# *Gravitational Waves: Theory*

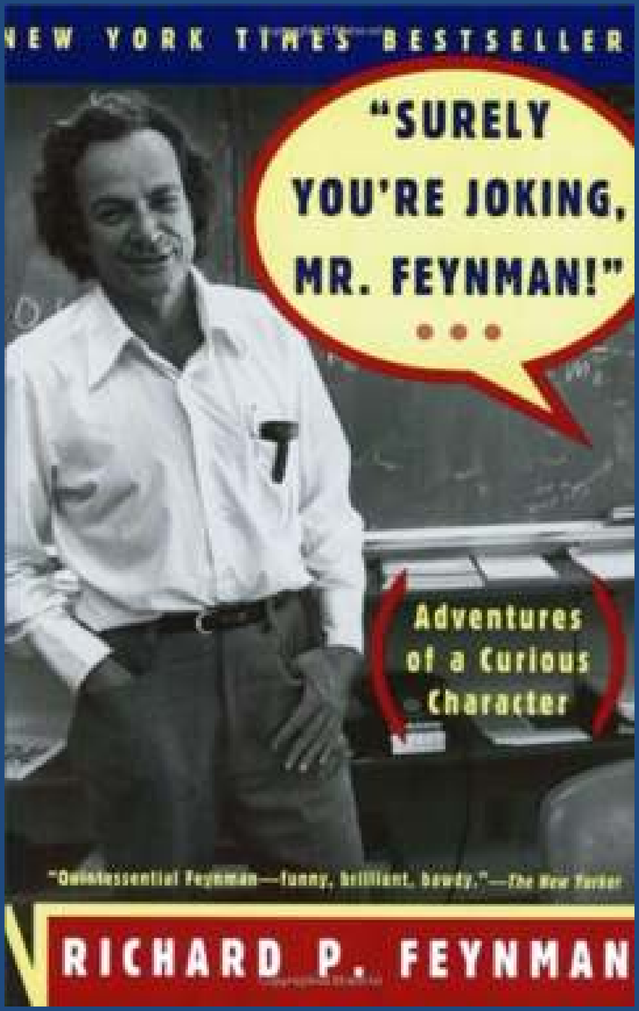
Luis Lehner

Perimeter Institute for Theoretical Physics

"I am not getting anything out of the meeting. I am learning nothing. Because there are no experiments, this field is not an active one, so few of the best men are doing work in it. The result is that there are hosts of dopes here (126) and it is not good for my blood pressure. Remind me not to come to any more gravity conferences!"

R. Feyman

(1962 Warsaw Conference)



# *Why go after gravitational waves?*

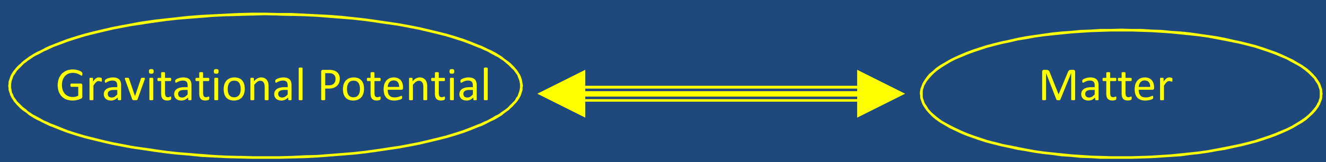
- Is GR consistent in systems with $M/R \sim 1$, $v/c \sim 1$?
- Population (and existence) of black holes, NSs.
  *masses, spins, location*
- Behavior of cold matter at nuclear densities
- Combine & complement astro-observations with EM and particle efforts
- Surprises!

# Gravity… < 1915

## Newtonian Gravity

- Absolute reference frame, preferred time

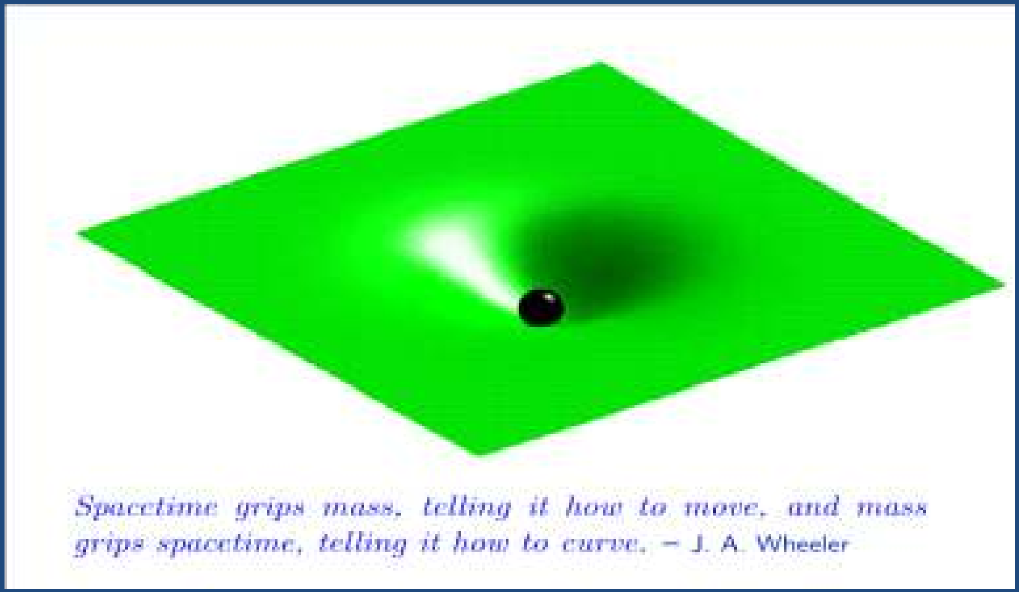
$$\nabla^2 \Phi = 4\pi\rho$$




- 1 Elliptic equation to solve (with well defined rhs)
- Potential $\Phi$ defined on an Euclidean manifold
  - Newtonian spacetime ($E^3$, $\Phi$) [Distances: $ds^2=dx^2+dy^2$]
- ‘Signals’ propagate at *infinite* speed
- Trajectories determined by forces
- Gravity is a force field

# *Einstein's new vision*

- Trajectories 'straightest paths' on curved manifold
- Matter/Energy curves spacetime and that in turn affects trajectories in it. For example:
  - Precession of Mercury's orbit
  - Deflection of light around the Sun




Gravity is a *manifestation* of the geometry

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

*This is a mess! &*
*K ~ $10^{-44}$ 1/N*

# *Black hole basics*

- Stationary BHs are uniquely described by 2 parameters: mass (M) and angular momentum parameter (a).

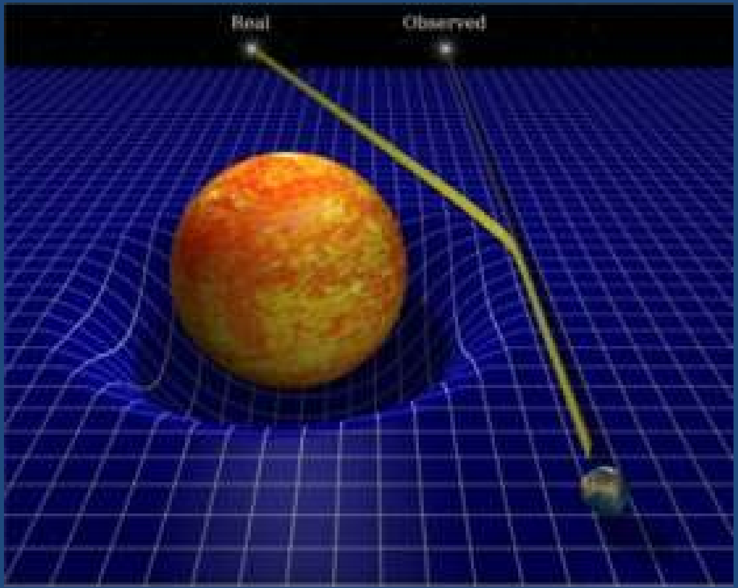
- 1-way membrane at R = 2M (a=0), R=M (a/M=1)

