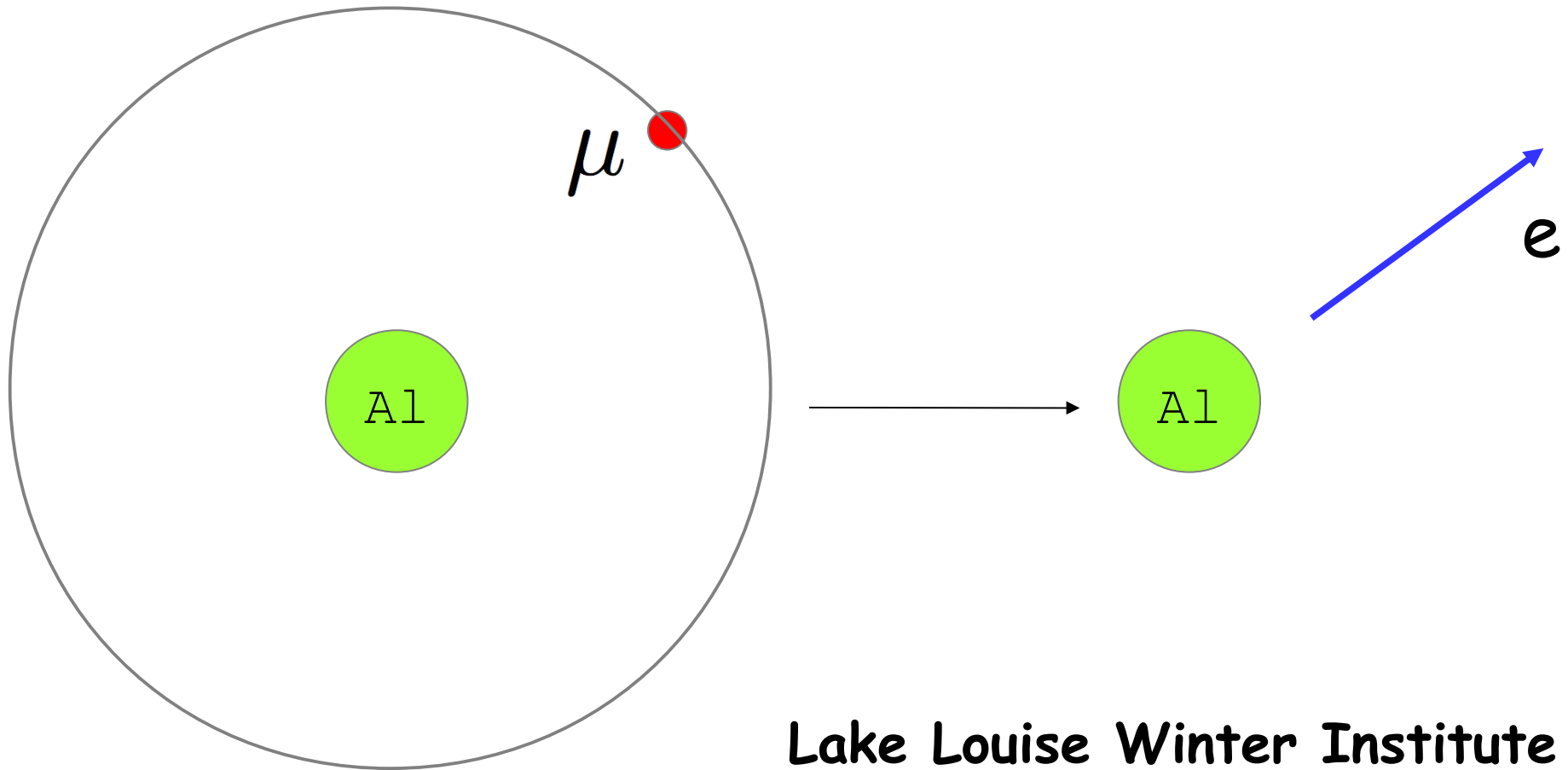


Looking for New Physics with muons



Lake Louise Winter Institute
February 22, 2016

Andrzej Czarnecki  University of Alberta
with M. Dowling, J. Piclum, R. Szafron

Outline

Muon decay

New era of experiments with muons

Muon-electron conversion: the rarest decay

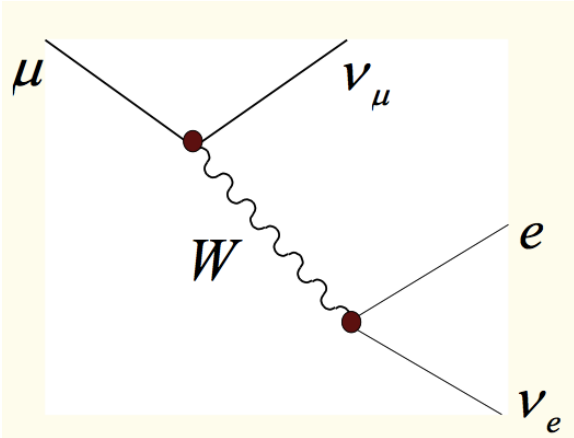
Muon decay in orbit: background for conversion

- approaches to radiative corrections

g -factor of a muon and of a bound electron

- binding effects at a new level

Free muon decay



A model process in particle physics
(tools for quark decays:
charm in b-decays, Nir 1989)

The first decay process known with one-
and two-loop QED effects.

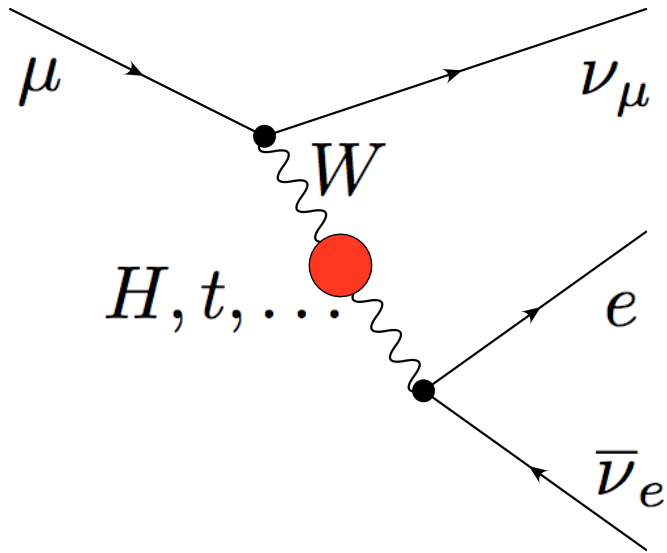
Anastasiou, Melnikov, Petriello, JHEP 0709 (2007) 014
van Ritbergen + Stuart, PRL 82 (1999) 488
Pak + Czarnecki, PRL 100 (2008) 241807

Also very thoroughly studied experimentally; most recently

* decay distributions ("Michel parameters") TWIST PRD 85 (2012) 092013

* total rate (1 ppm!) MuLan PRL 106 (2011) 041803

Fermi constant and tests of the SM

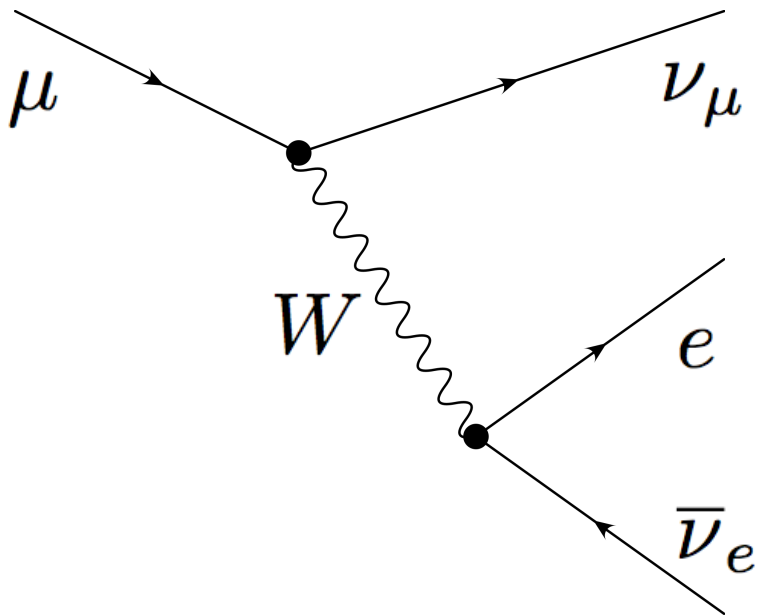


$$G_F \sim \frac{\alpha}{M_W \left(1 - \frac{M_W^2}{M_Z^2}\right)} (1 + \Delta r)$$

