

Inclusive Charmonium Production at Hadron Colliders

**Hee Sok Chung
CERN**

Based on

Geoffrey T. Bodwin, HSC, U-Rae Kim, Jungil Lee, PRL113, 022001 (2014)

Geoffrey T. Bodwin, HSC, U-Rae Kim, Jungil Lee, Yan-Qing Ma, Kuang-Ta Chao, PRD93, 034041 (2016)

**Particle and Astro-Particle Physics Seminar
27 May 2016, CERN**

OUTLINE

- Heavy quarkonia
- Inclusive charmonium production
- Polarization of prompt J/ψ
- Summary

HEAVY QUARKONIA

- Bound states of cc (charmonia), bb (bottomonia)
- $2m_b > 2m_c \gg \Lambda_{\text{QCD}}$.
- $m_{\text{Charmonium}} \approx 2m_c$, $m_{\text{Bottomonium}} \approx 2m_b$,
which allow nonrelativistic description :
 $v^2 \approx 0.3$ for charmonia, $v^2 \approx 0.1$ for bottomonia
- Typical energy scales : $m > \underset{\text{perturbative}}{mv} > \underset{\text{nonperturbative}}{mv^2} \approx \Lambda_{\text{QCD}}$

HEAVY QUARKONIA

- Quark model assignments :
 $c\bar{c}$, $b\bar{b}$ in colour-singlet states with same C, P, T

	Spin Triplet	Spin Singlet
S-wave	$J/\psi, \psi'$	η_c
P-wave	χ_{cJ}	h_c

Charmonia

	Spin Triplet	Spin Singlet
S-wave	$\Upsilon(nS)$	η_b
P-wave	χ_{bJ}	h_b

Bottomonia

HEAVY QUARKONIA

- Narrow widths, show up as sharp resonances

$$\Gamma_{J/\psi} = 92.9 \text{ keV}, \quad \Gamma_{\eta_c} = 31.8 \text{ keV},$$

- Well-known exclusive decay modes provide clean signals, e.g. $J/\psi \rightarrow \ell^+ \ell^-$, $\eta_c \rightarrow \gamma\gamma$
- Can provide information about heavy quarks; higgs-charm coupling could be measured from higgs decay into $J/\psi + \gamma$ at the HL-LHC

INCLUSIVE CHARMONIUM PRODUCTION

- Inclusive quarkonium production cross section at large p_T provides tests of QCD through
 - perturbative calculation of hard processes involving quarks and gluons
 - evolution of hadronic matrix elements such as PDFs and fragmentation functions
- Furthermore, long-distance nature of quarkonia can be investigated using Nonrelativistic QCD

INCLUSIVE CHARMONIUM PRODUCTION IN NRQCD

- Nonrelativistic QCD (NRQCD) effective theory can be used to investigate hadronic matrix elements.
- NRQCD matrix elements (operators defined in Coulomb gauge) scale with powers of v .

$$\langle \mathcal{O}^H(\dots) \rangle = \langle 0 | \chi^\dagger \dots \psi | H + X \rangle \langle H + X | \psi^\dagger \dots \chi | 0 \rangle$$

*covariant derivatives,
spin and colour matrices*

INCLUSIVE CHARMONIUM PRODUCTION IN NRQCD

- NRQCD factorization conjecture

$$d\sigma_{A+B \rightarrow H+X} = \sum_n \underbrace{d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}}_{\text{Short-distance cross section}} \underbrace{\langle \mathcal{O}^H(n) \rangle}_{\text{LDME}}$$

Bodwin, Braaten, and Lepage, PRD51, 1125 (1995)

- Short-distance cross sections (SDCSs) contain physics above the scale of the heavy-quark mass m
- Nonperturbative long-distance matrix elements contain physics below m , scale with powers of v
- No established proof of factorization for inclusive production, but can be tested experimentally.

INCLUSIVE J/ψ PRODUCTION

- NRQCD factorization conjecture

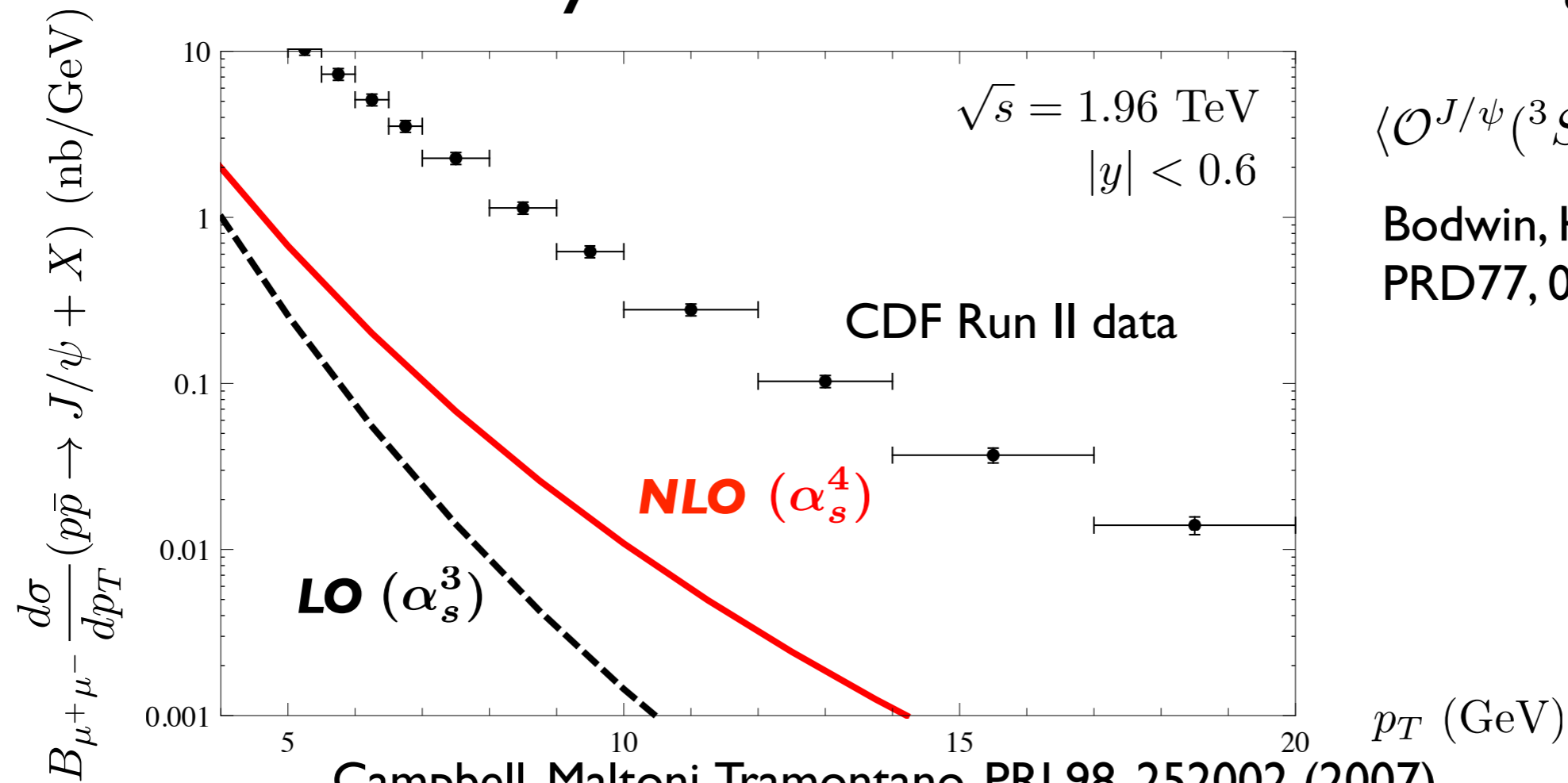
$$d\sigma_{A+B \rightarrow H+X} = \sum_n \underbrace{d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}}_{\text{Short-distance cross section}} \underbrace{\langle \mathcal{O}^H(n) \rangle}_{\text{LDME}}$$

Bodwin, Braaten, and Lepage, PRD51, 1125 (1995)

- At LO in v , the $c\bar{c}$ are produced with same colour and angular momentum as the J/ψ : colour-singlet channel
- Colour-singlet LDMEs can be determined from lattice NRQCD, potential models, or from decay rates

INCLUSIVE J/ψ PRODUCTION

- Colour-singlet channel underestimates the measured cross section by more than an order of magnitude



$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

Bodwin, HSC, Kang, Lee, Yu,
PRD77, 094017 (2008)

Campbell, Maltoni, Tramontano, PRL98, 252002 (2007)

Ma, Wang, Chao, PRL106, 042002 (2011) CDF, PRD71, 032001 (2005)

INCLUSIVE J/ψ PRODUCTION

- NRQCD factorization conjecture

$$d\sigma_{A+B \rightarrow H+X} = \sum_n \underbrace{d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}}_{\text{Short-distance cross section}} \underbrace{\langle \mathcal{O}^H(n) \rangle}_{\text{LDME}}$$

Bodwin, Braaten, and Lepage, PRD51, 1125 (1995)

- At LO in v , the $c\bar{c}$ are produced with same colour and angular momentum as the J/ψ : colour-singlet channel
- At higher orders in v , the $c\bar{c}$ in colour-octet states can evolve into a J/ψ through soft gluon emission

INCLUSIVE J/ψ PRODUCTION

- NRQCD factorization conjecture

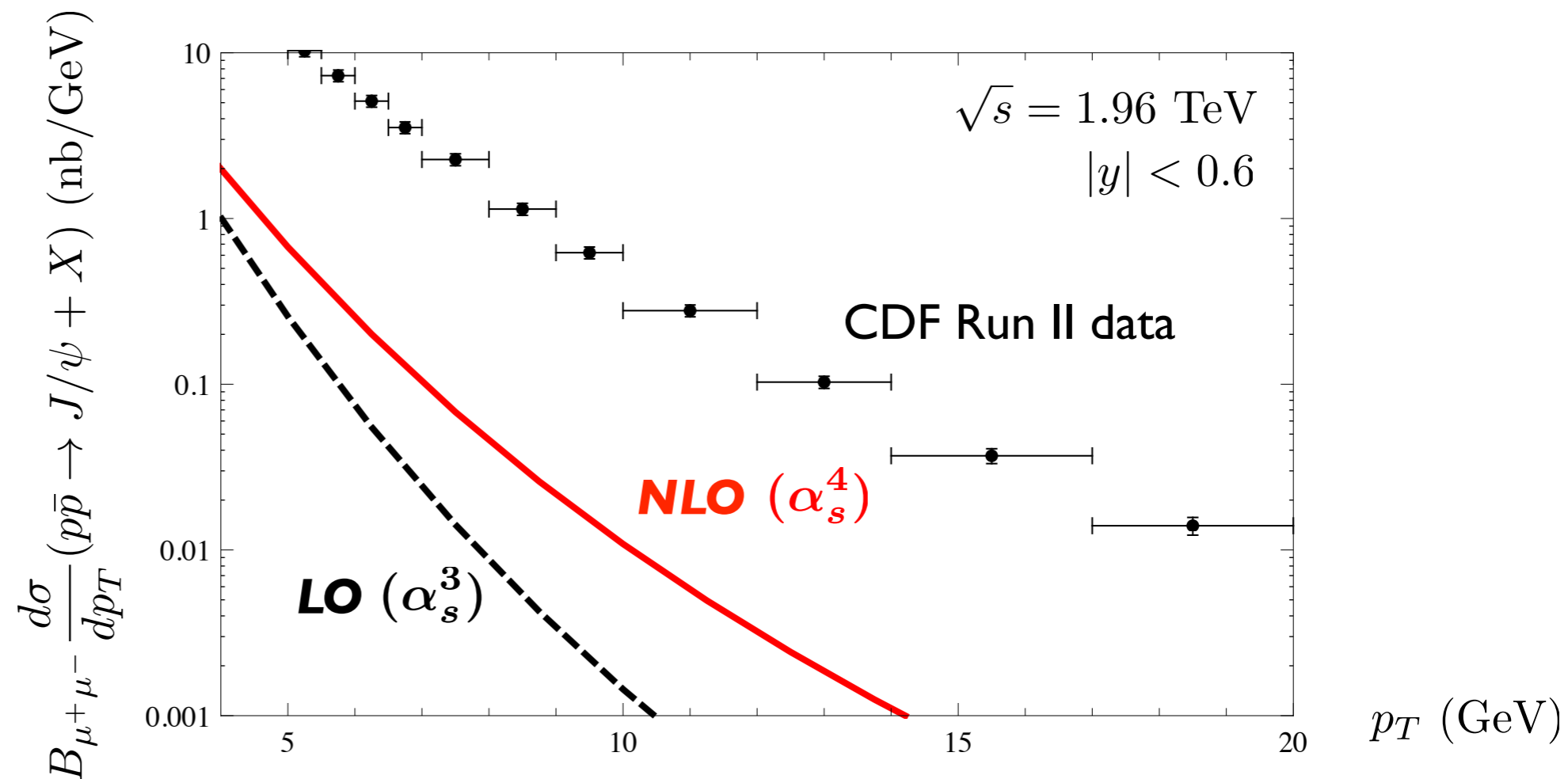
$$d\sigma_{A+B \rightarrow H+X} = \sum_n \underbrace{d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}}_{\text{Short-distance cross section}} \underbrace{\langle \mathcal{O}^H(n) \rangle}_{\text{LDME}}$$

Bodwin, Braaten, and Lepage, PRD51, 1125 (1995)

- Usually truncated at relative order v^4 :
 $^3S_1^{[1]}$, $^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$ channels for J/ψ
- Not known how to calculate colour-octet LDMEs, usually extracted from fits to measurements.
- Polarization of J/ψ was suggested as independent test of CO LDMEs

INCLUSIVE J/ψ PRODUCTION

- In order to accommodate the data, the CO channels should somehow fill in the gaps, even though the CO LDMEs are suppressed by $v^4 \sim 0.1$ compared to the CS LDME



INCLUSIVE J/ψ PRODUCTION

- Short-distance cross sections are perturbative, so in principle, they can be computed using perturbation theory. Then, the CO LDMEs can be extracted from the data by comparing the p_T dependence of the SDCSs to the data.
- It is therefore important to determine the shape of the cross sections against p_T well. This may not be simple because the short-distance cross sections depend on two scales (charm quark mass and p_T).