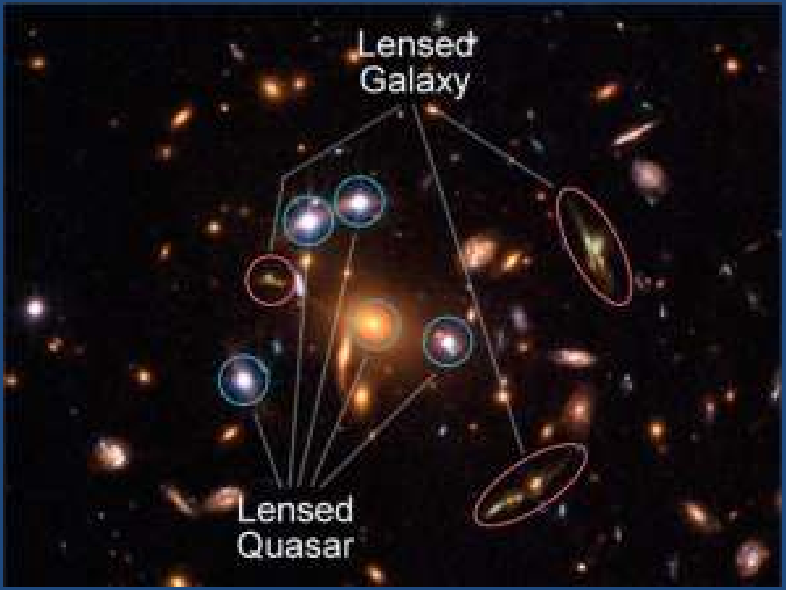
- No stable circular orbits if $r < R_{ISCO}$ (=[9M,6M,M] for a/M =[-1,0,1]

- Max energy extractable from a rotating BH: 29%M

# *Exploiting gravity to learn about our universe*

- An early prediction of GR : curved spacetime bending of light: ‘gravitational lens’



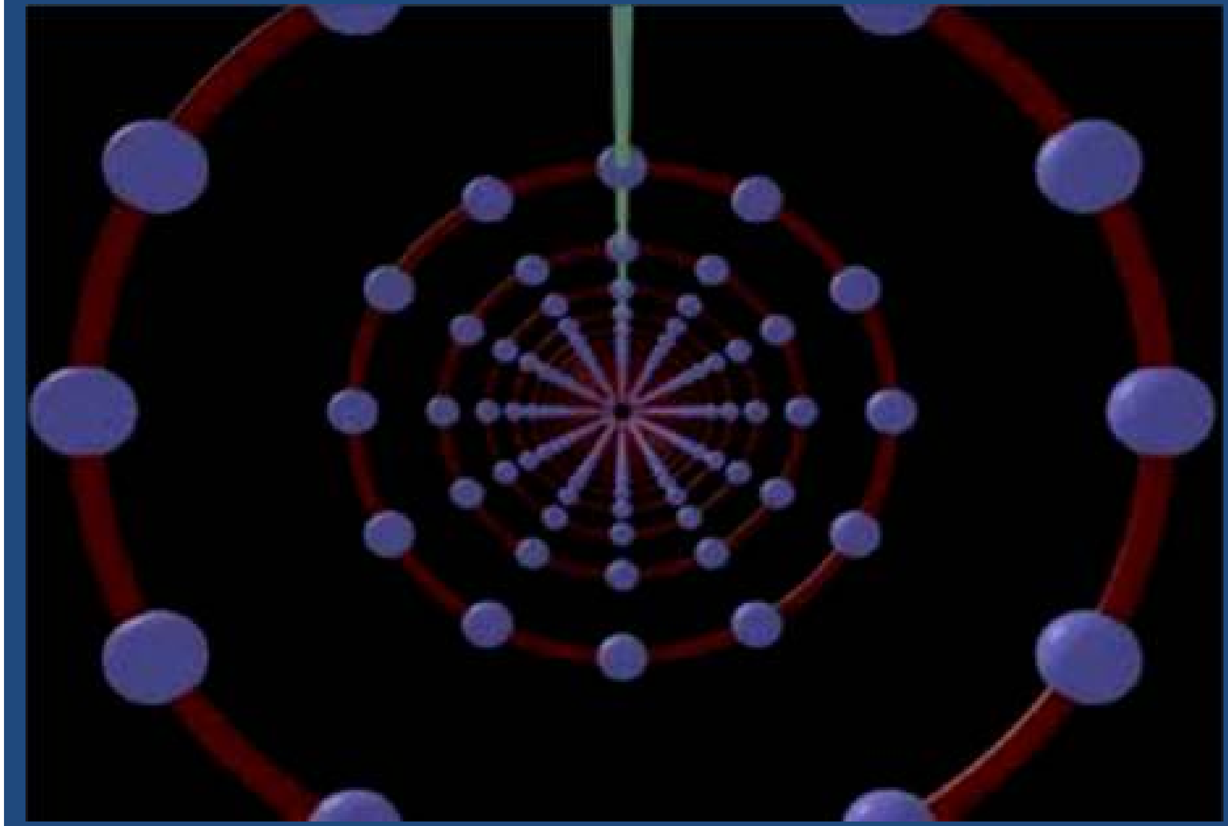




- E.g. ‘Dark Matter’ and exoplanets are inferred through lensing observations.

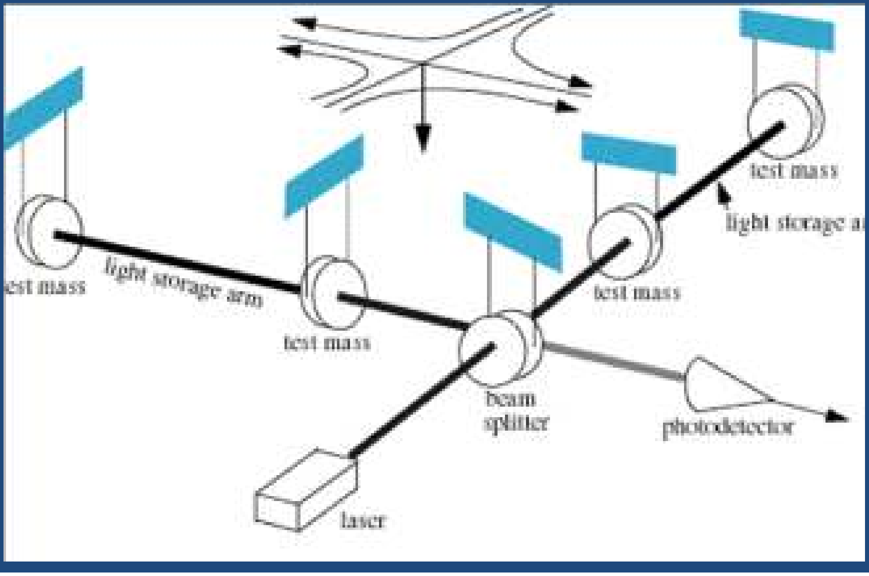
# *General Relativity, cont.*

- $G_{ab}$ is the Einstein tensor constructed out of the *metric of the spacetime.* $ds^2 = g_{ab}\, dx^a\, dx^b$
- For weakly curved spacetimes g = flat + h
- $G_{ab}$ Box(h) = -16π T (with T: stress energy tensor)
- Far from 'source' (T=0) solutions are travelling waves, which are transversal to propagating direction (only 2 polarization modes [massless graviton]

- Generation? Assume an expansion on (v/c) & 1/r and arrive at: $h \sim G/c^4\, Q_{,tt}$ with Q the source quadrupole:
- need 'accelerated' quadrupoles!
- (mass & momentum are conserved in GR)



[sky & telescope]



