Axions, supersymmetry, and all that: Emergent phenomena in condensed matter physics

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University of Alberta

Lake Louise Winter Institute

February 22, 2018

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What is more important, particle physics or condensed matter?

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“*Die Festkörperphysik ist eine Schmutzphysik*”

(*Solid state physics is dirt physics*)

“*One shouldn’t wallow in dirt*” (letter to Peierls)

“*squalid-state physics*”

Why study "dirt physics"?

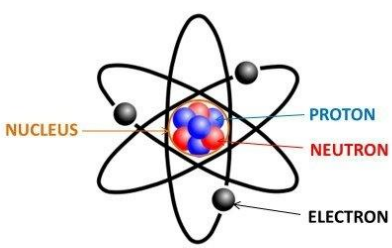
"*The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles*"

(P. W. Anderson, "More is Different", Science '72)

Collective phenomena

"*The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles*"

(P. W. Anderson, "More is Different", Science '72)



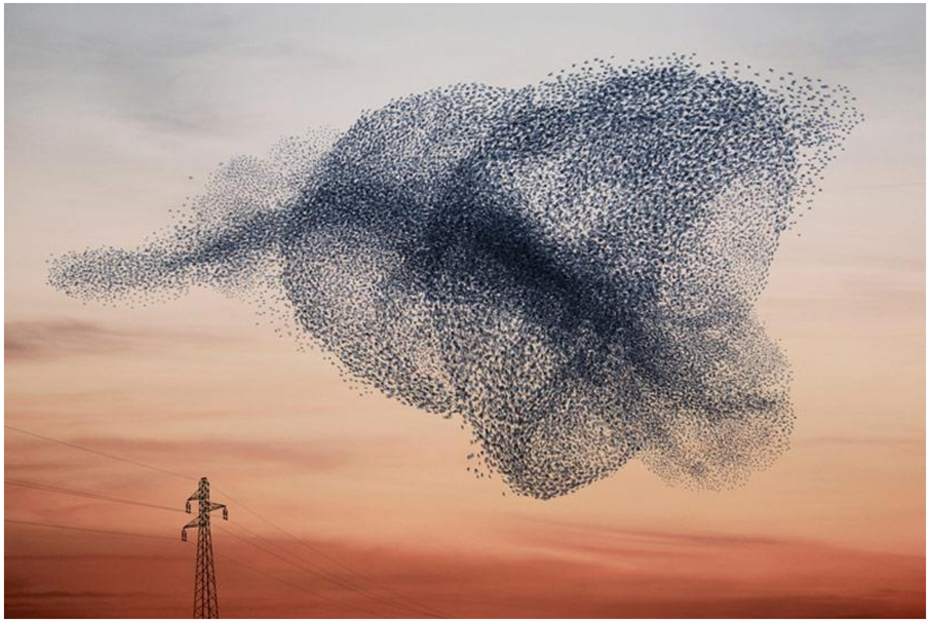
charge, spin, mass

$$N \gg 1$$

rigidity, resistance, specific heat…

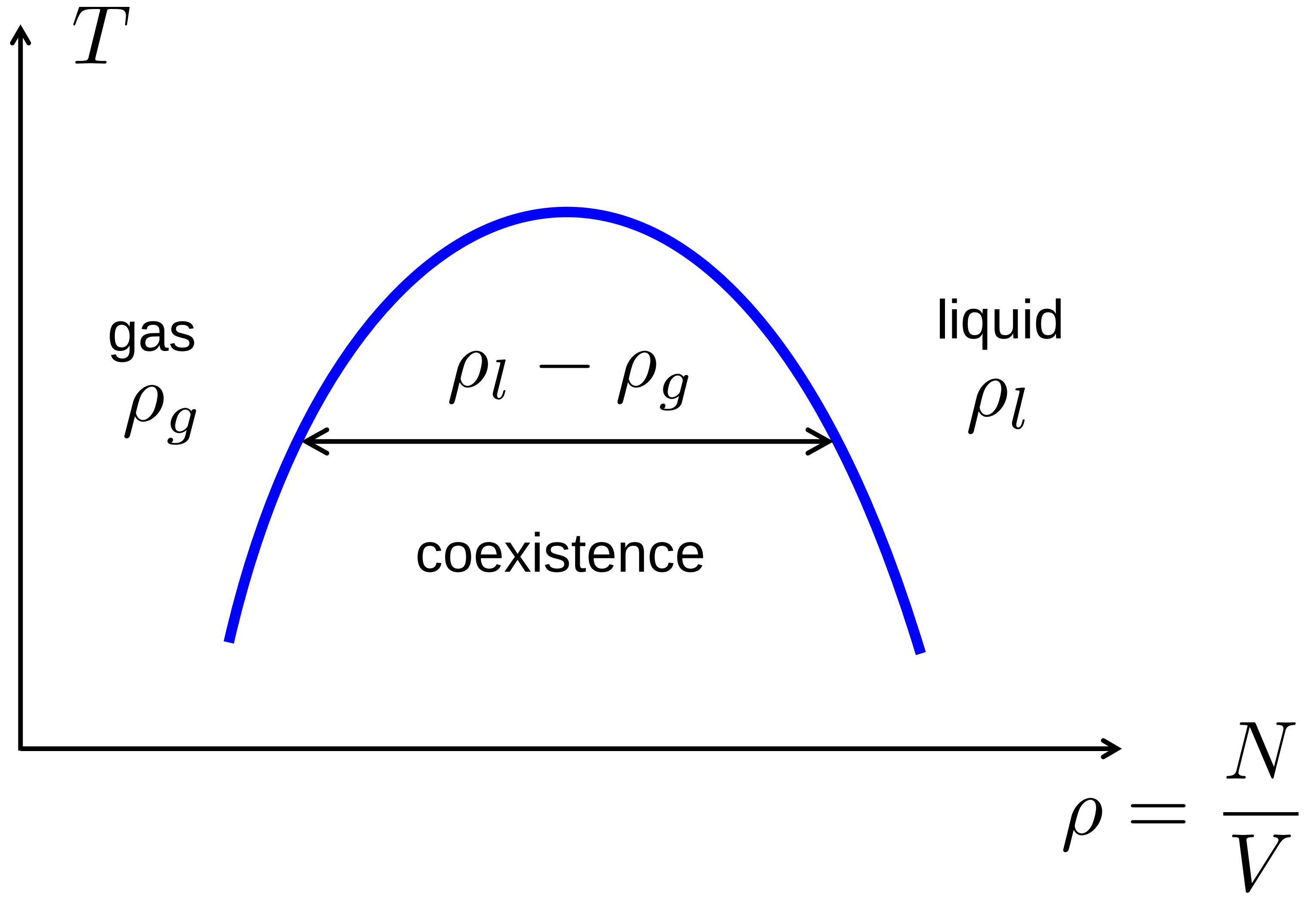
Emergence and universality

- Collective behavior can be qualitatively distinct from microscopic behavior (**emergence**)

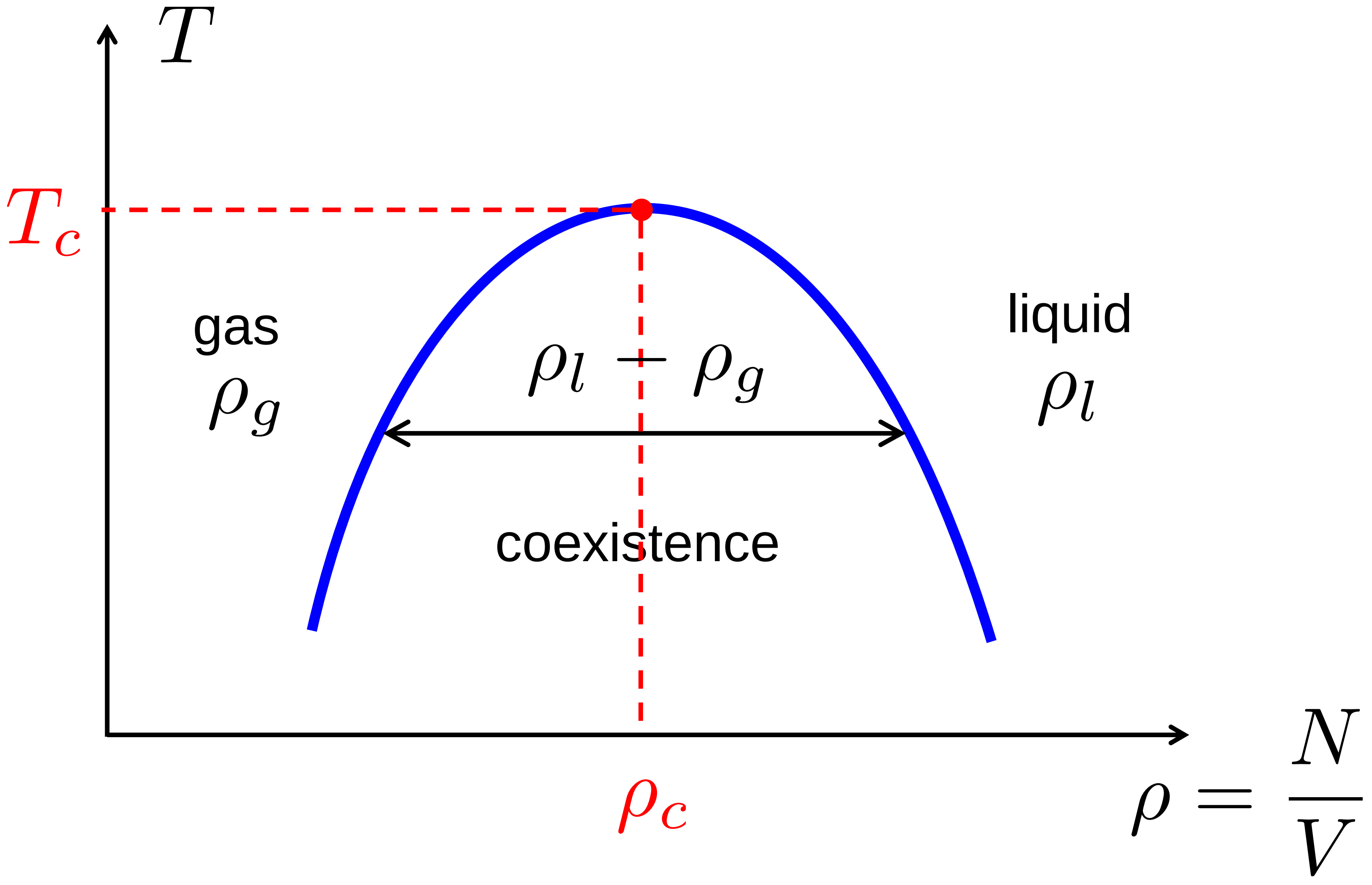


- The laws of physics that describe collective phenomena on long length/time scales are **universal**

Liquid-gas critical point

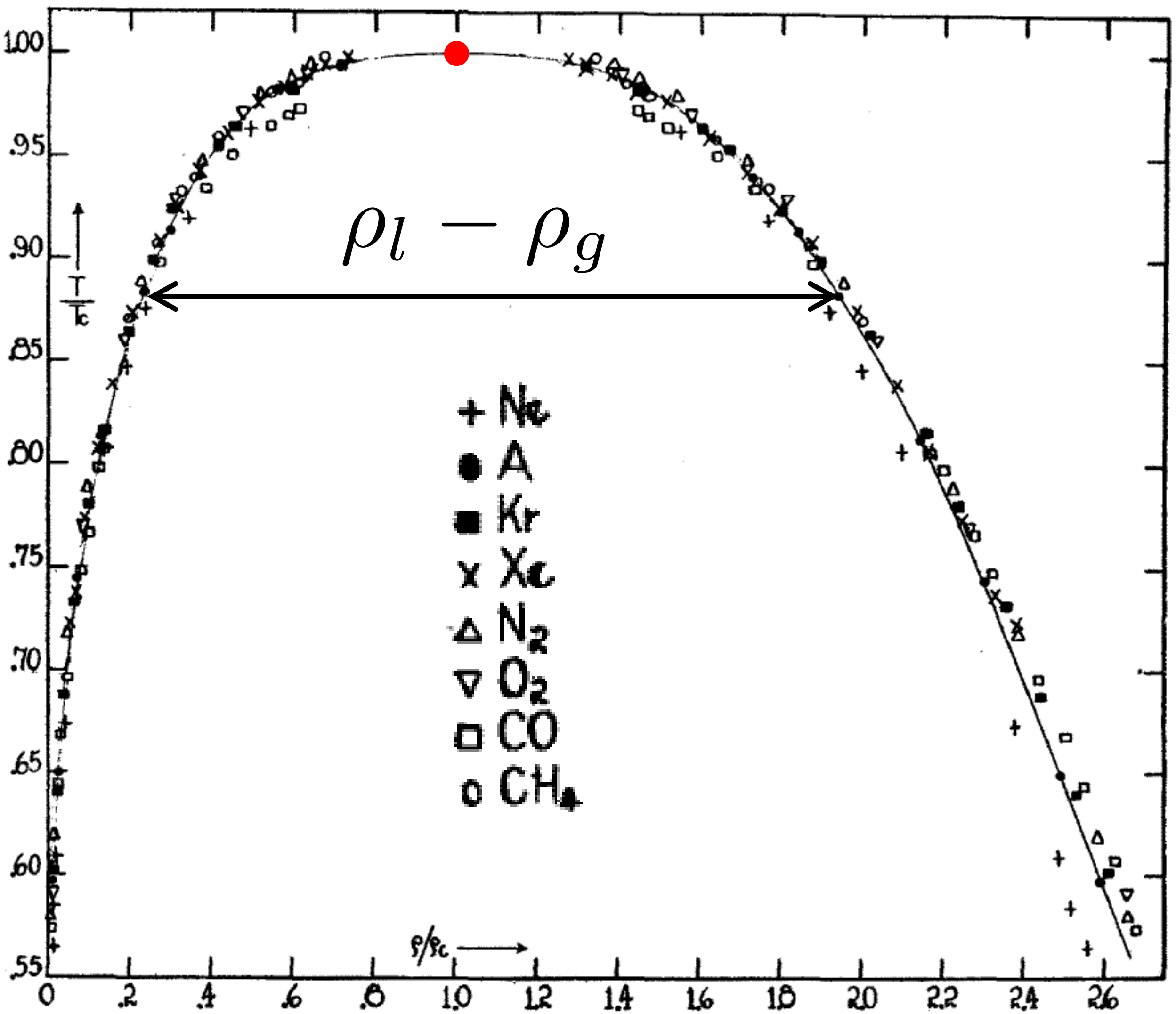


Liquid-gas critical point



T/T_c

$$|\rho_l - \rho_g| \propto |T - T_c|^\beta$$



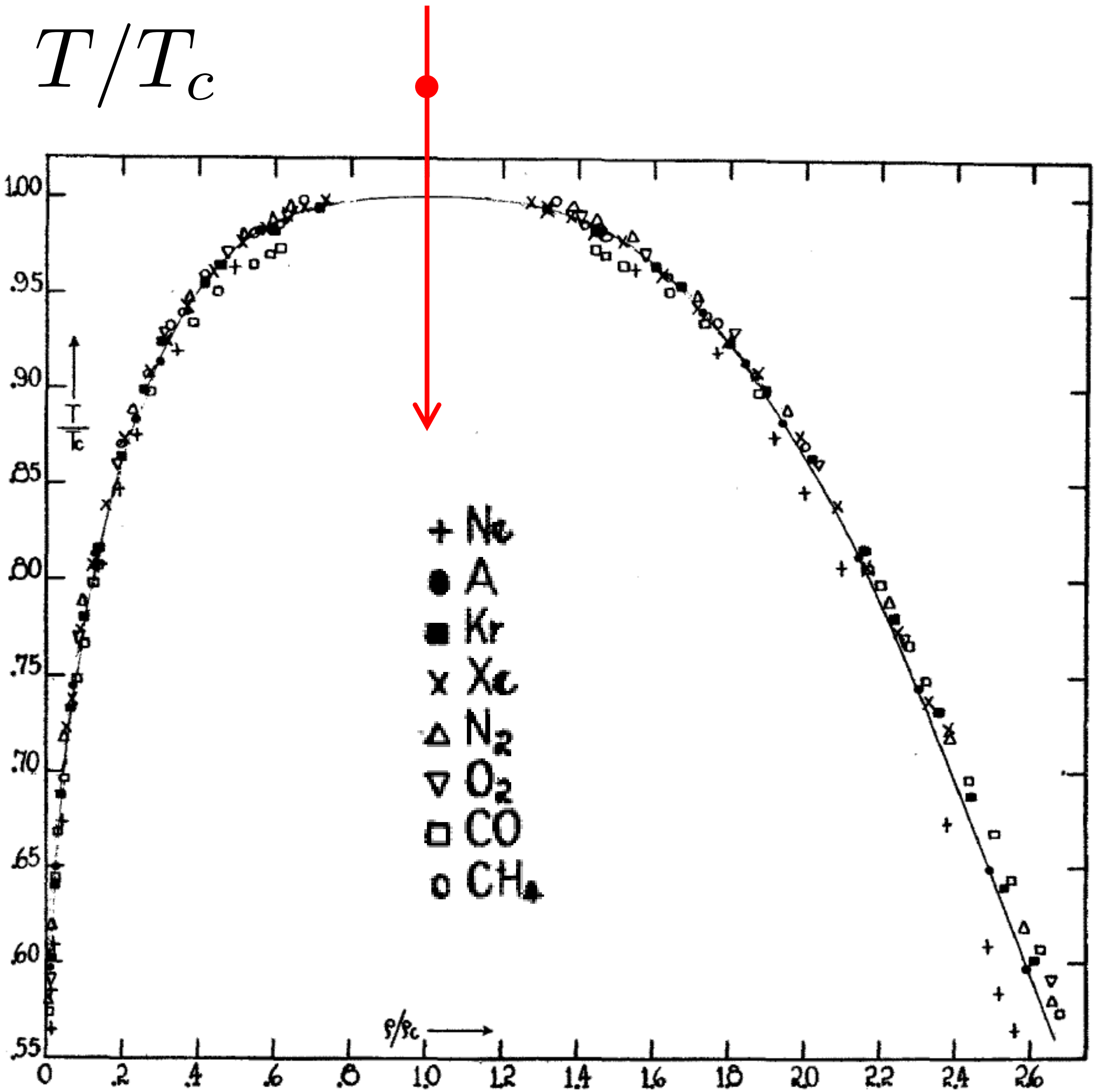
ρ/ρ_c

$\beta \approx 0.32$

for all substances!