INCLUSIVE J/ψ PRODUCTION

- Short-distance cross sections are perturbative, but "In determining the relative importance ... must take into account not only the size of the matrix element and the leading power of $\alpha_s(M)$... but also ... dimensionless ratios of kinematic variables."

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- Enhancements from powers of p_T/m_c can be more important than suppression from powers of α_s or v

INCLUSIVE J/ψ PRODUCTION

- We know from QCD factorization theorems that at leading power in $1/p_T$, cross section is given by single parton fragmentation

$$\sigma(ij \rightarrow H + X) = \sum_k \sigma(ij \rightarrow k + X) \otimes D_{k \rightarrow H}$$

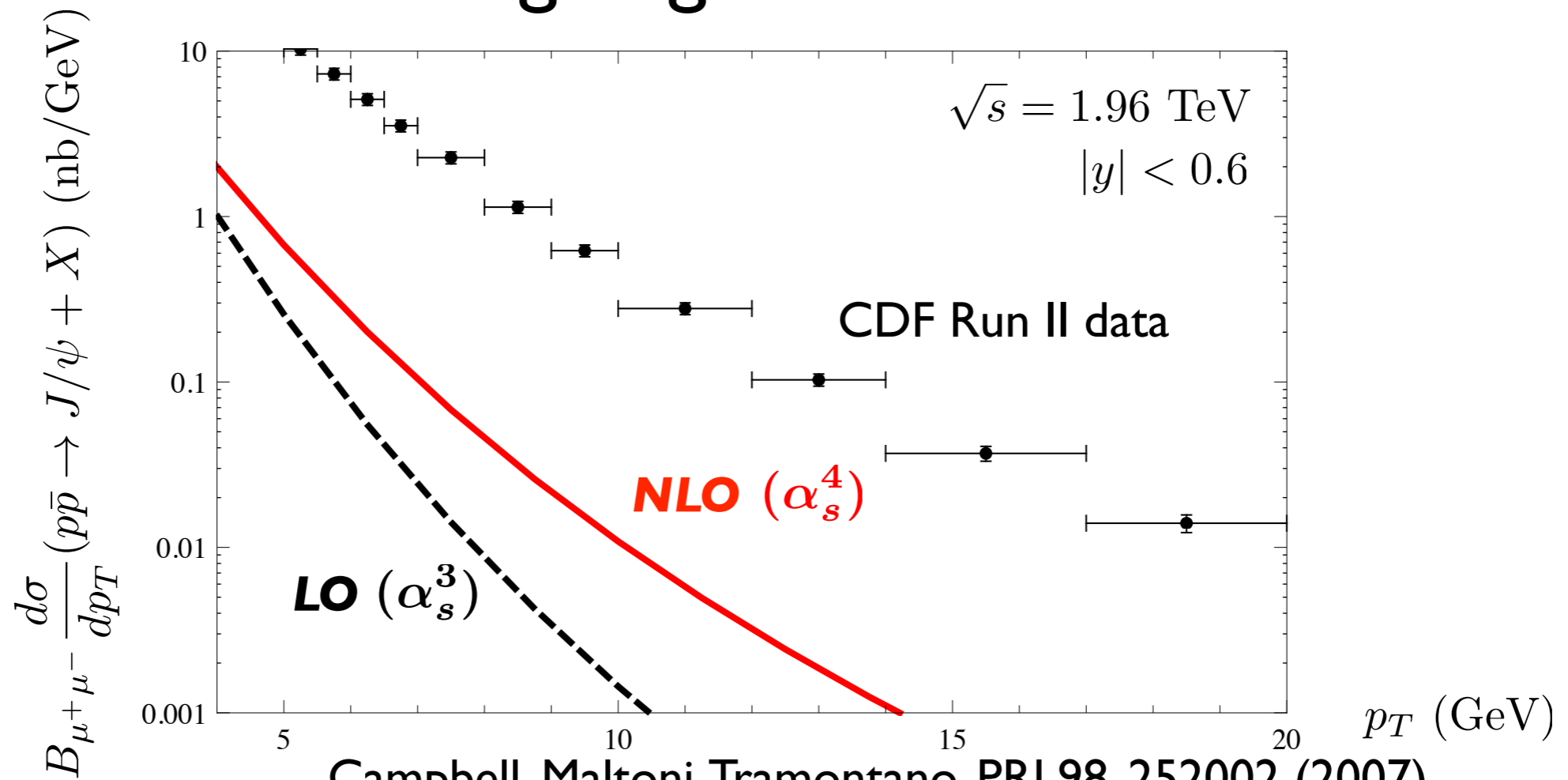
Collins and Soper, NPBI94, 445 (1982)

Nayak, Qiu, Sterman, PRD72, 114012 (2005)

- Fragmentation contribution scales like $1/p_T^4$
- Fragmentation contribution does not necessarily appear at LO in α_s

INCLUSIVE J/ψ PRODUCTION

- Colour-singlet channel changes shape and normalization when going from LO to NLO



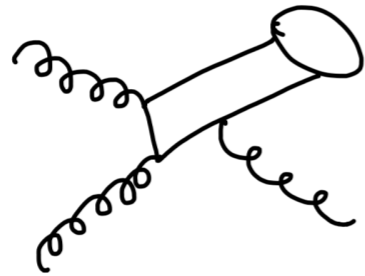
Campbell, Maltoni, Tramontano, PRL98, 252002 (2007)

Ma, Wang, Chao, PRL106, 042002 (2011) CDF, PRD71, 032001 (2005)

INCLUSIVE J/ψ PRODUCTION

- This is because fragmentation contributions appear from NLO. A few representative diagrams are :

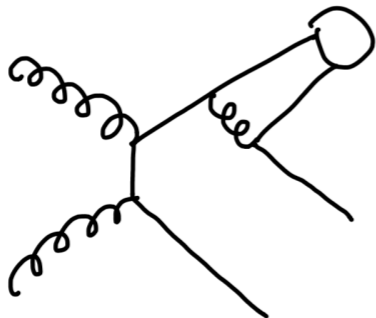
$$O(\alpha_s^3)$$



$$\sim 1/p_T^8$$

gluon fusion

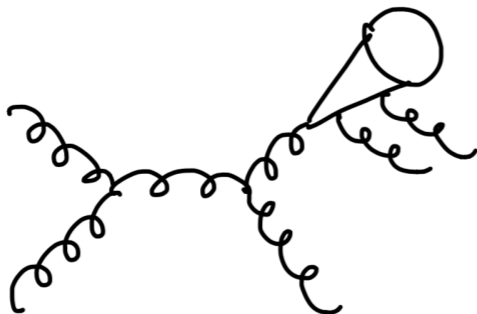
$$O(\alpha_s^4)$$



$$\sim 1/p_T^4$$

c quark fragmentation

$$O(\alpha_s^5)$$



$$\sim 1/p_T^4$$

gluon fragmentation

COLOUR OCTET SDCSs

- The enhancements are also dramatic in CO channels, which have been calculated to NLO (order α_s^4)

- Rather than using the fixed-order calculations, we use a different approach that has better control over fragmentation contributions.

Ma, Wang, Chao, PRD84, 114001 (2011)

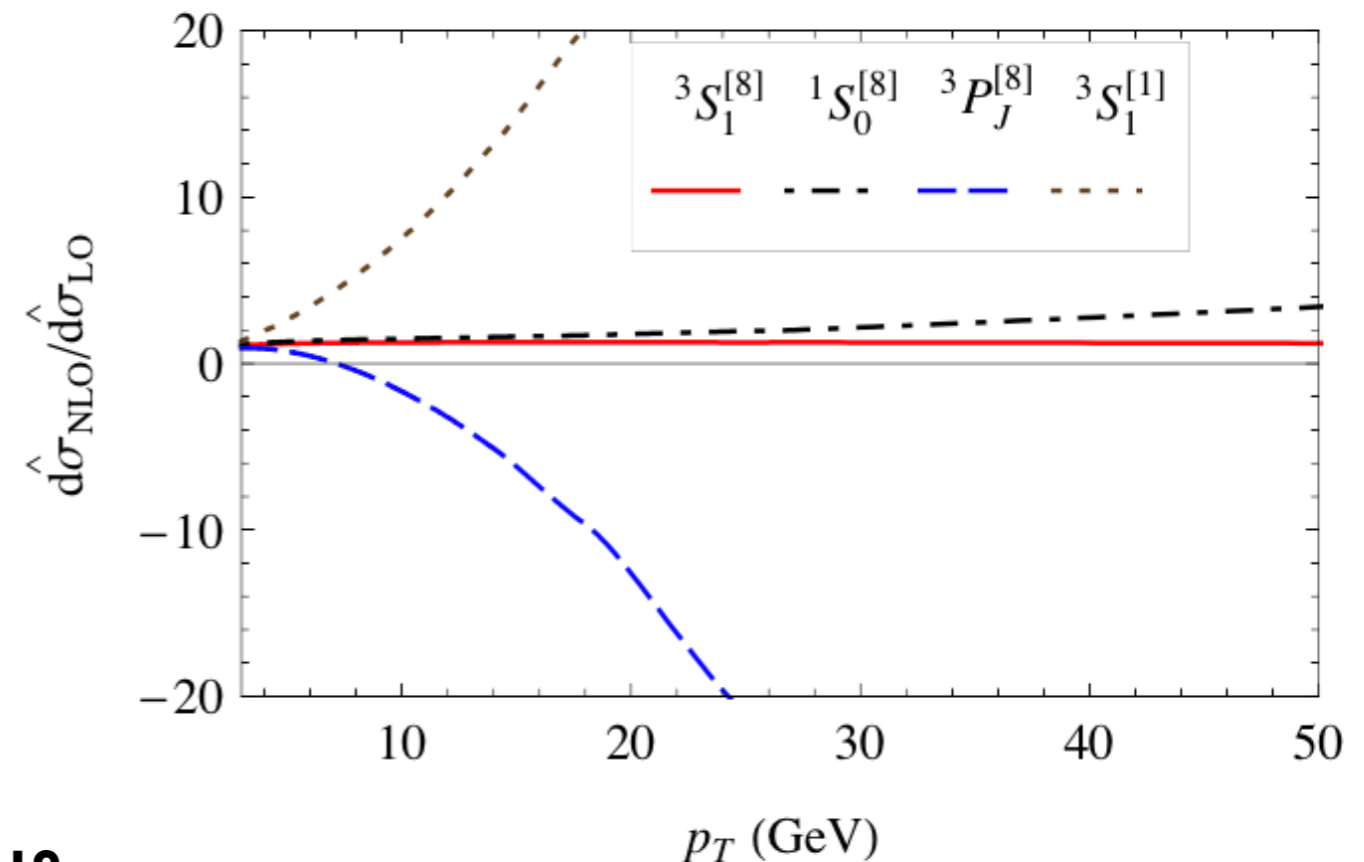
Butenschoen and Kniehl, PRL106, 022003 (2011)

Ma, Wang, Chao, PRL106, 042002 (2011)

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)

Butenschoen and Kniehl, PRL108, 172002 (2012)

Bong, Wan, Wang, Zhang, PRL110, 042002 (2013)



COLOUR OCTET SDCSs

- We take the following strategy to calculate the CO SDCSs.
- The leading-power (LP) fragmentation contribution is calculated from the fragmentation approximation

$$\sigma_{\text{LP}}(ij \rightarrow c\bar{c}(n) + X) = \sum_k \sigma(ij \rightarrow k + X) \otimes D_{k \rightarrow c\bar{c}(n)}$$

Parton cross sections, available publicly to NLO (α_s^3)
Aversa, Chiappetta, Greco, Guillet, NPB327, 105 (1989)

Fragmentation functions, computed to order α_s^2

This reproduces the fixed-order calculation to NLO (order α_s^4) at leading power in p_T , and also gives part of the NNLO corrections.