# *Source estimation*

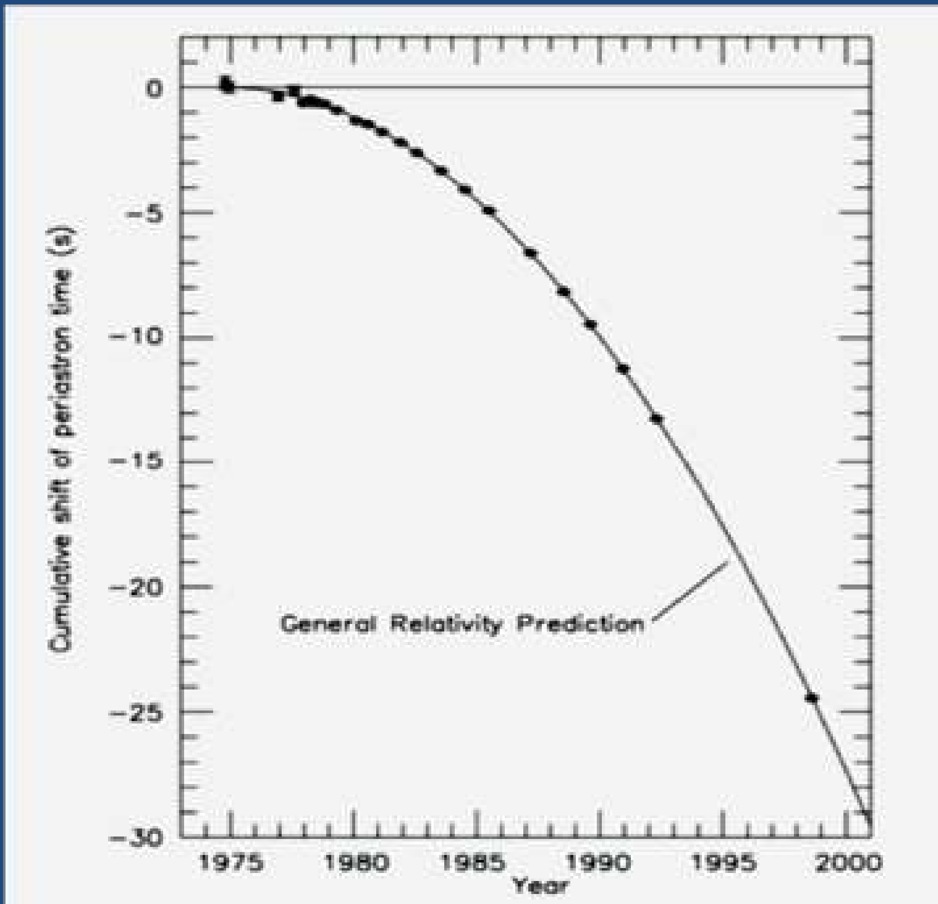
- Characteristic freqn of a density distribution:
  - $f \sim (G\,\rho)^{1/2}$ $\quad Q_{,tt} \sim f^2\, M\, R^2$
  - Thus, $h \sim (GM/Rc^2)\,(GM/rc^2)$
  - (i.e. h ~ grav potential from source x grav pot at observer)

- Luminosity: $L \sim (c^5/G)\,(G\,M/Rc^2)$
  - (ie. $10^{53}$ W x compactness of source - which is < 1 -)

- Example: equal mass binary
  - $h \sim 10^{-21}\,(15\text{Mpc}/r)\,(M/2.8M_O)^2\,(90\text{km}/R)$
  - $f \sim (M/2.8M_O)^{1/2}\,(90\text{km}/R)^{3/2}\,100$ Hz
  - $t_M/t_H \sim (M/M_o)\,(R/10^6\,R_s)^4$

# *Are they for real?*

– Taylor & Hulse measured variation in period of pulsar (PSR1913+16) in 1974. Excellent agreement with the prediction of G.R. ( Nobel prize in 1993).




$$\dot{P}_b^{\rm GR} = -\frac{192\pi G^{5/3}}{5c^5}\left(\frac{P_b}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) \times (1-e^2)^{-7/2}\, m_1 m_2 (m_1+m_2)^{-1/3} = -1.699451(8)\times 10^{-12}\left[\frac{m_1 m_2 (m_1+m_2)^{-1/3}}{M_\odot^{5/3}}\right]. \qquad (3)$$

[ ~ $(M/R)^5$ ]

$$\left\langle\frac{de}{dt}\right\rangle = -\frac{304}{15}e\frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^4 (1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right).$$

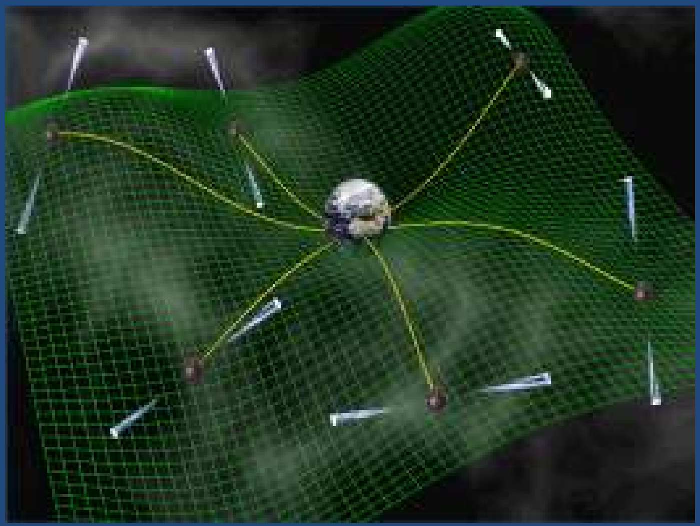
# Let's pause…some observations

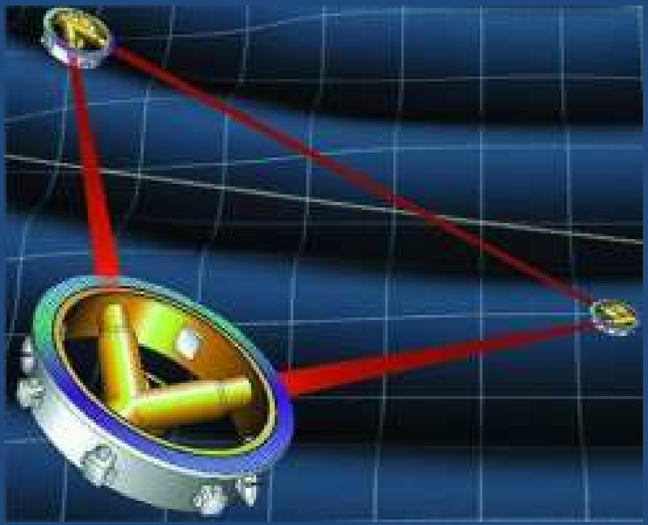
- Strain decays as 1/r, increases with mass & 1/separation
- Frequency decreases as 1/M [smallest R tied to M!]
- For m2/m1 = q ; h ~ $q/(1+q)^2\ M^2$

## How to detect them?

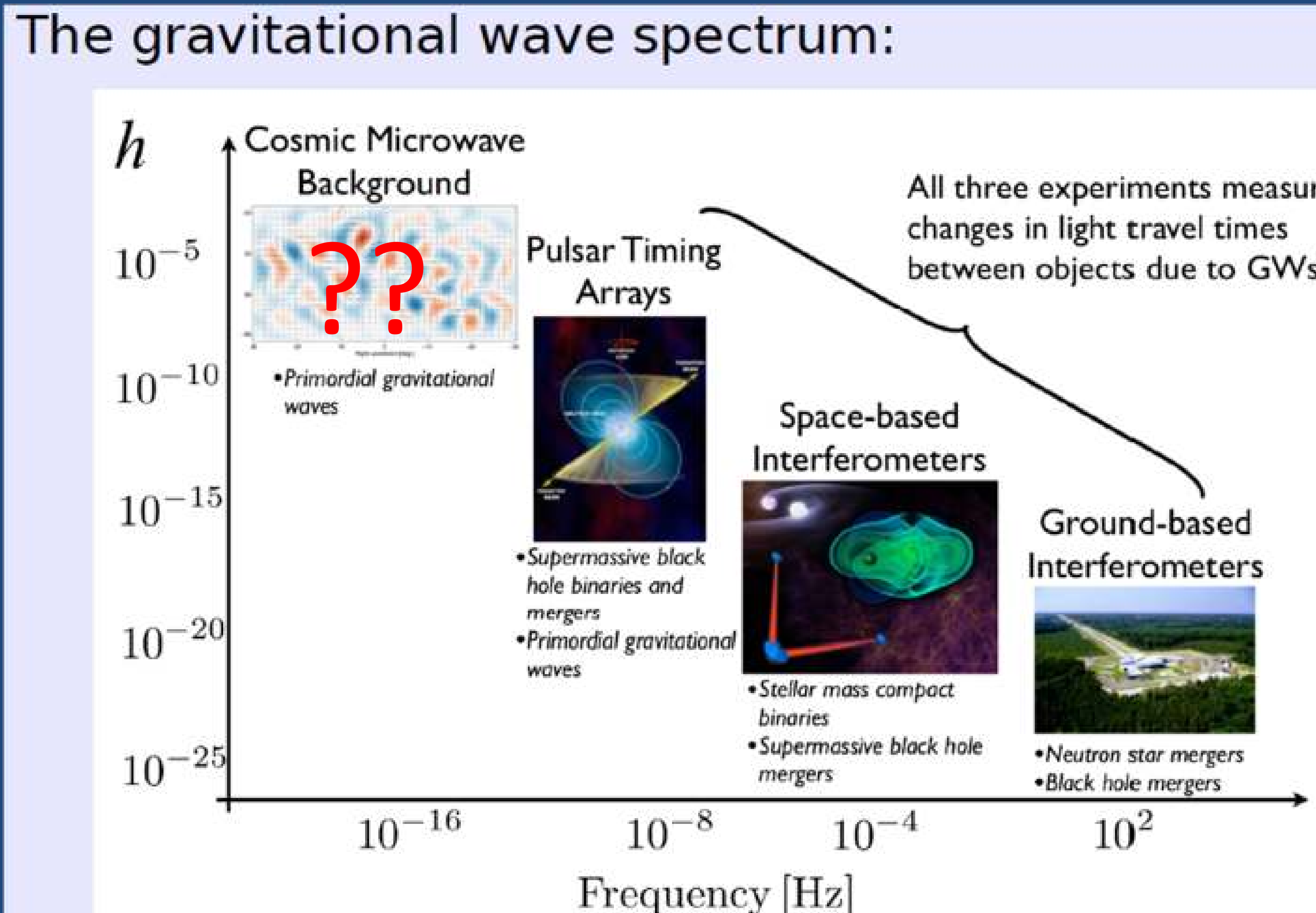
- – First, built an awesome instrument! (actually first find someone to pay for it case made based on NS-NS) *[Marka's talk]*
- – Second, prepare to dig signals from within the noise
- – Third, understand how to get the most science through multimessenger astronomy *[Bartos' talk]*