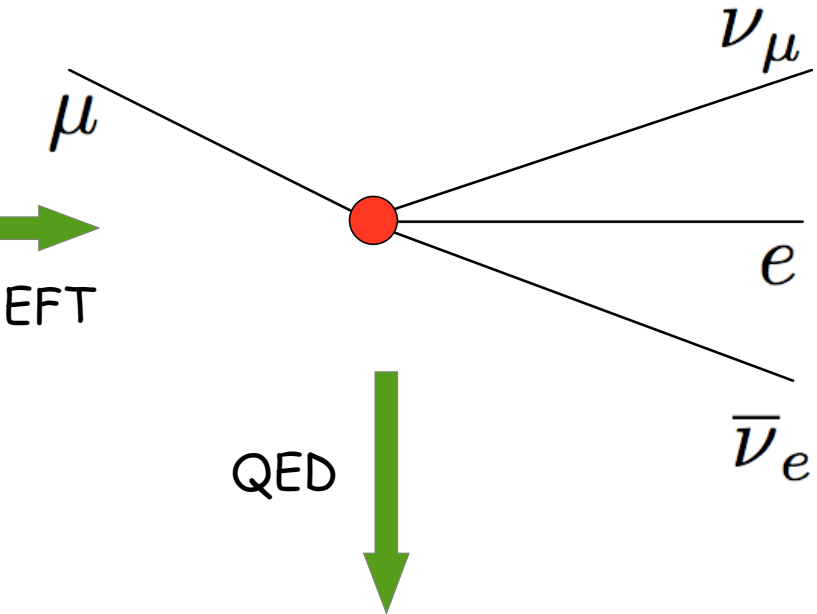
$$\Delta\alpha_{\text{had}} - cm_t^2 + c' \ln M_H + \dots$$

One of the pillars of electroweak precision tests.

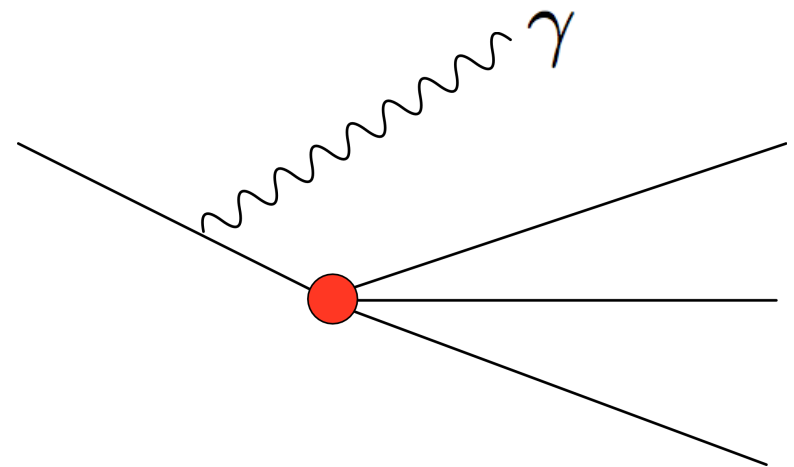
Determination of the Fermi constant (convention)



Fermi EFT



QED



$$\frac{1}{\tau} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + \Delta q)$$

Finite m_e and QED corrections
in Fermi EFT

Radiative effects in Fermi EFT

1956 one-photon, with finite m_e

Behrends, Finkelstein, Sirlin

1999 two-photon, with $m_e=0$

Stuart, van Ritbergen

2007 two-photon, spectrum of E_e

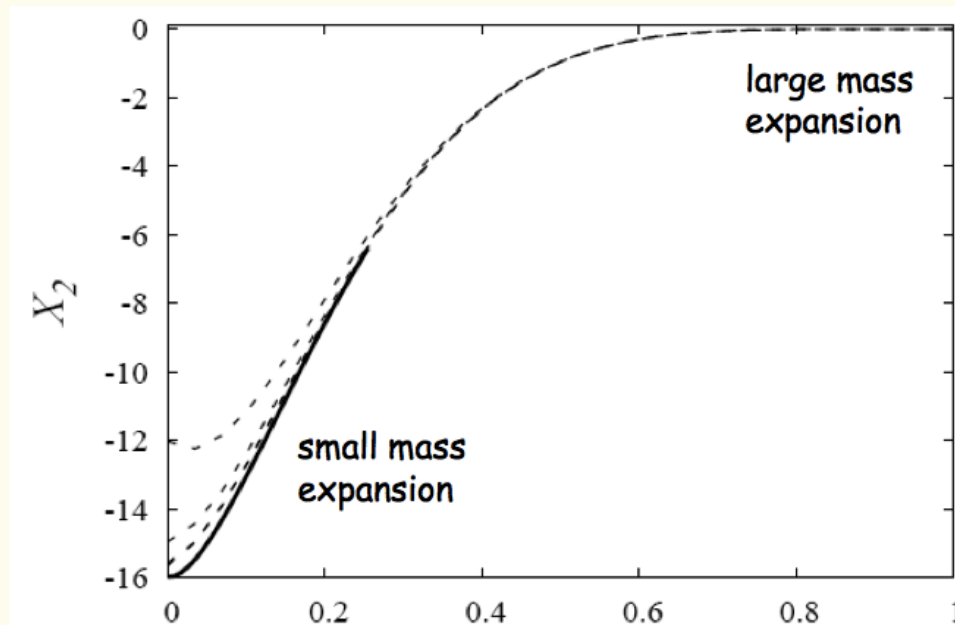
Anastasiou, Melnikov, Petriello

2008 two-photon, with finite m_e

Pak, AC

Can one go further: to three loops?

We have found an interesting way while checking the two-loop result: the calculation would be easier if the electron was very heavy, almost as heavy as the decaying muon.



From Dowling, Piclum, AC

"Electron to muon" mass ratio

Note: the plot actually for QCD.
QED given by a subset of QCD results.

Lepton Flavor Violation

New era of experiments with muons

PSI (Switzerland):

muonic atoms
mu \rightarrow e + gamma
mu + p scattering
mu \rightarrow eee

Fermilab (USA):

g-2
Mu2e

J-PARC (Japan):

g-2
DeeMe
COMET
muonium HFS

Muons are indeed a great tool for New Physics searches:
long-lived, just massive enough, easy to produce, with convenient spin properties.

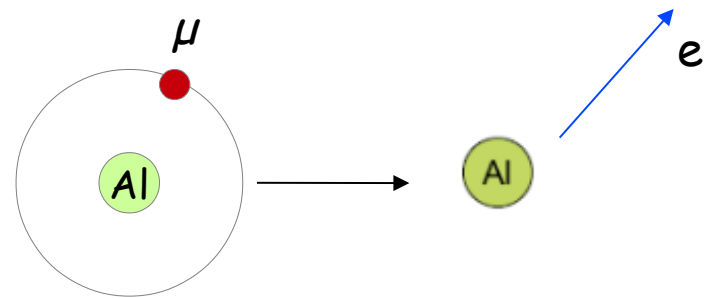
They are also mysterious. Some precise measurements disagree with expectations:
g-2, proton radius, B-decays.

$$B^0 \rightarrow K^{*0} \mu^+ \mu^-$$
$$R_K = \text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \text{BR}(B^+ \rightarrow K^+ e^+ e^-)$$

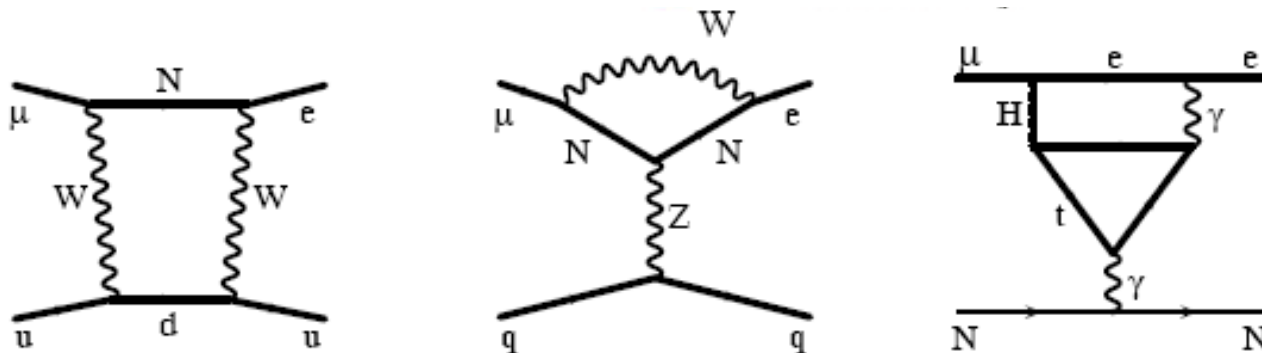
Muon-electron conversion: probes various types of interactions

Non-dipole interactions are not (directly) probed by processes with external photons, by gauge invariance requirements.