- Leading Logs of p_T/m are resummed using DGLAP evolution equations

FRAGMENTATION FUNCTIONS

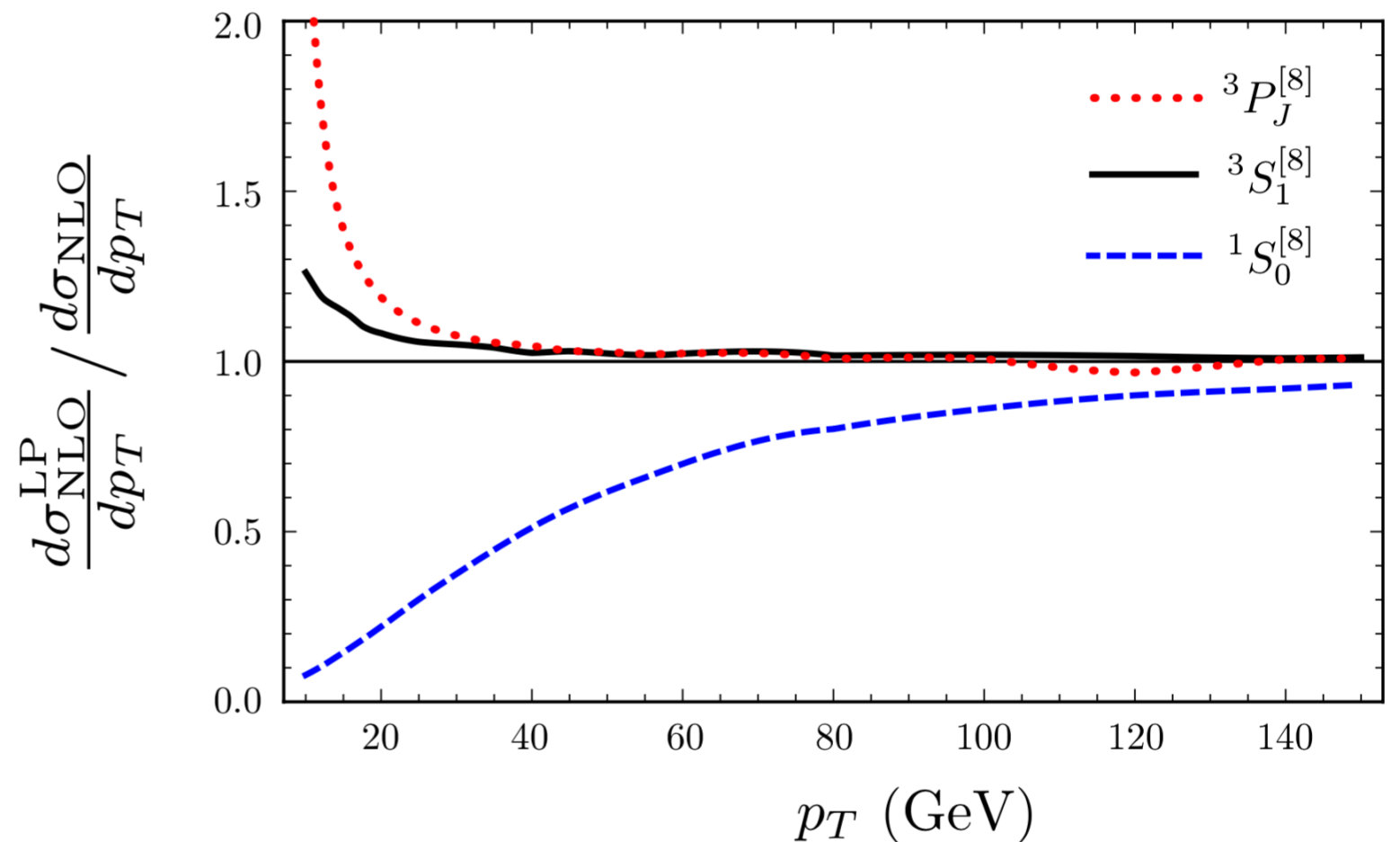
- Fragmentation functions (FFs) for production of $c\bar{c}$ can be computed using perturbative QCD
Collins and Soper, NPBI 94, 445 (1982)
- A gluon can produce a $c\bar{c}$ pair in ${}^3S_1^{[8]}$ state directly :
gluon FF for this channel starts at order α_s ,
involves a delta function at $z = 1$
- A gluon can produce a $c\bar{c}$ in ${}^3P_J^{[8]}$ state by emitting a soft gluon : gluon FF for this channel starts at order α_s^2 ,
involves distributions singular at $z = 1$
- A gluon can produce a $c\bar{c}$ in ${}^1S_0^{[8]}$ state by emitting a gluon : gluon FF for this channel starts at order α_s^2 ,
does not involve divergence at order α_s^2

FRAGMENTATION FUNCTIONS

- The fact that the FFs are distributions singular at $z = 1$ enhances the LP fragmentation contributions, which makes the CO SDCSs larger than the CS SDCSs by more than an order of magnitude.
- Because of the singularity at $z = 1$, DGLAP evolution must be handled with care, as the singularity does not disappear with evolution.

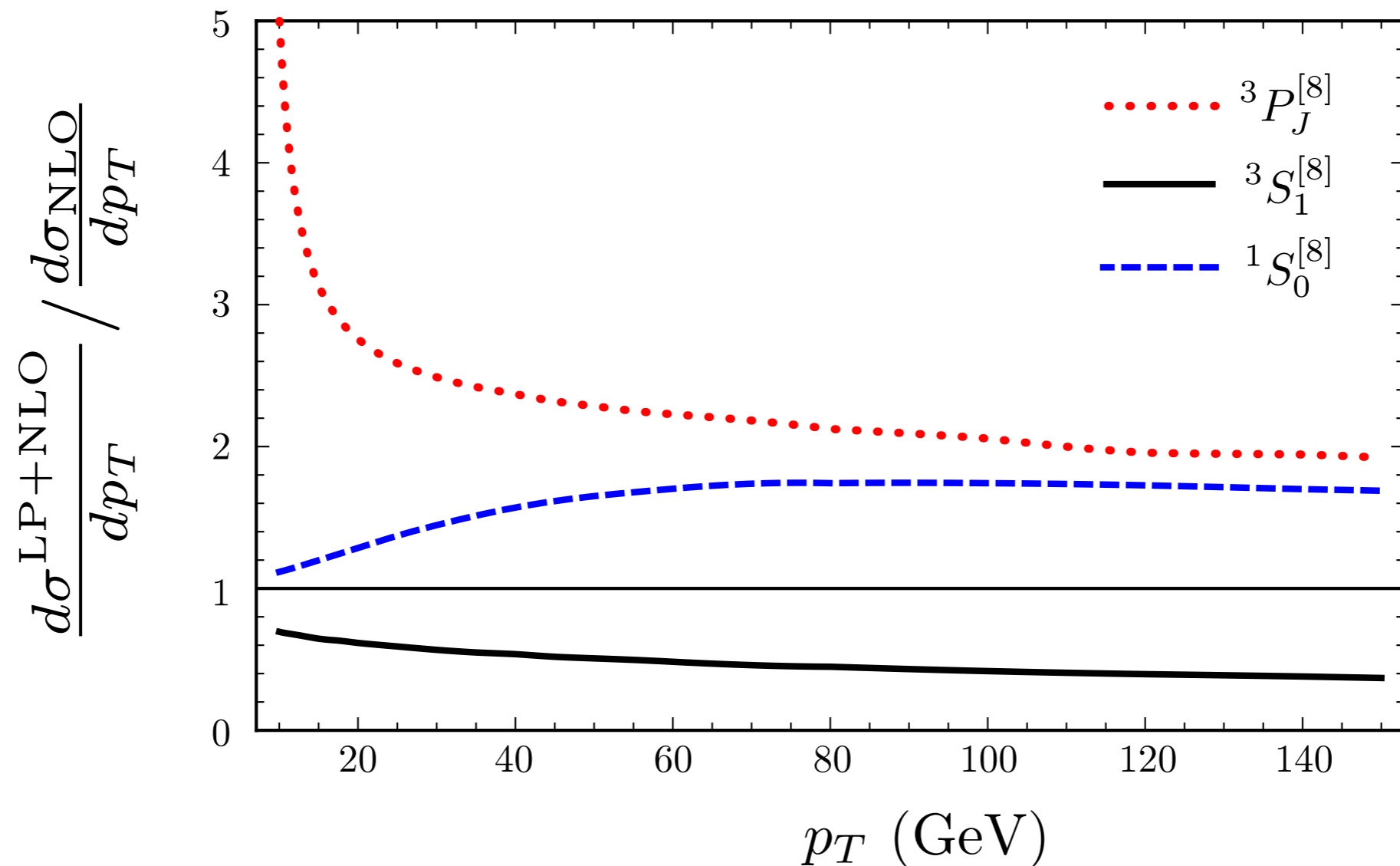
COLOUR OCTET SDCSS

- The corrections suppressed by powers of m^2/p_T^2 can be computed from fixed-order calculations available to order α_s^4 , after subtracting the fragmentation contributions.



LP+NLO

- The additional fragmentation contributions have important effects on the shapes in CO channels



J/ψ PRODUCTION

- We obtain good fits to data for $p_T > 10$ GeV ($\approx 3 \times m_{J/\psi}$)

CDF, PRD71, 032001 (2005)
 CMS, JHEP02, 011 (2012)
 CMS, PRL114, 191802 (2015)

- Cross section is dominated by the $^1S_0^{[8]}$ channel

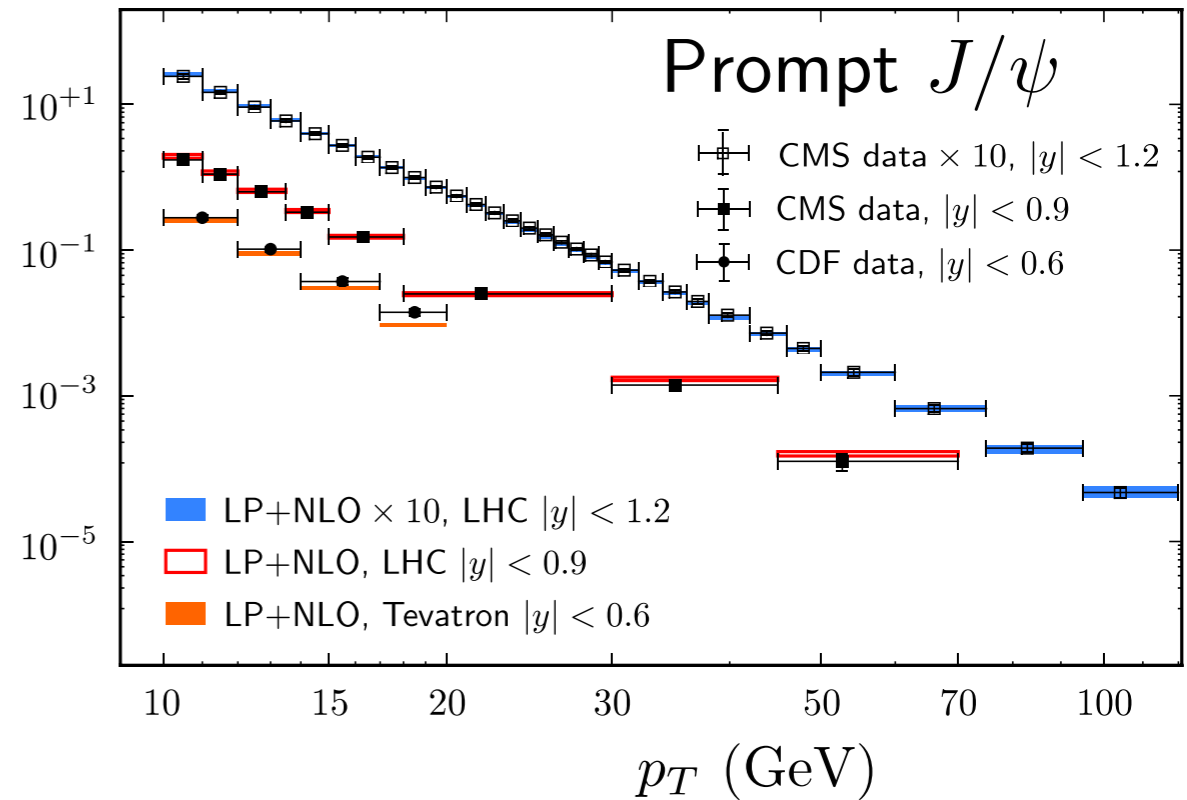
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle = (-7.13 \pm 3.64) \times 10^{-3} \text{ GeV}^3,$$