# Opening gravity wave 'bands'



- And others in concept stages…

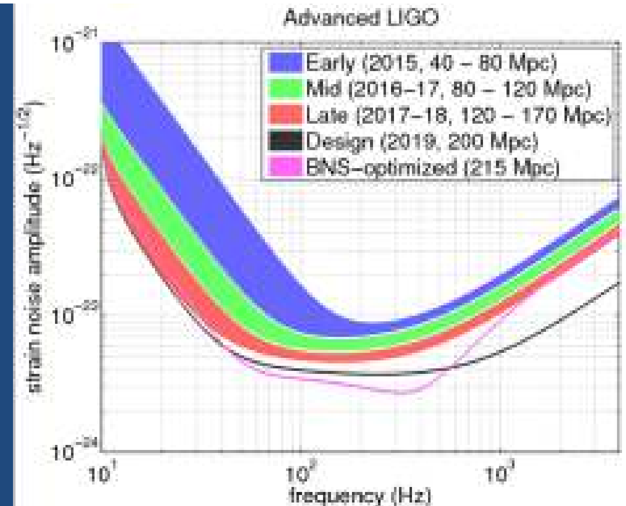



[Image: Nanograv]

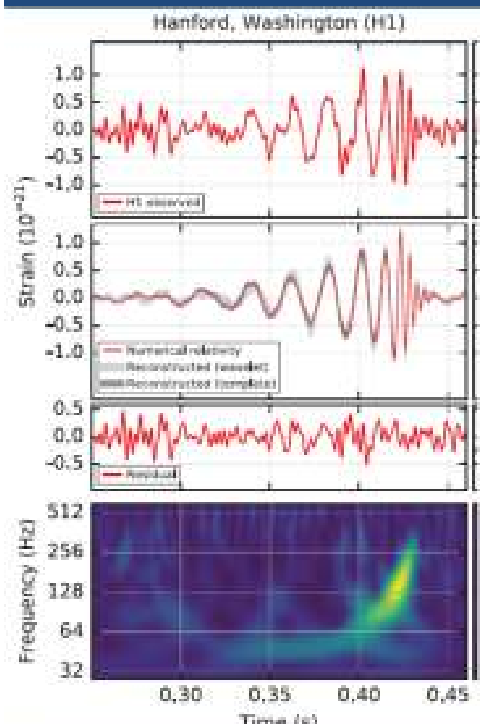
# *Detection strategies*

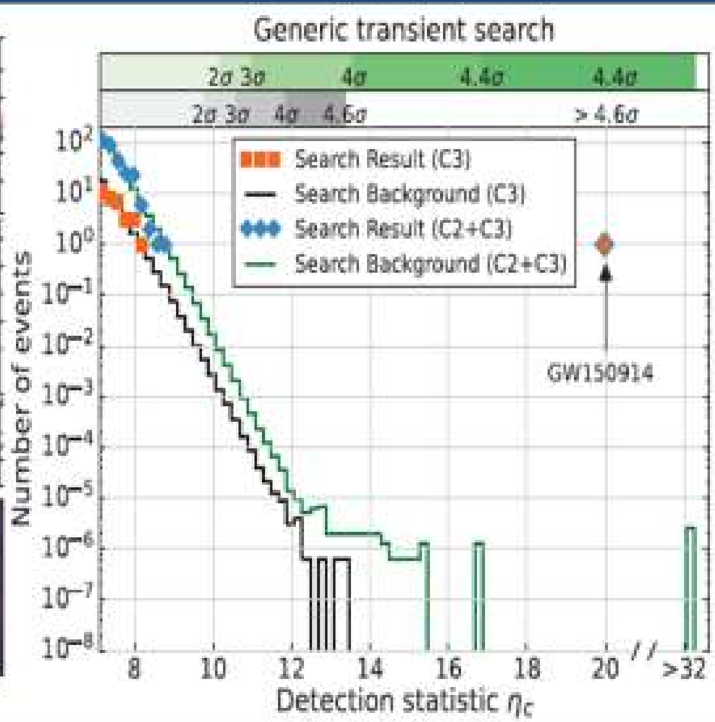
- Matched filtering

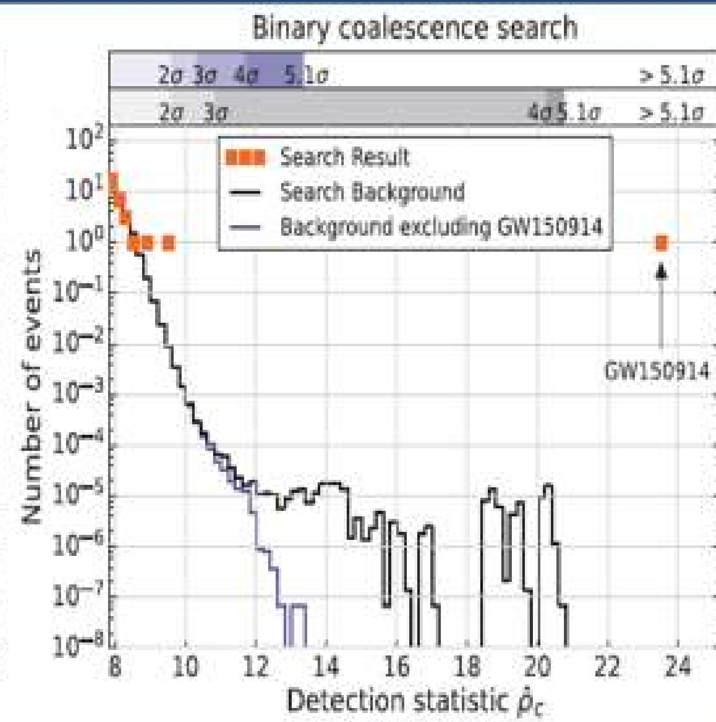
$$< d|h > = \int \frac{dh^* + c.c.}{Noise} df$$