New process: muon-electron conversion
(as well as $\mu \rightarrow eee$)

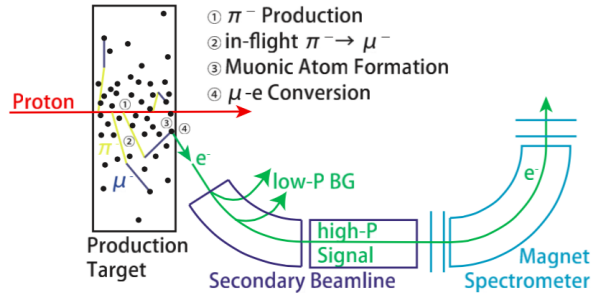


Variety of mechanisms:



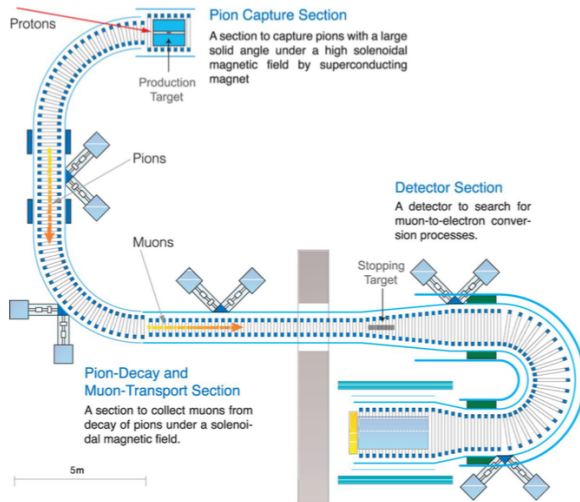
Muon-electron conversion plans (The Next Big Thing in muon physics)

DeeMe
J-PARC



starts 2016;
aims for $1e-13$ (graphite target),
followed by $1e-14$ (SiC target)

COMET
Phase 1
J-PARC

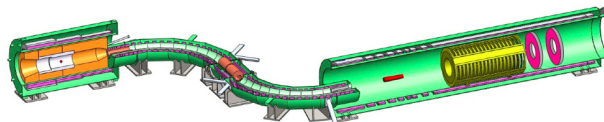


$7e-15$

COMET
Phase 2
J-PARC

$2.6e-17$

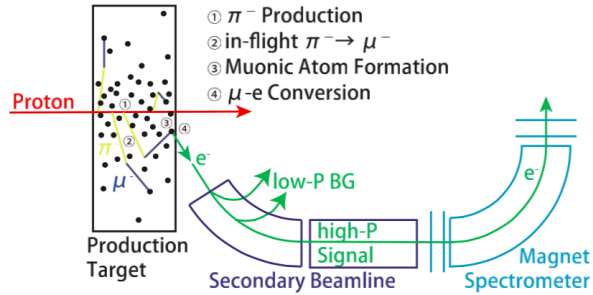
Mu2e
Fermilab



$2e-17$

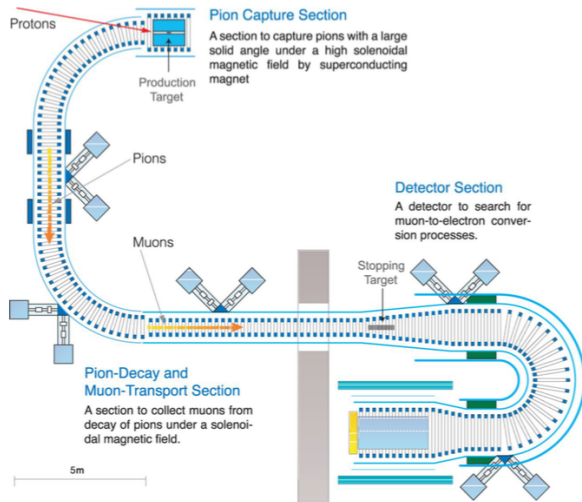
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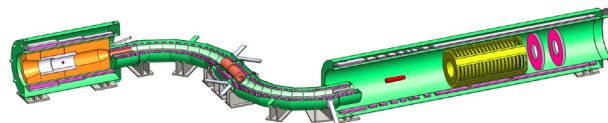
$7e-15$

For comparison,
 $BR(\mu \rightarrow e\gamma) < 4e-13$

COMET
Phase 2
J-PARC

$2.6e-17$

Mu2e
Fermilab



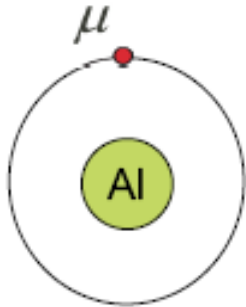
$2e-17$

Comparison with scattering experiments

Highest luminosity in fixed-target experiments

$$\sim 10^{37 \dots 38} / (\text{cm}^2 \cdot \text{s})$$

In a single muonic atom



flux = density \times velocity

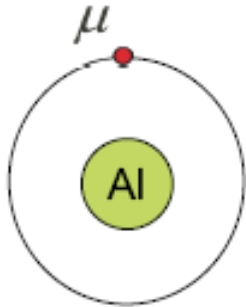
$$= |\psi(0)|^2 \times Z\alpha = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \sim Z^4 \times 4 \cdot 10^{39} / (\text{cm}^2 \cdot \text{s})$$

Comparison with scattering experiments

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Many atoms are studied in parallel: $\sim 10^{11}$ muons stopped per second;
each lives about 10^{-6} seconds: 10^5 atoms present:

$$\sim 10^{49} / (\text{cm}^2 \cdot \text{s})$$

What does the conversion rate mean?

Three fates of a bound muon:

- decay in orbit (dominates for $Z < 11$)
- nuclear capture (most likely for $Z > 11$)
- muon-electron conversion (less than $4.3e-12$ cases)

$$\Gamma(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) / \Gamma(\mu^- \text{Ti} \rightarrow \text{all}) < 4.3 \times 10^{-12} \quad \text{PDG}$$

On the other hand, for free muons: $\Gamma(\mu \rightarrow e\gamma) / \Gamma(\mu \rightarrow \text{all}) < 4.2 \times 10^{-13}$

Which of these bounds is "better"?

Two types of operators contribute to LFV

$$O_L^D = e m_\mu (\bar{e} \sigma^{\mu\nu} P_L \mu) F_{\mu\nu} \leftarrow \begin{array}{l} \text{connects opposite chiralities} \\ \text{of } e, \mu \end{array}$$
$$O_{ff}^{V LL} = (\bar{e} \gamma^\mu P_L \mu) (\bar{f} \gamma_\mu P_L f) \leftarrow \begin{array}{l} \text{equal chiralities} \\ \text{equal chiralities} \end{array}$$
$$O_{ff}^{V LR} = (\bar{e} \gamma^\mu P_L \mu) (\bar{f} \gamma_\mu P_R f) \leftarrow \begin{array}{l} \text{equal chiralities} \\ \text{equal chiralities} \end{array}$$

...

$$P_{L/R} = (\mathbb{I} \mp \gamma^5) / 2$$

Crivellin, Davidson, Pruna, Signer
1702.03020.

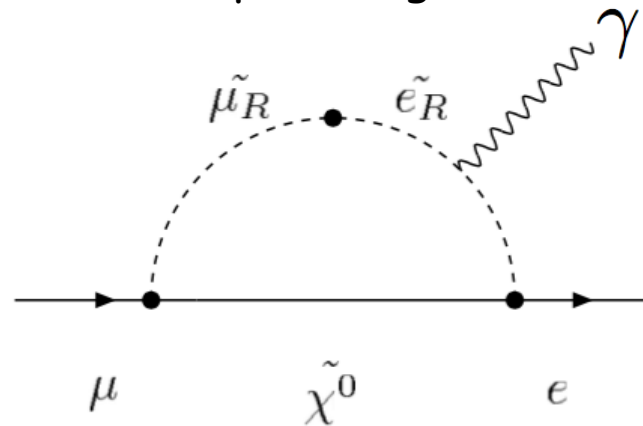
Only the chirality-flipping operators O_L^D and O_R^D contribute to $\mu \rightarrow e \gamma$

All operators contribute to the muon-electron conversion
(because there are no external photons \rightarrow no gauge constraints).

Take-home message #1: conversion probes a broader range of physics.

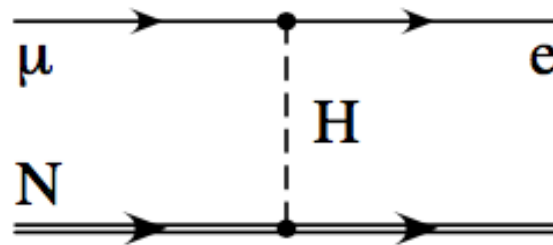
Examples of New Physics scenarios

Dipole operators: closed SUSY loops (like $g-2$):



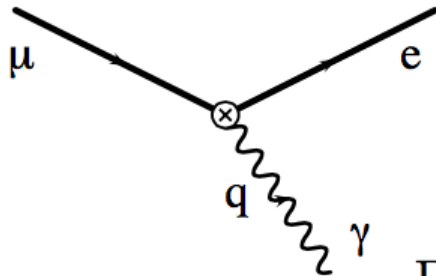
Kuno, Okada
RMP 73, 151

Scalar four-fermion operator, in case of a flavor-offdiagonal Higgs coupling



Contributes mainly to the conversion

How fast is the conversion induced by dipoles?