E. A. Guggenheim, J. Chem. Phys. ‘45

T/T_c



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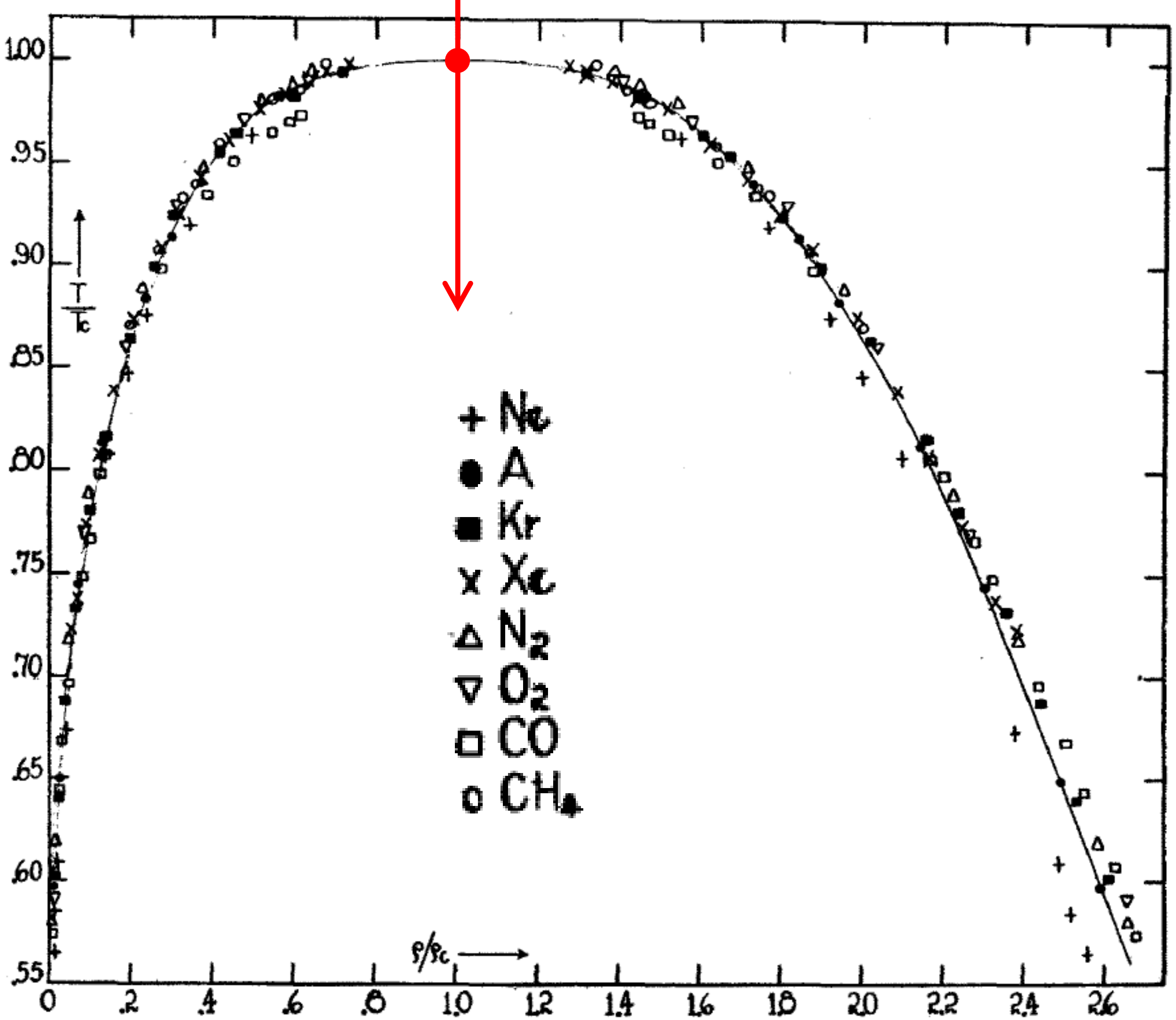
□ CO
o CH4

ρ/ρ_c

T/T_c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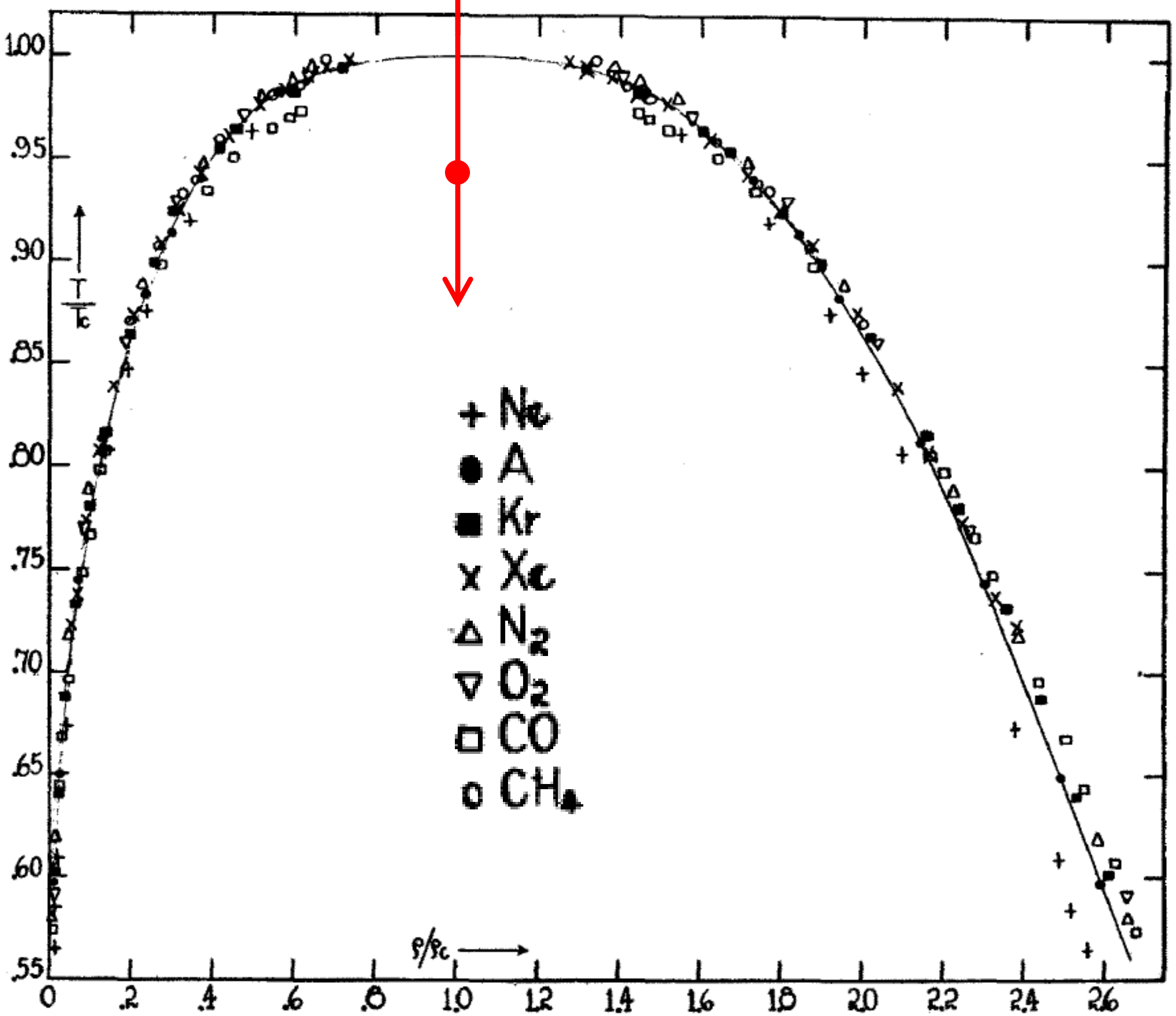
ρ/ρ_c

T/T_c



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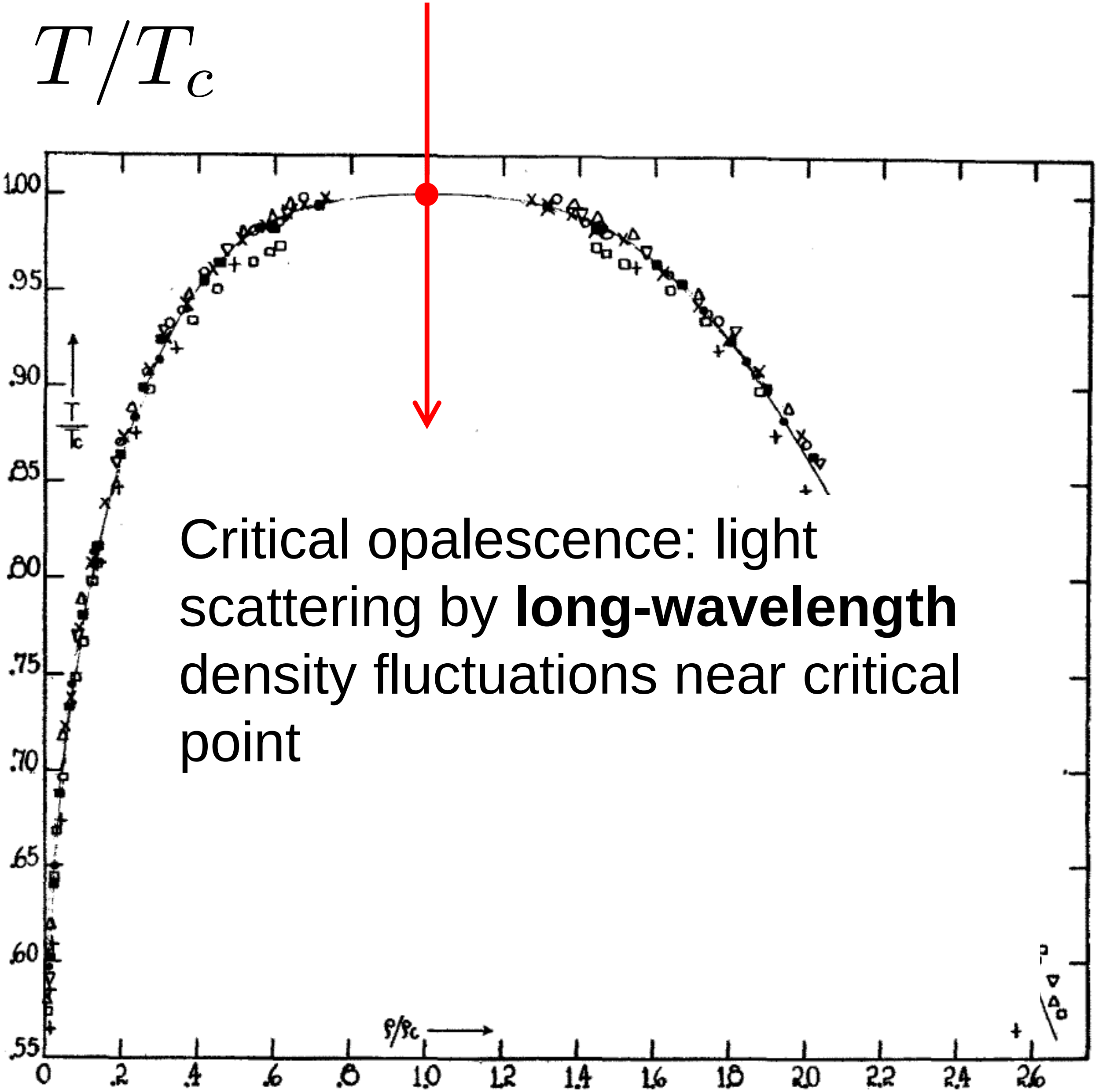
ρ/ρ_c

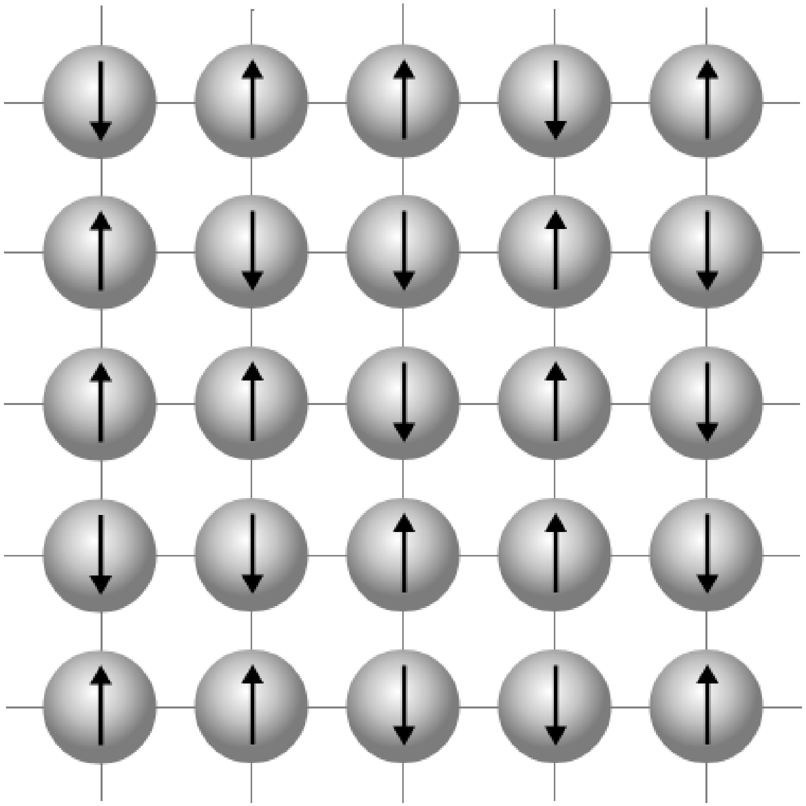
T/T_c



Critical opalescence: light scattering by **long-wavelength** density fluctuations near critical point