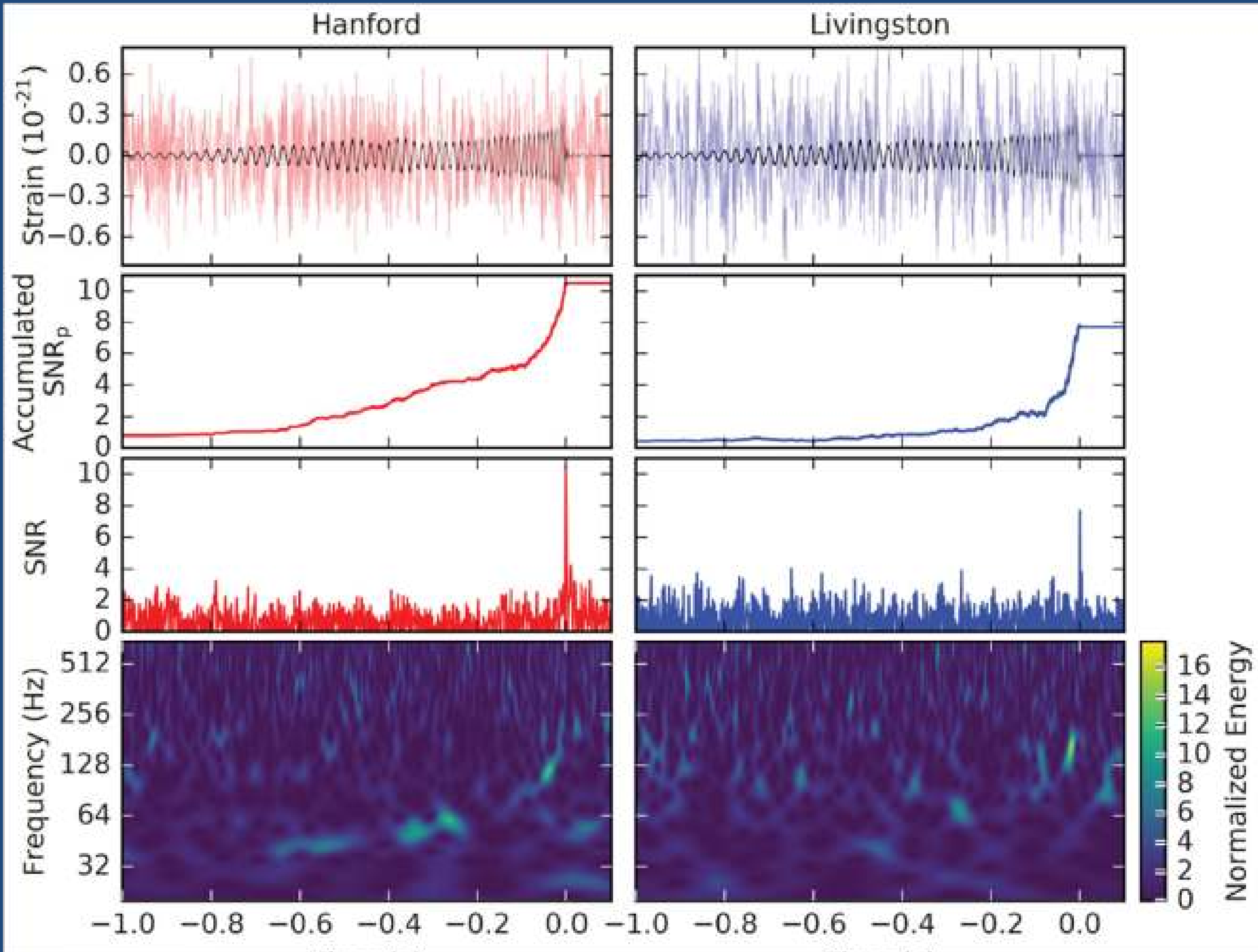

- Coincidence+wavelets decomposition



# GW151226



Hanford
Livingston
Strain ($10^{-21}$)
0.6
0.3
0.0
−0.3
−0.6
Accumulated SNR$_p$
10
8
6
4
2
0
SNR
Frequency (Hz)
512
256
128
64
32
Normalized Energy
16
14
12
10
8
6
4
2
0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0

# *Anatomy of a binary merger*

*4 stages: newtonian, inspiral, plunge/merger, after-merger*

*Newtonian*: $t_M < t_H$ : other physics is needed to induce merger: dynamical friction, n-body encounters, etc.

*Inspiral:* energy/ang. mom. Loss through GWs is the dominant mechanism.

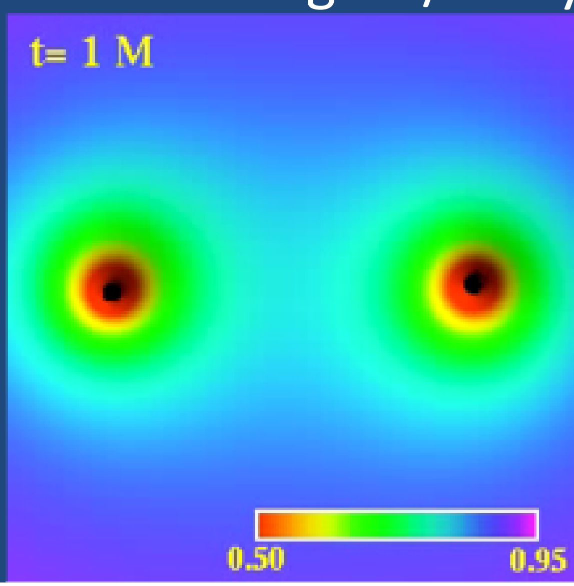
*Post-Newtonian techniques, or Effective field theory can be called for obtaining analytical expressions for the orbit/GWs.*

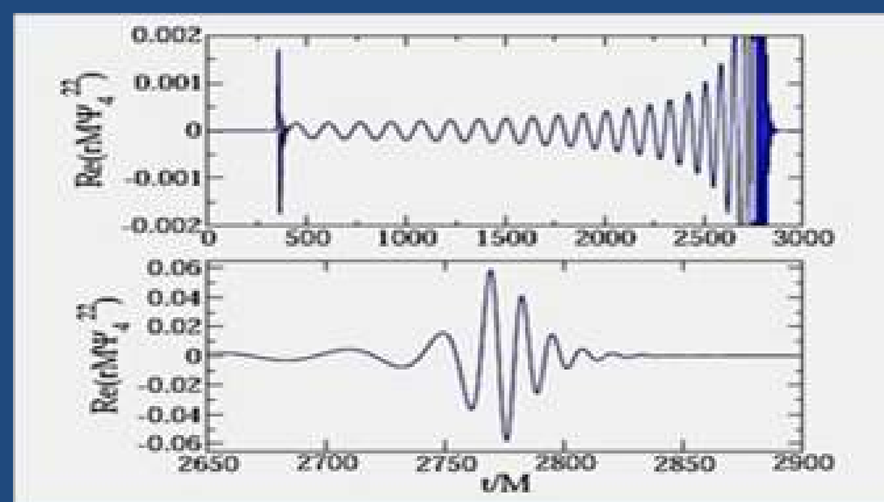
*Rely on: separation of scales! (v/c), M/R, etc*

$a_i \sim Newt + \{SpinOrbit\} + .... + RADN\ (M/R)^5\ + .... + tidal\_effects\ (M/R)^{10}$

*Eccentricity is most often removed before LIGO freqns quasi-circular trajectories by the time signal enters LIGO band.*

- *Merger/plunge:*
  - 2 black holes merge into one *if cosmic censorship holds.*
  - 2 NS will form another one which may collapse to a BH
  - BH-NS. The BH will disrupt or swallow the NS depending on typical radii involved
  - Numerical Relativity required, full Einstein equations are to be solved as there is no intrinsically perturbative scale (a priori!)
  - In practice: short duration (few cycles), and different phenomenological/analytical approximants can be devised.