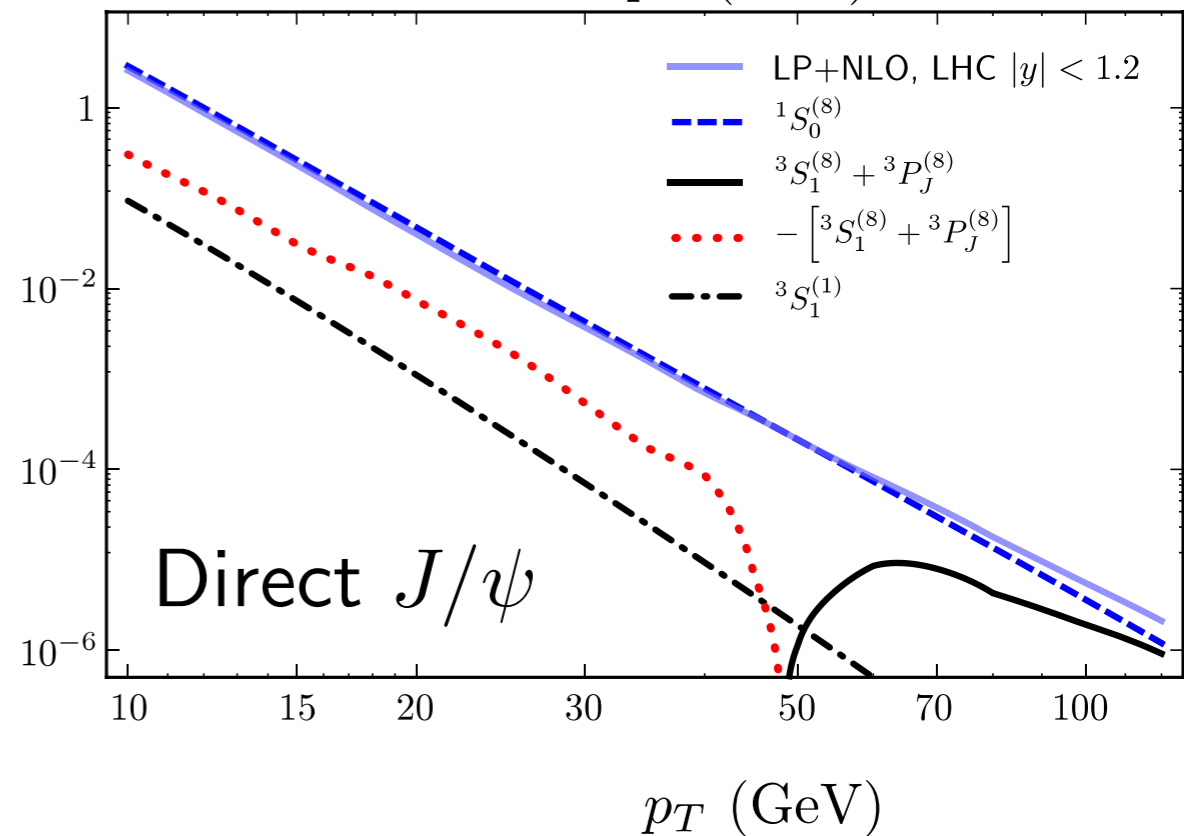
$$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle = (+1.10 \pm 0.14) \times 10^{-1} \text{ GeV}^3,$$

$$\frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle}{m_c^2} = (-3.12 \pm 1.51) \times 10^{-3} \text{ GeV}^3.$$

$$B_{J/\psi} \times \frac{d\sigma}{dp_T} \text{ (nb/GeV)}$$



$$B_{J/\psi} \times \frac{d\sigma}{dp_T} \text{ (nb/GeV)}$$

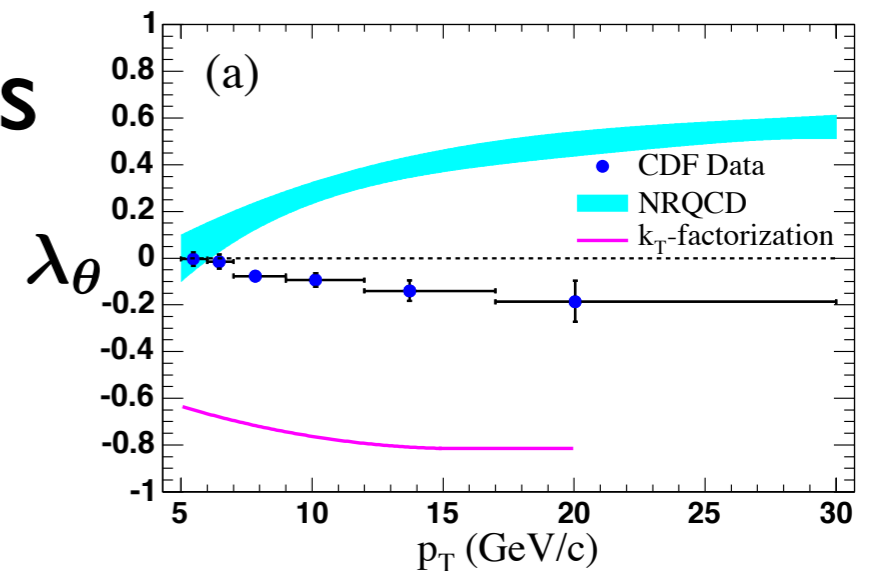


J/ψ POLARIZATION

- Polarization of J/ψ was suggested as a test of the colour-octet channels

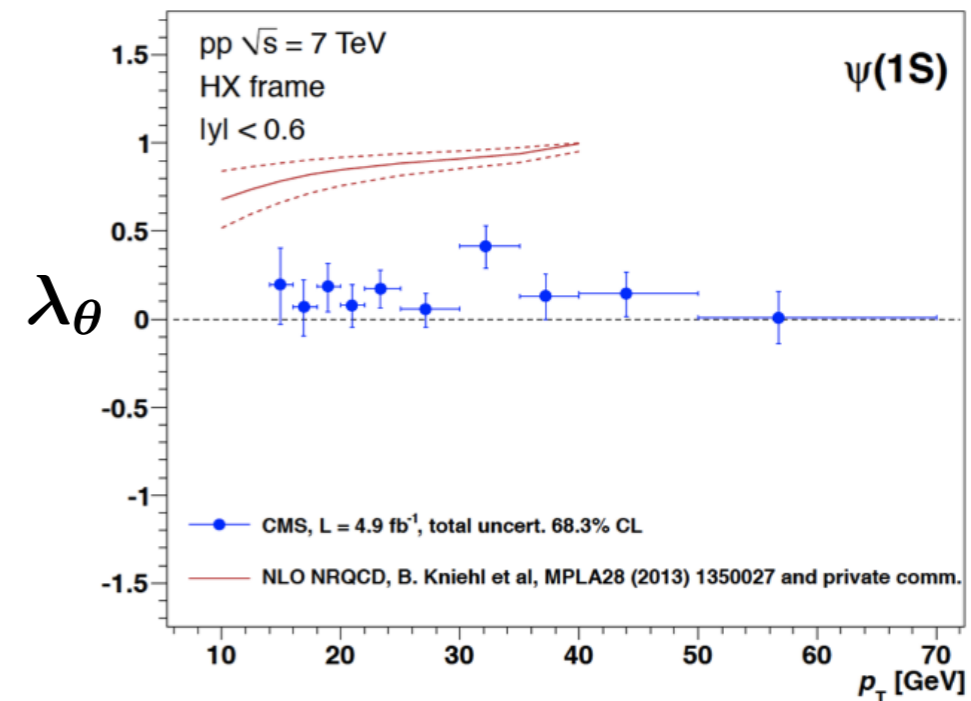
$$\lambda_\theta = \begin{cases} +1 & : \textit{Transverse} \\ 0 & : \textit{Unpolarized} \\ -1 & : \textit{Longitudinal} \end{cases}$$

- NRQCD at LO in α_s predicts transverse polarization at large p_T
- Disagrees with measurement
- NRQCD at NLO still predicts transverse polarization



CDF, PRL99, 132001 (2007)

Braaten, Kniehl, and Lee, PRD62, 094005 (2000)



CMS, PLB727, 381 (2013)

Butenschoen and Kniehl, PRL108, 172002 (2012)

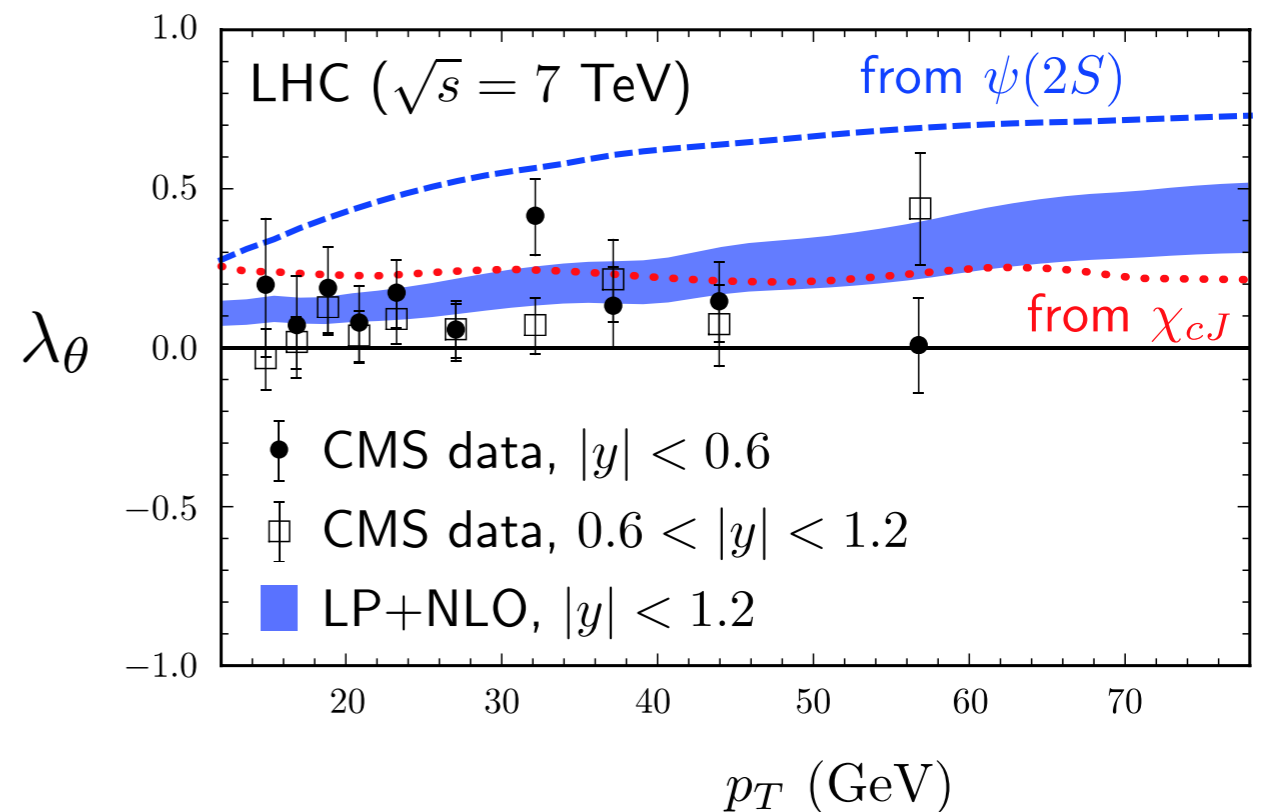
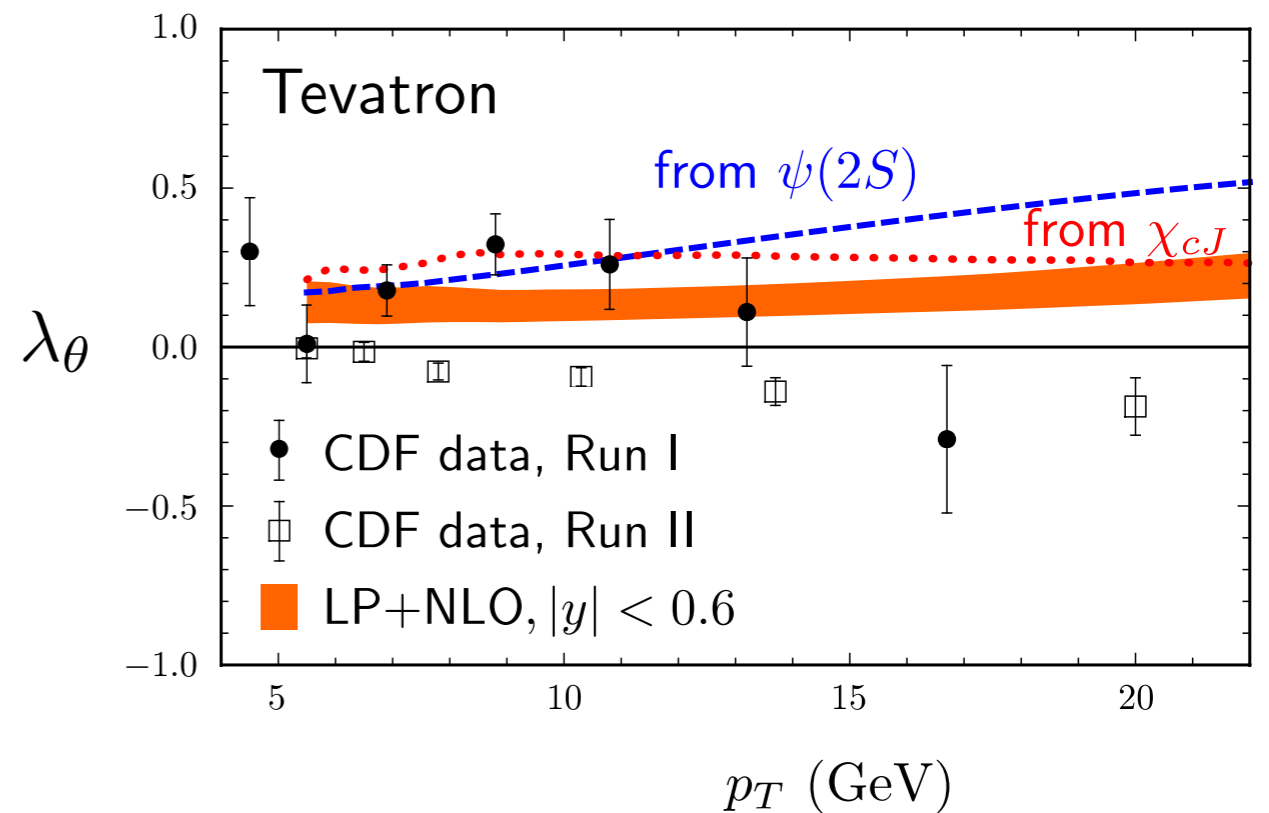
POLARIZATION IN CO CHANNELS