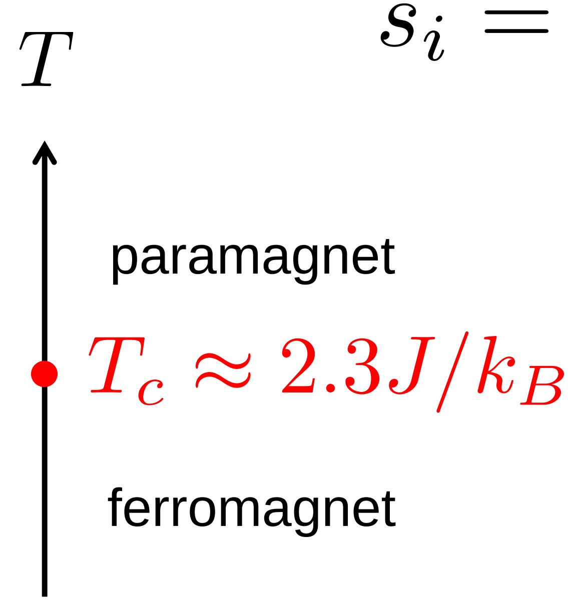
ρ/ρ_c

Ising model

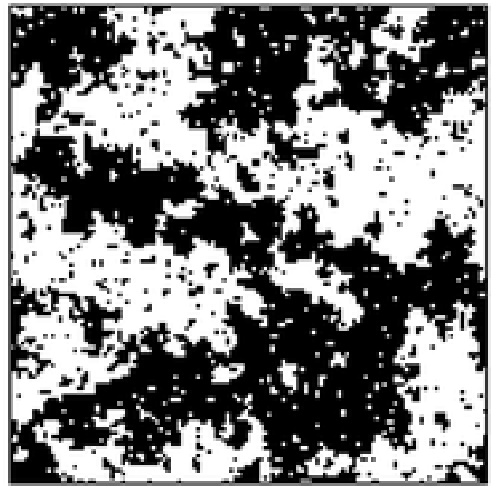


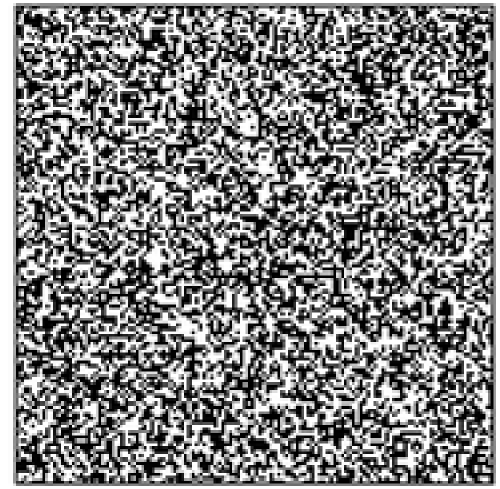
$$H = -J \sum_{\langle ij \rangle} s_i s_j ,$$

$$s_i = \pm 1$$

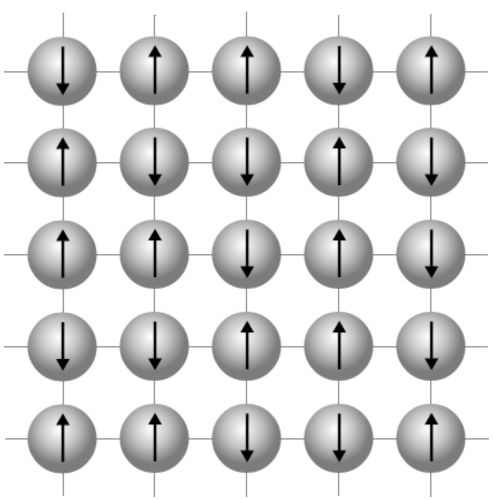


$$T \ll T_c \qquad T \approx T_c \qquad T \gg T_c$$





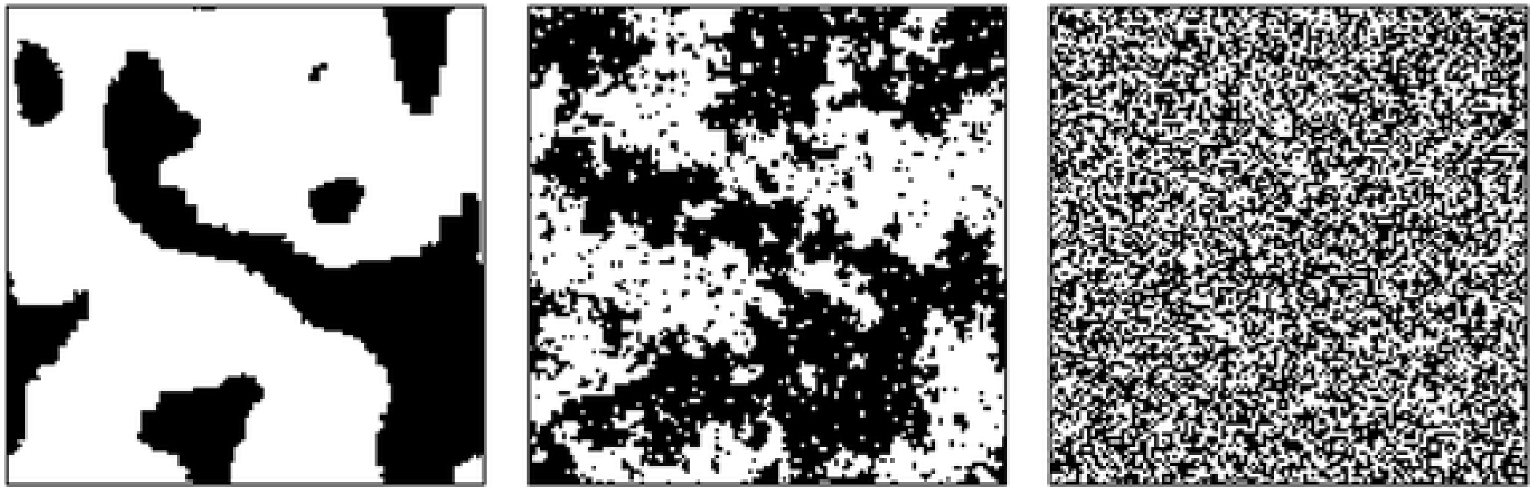
$$\xi \sim a \qquad \xi \gg a \qquad \xi \sim a$$



a

- Correlation length ξ for fluctuations **diverges** at criticality

$T \ll T_c$ $T \approx T_c$ $T \gg T_c$



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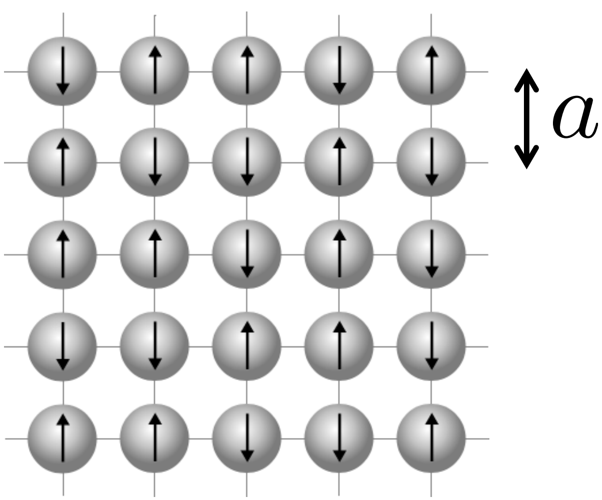
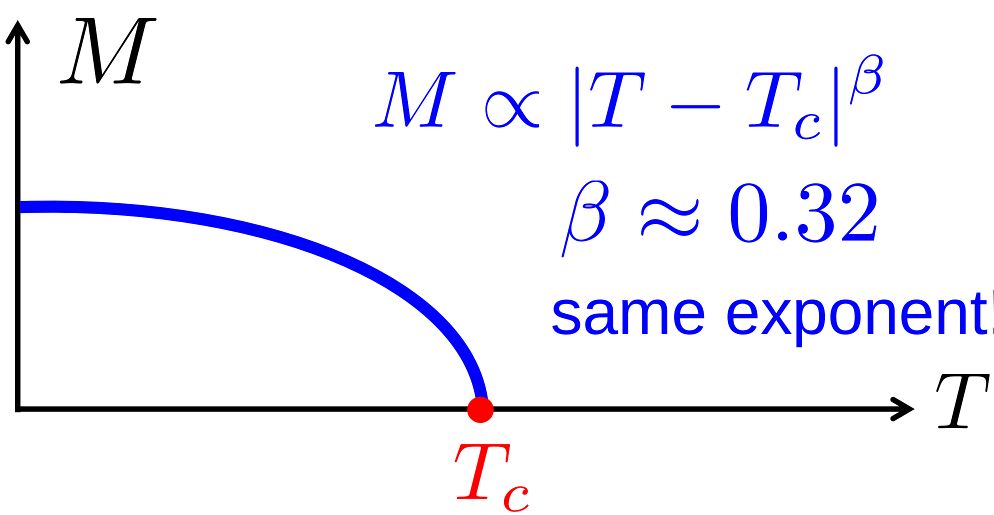
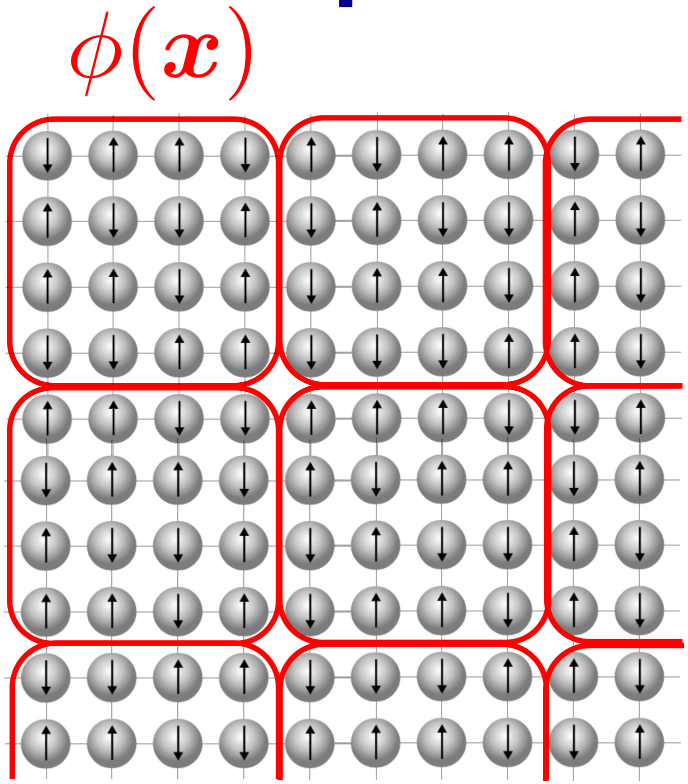




From spins to fields



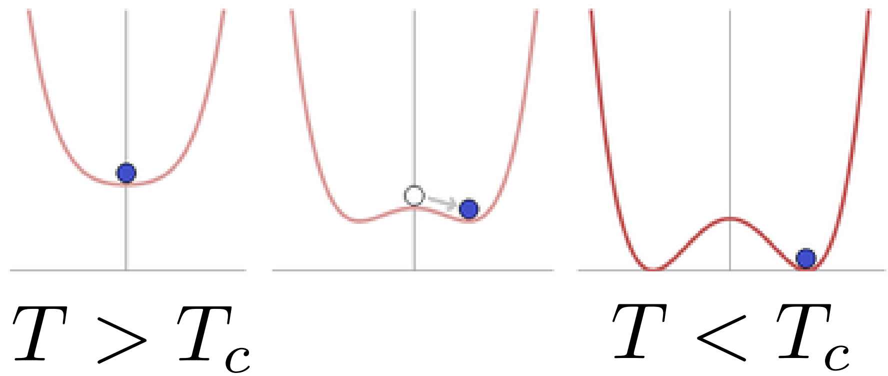
- Critical properties governed by long-wavelength (IR) fluctuations, for which microscopic (UV) details are unimportant: **universality**

- Coarse-grained description is sufficient: statistical **field** theory

$$Z = \sum_{\{s_i\}} e^{-\beta H} \approx \int \mathcal{D}\phi(\boldsymbol{x}) e^{-S}$$

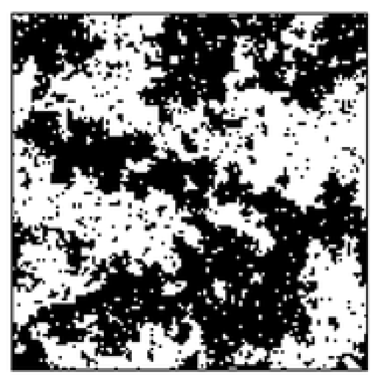
$$S = \int d\boldsymbol{x} \left(\frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right)$$

$$m^2 \propto T - T_c$$



- At long wavelengths, emergent **rotational** invariance

- At critical point, also emergent **scale** invariance: universal critical exponents (*Wilson, Fisher, Polyakov…*)



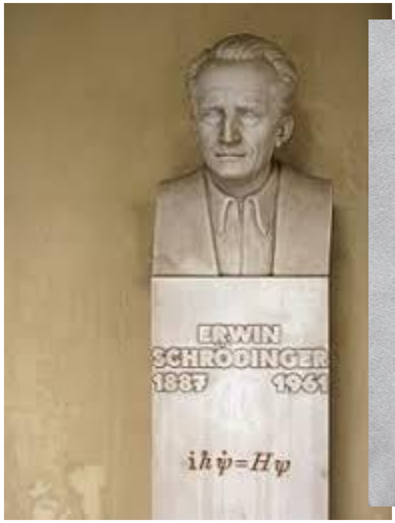
$$S = \int dx \left(\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \right)$$

- Lookes like “relativistic” QFT for scalar particle… but **static**!

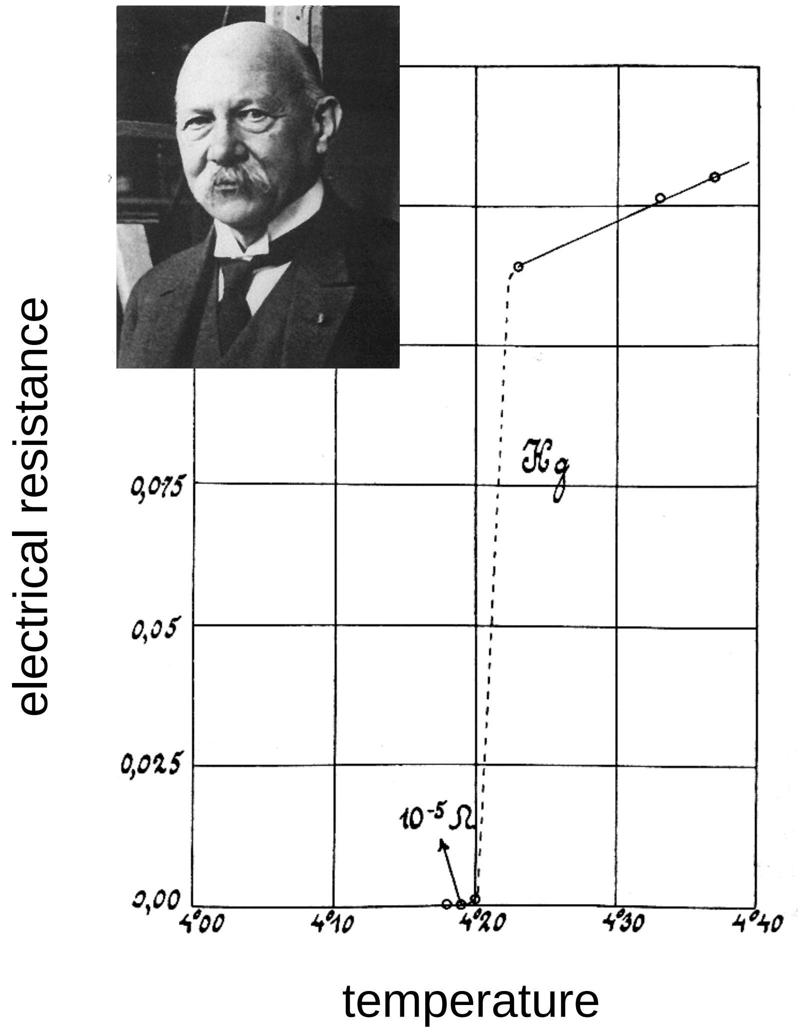
- Describes spatial correlations, no time direction

- No time = no particles

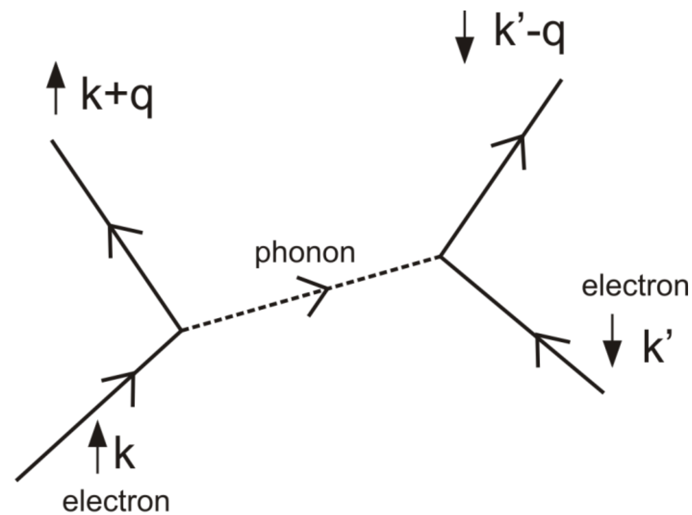
- What about **quantum** statistical mechanics?