- For BH-BH and BH-NS, the final object so far always settles into a rotating black hole (cosmic censorship stands its ground).
  - During the transition, the ''BH looses its hair''.
  - Linearized analysis wrt to such a BH indicates the decay is described by waveforms of the form h ~ exp(i $\omega_{lmn}$ t) f(r) $Y_{lm}$
  - With $\omega_{lmn}$ *being complex exponential/oscillatory decay*

  - Fundamental mode: $\omega \sim 32$ kHz ($M_o$/M) (1-0.63 $(1\text{-}j)^{0.3}$ )
  - Associated decay rate: $\tau \sim 20\ \mu$s (M/$M_o$) g(j)

  - Measurement of 2 modes strong constraint to GR [and doable with advanced LIGO [Yang etal '17] ]

# *Directional bias….*

## "Antenna pattern"

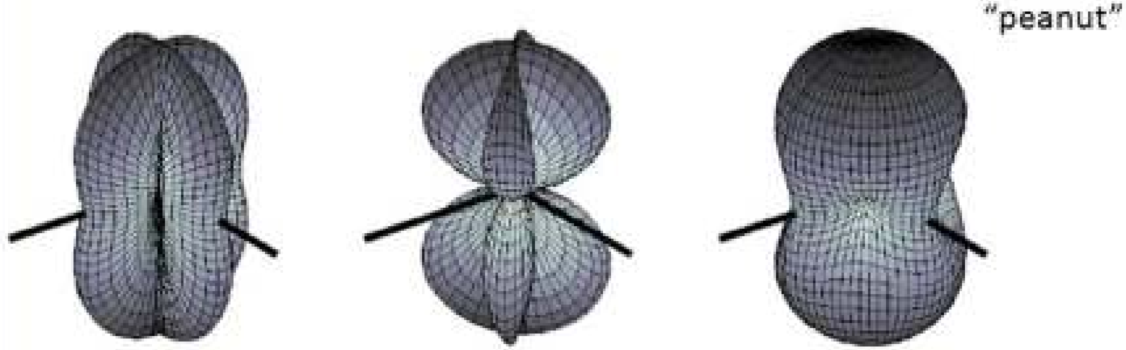
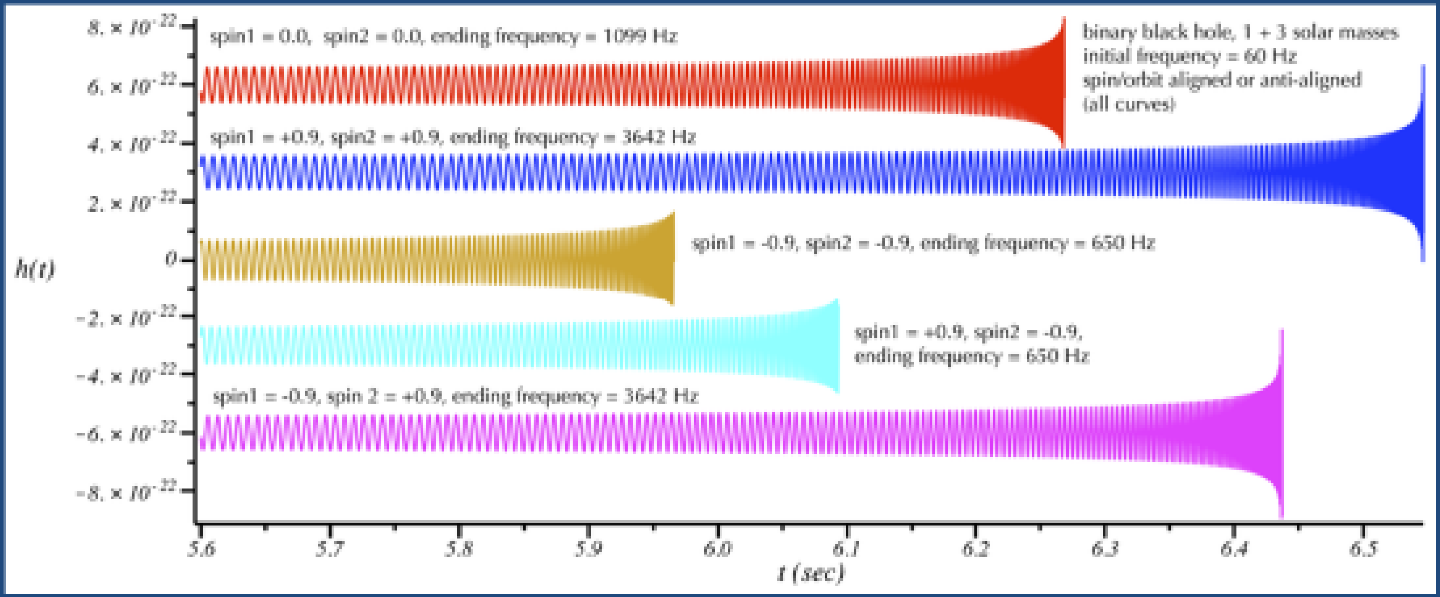


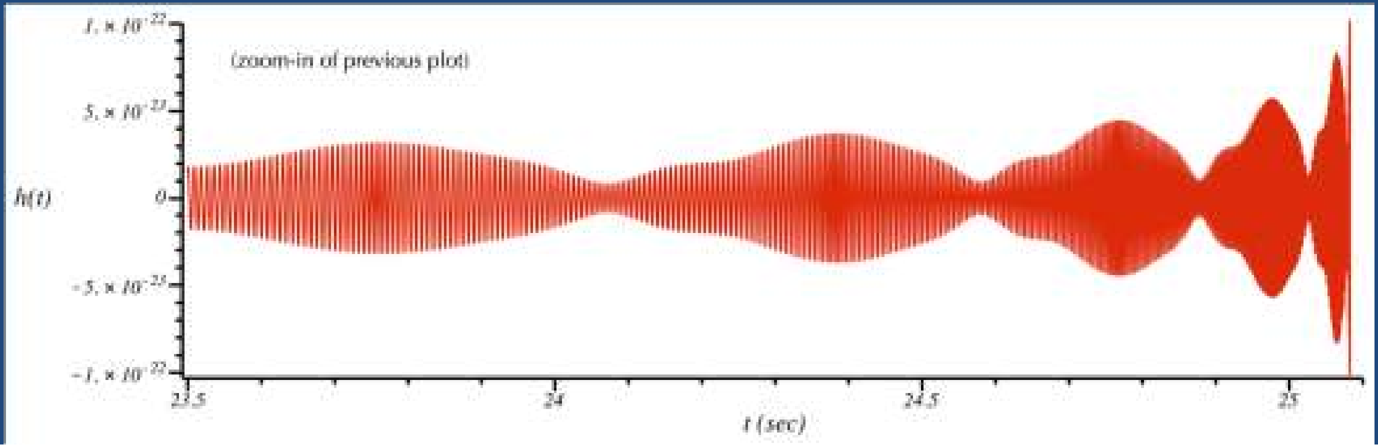

**Figure 5.** Antenna response pattern for a LIGO gravitational wave detector, in the long-wavelength approximation. The interferometer beamsplitter is located at the center of each pattern, and the thick black lines indicate the orientation of the interferometer arms. The distance from a point of the plot surface to the center of the pattern is a measure of the gravitational wave sensitivity in this direction. The pattern on the left is for + polarization, the middle pattern is for × polarization, and the right-most one is for unpolarized waves.

- Dependence on: 7 parameters m1,m2, s1, s2 (with a 'free scale' total mass!). For m1 ~ m2 much of the final spin is defined by orbital angular momentum contribution
- Also, we observe *luminosity distance*

# *Individual spins*

- Aligned (+,-) with orbital ang. Momentum higher/lower final spin
- Misaligned -> waveform modulation (spin-orbit and spin-spin coupling) but strong dependence on observation direction