- For the ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels, gluon polarization is transferred to the $c\bar{c}$ pair, and therefore the $c\bar{c}$ is mostly transverse.
- For the ${}^1S_0^{[8]}$ channel, the $c\bar{c}$ is unpolarized because the state is isotropic. ${}^1S_0^{[8]}$ dominance would naturally explain small polarization of J/ψ .

J/ψ POLARIZATION

- **PROMPT J/ψ HAS SMALL POLARIZATION**
- This is in *reasonably good agreement with CMS data*

CDF, PRL85, 2886 (2000), PRL99, 132001 (2007)
CMS, PLB727, 381 (2013)



SUMMARY

- Heavy quarkonia are simple bound states that allow nonrelativistic treatment, but reliable description of inclusive production rates proved difficult.
- We obtained good agreement with J/ψ polarization data after careful reorganization of perturbative calculation that captures important fragmentation contributions.
- Further theoretical study and more experimental data may improve our understanding of strongly interacting bound states.

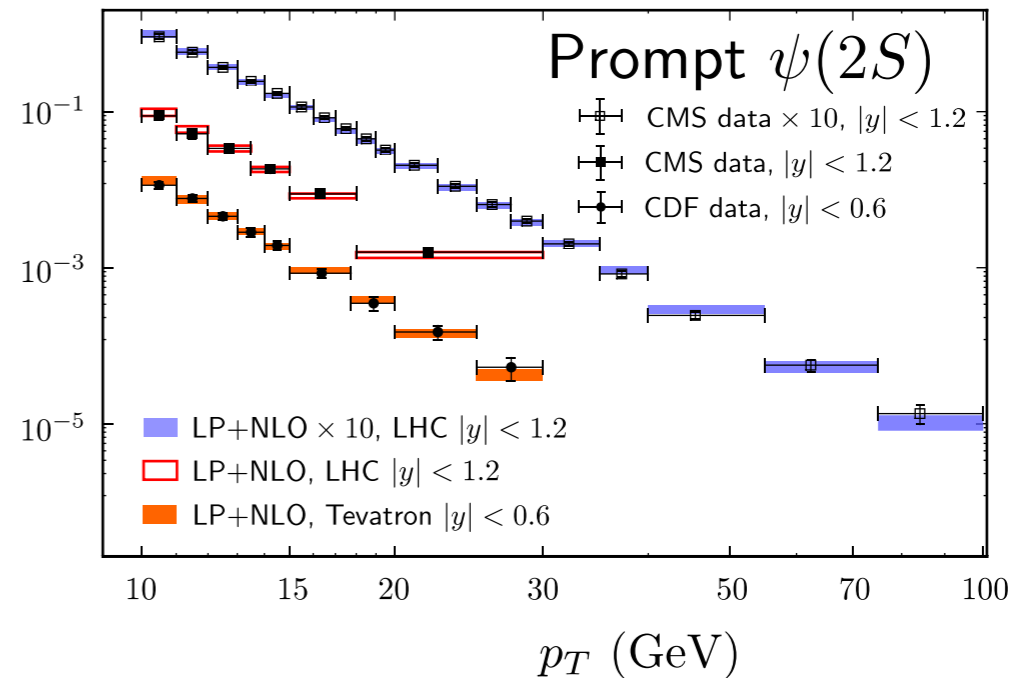
SUPPLEMENTARY

$\psi(2S)$ PRODUCTION

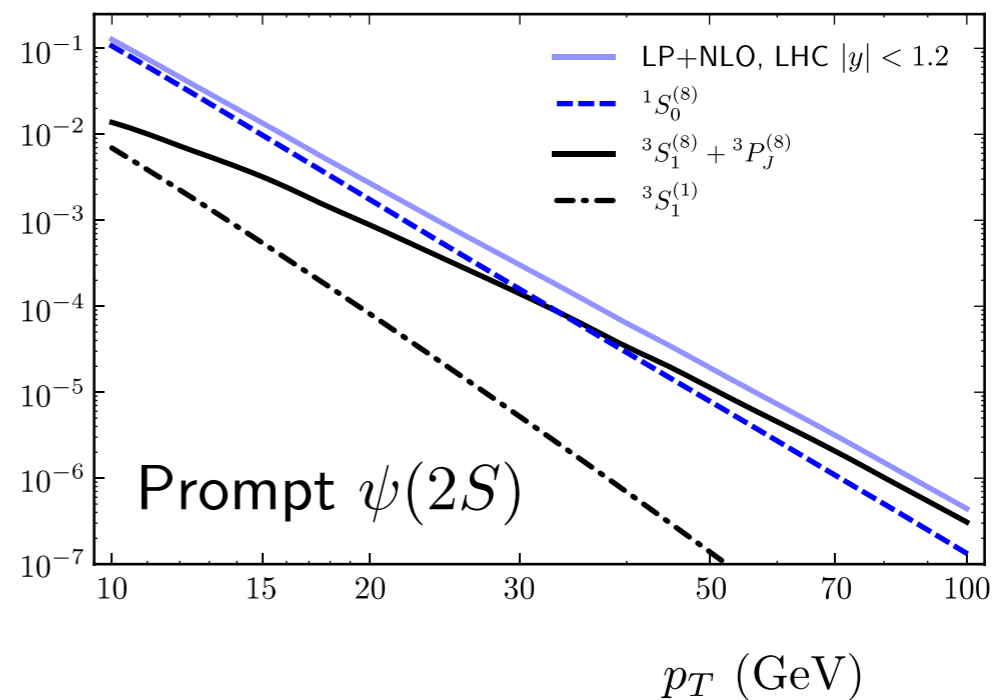
- $\psi(2S)$ LDMEs from fit to CMS and CDF cross section data for $p_T > 11\text{GeV}$ ($\approx 3 \times m_{\psi(2S)}$)

CDF, PRD80, 031103 (2009)
 CMS, JHEP02, 011 (2012)
 CMS, PRL114, 191802 (2015)

$$B_{\psi(2S)} \times \frac{d\sigma}{dp_T} \text{ (nb/GeV)}$$

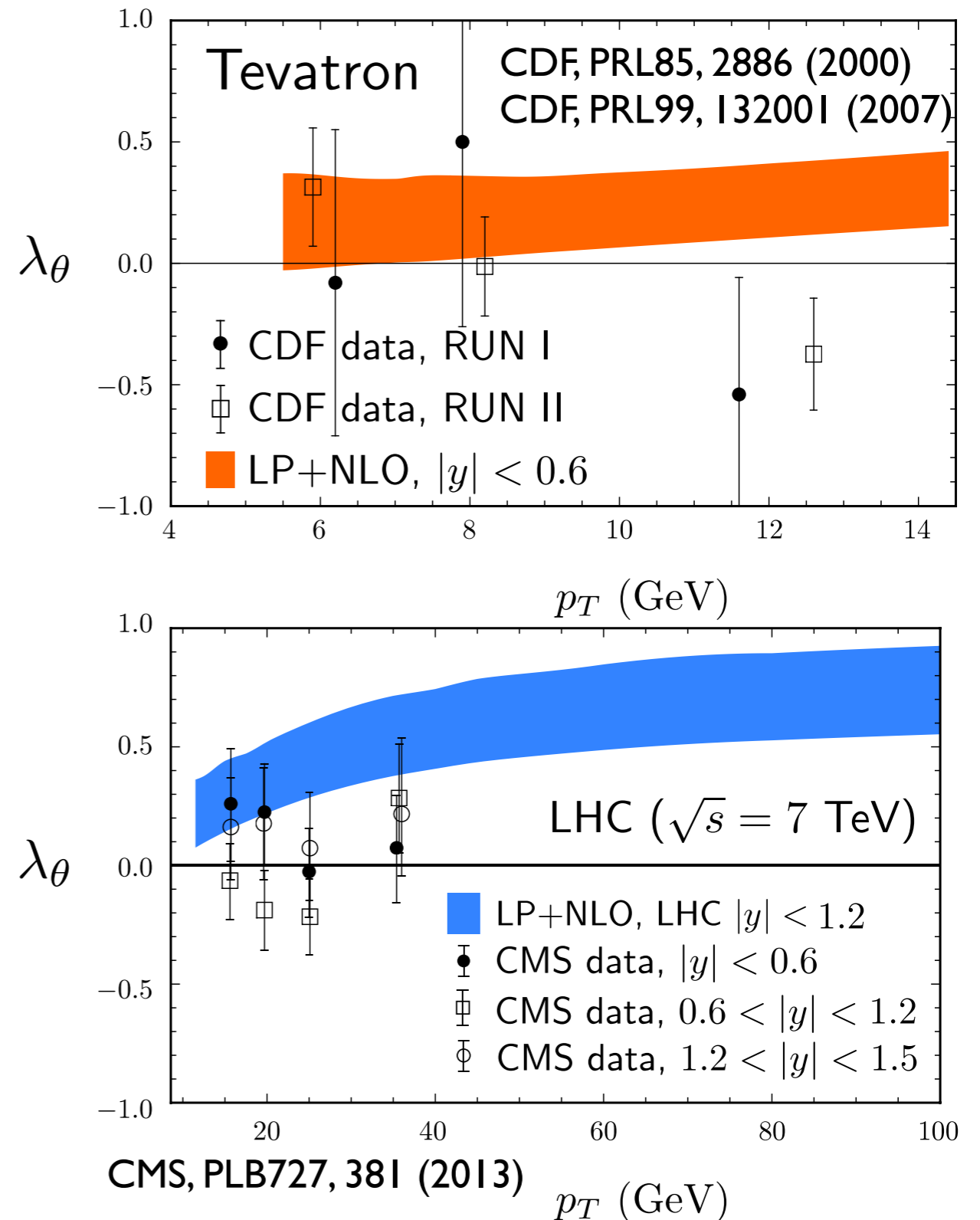


$$B_{\psi(2S)} \times \frac{d\sigma}{dp_T} \text{ (nb/GeV)}$$



$\psi(2S)$ POLARIZATION

- Slightly transverse at the Tevatron and the LHC
- Agrees with CMS data within errors



χ_{cJ} PRODUCTION

- $^3S_1^{[8]}$ and $^3P_J^{[1]}$ channels contribute at leading order in v
- We obtain good fits to ATLAS data ATLAS, JHEP1407, 154 (2014)
- The $^3P_J^{[1]}$ matrix element obtained from fit agrees with the potential model calculation