Superconductivity



- Quantum phenomenon: Cooper pairing mediated by phonon exchange (*BCS ‘57*)

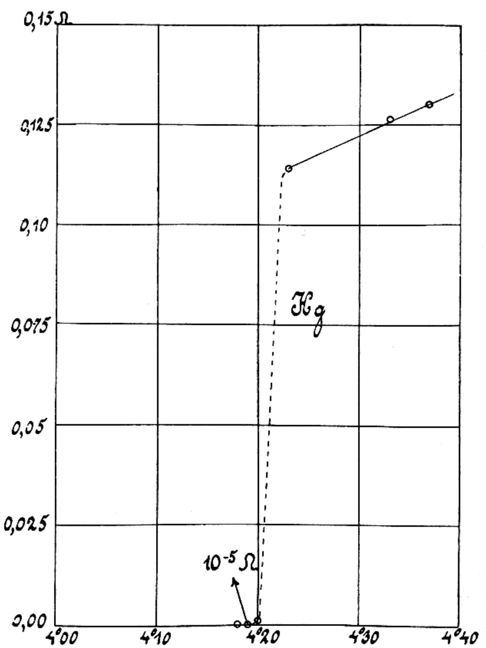


K. Onnes, Comm. Phys. Lab. Univ. Leiden 1911

- Near superconducting transition, coarse-grained Cooper pair field = complex scalar ϕ
- Long-wavelength description is again a **static** field theory! (*Ginzburg & Landau, ‘50*)

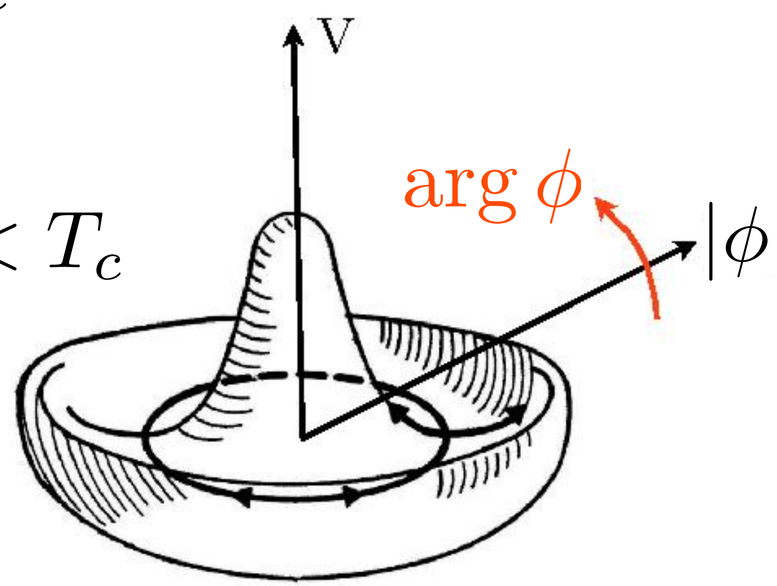
$$S = \int d\boldsymbol{x} \left(\frac{1}{2m} |(-i\hbar\nabla - 2e\boldsymbol{A})\phi|^2 + \alpha|\phi|^2 + \beta|\phi|^4 \right)$$

$$\alpha \propto T - T_c$$



T_c

$T < T_c$



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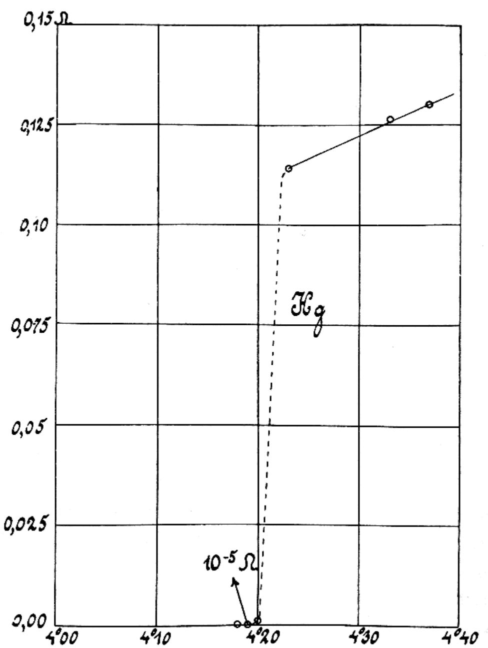

T_c

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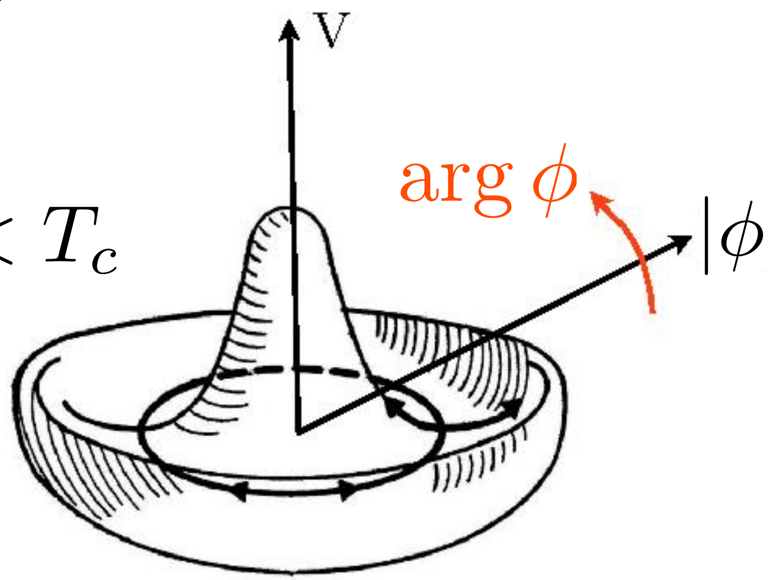


From Euclid to Minkowski

Can “real” QFTs emerge in condensed matter?

From Euclid to Minkowski

Can “real” QFTs emerge in condensed matter? Yes!

- Relativistic QFTs: Weyl/Dirac fermions and axion electrodynamics in CMP
- Supersymmetric QFTs: Higgsinos and gauginos in CMP?

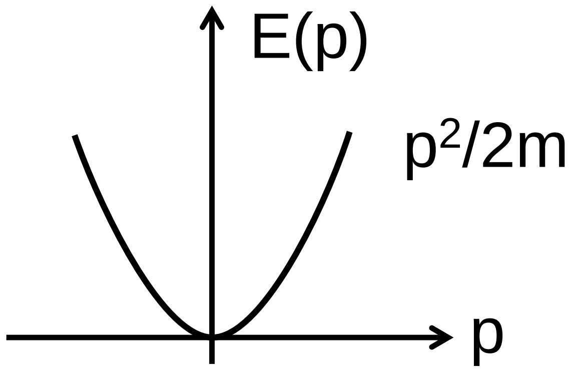
Banff International Research Station

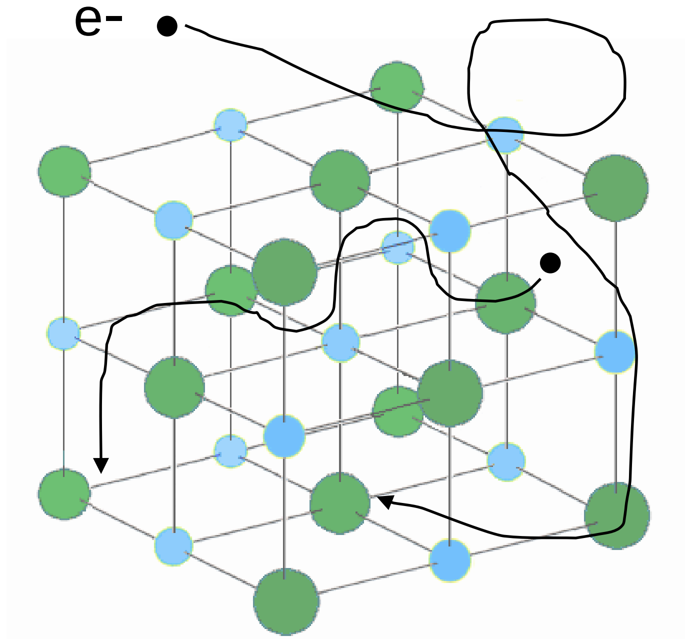
for Mathematical Innovation and Discovery

Relativistic Fermions and Nodal Semimetals from Topology

February 11-16, 2018

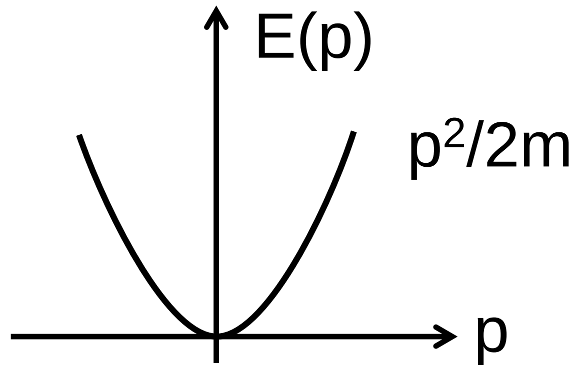
Band theory of solids

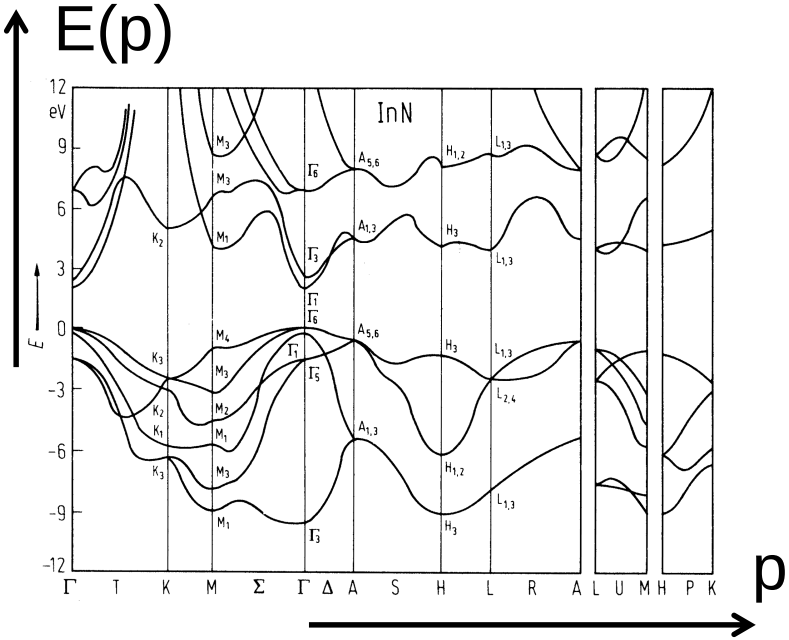


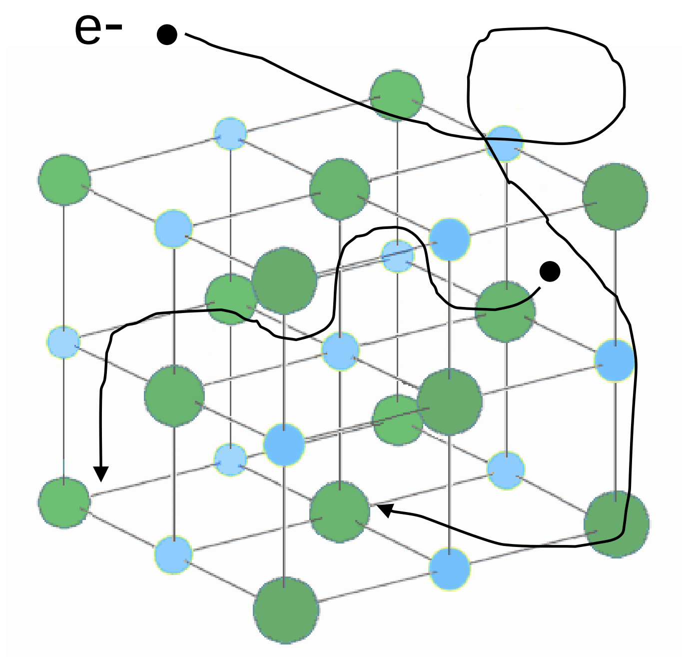


Bloch, Wilson, Peierls, Wigner… ('20-'30s)

Band theory of solids







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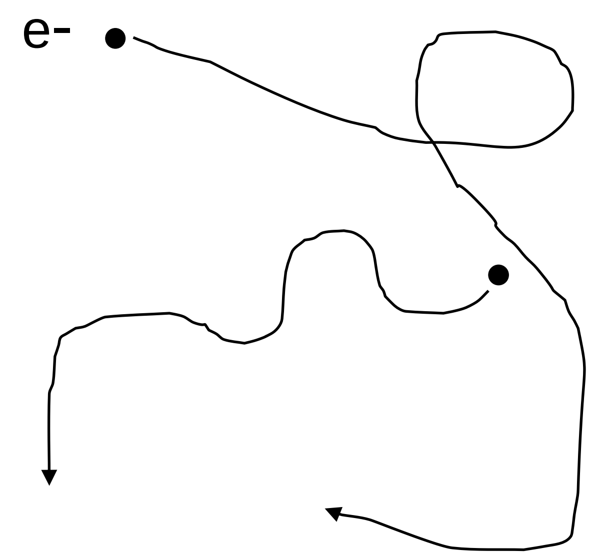

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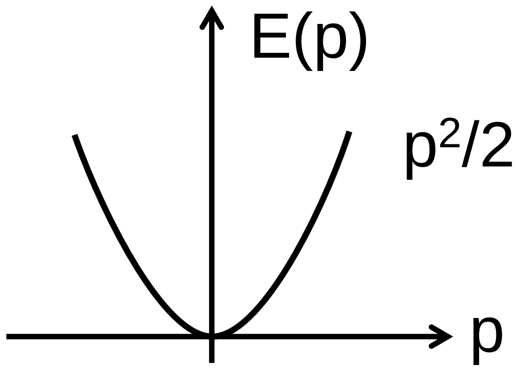