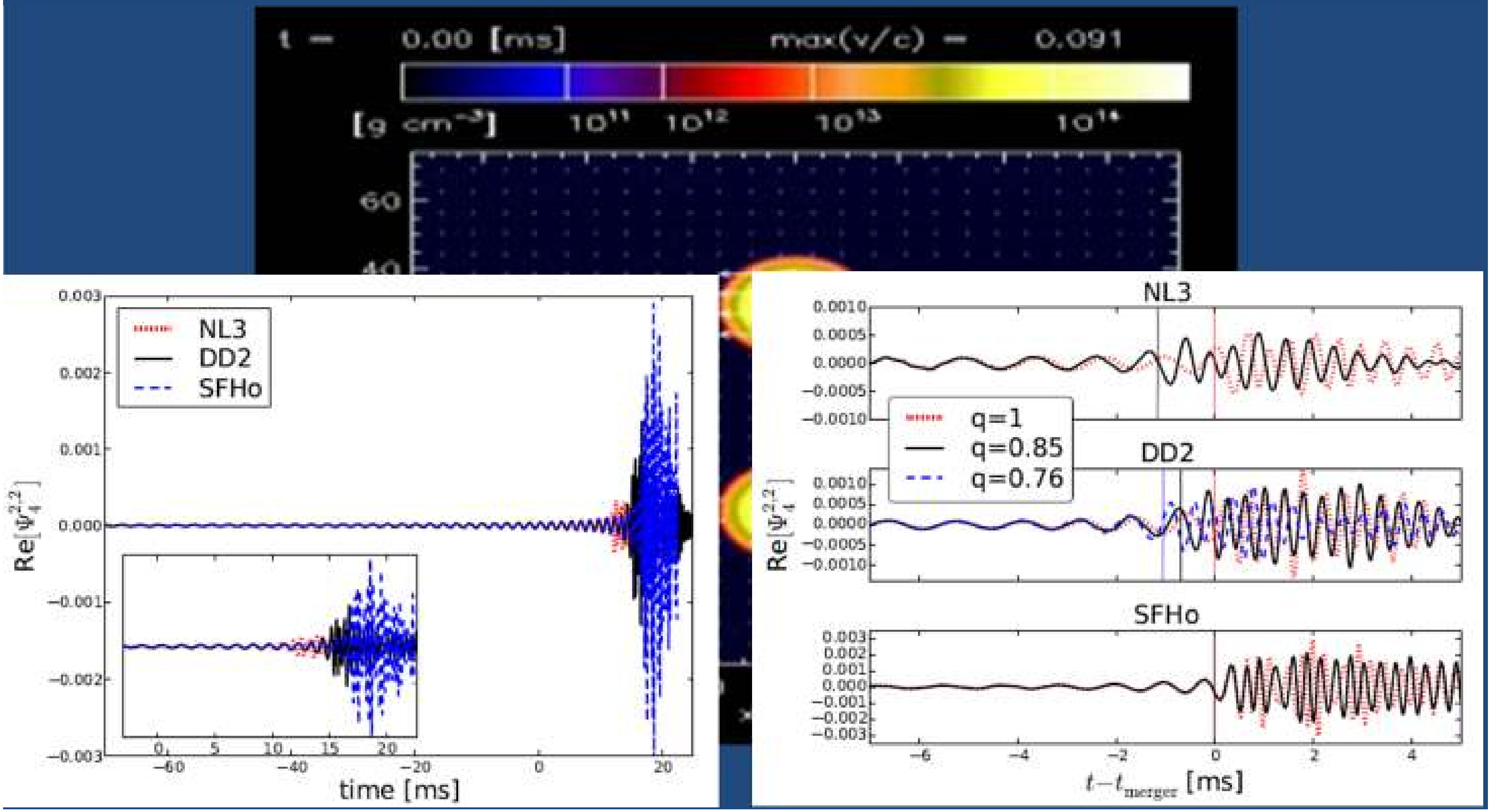

[60$^o$ misalinged)

# BH-BH Main outcomes/surprises

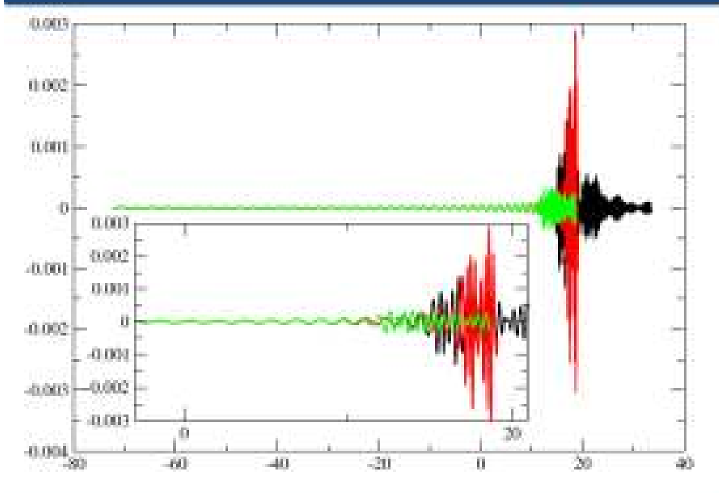
- @ largest strain! two 10 $M_O$ BHs at 10Mpc $\Delta L/L \sim 5\ 10^{-17}$
- Peak luminosity only 1/100$^{th}$ of Planck Lum of $10^{59}$ erg/s
- Very efficient mass-to-energy conversion: ~3 – 12 % $M_{total}$
- Very large recoils of final object possible ~ several 1000s km/s. large enough to induce:
  - Galaxies without BHs
  - Offset AGNs
  - Off-centered TDEs….
  - (may be nature doesn't like these configurations!)

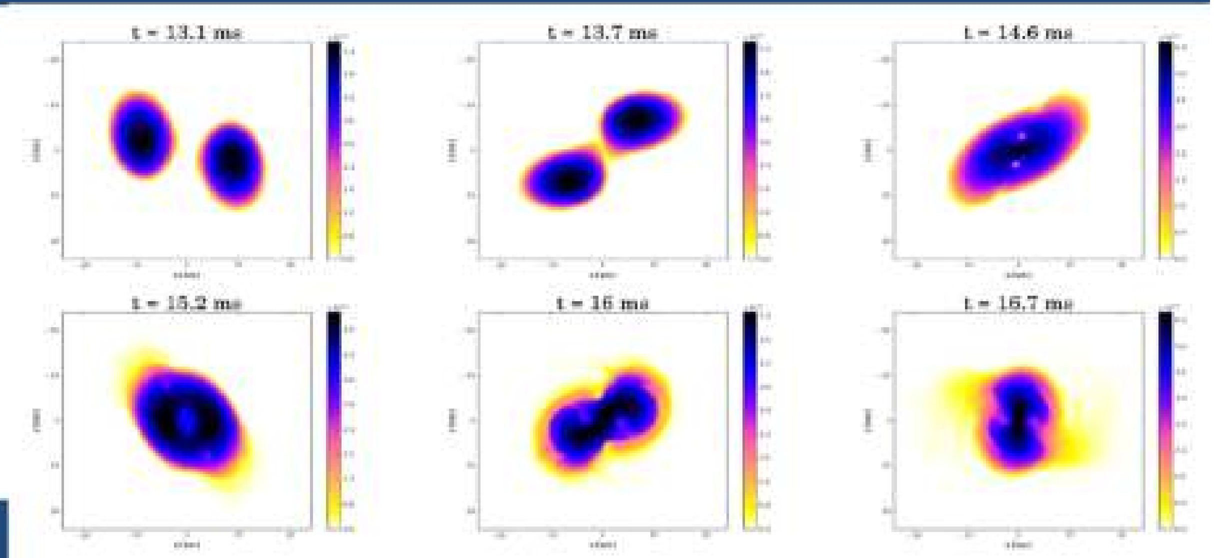
# *Non-vacuum binaries*

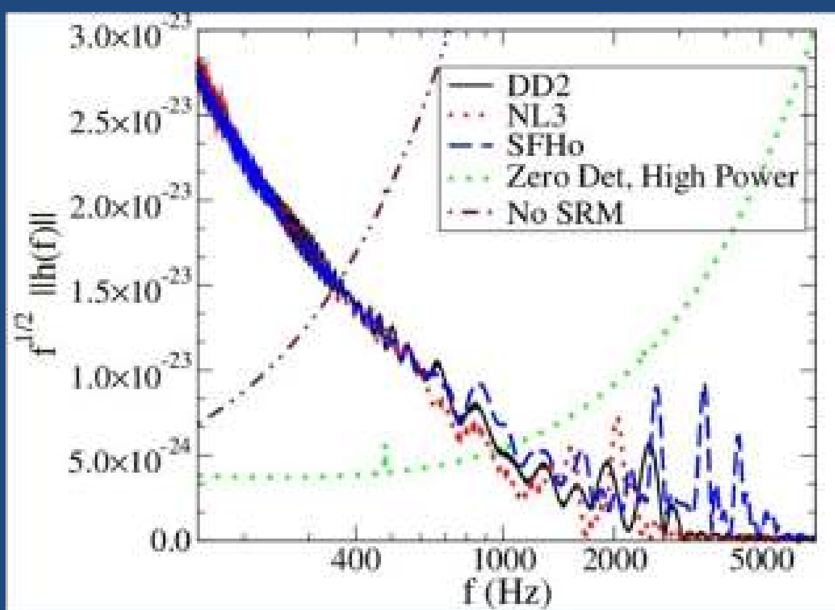
- No-rescaling of mass possible, though constrained masses
- Recall tidal effects F ~ $(R_s/M)^5\ (M/R)^{10}$