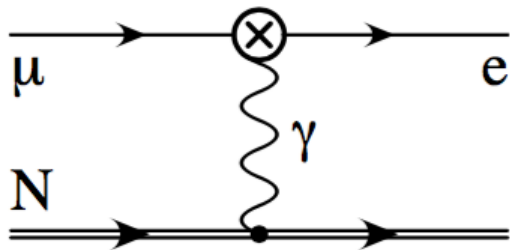


$$= \bar{e} \sigma^{\mu\nu} (f_M + f_E \gamma_5) \mu \cdot q_\mu A_\nu$$

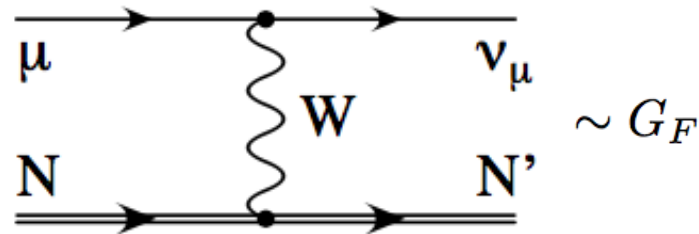
$$\Gamma(\mu \rightarrow e\gamma) = \frac{m_\mu^3}{8\pi} (|f_M|^2 + |f_E|^2) \equiv \frac{m_\mu^3 f^2}{8\pi}$$

$$BR(\mu \rightarrow e\gamma) = \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow \text{all})} \simeq \frac{m_\mu^3 f^2}{8\pi} \cdot \frac{192\pi^3}{G_F^2 m_\mu^5} = 24\pi^2 \left(\frac{f}{G_F m_\mu} \right)^2$$

The same operator induces conversion, competing with capture:



$$\sim \frac{m_\mu f}{m_\mu^2}$$



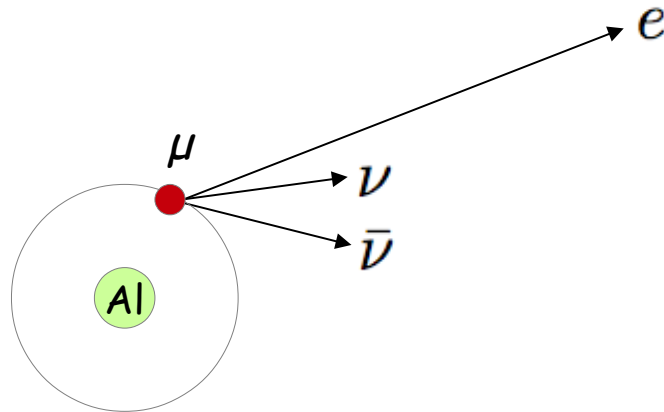
$$\sim G_F$$

Take-home message #2: in case of dipole operators,

$$BR(\text{conversion}) \sim \frac{BR(\mu \rightarrow e\gamma)}{200(\text{Ti}) - 400(\text{Al, Pb})}$$

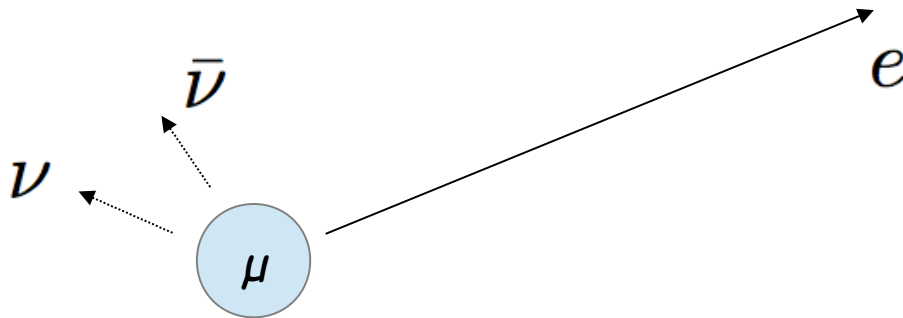
Background for the conversion search

Normal decay of the muon bound in the atom can produce high-energy electron,

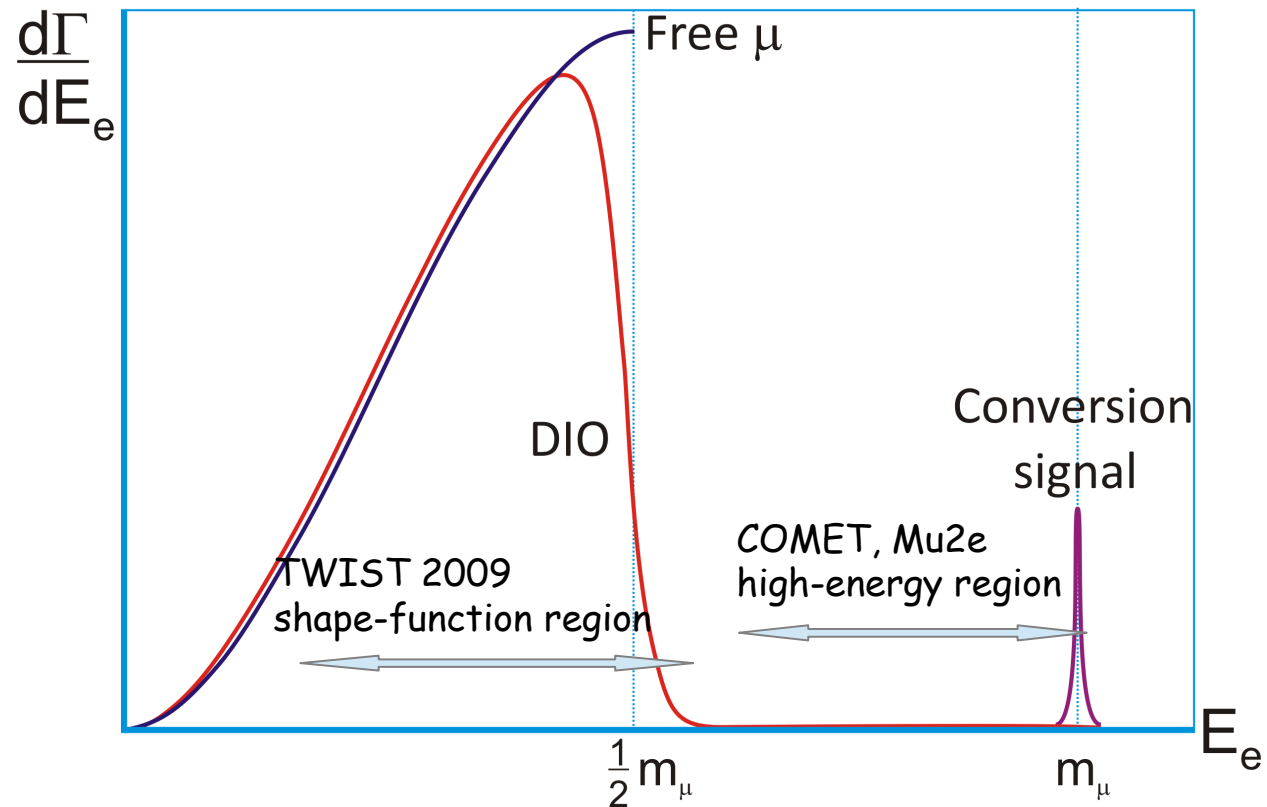


Spectrum has to be well understood.

Electron spectrum in a bound muon decay



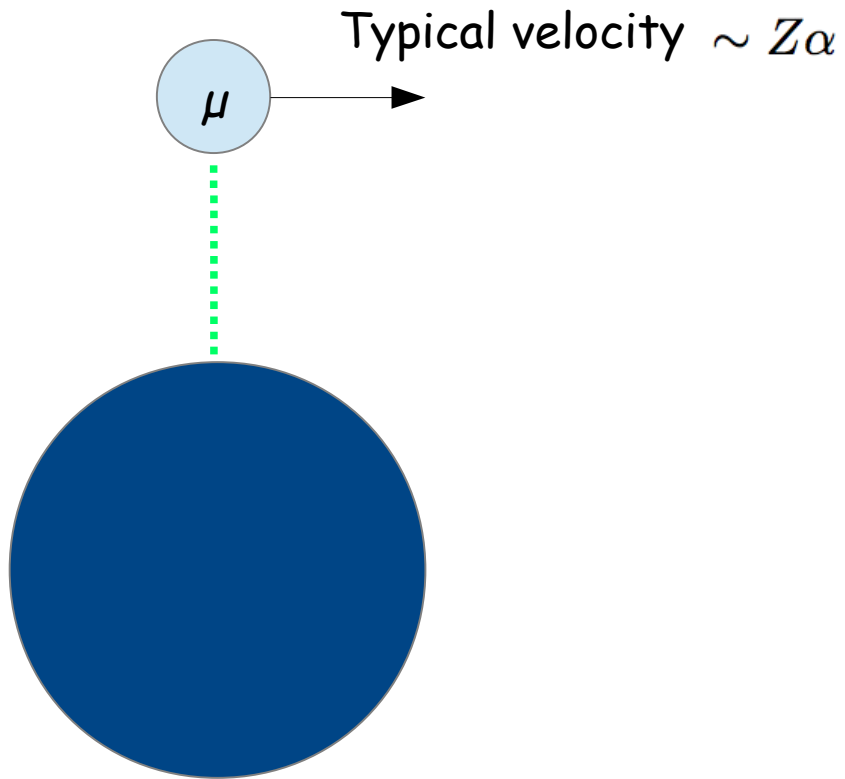
Electron energy can be as large as the whole muon mass



