the electrons and 1A for the positrons are stored.
- Find the maximum achievable peak luminosity and the luminosity if the beams collide with a total angle of 30 mrad, knowing that the bunch length is $\sigma_s = 2$ cm

- $k = 100$
- $\sigma_x = sqrt(\beta_x \varepsilon_x) = 0.001$ m
- $\varepsilon_y = 0.8\ 10^{-2} * \varepsilon_x = 2.4\ 10^{-09}$ m rad
- $\sigma_y = sqrt(\beta_y \varepsilon_y) = 3.46\ 10^{-06}$ m
- $I^+_{bunch} = I^+ /k = 0.01$ A
- $I^-_{bunch} = I^- /k = 0.015$ A
- $f = c/C = 3.06$ MHz
- $N^+ = I^+_{bunch}/ef = 2.04\ 10^{10}$
- $N^- = I^-_{bunch}/ef = 3.06\ 10^{10}$
- $L_{max} = 4.4\ 10^{36}$ m^{-2} sec^{-1}=4.4 10^{32} cm^{-2} sec^{-1}
- $L_{max+Cross} = L_{max}\ F = 3.65\ \ 10^{32}$ cm^{-2} sec^{-1}

- Dipoles at the Alba Synchrotron have a magnetic field of 1.4 T. The beam energy is 3 GeV.
What is the energy emitted in the form of synchrotron radiation per turn?
What would be the energy emitted by turn if the electron beam had an energy of 3.5 GeV, maintaining the same orbit?

$B\rho$ = 3.33 E (GeV) = 9.99 Tm
$\rho = B\rho/B$ = 7.14 m
$U = C_g E^4/\rho$ = 1 MeV
If E = 3.5 GeV, U =1.9 MeV with the same ρ

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad = \frac{2}{3} r_e m_o c^2 \beta^3 \gamma^4 \oint \frac{ds}{\rho^2} = C_\gamma \frac{E^4 \left(GeV^4\right)}{\rho(m)}$$

$$C_\lambda = \frac{4\pi}{3} \frac{r_c}{\left(mc^2\right)^3} = 8.846 \cdot 10^{-5} \frac{m}{GeV^3} \qquad for\ e^-, e^+$$

An undulator in a synchrotron used to produce synchrotron light emits in first harmonic at 12 keV. To increase the wavelength of the first harmonic

X - the magnetic field is increased in the orbit plane

B - the magnetic field is decreased in the orbit plane

C - Do not depend on the magnetic field in the plane of the orbit

$$\varepsilon_n(eV) = 9.496 \frac{nE[GeV]^2}{\lambda_u[m]\left(1+\frac{K^2}{2}\right)}$$

Compare two configurations at LHC colliding beam proton (p), with the same normalized emittance per bunch and the same crossing angle.

I. Proton energy 6.5 TeV. Number of bunches in interaction $k = 2000$, total current $I+ = 300$mA, $\beta^* = 0.5$ m
II. Proton energy 4.5 TeV. Number of bunches in interaction $k = 1000$, total current $I+ = 400$mA, $\beta^* = 1$ m

The luminosity, without taking into account beam-beam effects, is
A - higher for I
B - greater in the case II
C - the same in both cases
The brightness is proportional to k, energy, the square of the current per bunch, and inversely proportional to β^*: LII / LI = 720/585 = 1.23

In a synchrotron for hadronterapy with a circumference of C = 70 m, the energy of protons in a treatment varies from 180 MeV to 210 MeV. Considering that h = 1, the rf frequency between maximum power and minimum varies

X - less than 1 MHz

B - 1MHz

C - More than 1 MHz

The revolution frequency is proportional to the velocity and 2.33 and 2.47 MHz at the two energies.

Consider Petra III synchrotron where the electrons have energy of 6 GeV. The operating current is 100mA, the circumference is 2304 m, the dipoles have an arc of 4 m and an angle θ= 2.25^o. The lattice structure is a double bend achromat.

Calculate the effective emittance knowing that is 5 times greater than the minimum that can be obtained with existing dipoles.

Determine the critical energy synchrotron radiation emitted in the dipole and a wiggler whose maximum field is 2.5 T.

If the beam lifetime is 24 hours, how often should you inject to maintain the photon flux within 0.5% of maximum?

γ = 11742

θ = 0.03927

Minimum emittance = 206 pm rad

Emittance = 1.03 nm rad

$\boldsymbol{\varepsilon_c(keV)=0.66503E^2(GeV^2)B(T)}$

Bρ = 19.98 Tm

$\rho = L_{arc}/\theta$ = 101.86m

B_{dipole} = 0.196 T

Dipole: ε_c = 4.7 keV

Wiggler ε_c = 59.9 keV

$N=N_o e_{-t\tau/}$ $\quad\quad$ $t=-\tau \ln N N_o$

N = 0.995 No

Ln (N/No) = -0.005 $t=-\tau \ln N N_o = 0.005*24*3600=7.2\ min$