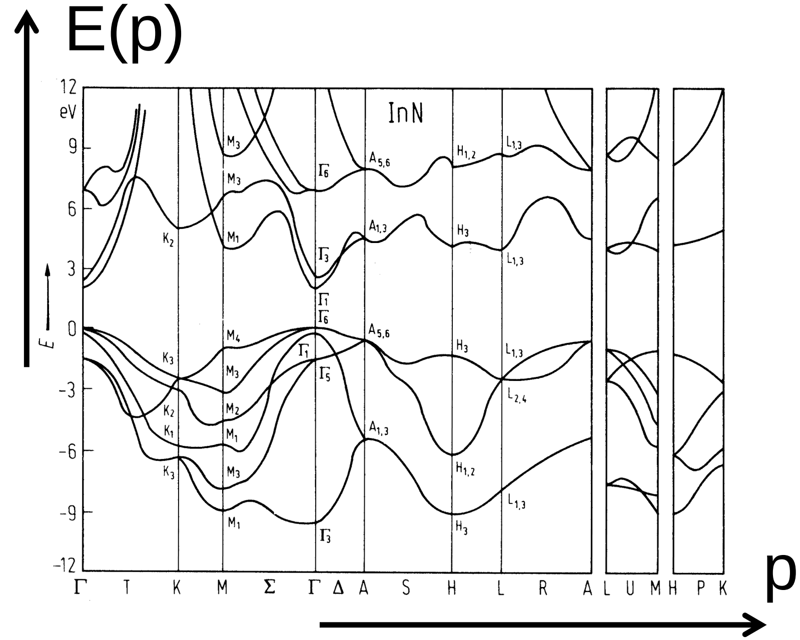

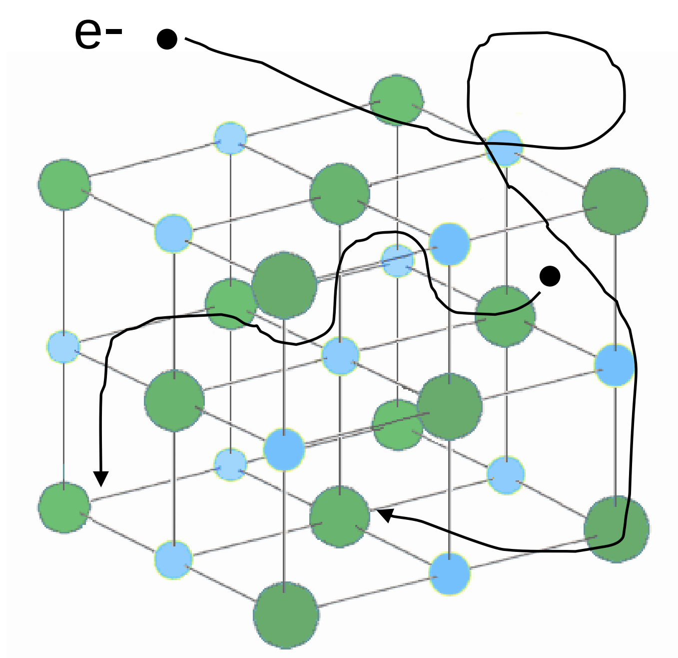

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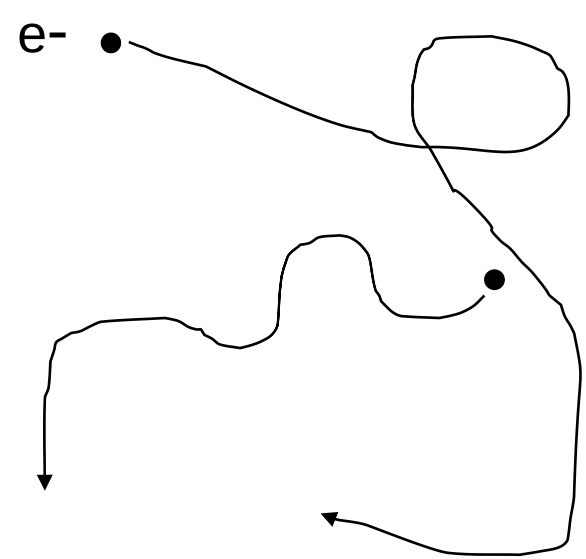
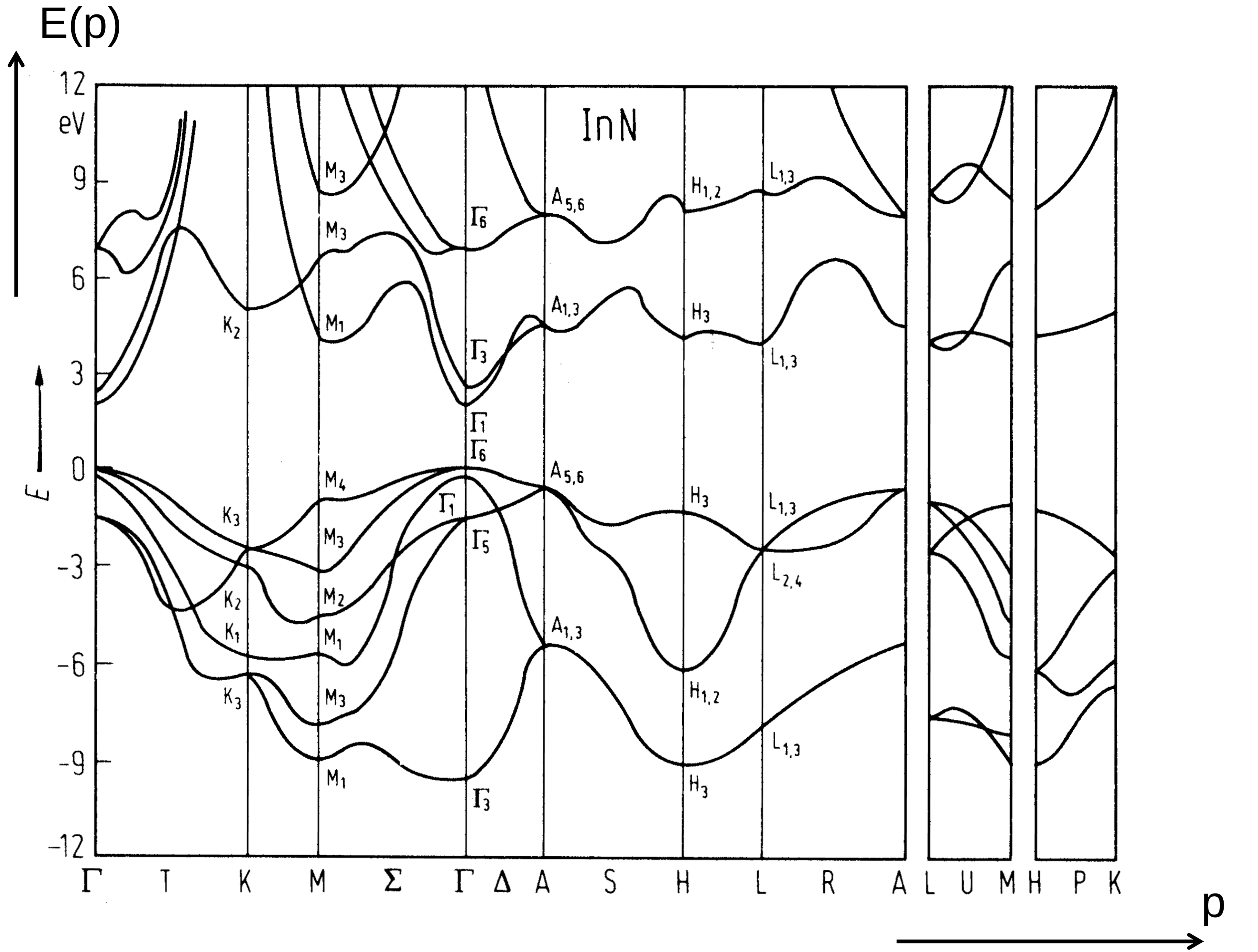


Each crystal is a new “vacuum” with different properties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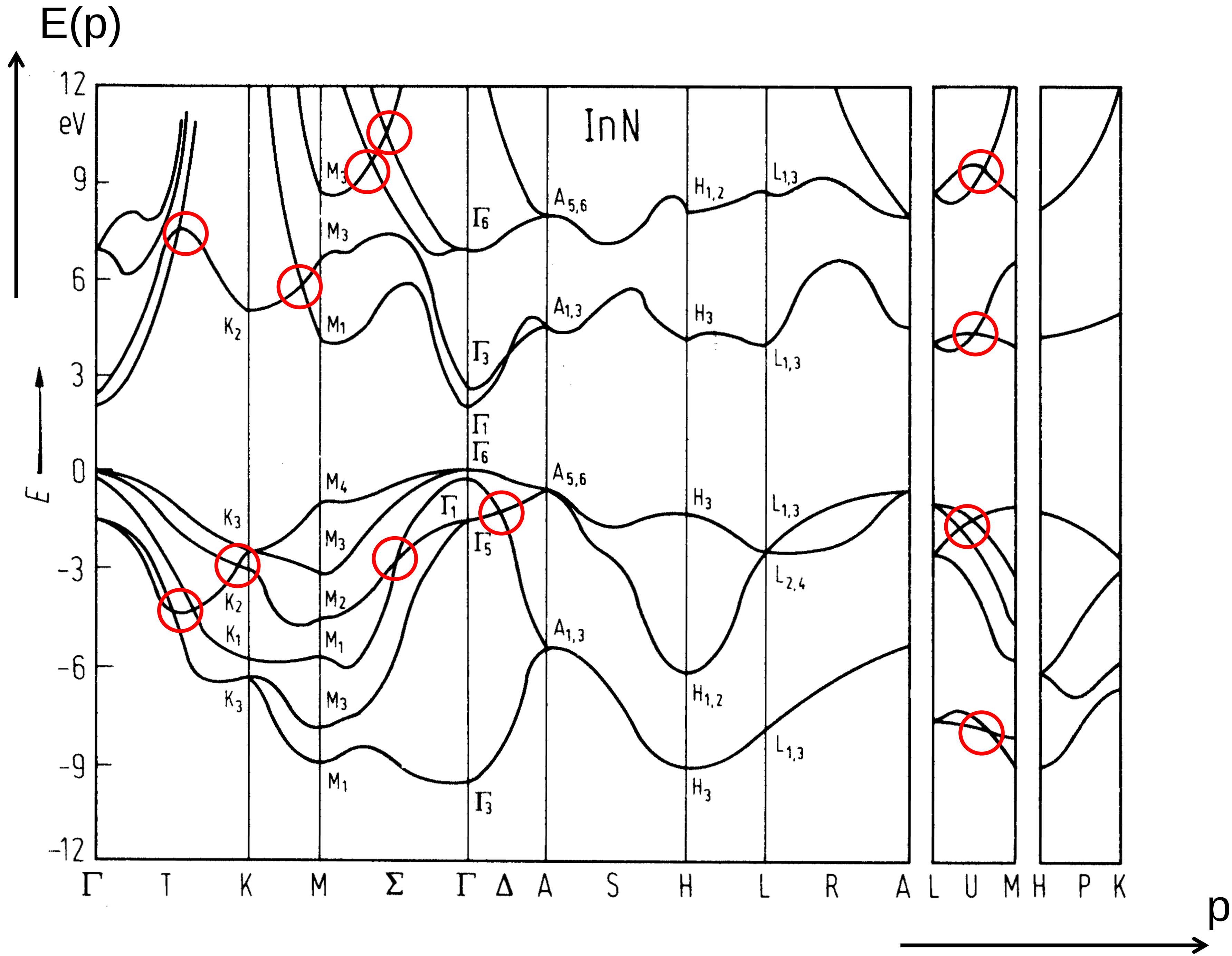
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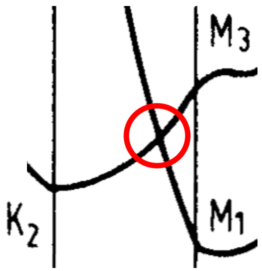
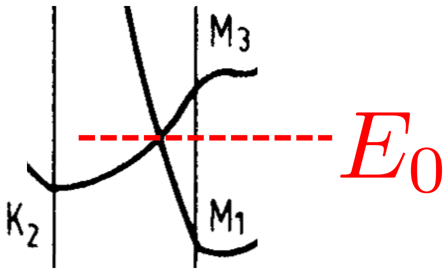
von Neumann-Wigner theorem



- “Level repulsion” in QM… why so many accidental degeneracies?

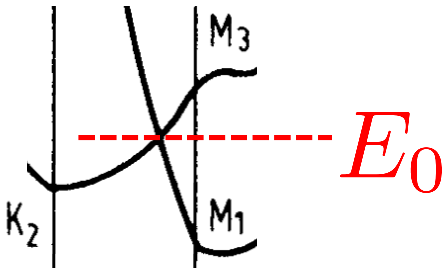
von Neumann-Wigner theorem



- "Level repulsion" in QM… why so many accidental degeneracies?

$$H = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}$$

von Neumann-Wigner theorem

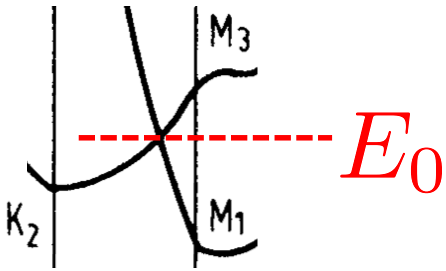


- "Level repulsion" in QM… why so many accidental degeneracies?

$$H = \begin{pmatrix} E_0 + \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & E_0 - \Delta_3 \end{pmatrix}$$

$$E_\pm = E_0 \pm \sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}$$

von Neumann-Wigner theorem

- "Level repulsion" in QM… why so many accidental degeneracies?

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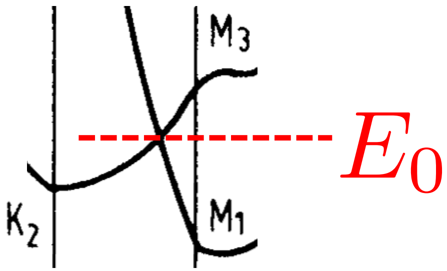
$$E_\pm = E_0 \pm \sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}$$

- In 3D, can adjust p_x, p_y, p_z to set $\Delta_1=\Delta_2=\Delta_3=0$
- 2-fold degenerate **points** with **linear** dispersion are generic ("topologically stable")

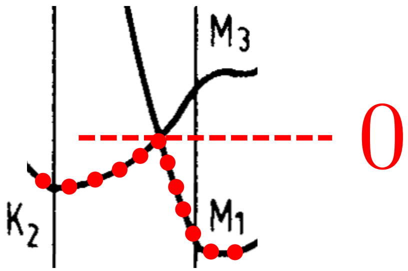
Weyl fermions in CMP



$$H = E_0 + v\boldsymbol{\sigma} \cdot \boldsymbol{p}$$

- Weyl fermion with “speed of light” = |v| and chirality = sign(v)

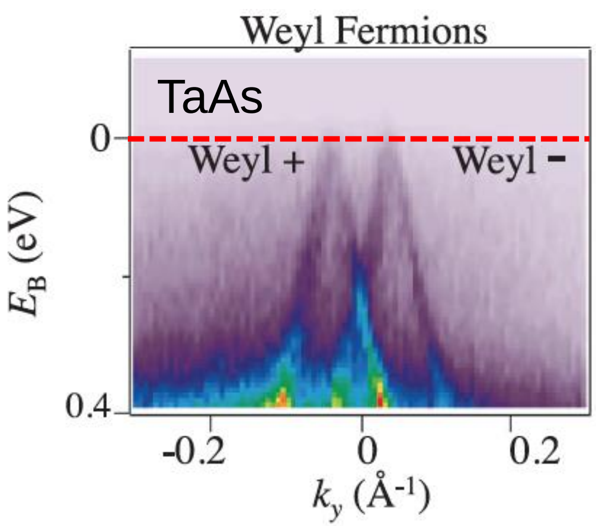
Weyl fermions in CMP



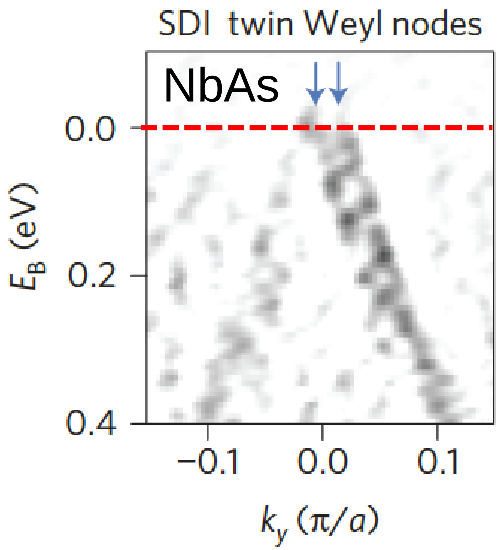
$$H = v\boldsymbol{\sigma}\cdot\boldsymbol{p}$$

- Weyl fermion with "speed of light" = |v| and chirality = sign(v)

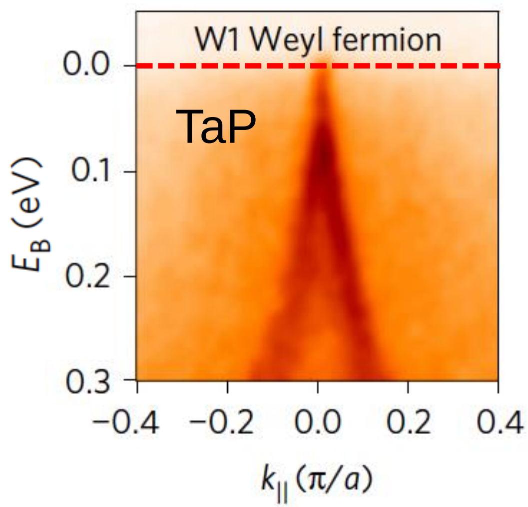
- To mimic relativistic vacuum, Fermi level must be at the Weyl point: "Weyl semimetal"



Xu et al., Science '15



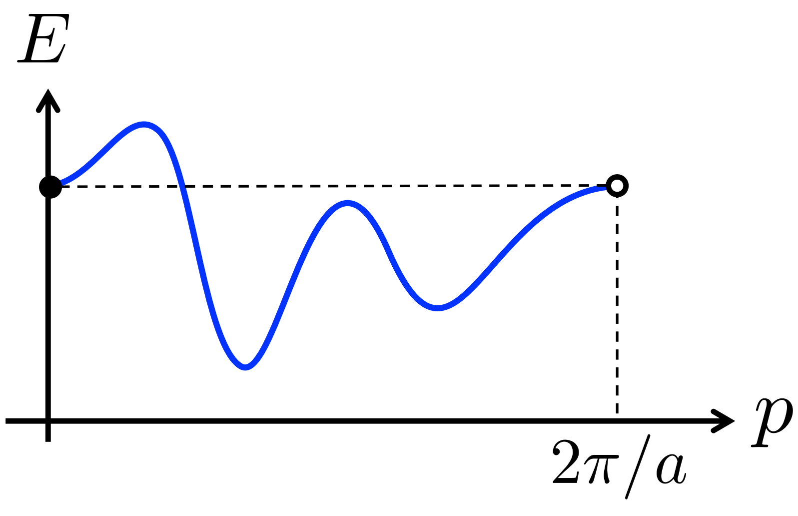
Xu et al., Nature Phys. '15



Zhang et al., Nature Phys. '17

Chiral fermions on a lattice?