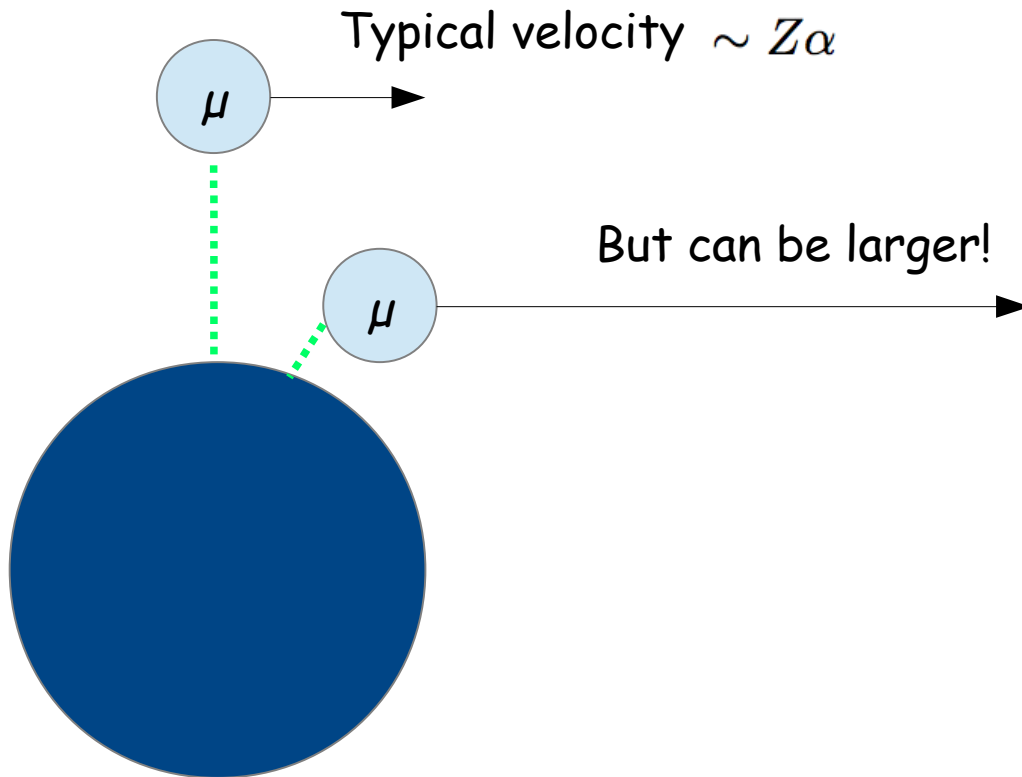
**Muon decay-in-orbit spectrum:
the shape-function region**

Experiment: TWIST

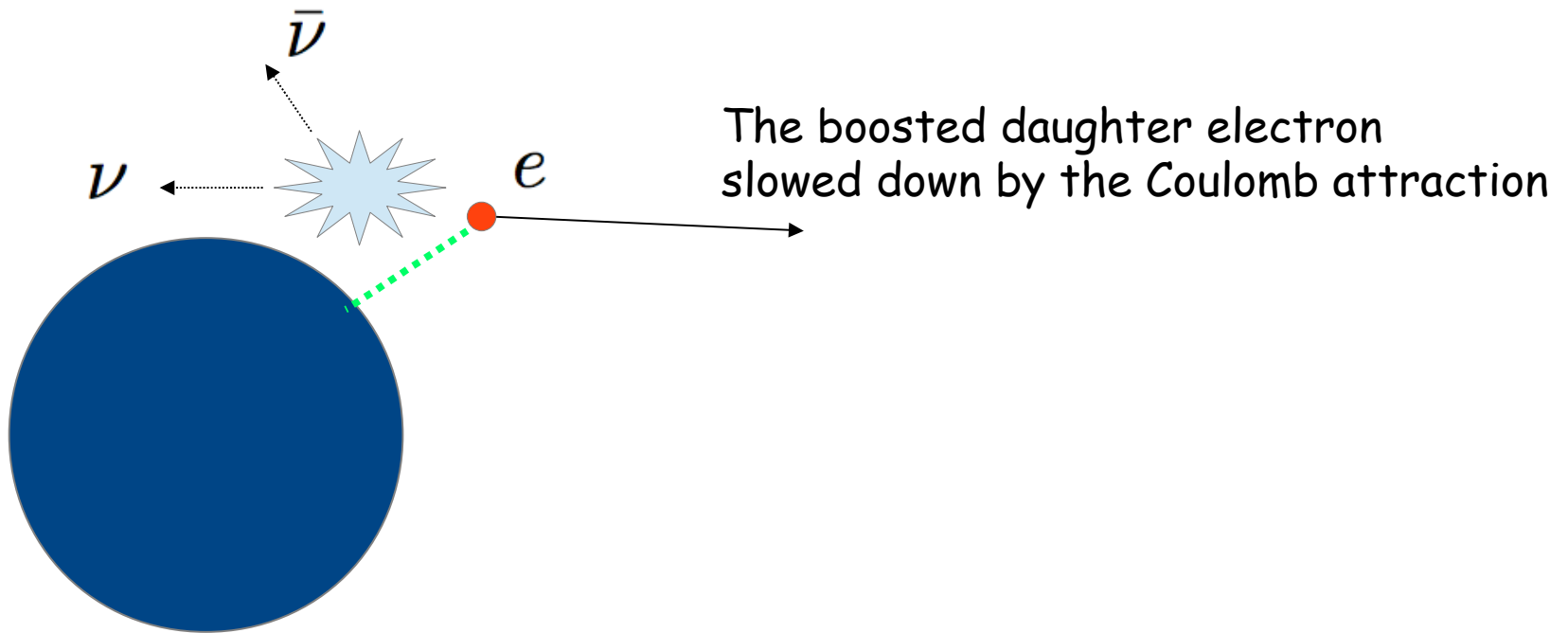
Two effects: muon motion & Coulomb attraction



Two effects: muon motion & Coulomb attraction



Two effects: muon motion & Coulomb attraction



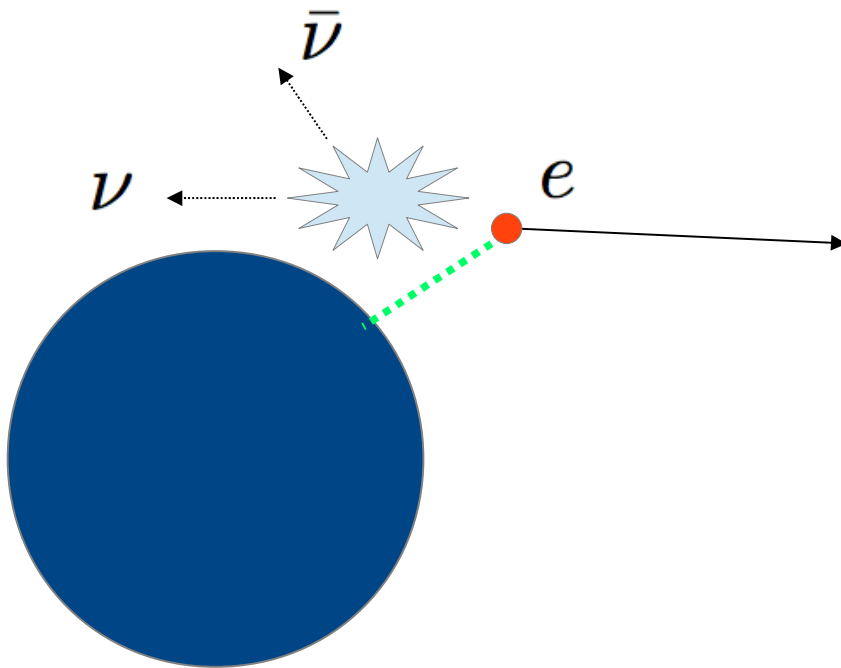
Two effects: muon motion & Coulomb attraction

Net effect:

- In the decay rate: almost none;
only time dilation

$$\Gamma \rightarrow \left(1 - \frac{(Z\alpha)^2}{2}\right) \Gamma$$

Überall, Phys. Rev. 119, 365 (1960)