Potential model

$$|R'(0)|^2 = 0.075 \text{ GeV}^5$$

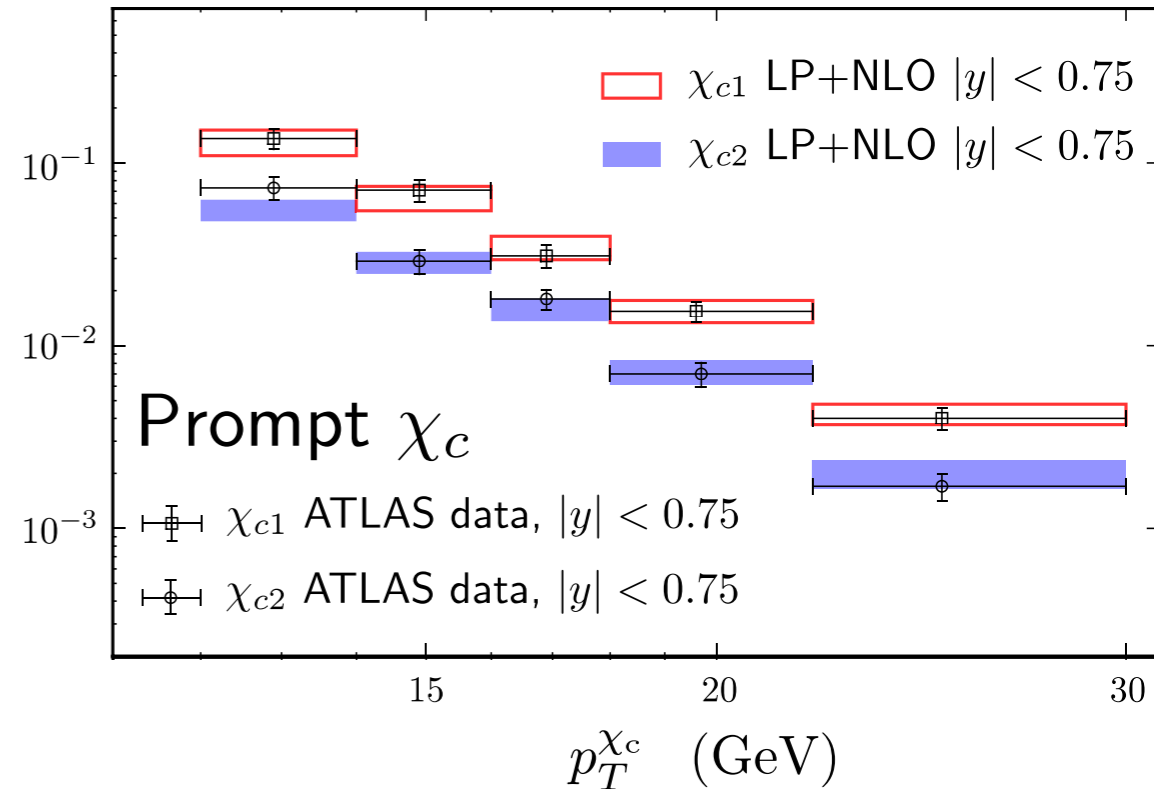
Eichten and Quigg, PRD 52, 1726 (1995)

Our fit

$$|R'(0)|^2 = 0.055 \pm 0.025 \text{ GeV}^5$$

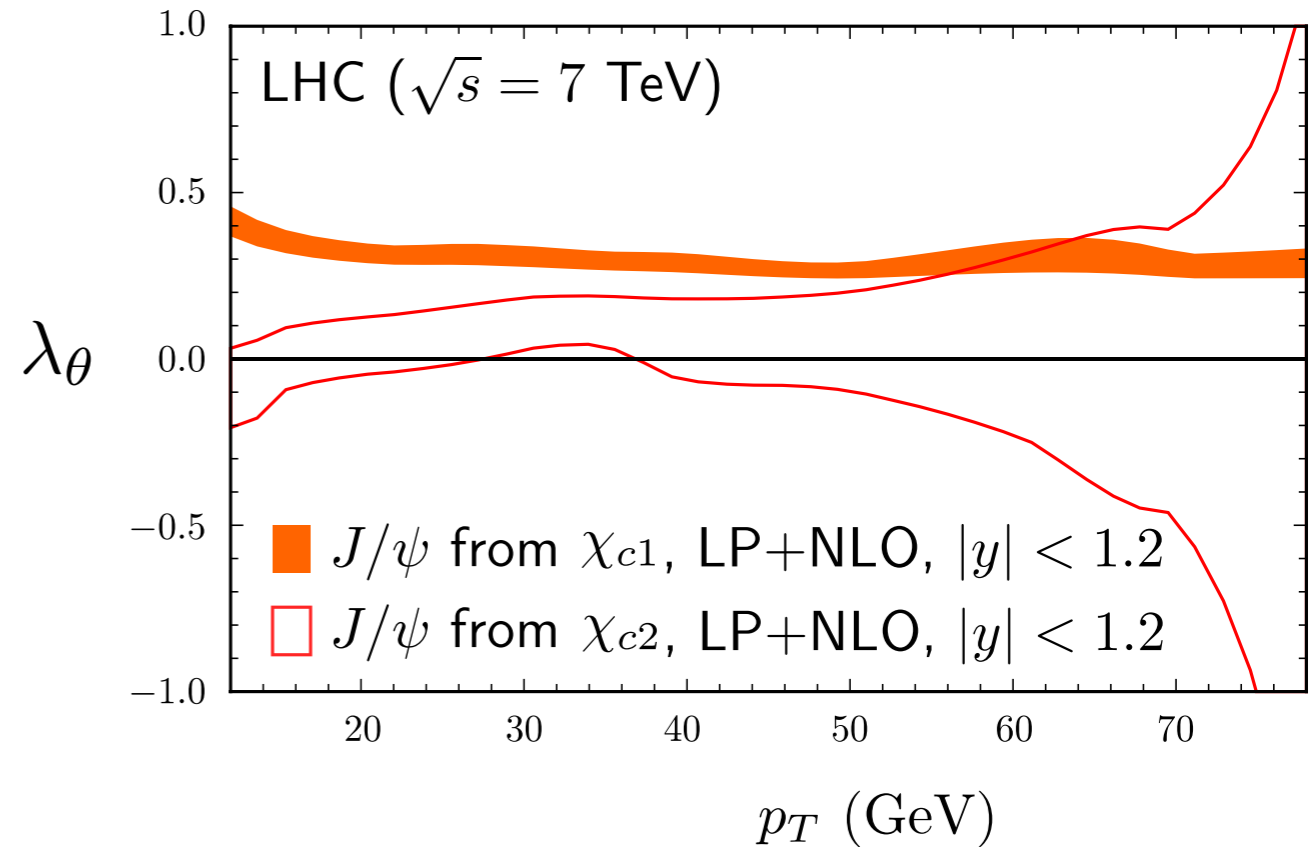
→ Suggests that NRQCD factorization works

$B_{\chi_c} \times \frac{d\sigma}{dp_T}$ (nb/GeV)



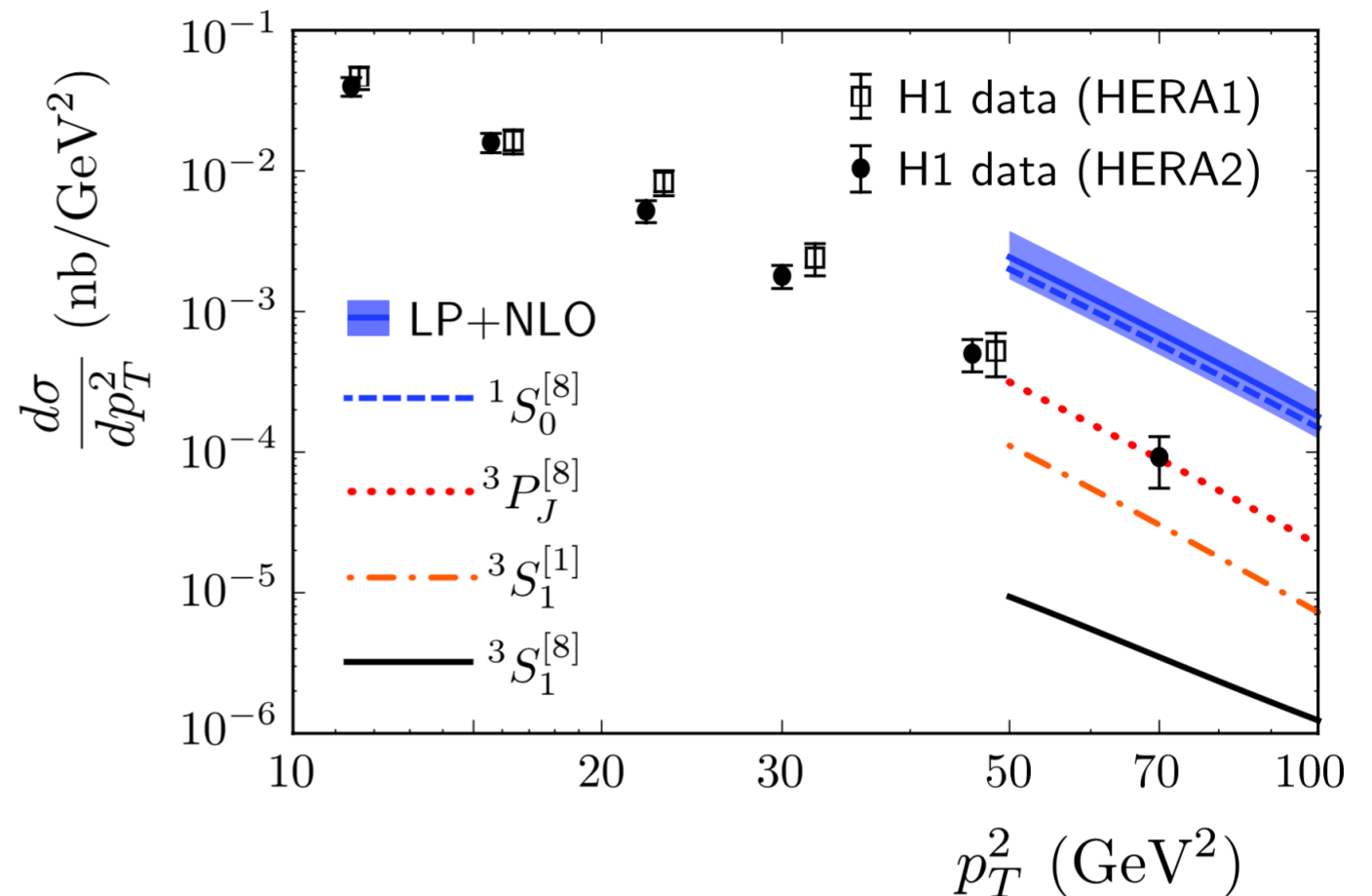
χ_{cJ} POLARIZATION

- Polarization of J/ψ from $\chi_{cJ} \rightarrow J/\psi + \gamma$
- No measurement available



J/ψ PHOTOPRODUCTION

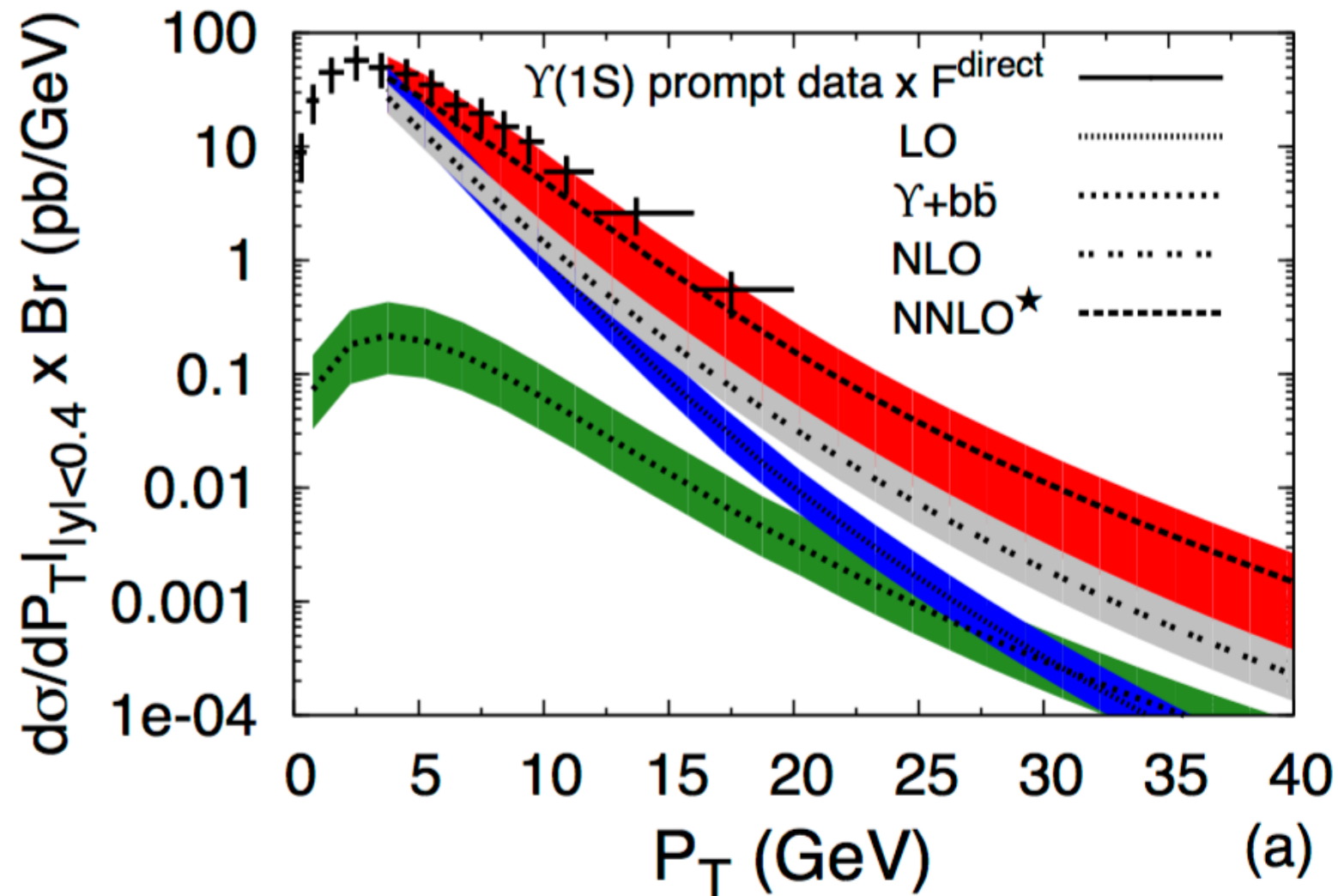
- $1S_0^{[8]}$ dominance overestimates $\gamma p \rightarrow J/\psi + X$ cross section