- In CMP, Weyl fermions always appear in R/L pairs
- Well-known from lattice gauge theory! “Fermion doubling problem”
- E(p) must be **periodic** on a lattice, e.g. in 1D:



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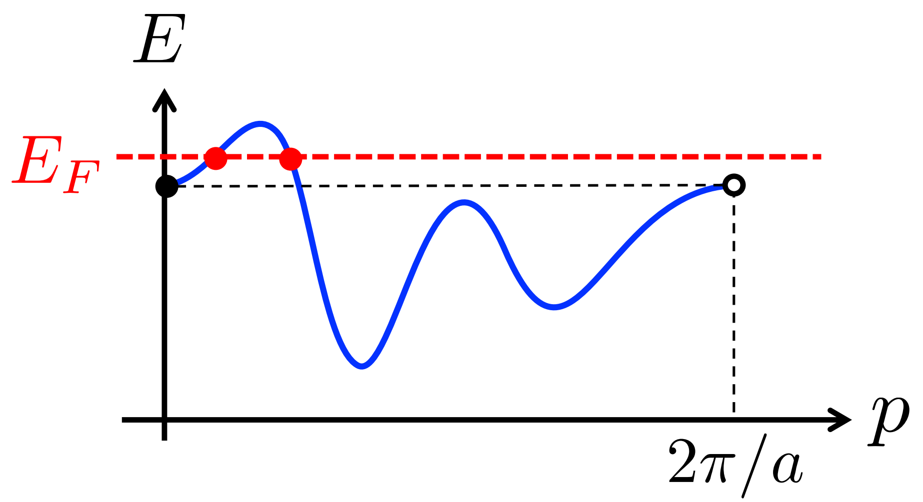

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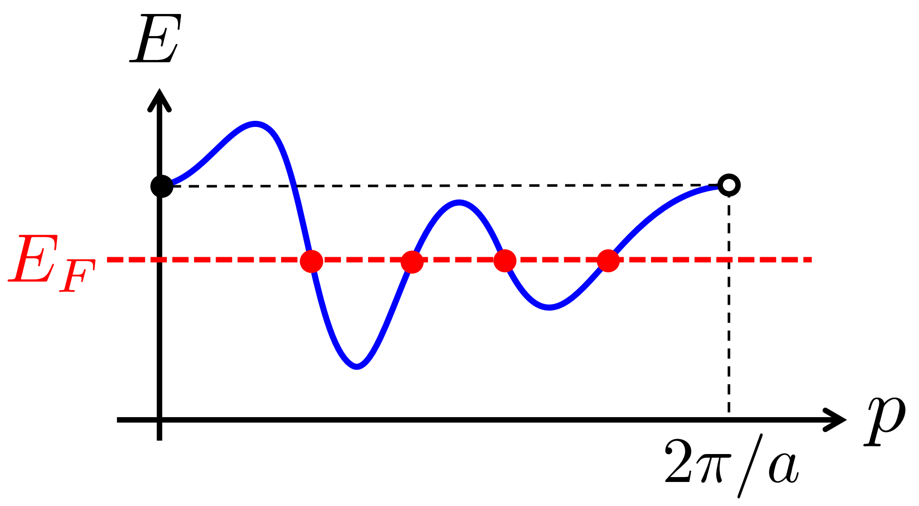

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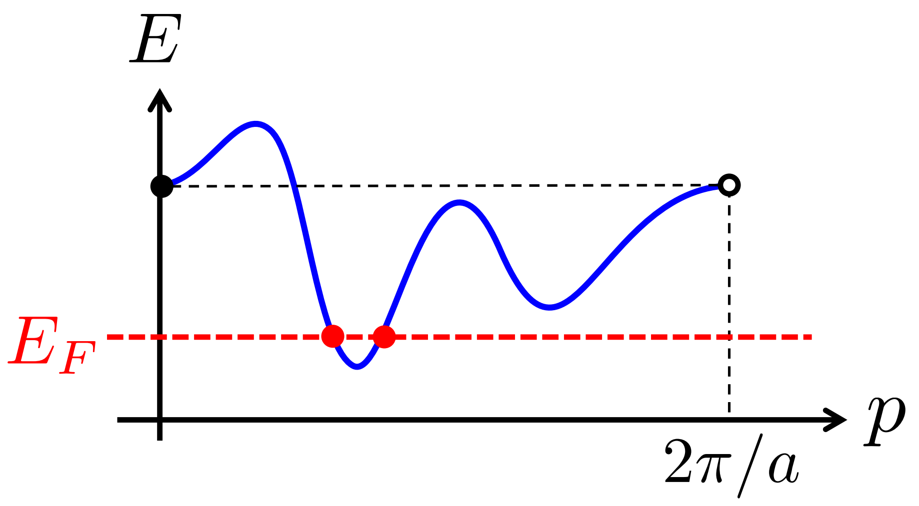
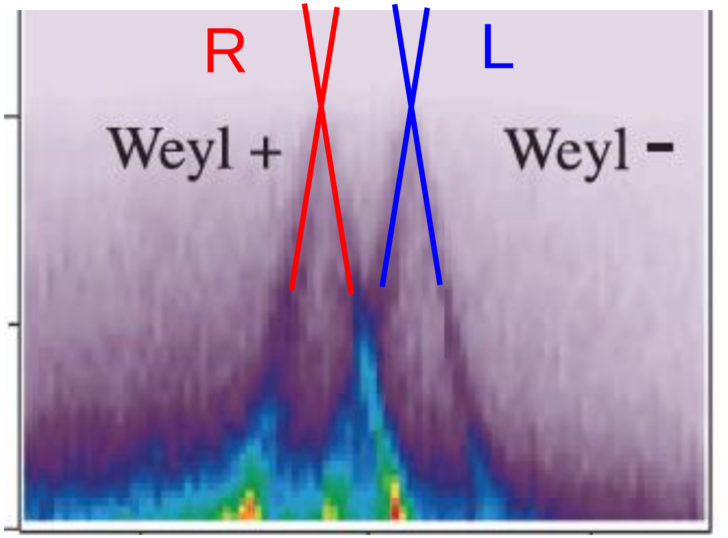


- Analogous in 3D (*Nielsen & Ninomiya, NPB ’81*)

Emergent chiral symmetry



- For energies near E_F, **emergent** chiral symmetry protected by vN-W theorem

$$\psi_R \to e^{i\alpha}\psi_R$$
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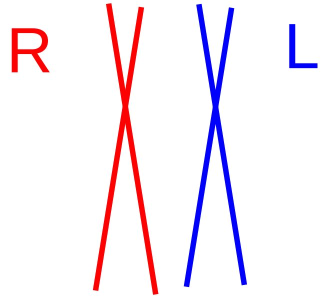

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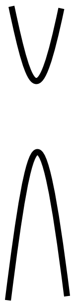
Broken chiral symmetry



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$$\psi_L \rightarrow e^{-i\alpha}\psi_L$$

- What if R/L Weyl fermions at the same momentum point?
- Fourfold degeneracy, not stable! Dirac mass generally allowed, (explicitly) broken chiral symmetry

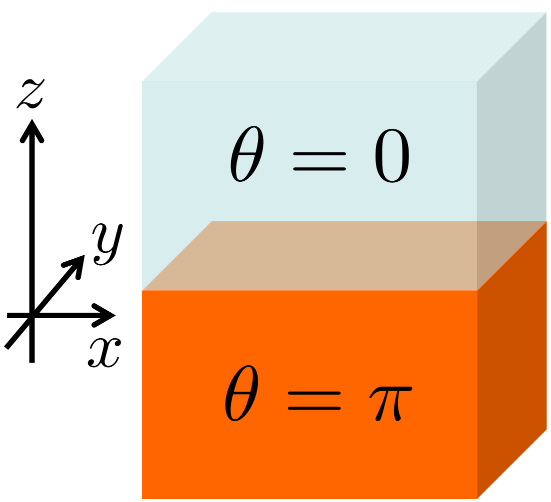
$$m\bar{\Psi}e^{i\theta\gamma^5}\Psi = m\cos\theta\bar{\Psi}\Psi + im\sin\theta\bar{\Psi}\gamma^5\Psi$$

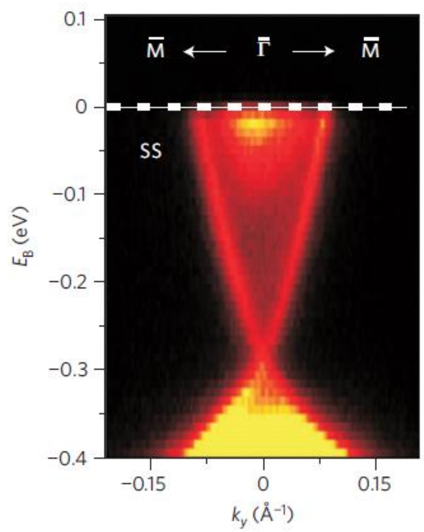
normal mass   axial mass

θ and T

$$m\bar{\Psi}e^{i\theta\gamma^5}\Psi = m\cos\theta\,\bar{\Psi}\Psi + im\sin\theta\,\cancel{\bar{\Psi}\gamma^5\Psi}$$

- Most materials have T symmetry: only $\theta=0$ or $\theta=\pi$
- Interface between $\theta=0$ and $\theta=\pi$: mass domain wall, traps fermion zero mode (*Jackiw & Rebbi, PRD '76*)



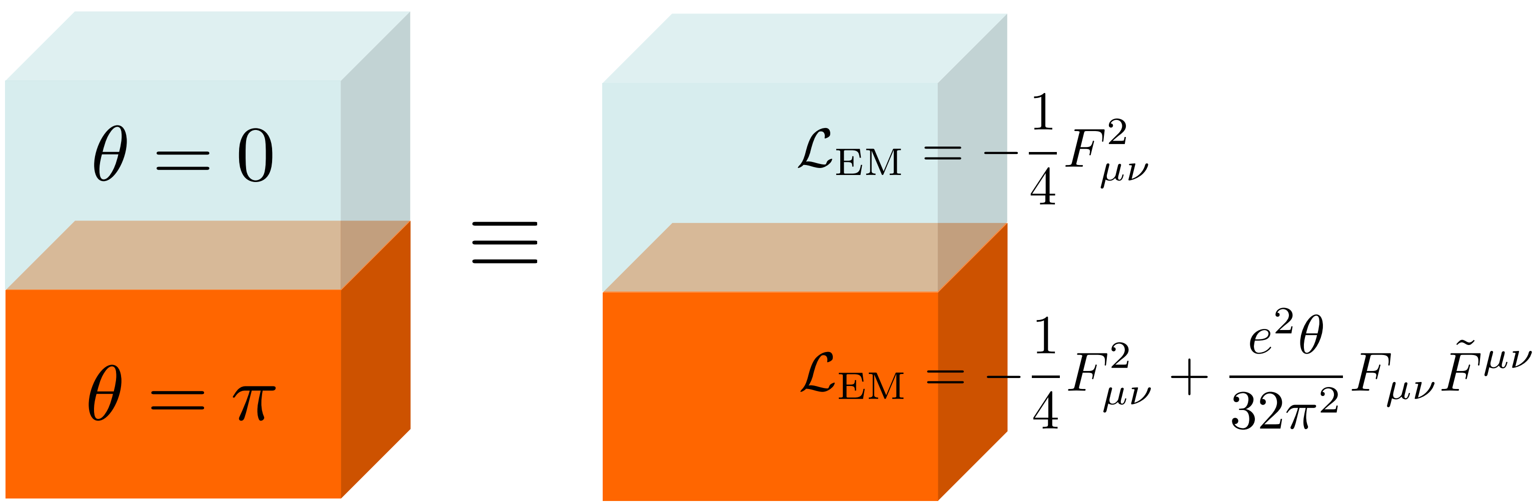


Xia et al., Nature Phys. '09

- Zero mode = massless **(2+1)D** Dirac fermion!
- $\theta=\pi$ materials = "topological insulators"

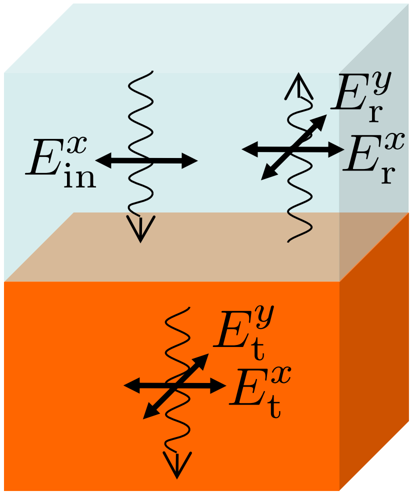
Axion electrodynamics

- Coupling to E&M: chiral rotation converts the mass phase to an electromagnetic **theta angle** $\theta=\pi$



Axion electrodynamics

- Theta angle can be measured via Kerr/Faraday effect

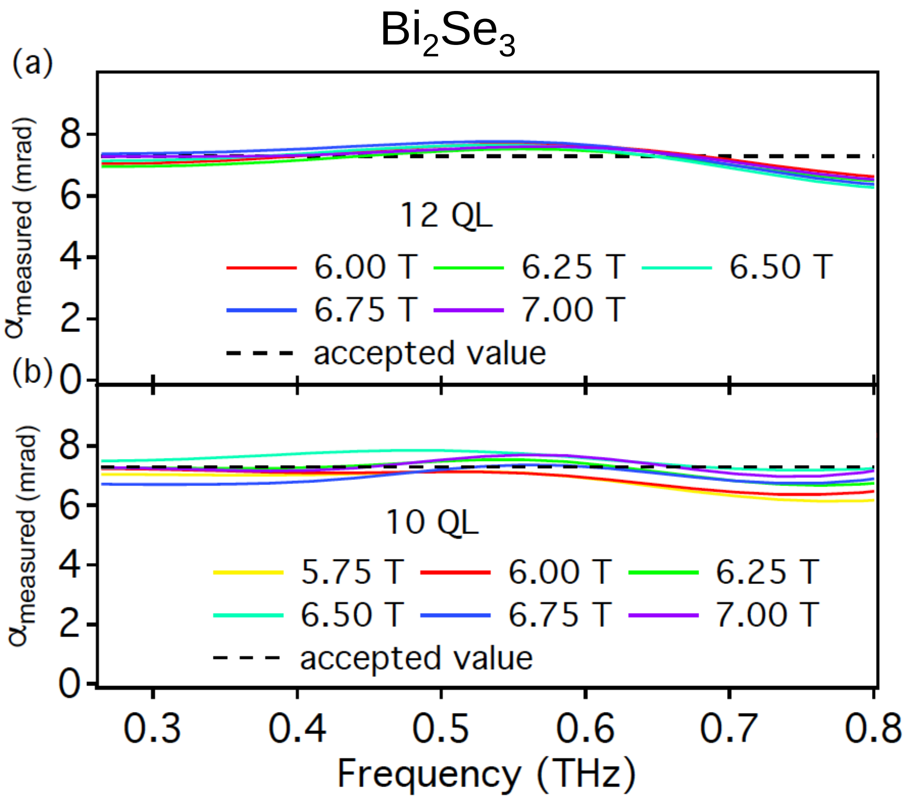


$$\tan\theta_K = \frac{E_{\rm r}^y}{E_{\rm r}^x}$$

$$\tan\theta_F = \frac{E_{\rm t}^y}{E_{\rm t}^x}$$

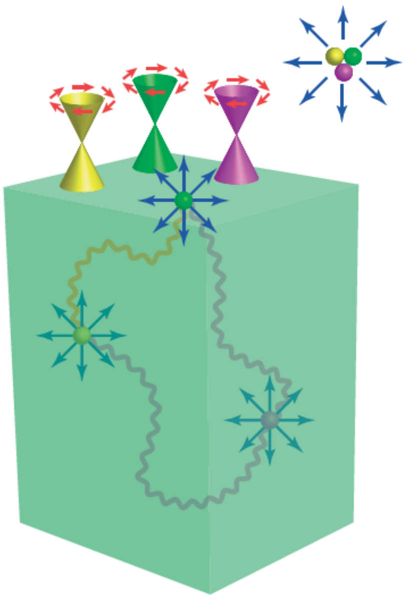
$$\frac{\cot\theta_F + \cot\theta_K}{1+\cot^2\theta_F} = \alpha$$