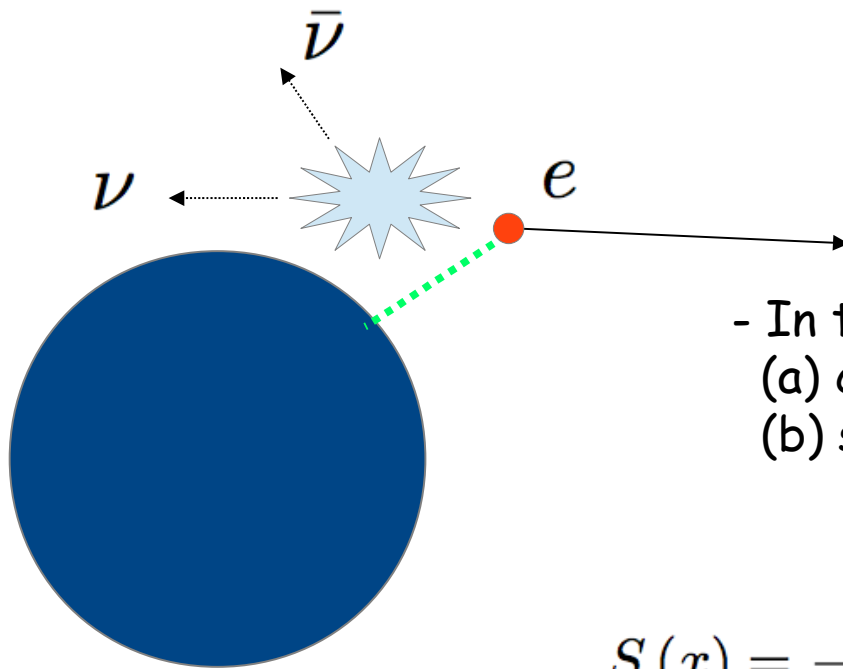
Two effects: muon motion & Coulomb attraction

Szafron, AC, PRD 92 (2015) 053004

Net effect:

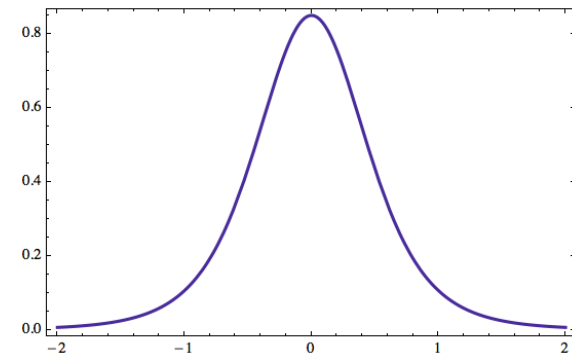
- In the decay rate: almost none; only time dilation

$$\Gamma \rightarrow \left(1 - \frac{(Z\alpha)^2}{2}\right) \Gamma$$



- In the electron energy spectrum:
 - (a) computable shift
 - (b) smearing \rightarrow "shape function"

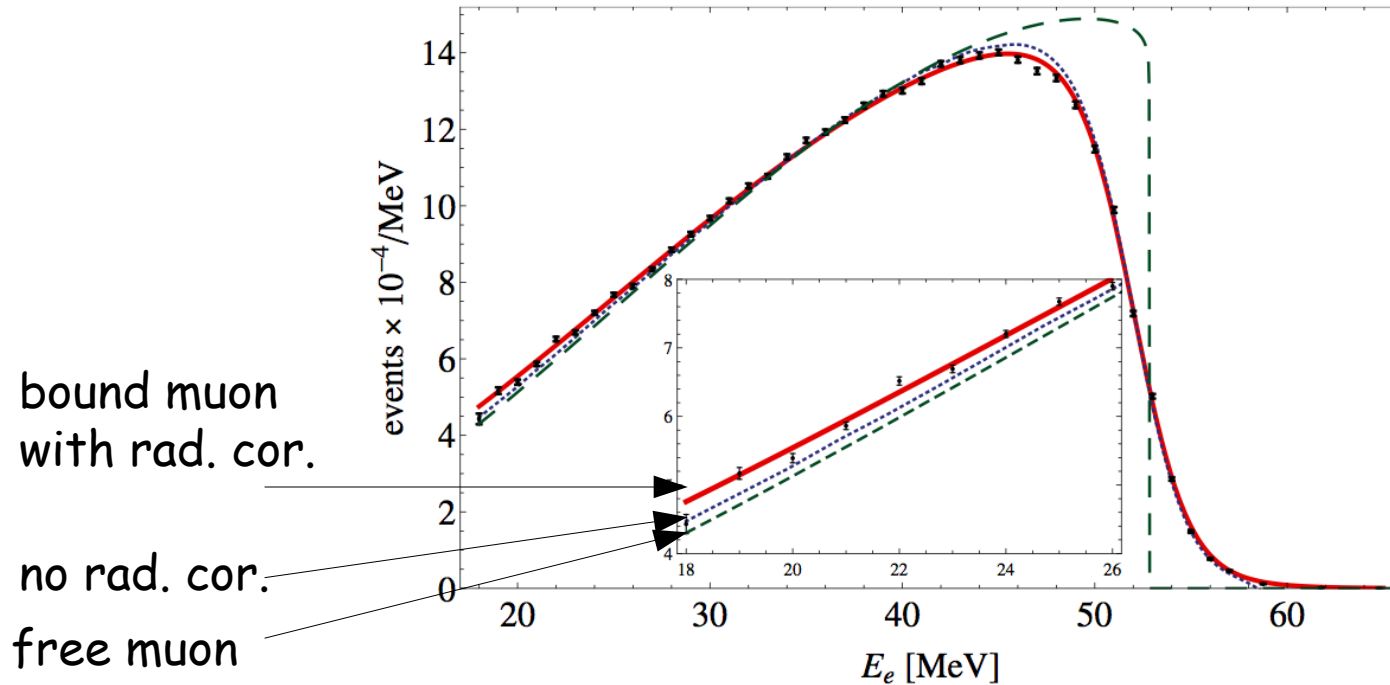
$$S(x) = \frac{8}{3\pi [1 + x^2]^3}$$



Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert,
Bigi, Shifman, Uraltsev, Vainshtein