Bodwin, HSC, Kim, Lee, PRD92, 074042 (2015)

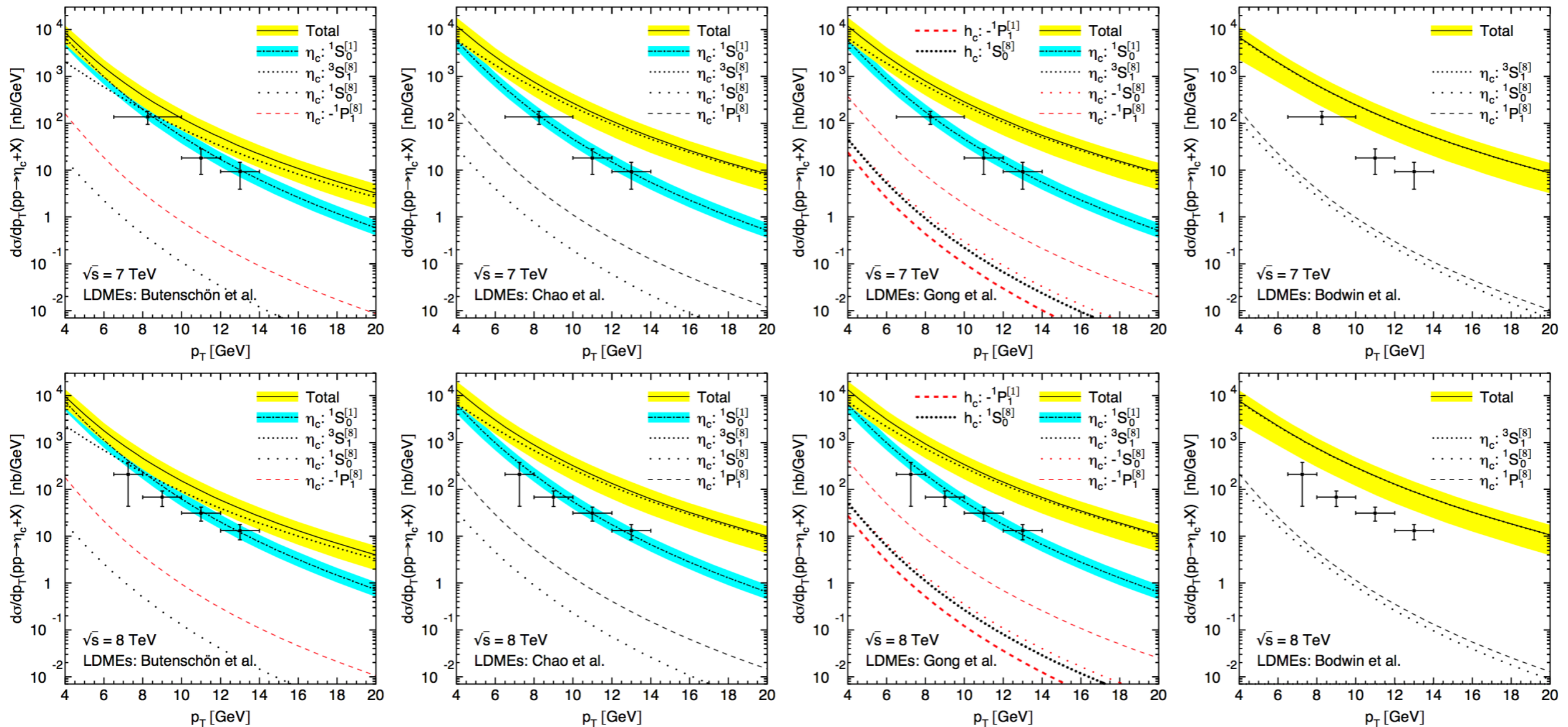
Υ PRODUCTION

- For Υ , the colour-singlet channel makes up for most of the cross section. This is consistent with small v^2 ($v^2 \approx 0.1$), which strongly suppresses colour-octet channels.



η_c HADROPRODUCTION

- J/ψ LDMEs constrain η_c LDMEs through heavy quark spin symmetry, but this overestimates the cross section



LHCb, EPJC75 (2015), 311

Butenschoen, He, Kniehl, PRL114, 092004 (2015)

RESUMMATION OF LEADING LOGARITHMS

- The leading logarithms can be resummed to all orders by solving the LO DGLAP equation

$$\frac{d}{d \log \mu_f^2} \begin{pmatrix} D_S \\ D_g \end{pmatrix} = \frac{\alpha_s(\mu_f)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_S \\ D_g \end{pmatrix}$$

$$D_S = \sum_q (D_q + D_{\bar{q}})$$

- This equation is diagonalized in Mellin space; the inverse transform can then be carried out numerically
- Because the FFs are singular at the endpoint, the inversion is divergent at $z = 1$; special attention is needed for contribution at $z \approx 1$

RESUMMATION OF LEADING LOGARITHMS

- We split the z integral :

$$\int_0^1 dz \hat{\sigma}(z) D(z) = \int_0^{1-\epsilon} dz \hat{\sigma}(z) D(z) + \int_{1-\epsilon}^1 dz \hat{\sigma}(z) D(z)$$

$$\approx \int_0^{1-\epsilon} dz \hat{\sigma}(z) D(z) + \hat{\sigma}(z=1) \int_{1-\epsilon}^1 dz z^N D(z)$$

- N is chosen so that $\hat{\sigma}(z) \approx \hat{\sigma}(1)z^N$ near $z \approx 1$

$$\int_{1-\epsilon}^1 dz z^N D(z) = \int_0^1 dz z^N D(z) - \int_0^{1-\epsilon} dz z^N D(z)$$