JM et al., PRL '10



Wu et al., Science '16

θ and T, revisited



- EM θ angle different from 0, π can be consistent with T, if there exist **deconfined, fractionally charged** collective excitations

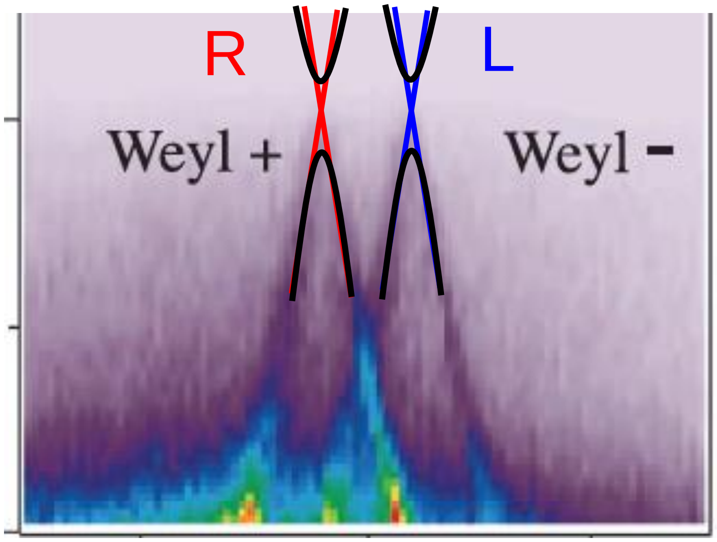
- Example: e/3 quasiparticles in the fractional quantum Hall effect (*Laughlin, PRL ’83*)

- Quasiparticles with charge e/q: $\theta = \pi \frac{p}{q}$, p, q odd

- Reason: modifies Dirac quantization condition = changes periodicity of θ angle
- Ongoing search for “fractional” topological insulators

JM et al., PRL ‘10; PRB ‘12; Nature Phys. ’15; Ye, Hughes, JM, Fradkin, PRB ‘16

Dark matter on a chip?



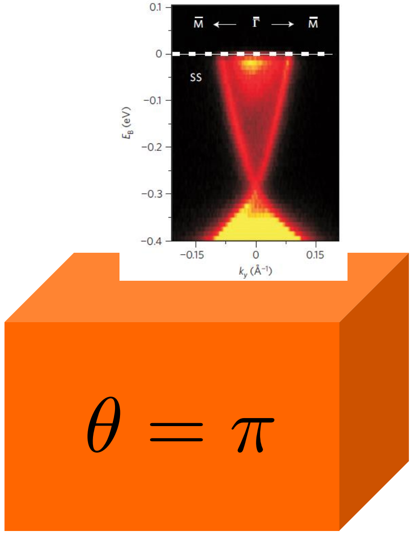
- Chiral symmetry in Weyl semimetals can in principle be broken **spontaneously** as a result of electron-electron interactions

- Pseudoscalar Goldstone mode = "dynamical axion field"
- Topological defects in the broken symmetry phase = "axion strings", trap chiral fermion zero modes (*Callan & Harvey, NPB '85*)
- Search for interacting Weyl materials ($CeRu_4Sn_6$, CeSb, …)

Wang & Zhang, PRB '13; JM & Nandkishore, PRB '14

SUSY in condensed matter?

Superconducting Dirac fermions



nature COMMUNICATIONS

ARTICLE

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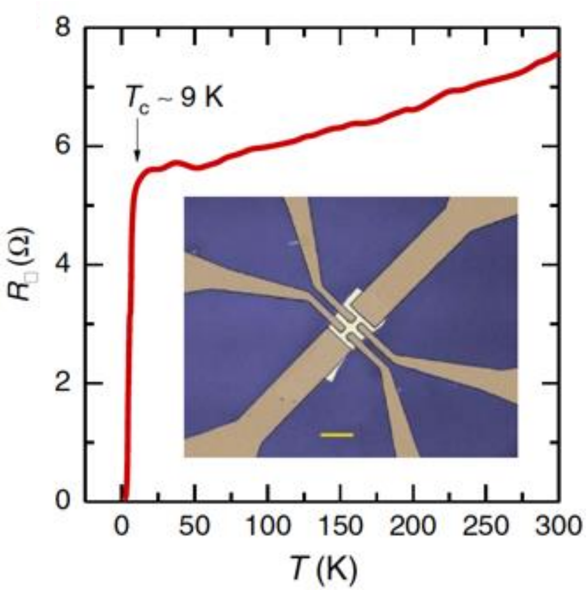
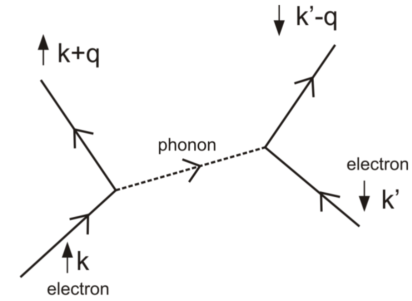
Emergent surface superconductivity in the topological insulator Sb_2Te_3

Lukas Zhao[1], Haiming Deng[1], Inna Korzhovska[1], Milan Begliarbekov[1], Zhiyi Chen[1], Erick Andrade[2], Ethan Rosenthal[2], Abhay Pasupathy[2], Vadim Oganesyan[3,4] & Lia Krusin-Elbaum[1,4]

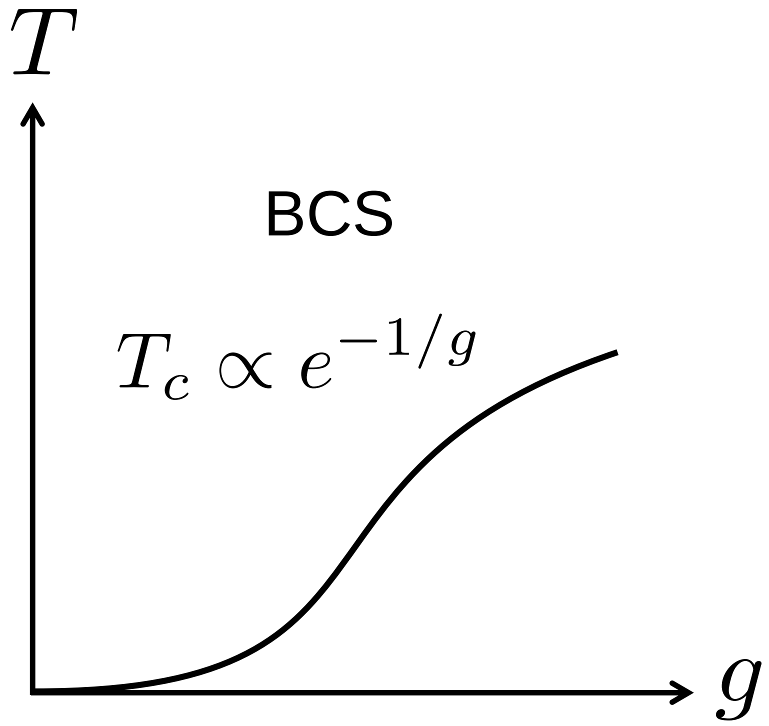


- What is special about superconducting Dirac fermions in 2+1 dimensions?

BCS vs Dirac



$$\approx g$$



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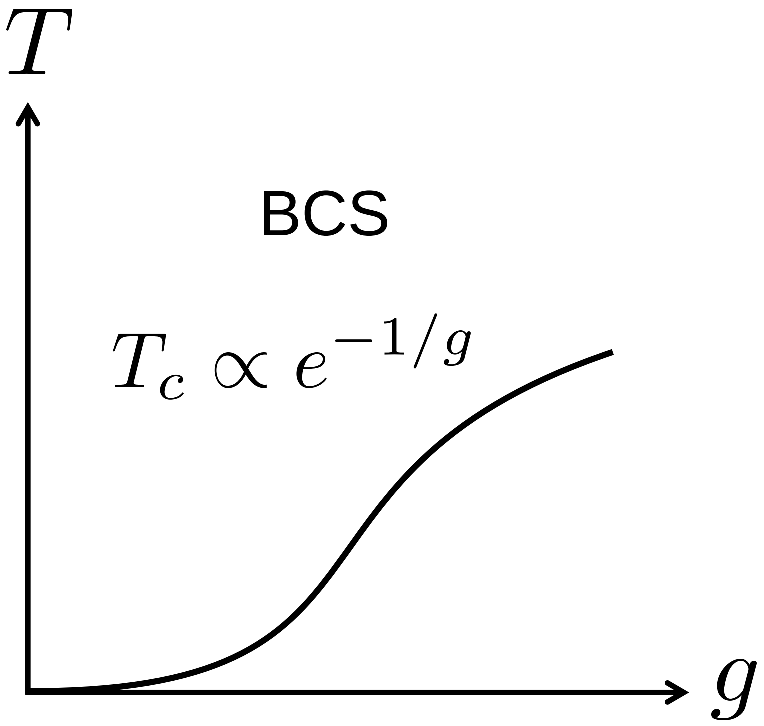

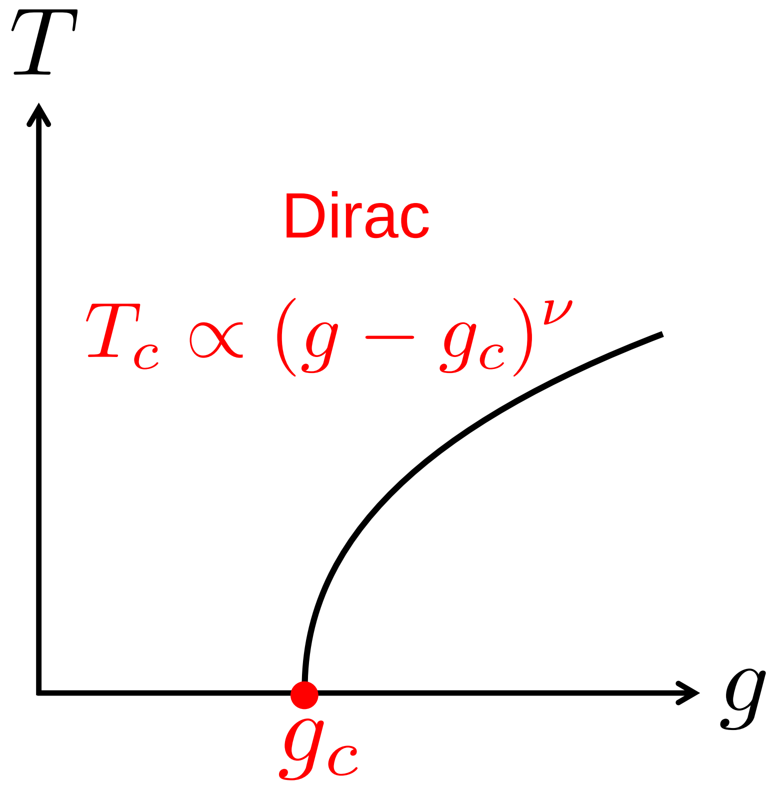

Roy, Juričić, Herbut, PRB ‘13; Nandkishore, JM, Huse, Sondhi, PRB ‘13

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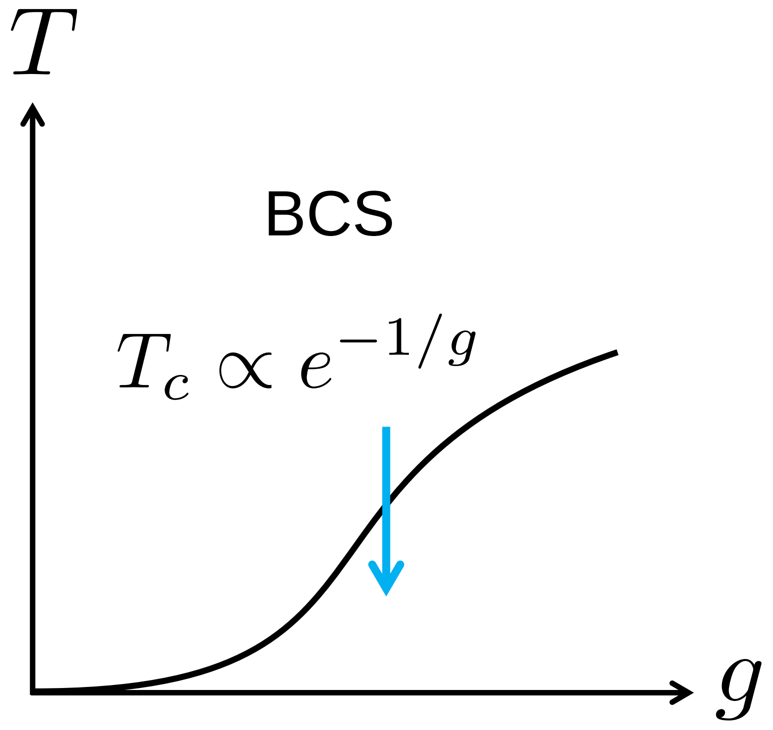

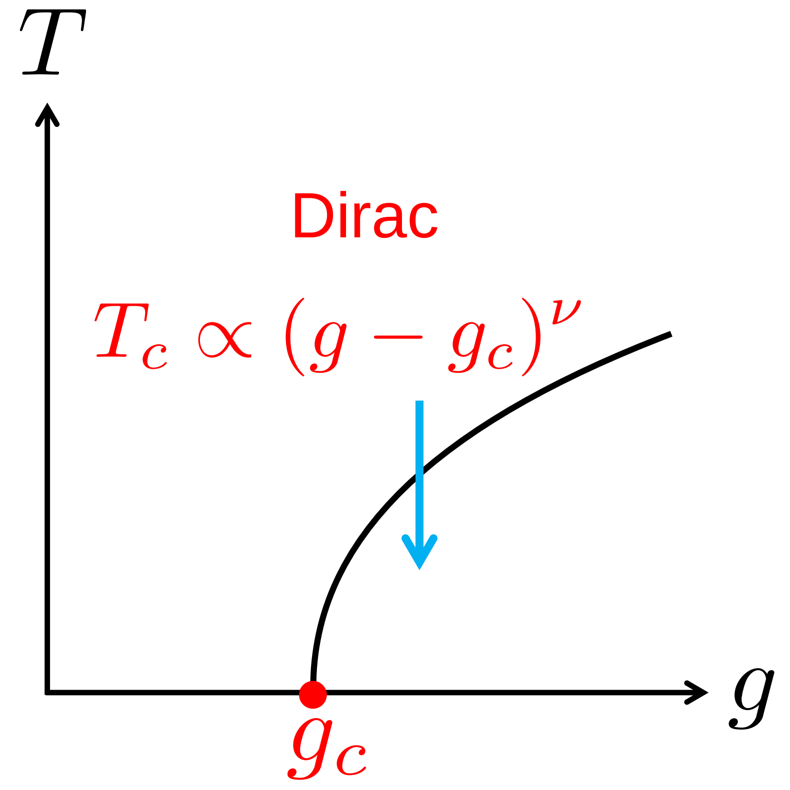

- Thermal phase transition similar to BCS (static GL)