Comparison with measurement: TWIST

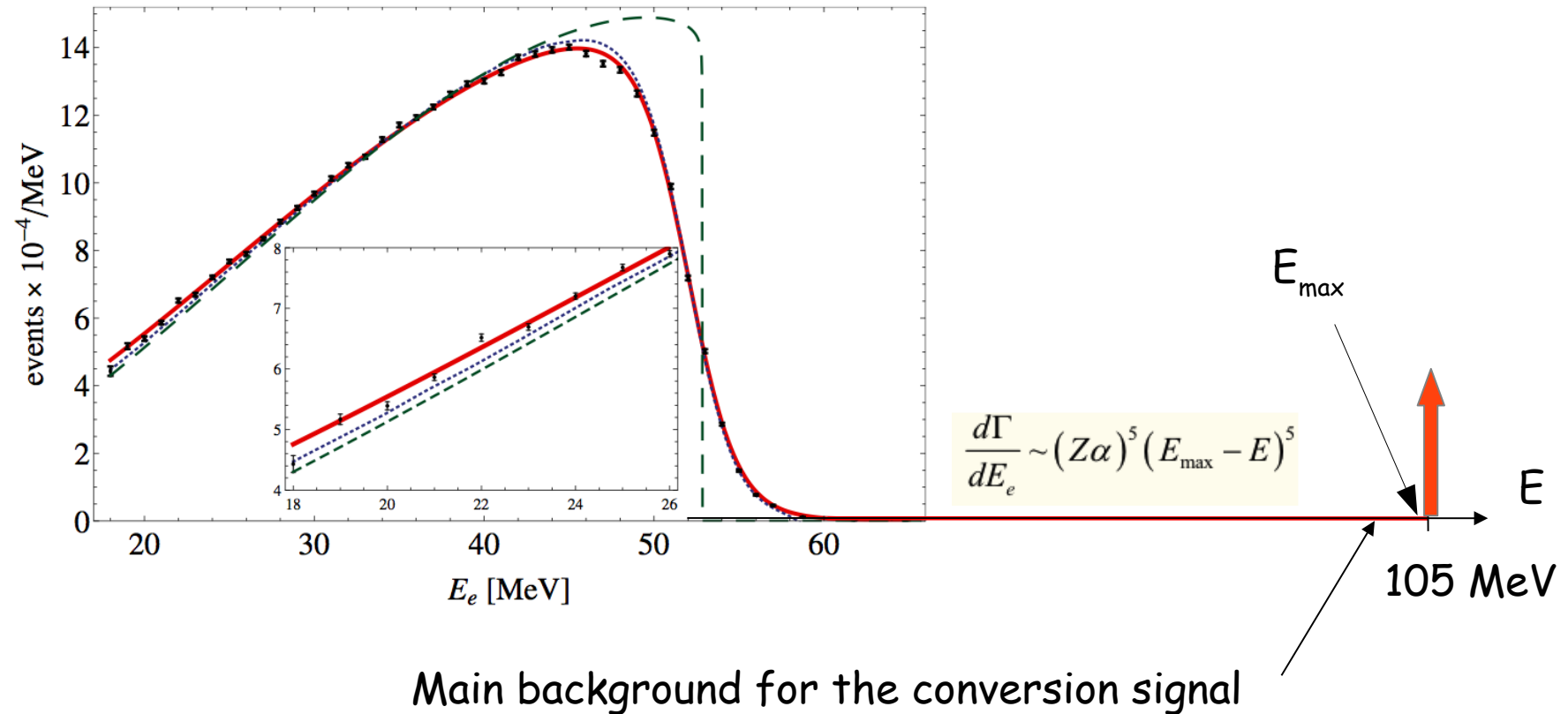


The spectrum is modified very significantly: effects $\sim 1/Z\alpha$

**Muon decay-in-orbit spectrum:
the high-energy region**

Experiments: Mu2e and COMET

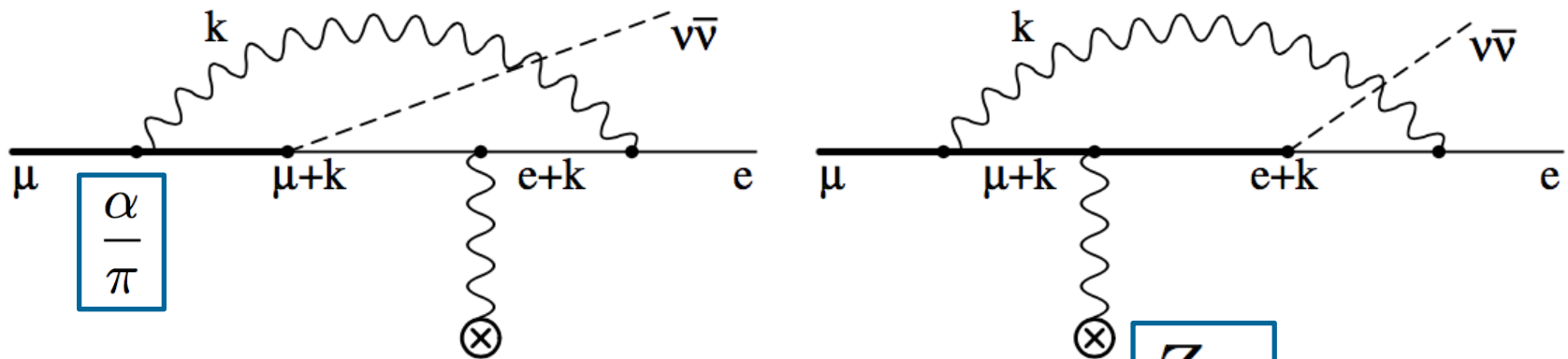
Spectrum of the bound muon decay



Radiative corrections to the electron spectrum

Expansion near the end-point $\frac{m_\mu}{\Gamma_0} \frac{d\Gamma}{dE} = \sum_{ijk} B_{ijk} \Delta^i (\pi Z\alpha)^j \left(\frac{\alpha}{\pi}\right)^k$

Three "small" parameters: $\Delta = \frac{E_{\max} - E}{m_\mu}$



$$B_{550} + \frac{\alpha}{\pi} B_{551} \rightarrow B_{550} \left[\Delta^{\frac{\alpha}{\pi} \delta_S} + \frac{\alpha}{\pi} \delta_H \right]$$

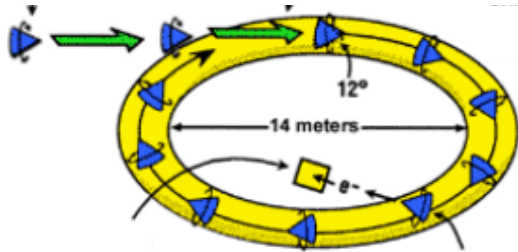
$$\delta_S = 10.1$$

number of electrons in the end-point bin of 1 (0.1) MeV is reduced by 11% (16%)

Anomalous magnetic moment

The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy,



$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11}$$

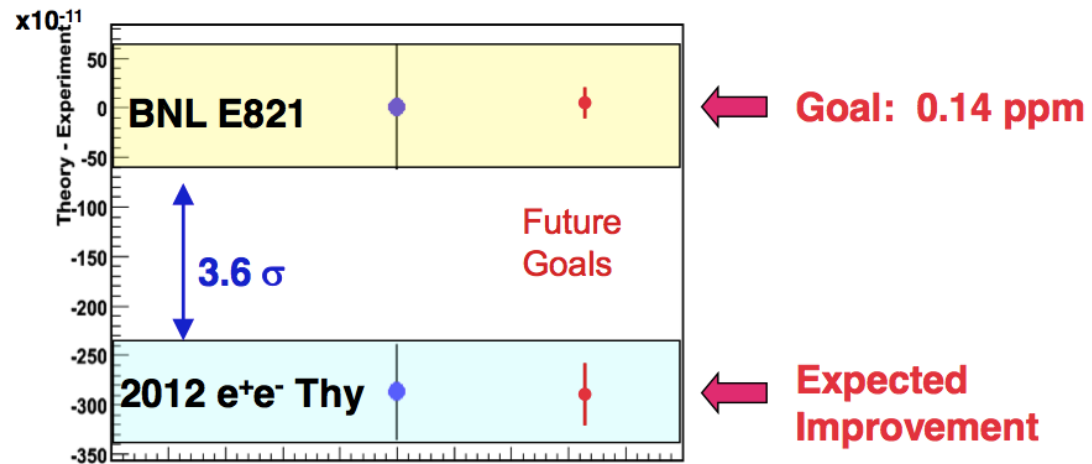
PRD 86, 095009 (2012)

0.7 ppm

is rather large when compared with other bounds on New Physics.

How can $g_{\mu}-2$ be checked?

New experiment at Fermilab



New experimental concept at J-PARC

Can we use g_e-2 ?

New approach to $g_\mu - 2$ at J-PARC

Slower muons 300 MeV (instead of the "magic" 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring $r = 33$ cm (instead of 7 m)

$$r \text{ [in meters]} \simeq \frac{\gamma}{3B \text{ [in Tesla]}}$$

Strong, very precisely controlled magnetic field.

~ 10 times more muons than at Fermilab (compensates shorter lifetime).

	Brookhaven	Fermilab	J-PARC
Muon momentum	3.09 GeV/c		0.3 GeV/c
gamma	29.3		3
Storage field	B=1.45 T		3.0 T
Focusing field	Electric quad		None
# of detected μ^+ decays	5.0E9	1.8E11	1.5E12
# of detected μ^- decays	3.6E9	-	-
Precision (stat)	0.46 ppm	0.1 ppm	0.1 ppm

$$\simeq \sqrt{\frac{2\pi}{\alpha}}$$

How to check $g_\mu - 2$?

Electron $g-2$ is likely sensitive to the same New Physics; but at present it is used to determine the fine-structure constant.

A new source of α is needed.

How to check $g_\mu - 2$?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$$14 \cdot 10^{-11}$$

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

$$7 \cdot 10^{-12}$$

(but is it
for sure?)

$$8 \cdot 10^{-11}$$

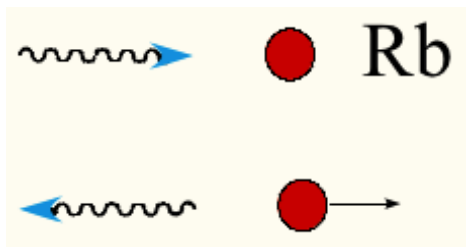
Nature 2014
Sturm et al

$$12 \cdot 10^{-11}$$

for Rb
(better for He)

$$124 \cdot 10^{-11}$$

improvement
needed by
factor ~ 10

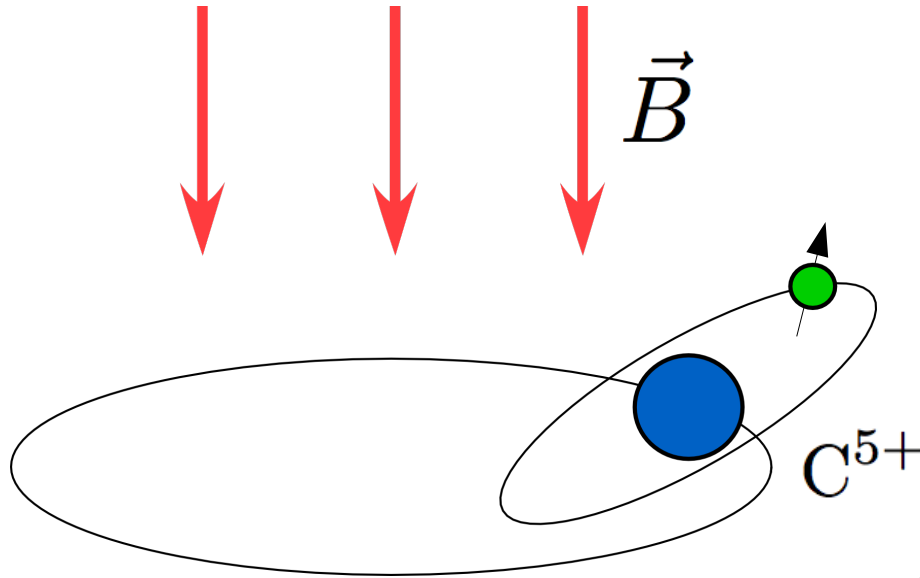


gives h/m

$$\alpha(\text{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

Fine structure constant from bound-e g-factor



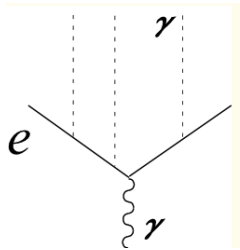
Larmor frequency $\omega_L = \frac{geB}{2m_e}$

Cyclotron frequency $\omega_{\text{cycl}} = \frac{ZeB}{M}$

$$m_e = \frac{g}{2Z} \frac{\omega_{\text{cycl}}}{\omega_L} M$$

Bound-electron $g-2$: the leading effect

Breit 1928: energy correction due to magnetic field in the hydrogen ground state.