*Well defined in Mellin space
(Mellin transform of $D(z)$)*

*Well behaved
numerically*

POLARIZATION AT NLO

Bernd Kniehl's group

□ / • CDF data: Run I / II

Helicity frame

..... CS, LO

—•— CS, NLO

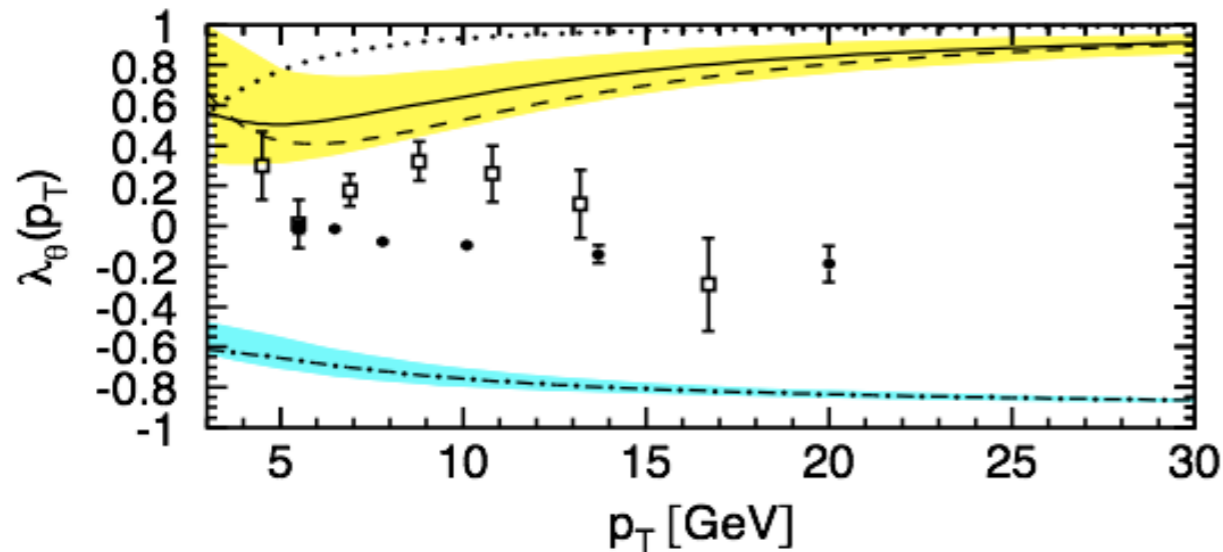
--- CS+CO, LO

— CS+CO, NLO

$|y| < 0.6$

$\sqrt{s} = 1.96$ TeV

$p\bar{p} \rightarrow J/\psi + X$



• ALICE data

Helicity frame

..... CS, LO

—•— CS, NLO

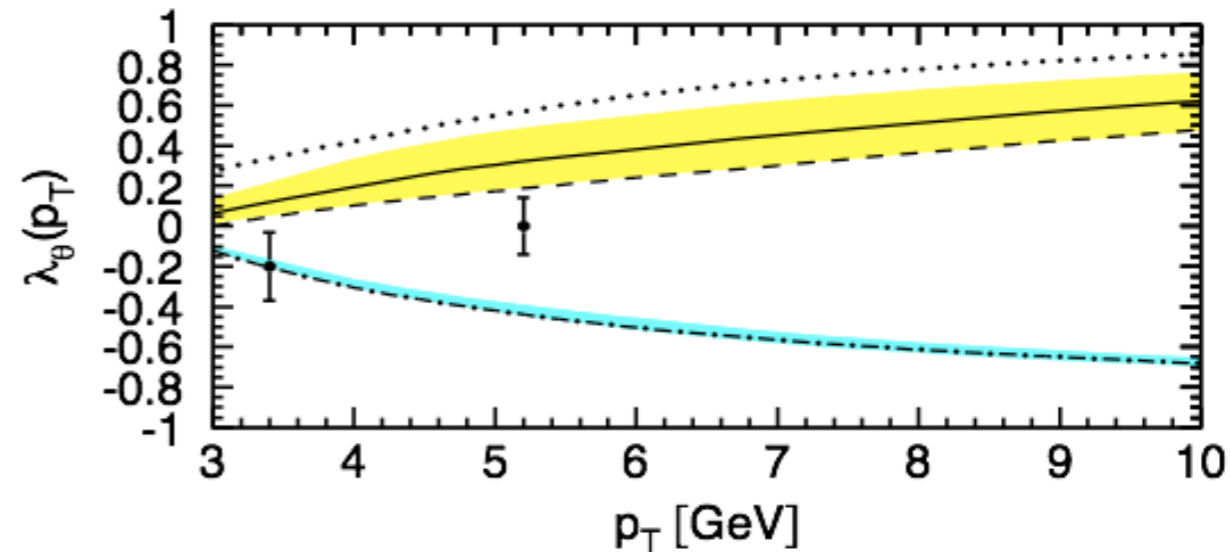
--- CS+CO, LO

— CS+CO, NLO

$2.5 < y < 4$

$\sqrt{s} = 7$ TeV

$pp \rightarrow J/\psi + X$



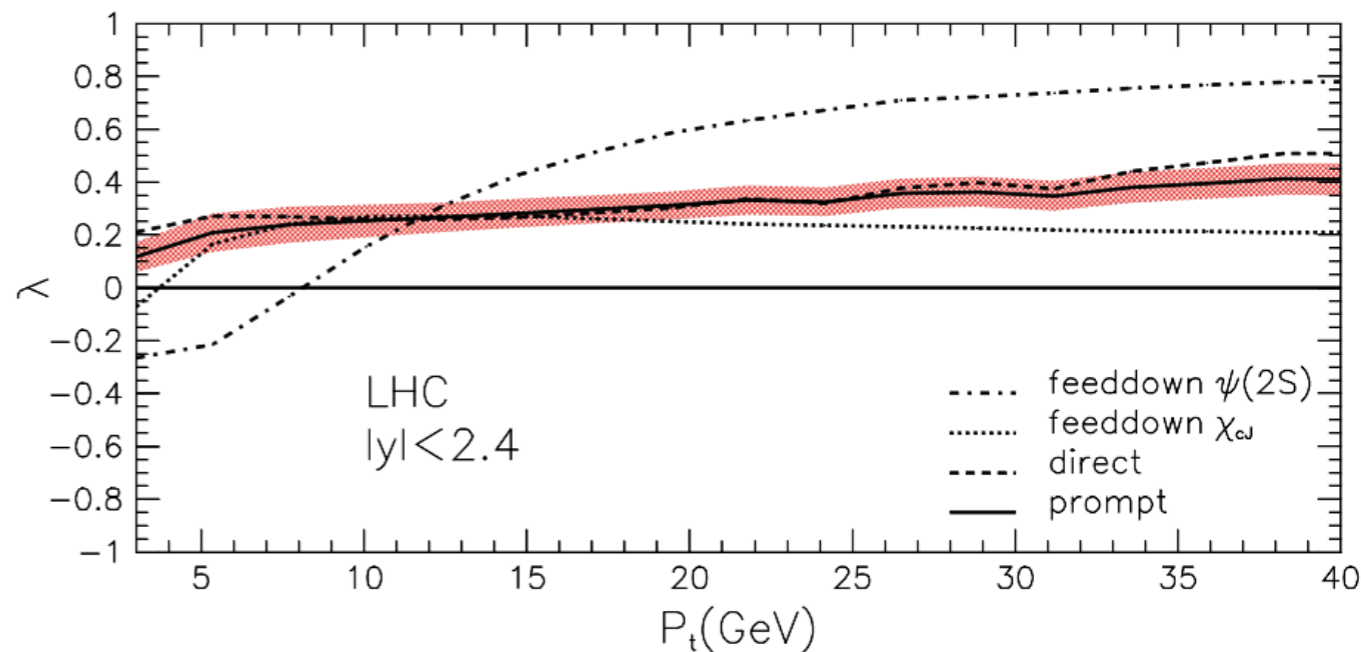
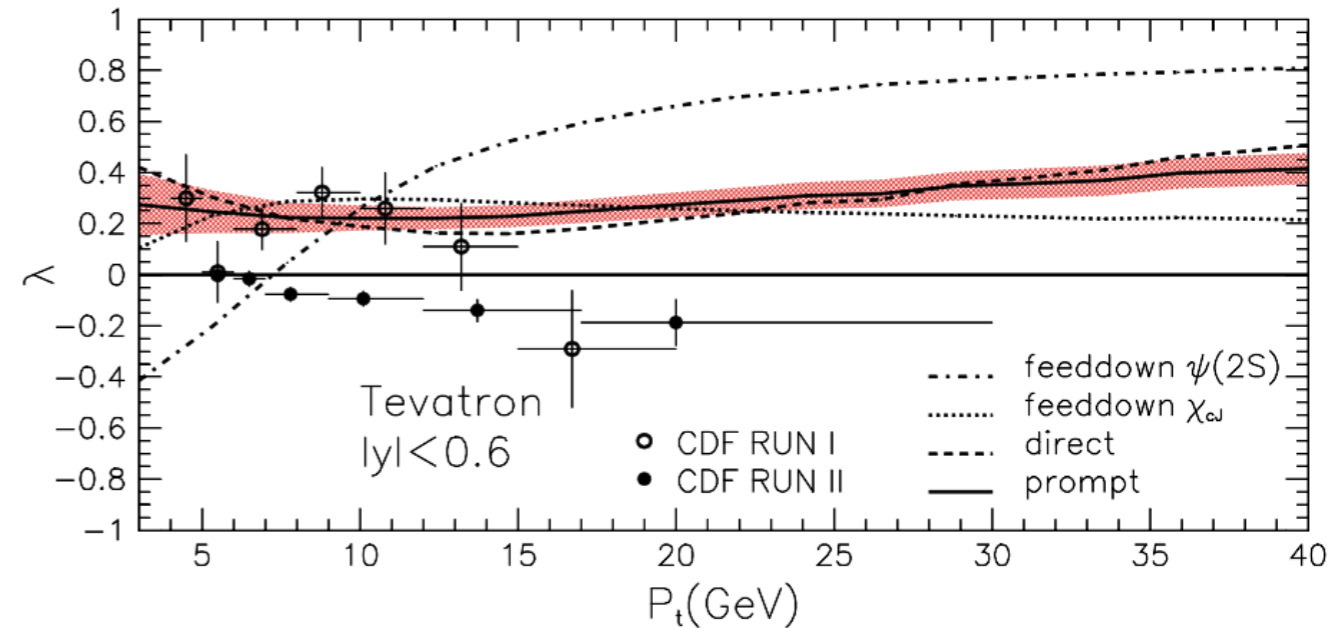
Butenschoen and Kniehl, Mod.Phys.Lett.A28, I350027 (2013)

- Used cross section measurements at HERA and Tevatron to fix LDMEs, predicts transverse polarization at large p_T
- H1 and ZEUS data are at small p_T
- Does not include feeddown

POLARIZATION AT NLO

Jianxiong Wang's group

- Used CDF and LHCb cross section data to fit LDMEs
- Includes feeddown
- Prediction still more transverse than measurement

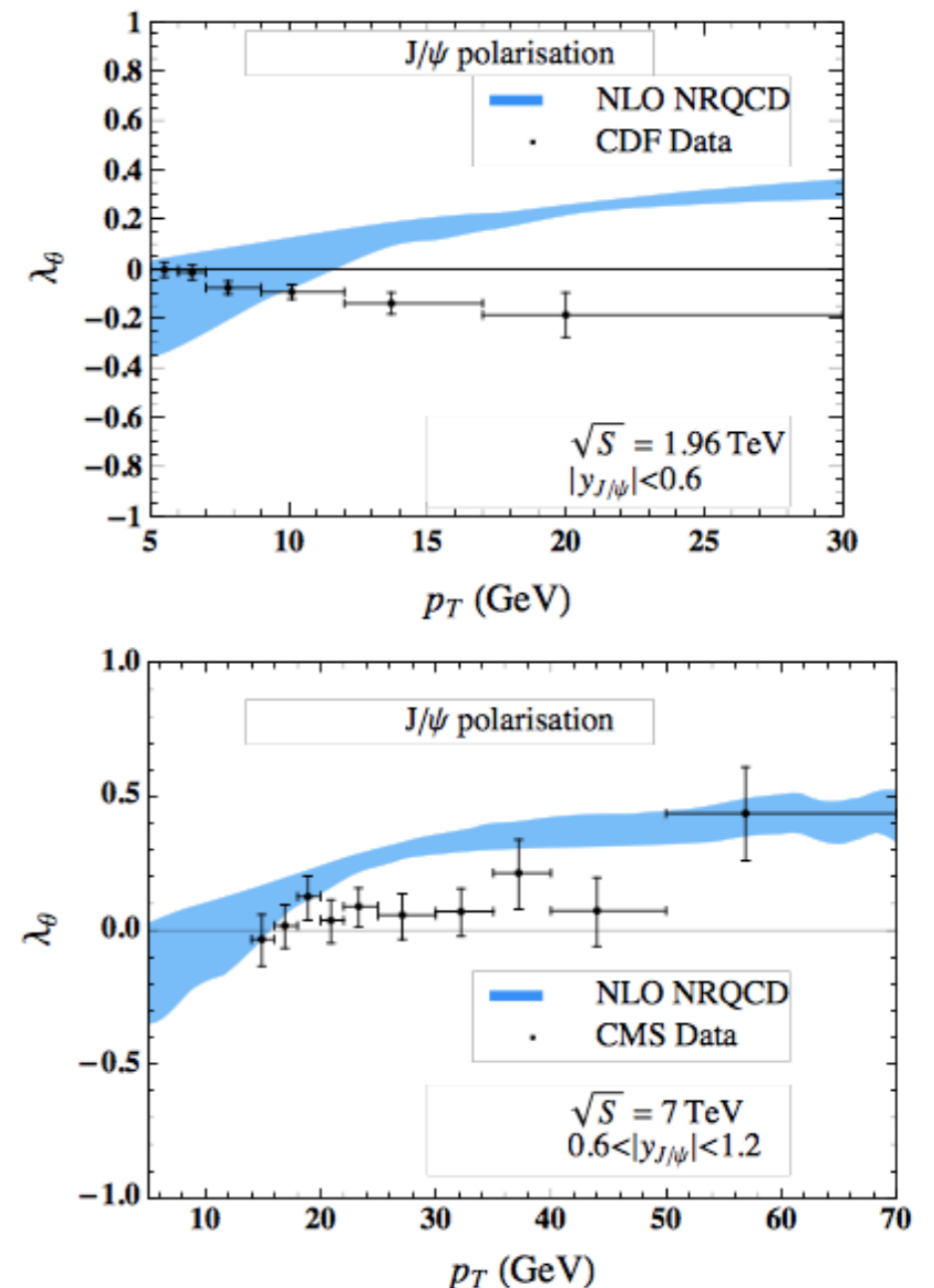


Gong, Wan, Wang, Zhang, PRL 110, 042002 (2013)

POLARIZATION AT NLO

Kuang-Ta Chao's group

- Used CDF, ATLAS, CMS, LHCb cross section data to fit LDMEs
- Included feeddown
- Assumed positivity of all LDMEs, although $^3P_J^{[8]}$ LDME has strong factorization scale dependence
- Prediction still more transverse than data

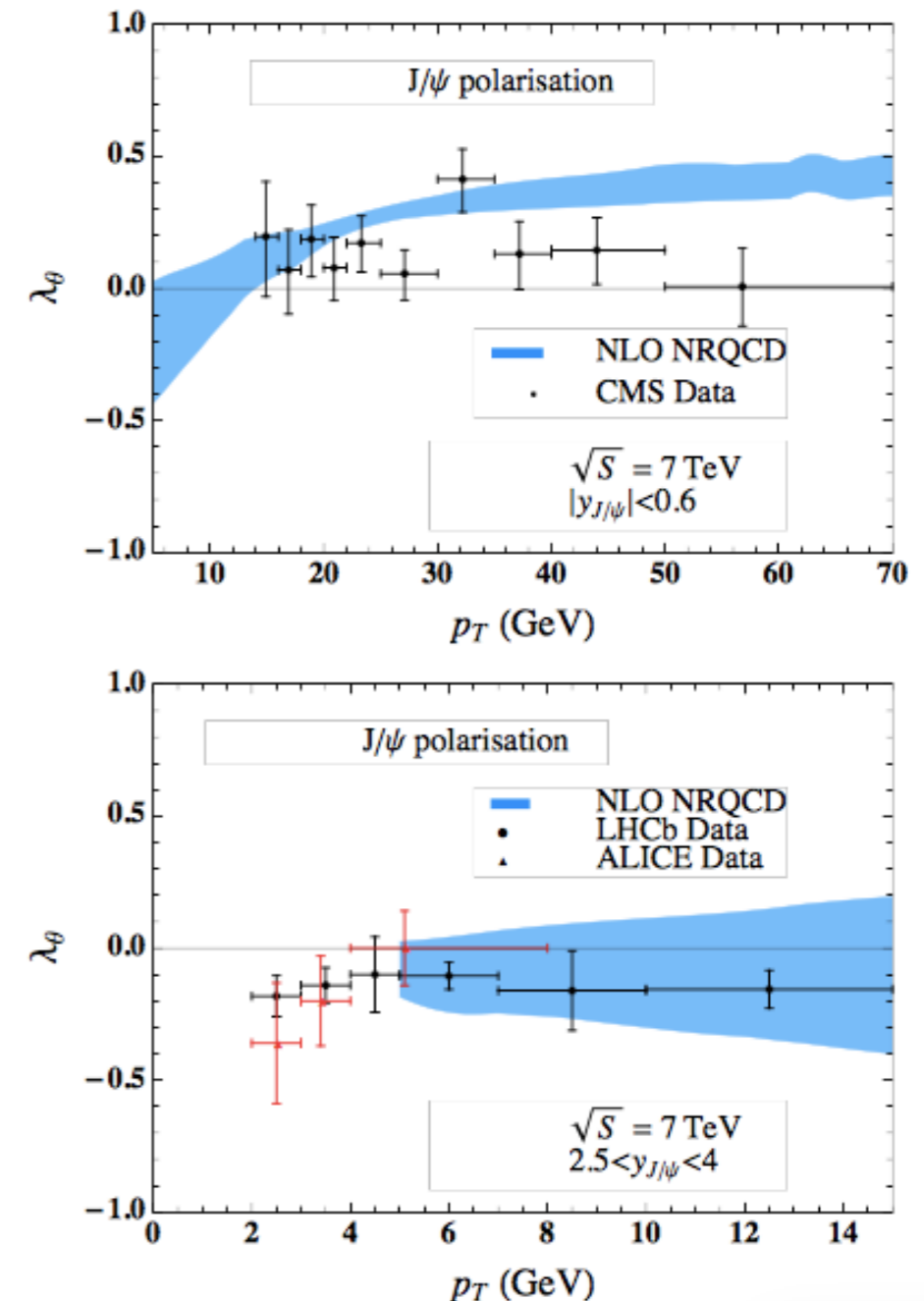


Shao, Han, Ma, Meng, Zhang, Chao, JHEP05 (2015) 103

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Kuang-Ta Chao's group



Shao, Han, Ma, Meng, Zhang, Chao, JHEP05 (2015) 103