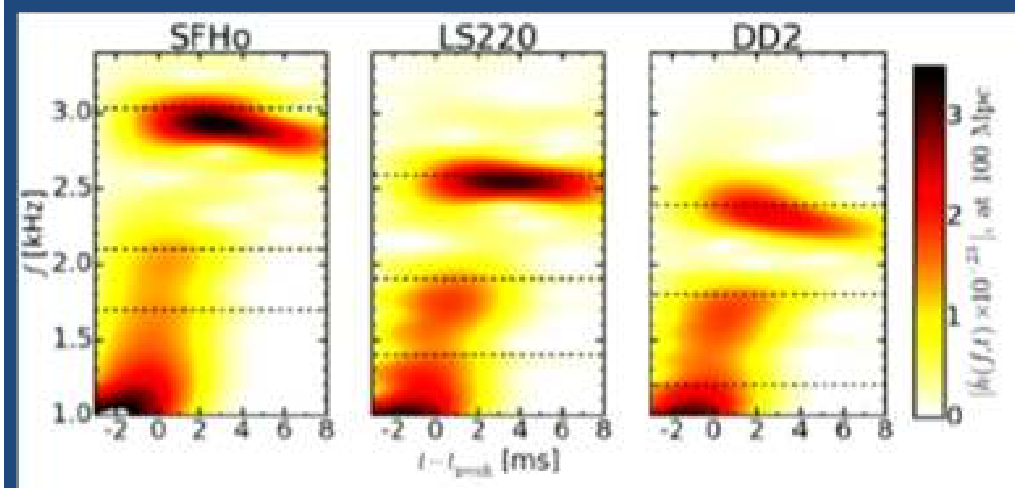
# *Cold matter at high densities, EoS?…*






[Palenzuela,LL,Liebling,Neilsen,Caballero '15]




[Foucart etal '15]

*Further info will be available*

- Stars collide at (v/c) ~ 0.3. Strain 'almost' as high as BH-BH (especially for 'soft' EoS)
- Large available mechanical energy! possible engine of sGRBs
- Ejected material can undergo r-processes and radiate in particular bands tied to EoS [NS-NS/BH-NS can explain high atomic mass numbers!]
- Pre-merger can induce magnetosphere interactions and have the system looks like a pulsar on steroids….
- Etc.

- All this is key, as there could be degeneracies between EoS & extensions to General Relativity [e.g. those with energy loss d.o.f]

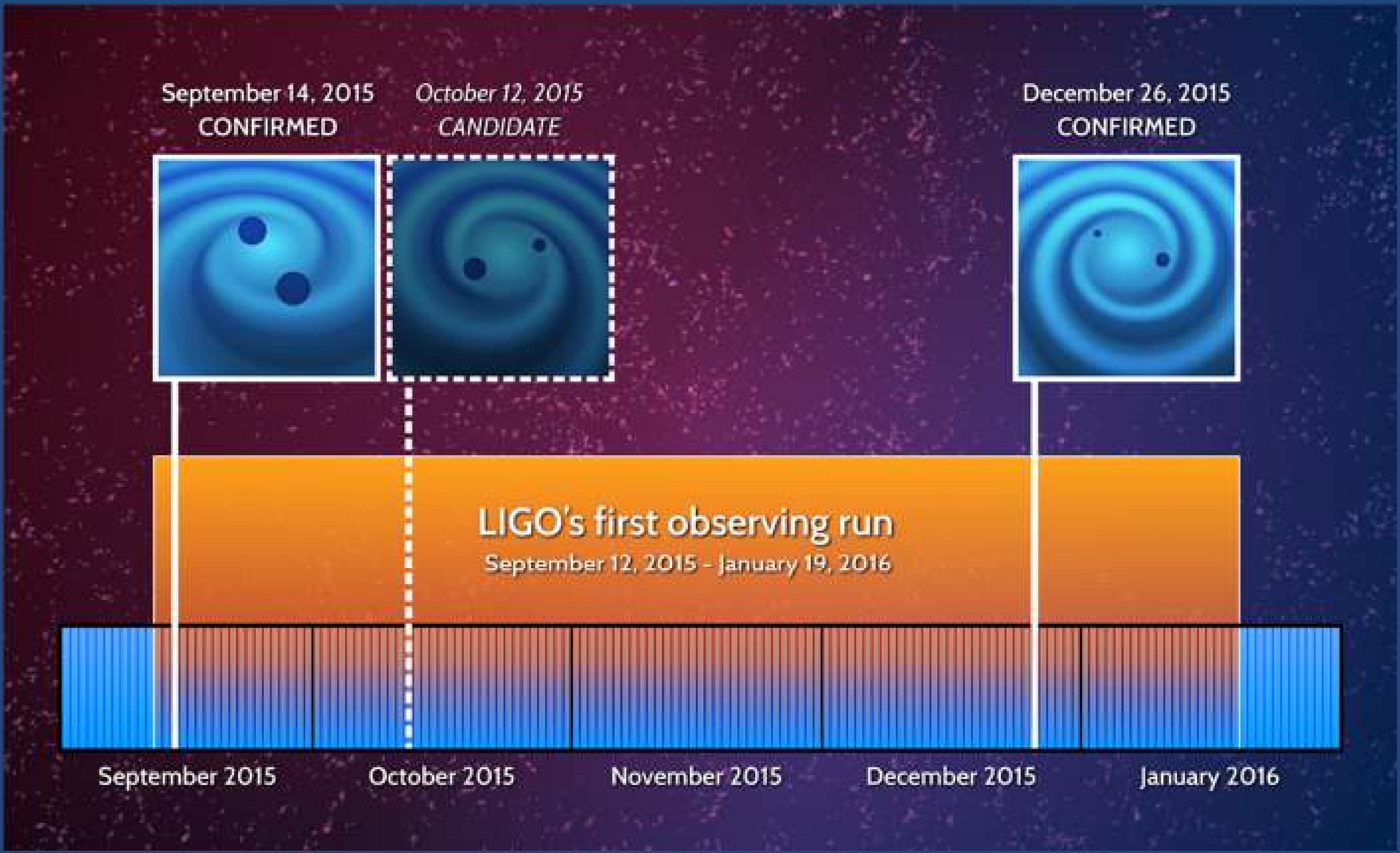
# *Open fronts*

- Template construction for BH-BH ‘under control’ but too costly for some configurations. Expediting data analysis is a high priority: Machine learning, Singular Value Decomposition, Reduced Order Methods, and related all being scrutinized/implemented

- BH-NS & NS-NS larger parameter space of physics ingredients, slower codes, more variation in possible outcomes.  Still in exploratory mode: *the good*: LIGO won’t need much more for *detection.*
  *the bad:* parameter and physics extraction is a different story.

- What if not GR? Phenomenological models (ppE) & thorough studies in (some) extensions are being produced. Strategy to search for deviations continuously being reassessed and improved. Further, EoS effects can be degenerate with GR modifications

# Putting all together




[ligo.org]

# *Parameters inferred*

| Event | Prob | m1 ($M_O$) | m2 ($M_O$) | $\chi_{eff}$ | $D_L$ (Mpc) | Mrad ($M_O$) |
|---|---|---|---|---|---|---|
| GW150914 | $> 5.1\sigma$ | 36 (5,-4) | 29 (4,-4) | -0.06 (0.17, -0.18) | 410 (160,-180) | 3 |
| LVT151012 | $2.1\sigma$ | 23 (18,-5) | 13 (4,-5) | 0.0 (0.3,-0.2) | 1100 (500,-500) | 2 |
| GW151226 | $> 5\sigma$ | 14.2 (8.3,-3.7) | 7.5 (2.3,-2.3) | 0.2 (>) | 440 (180,-190) | 1 |

Some implications & qns:

- Rate: ~50 (+111,-40) $Gpc^{-3}\ yr^{-1}$
- Why these distances? Not surprising (volume!)
- No clear precession?   preference of face-on, significantly less relevant in that direction
- DM candidate? Still few to make an argument [peak in distribution?]
- Large masses in GW150914 not 'first bet'   population implication?
- Spins? Why are they consistent with very low values in individual BHs (assuming alignment takes place)

# *Final thoughts*

- We are in a new era. Still to be decided if we have
  - More than solid new tool for astrophysics, a way to obtain guidance for what replaces GR. Ripe time to think new ideas and explore new prospects
- Detections will spur new developments, remove (some) serendipity from EM observations & hopefully bring surprises.
- It's taken lots of efforts through ~ 4 decades to get to this point. Now what? To think what else can GWs and the technology to get us here can do for you!