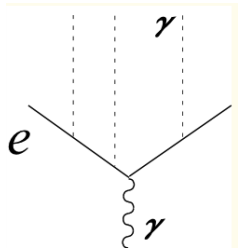


$$\delta E = e \int d^3x f^2 v^* [1 - i\gamma \Sigma \cdot \hat{r} \gamma^5] \gamma^5 \mathbf{A} \cdot \Sigma [1 + i\gamma \Sigma \cdot \hat{r} \gamma^5] v$$

$$g = 2 \cdot \frac{1}{3} \left(1 + 2\sqrt{1 - (Z\alpha)^2} \right) \simeq 2 \left(1 - \frac{(Z\alpha)^2}{3} \right)$$

Bound-electron g-2: the leading effect

Breit 1928: energy correction due to magnetic field in the hydrogen ground state.

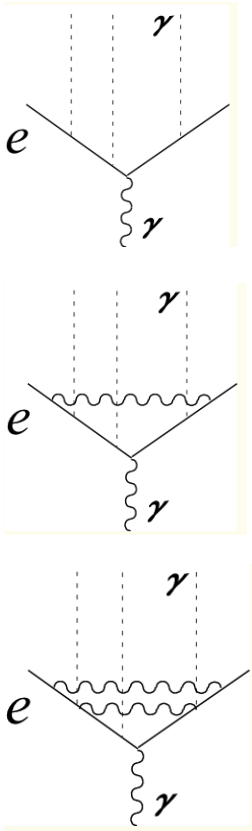


$$\delta E = e \int d^3x f^2 v^* [1 - i\gamma \Sigma \cdot \hat{r} \gamma^5] \gamma^5 \mathbf{A} \cdot \Sigma [1 + i\gamma \Sigma \cdot \hat{r} \gamma^5] v$$

$$g = 2 \cdot \frac{1}{3} \left(1 + 2\sqrt{1 - (Z\alpha)^2} \right) \simeq 2 \left(1 - \frac{(Z\alpha)^2}{3} \right)$$

Important: dependence on alpha; may be exploited to determine its value.
(Use ions with various Z)

Bound-electron g-2: theory

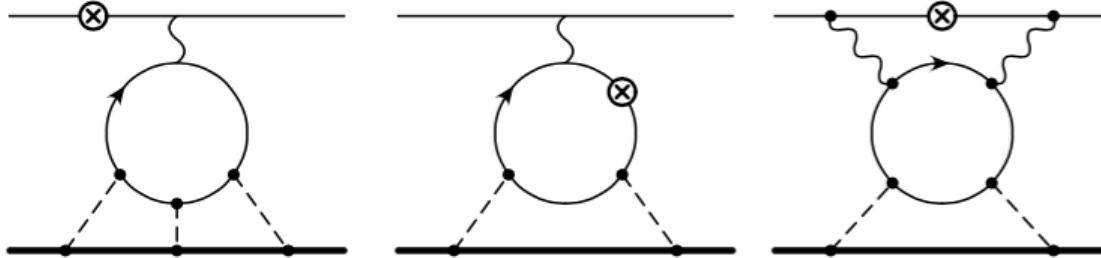


$$\begin{aligned}
 g = & 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots \\
 & + \frac{\alpha}{\pi} \left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \dots \right] \\
 & + \left(\frac{\alpha}{\pi} \right)^2 \left[-0.65.. \left(1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \dots \right]
 \end{aligned}$$

Pachucki,
AC
Jentschura,
Yerokhin

Next goal: $\alpha^2(Z\alpha)^5$ corrections to g

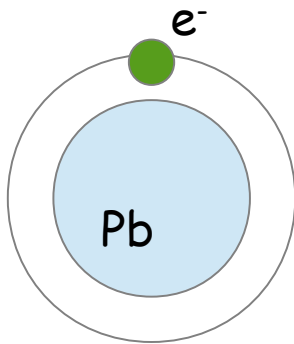
Examples:



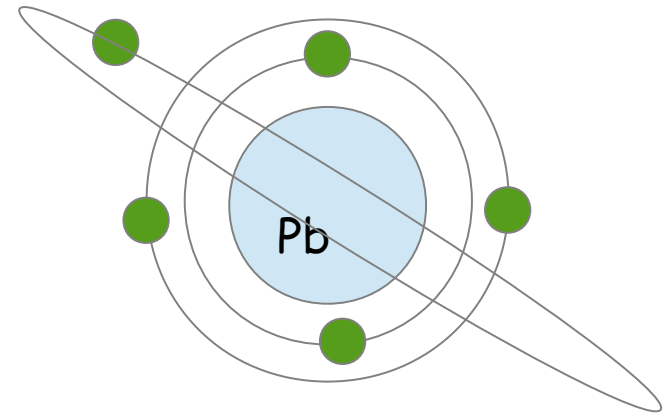
More than 300 contributions.

A new source of alpha: highly-charged ions

$$g \simeq 2 - \frac{2(Z\alpha)^2}{3} \longrightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2} \sqrt{(\delta g_{\text{exp}})^2 + (\delta g_{\text{th}})^2} \quad \text{large } Z \text{ favorable}$$



Hydrogen-like lead

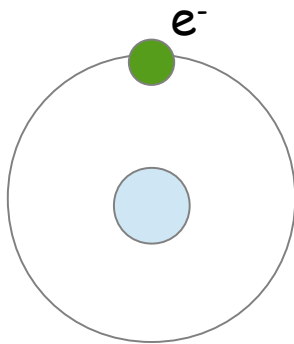


Boron-like lead

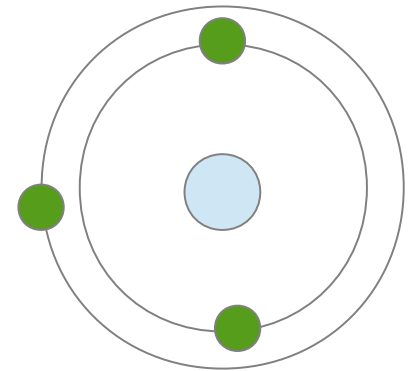
There is a combination of g -factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

New idea: medium-charged ions

$$g \simeq 2 - \frac{2(Z\alpha)^2}{3}$$



Hydrogen-like ion



Lithium-like ion

Combine H-like and Li-like to remove nuclear dependence;
then combine with a different nucleus, to remove free- g dependence!

Much interesting theoretical work remains to be done!

Summary

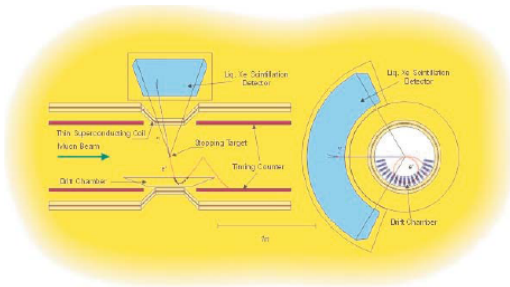
- * New era of muon studies just starting
- * Muon-electron conversion will probe very high mass scales
- * Binding modifies the muon decay and the electron g -factor
- * Theory of both effects: more fun than for free particles
- * Synergy with beautiful experiments: lepton-flavor violation, mass of the electron and, in future, the fine structure constant.
- * For g : $\alpha(Z\alpha)^5$ effects almost finished; $\alpha^2(Z\alpha)^5$ hopefully soon.
- * Opportunities for more theoretical improvement...

Lepton flavor violation: $\mu \rightarrow e\gamma$

New bound (MEG @ Paul Scherrer Institute)

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \cdot 10^{-13}$$

arXiv:1605.05081



This corresponds to the transition dipole moment

$$d_{\mu \rightarrow e} \lesssim 3.5 \cdot 10^{-27} e \cdot \text{cm}$$

Sensitive to
the heaviest
"new physics"

For comparison: electron EDM $d_e < 0.87 \cdot 10^{-28} e \cdot \text{cm}$

10.1126/science.1248213

muon g-2

$$d_\mu < 3 \cdot 10^{-22} e \cdot \text{cm}$$