Roy, Juričić, Herbut, PRB '13; Nandkishore, JM, Huse, Sondhi, PRB '13

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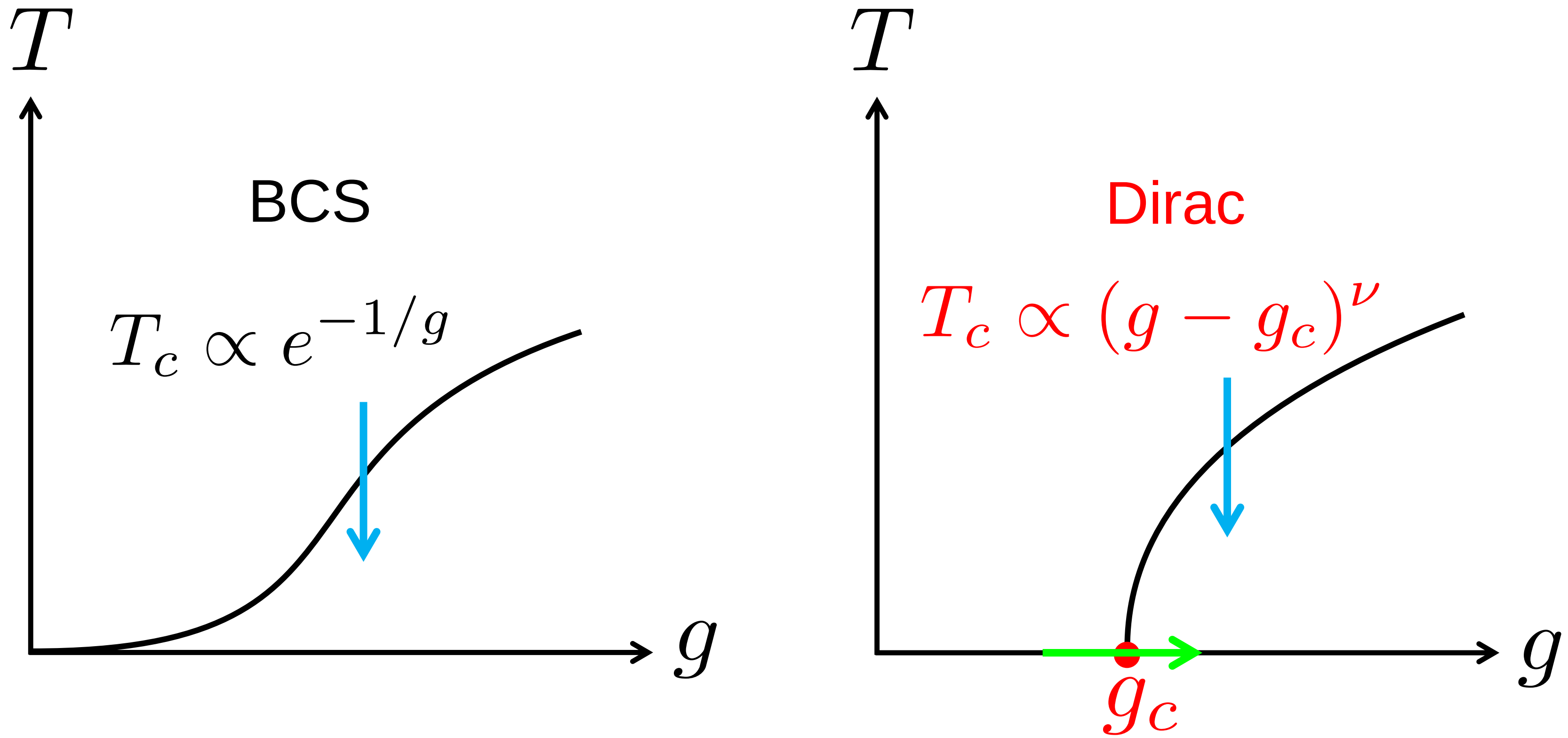
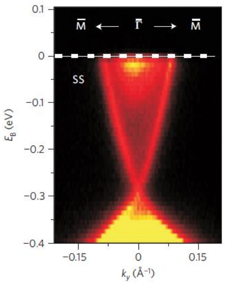


- Thermal phase transition similar to BCS (static GL)
- Dirac: additional **quantum phase transition** (T=0)!

Roy, Juričić, Herbut, PRB ‘13; Nandkishore, JM, Huse, Sondhi, PRB ‘13

Quantum Ginzburg-Landau theory

- Quantum phase transition: driven by **quantum** rather than thermal fluctuations
- Described by **time-dependent** GL action:



$$S = \int dx\, dt \Big[i\bar{\psi}(\gamma^0 \partial_t + c_f \boldsymbol{\gamma} \cdot \nabla)\psi$$

Dirac fermion ψ

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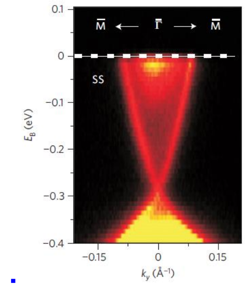

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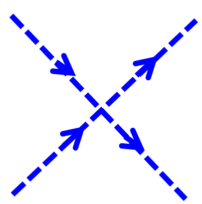
Cooper pair field ϕ

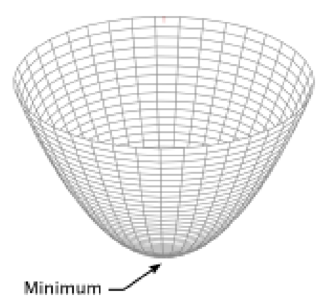
$$+ |\partial_t \phi|^2 - c_b^2 |\nabla \phi|^2 - \alpha |\phi|^2 - \beta |\phi|^4$$

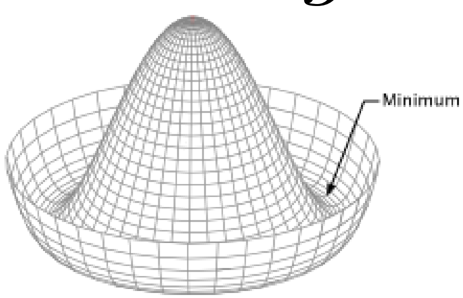


$$\alpha \propto g_c - g$$

g

g_c





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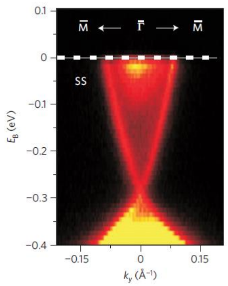

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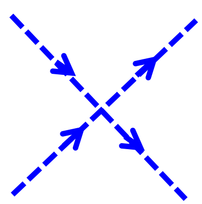
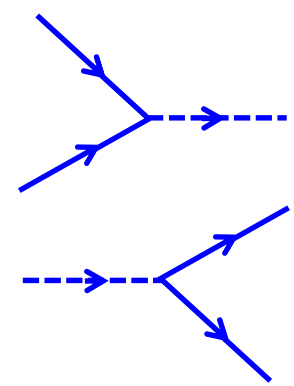


$$+h(\phi^* \psi_\uparrow \psi_\downarrow + \mathrm{c.c.})\Big]$$

pair formation/breaking



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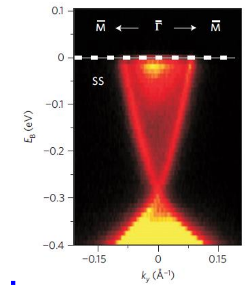

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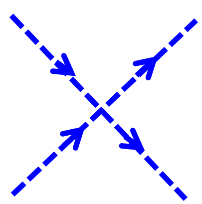
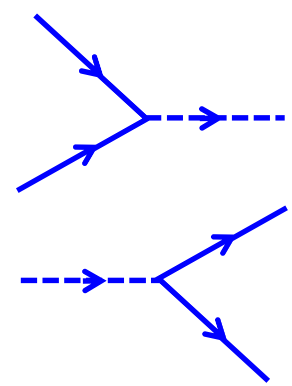
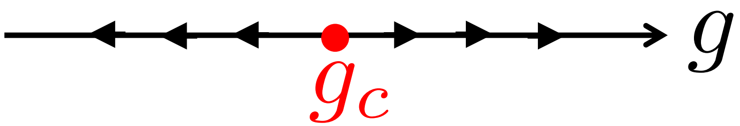




- **True QFT**, but not (yet) relativistic… two velocities

Quantum critical point

- Quantum critical point =
  (IR unstable) RG fixed point,
  scale invariance in space **and** time



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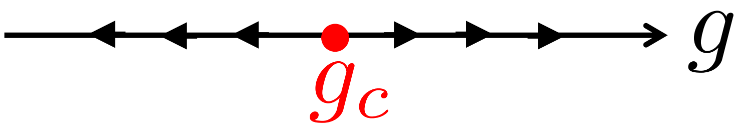
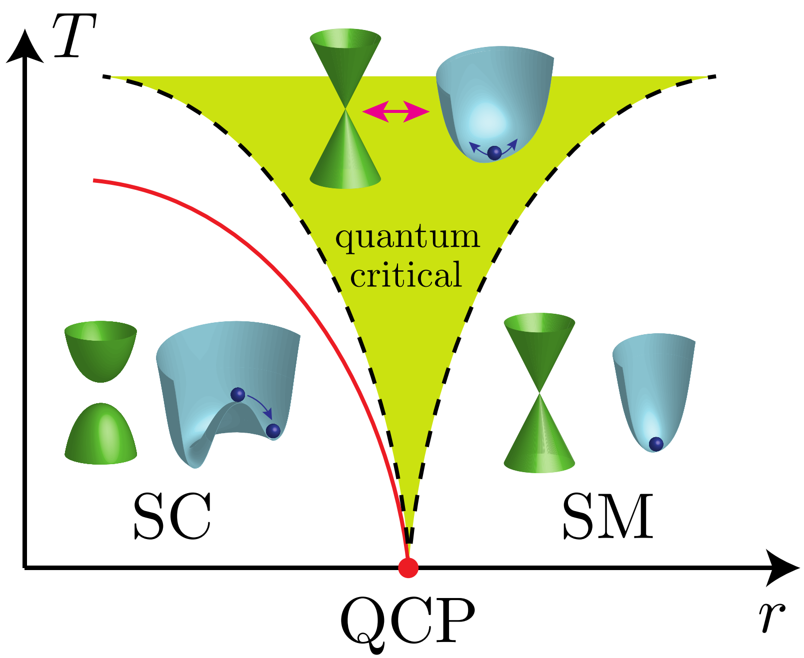


- 1-loop RG: fermion/boson velocities flow to a common value = **emergent Lorentz invariance**
- In fact, $|\phi|^4$ and Yukawa couplings also flow to common value: **emergent N=2 SUSY**! (*Grover, Sheng, Vishwanath, Science '14; Ponte & Lee, NJP '14*)
- Remains true at 3-loop level (*Zerf, Lin, JM, PRB '16*)

A new universality class

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + |\partial_\mu\phi|^2 + r|\phi|^2 + h^2|\phi|^4 + h(\phi^*\psi^T i\sigma^y\psi + \text{h.c.})$$

- N=2 Wess-Zumino model = 1 chiral multiplet X with W=X^3. Dirac fermion and Cooper pair are superpartners!



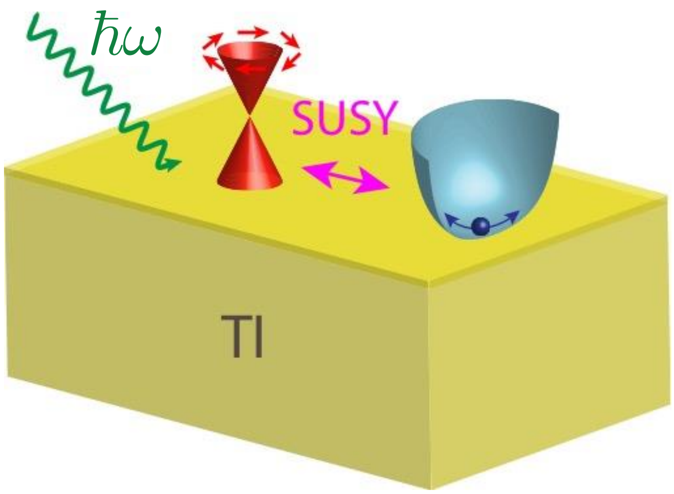
- "SUSY version" of usual Ginzburg-Landau theory

- Thermodynamic critical exponents known at 4 loops (*Zerf, Lin, JM, PRB '16; Zerf et al., PRD '17*)

A new quantum universality class

- **Quantum** phase transition: besides (static) critical exponents, also universal **dynamical** properties

- In (2+1)D, AC conductivity $\sigma(\omega)$ is a **universal constant** at quantum critical points (*Damle & Sachdev, PRB '97*)



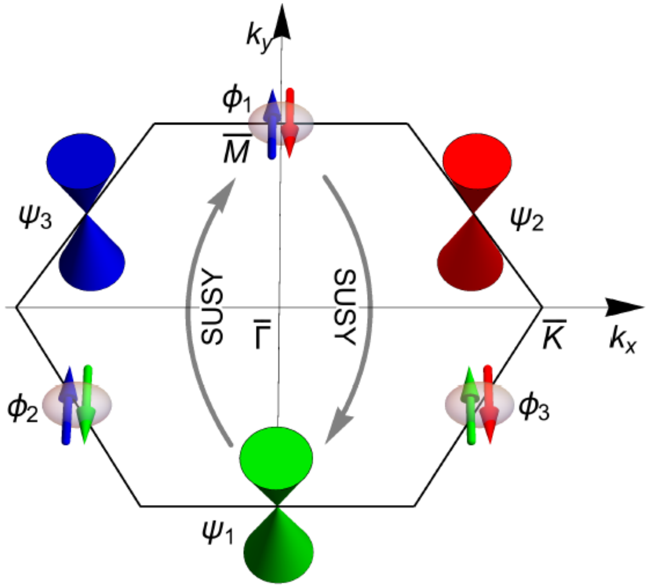
- Can be computed exactly for this transition despite strong coupling, unprecedented in D > 1+1

$$\sigma(\omega) = \frac{5(16\pi - 9\sqrt{3})}{243\pi}\frac{e^2}{\hbar} \approx 0.2271\frac{e^2}{\hbar}$$

Witczak-Krempa & JM, PRL '16

SUSY and mirror symmetry

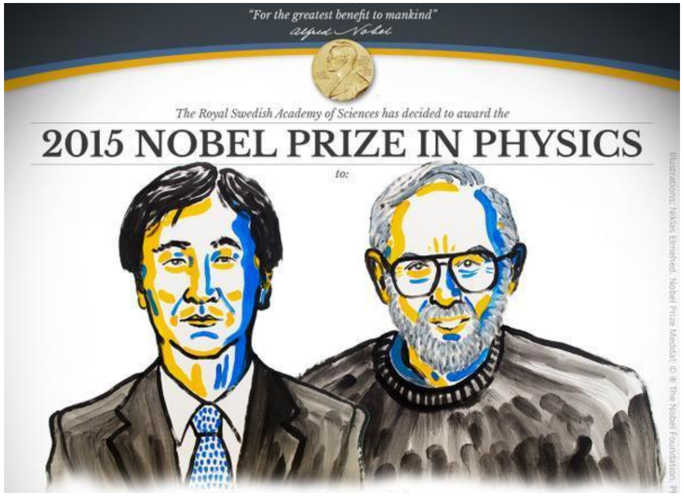
- Topological insulators have **odd #** of surface Dirac fermions
- **3** Dirac fermions: different universality class = XYZ model (*Aharony et al., NPB '97*)
- Infrared dual to N=2 $SQED_3$ with one "flavor" of matter fields
- Bosonic Cooper pairs & Dirac fermions are dual to U(1) vortices, "photons" and "photinos"



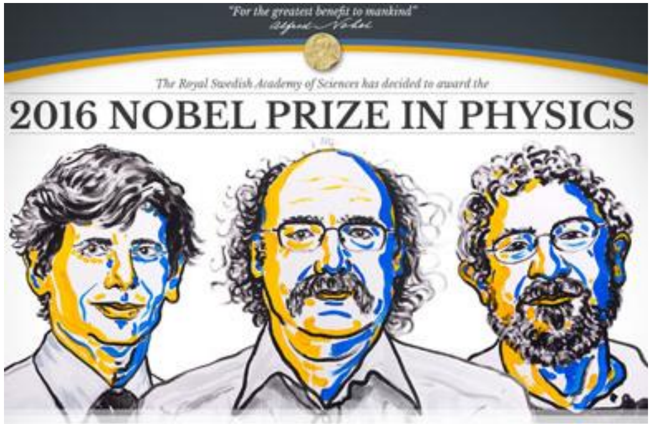
Jian, Lin, JM, and Yao, PRL '17

Conclusion

- An exciting time for HEP and CMP alike!
- Theory predictions inspired by HEP have led to “topological revolution” in CMP
- “Dirt” as potential platform to observe exotic (emergent) HEP phenomenology?



neutrino oscillations



topological phases of matter

To see a World in a Grain of Sand
And a Heaven in a Wild Flower
Hold Infinity in the palm of your hand
And Eternity in an hour

(William Blake, *Auguries of Innocence*, 1863)