Non-Gaussian cosmological perturbations: The inflationary trispectrum

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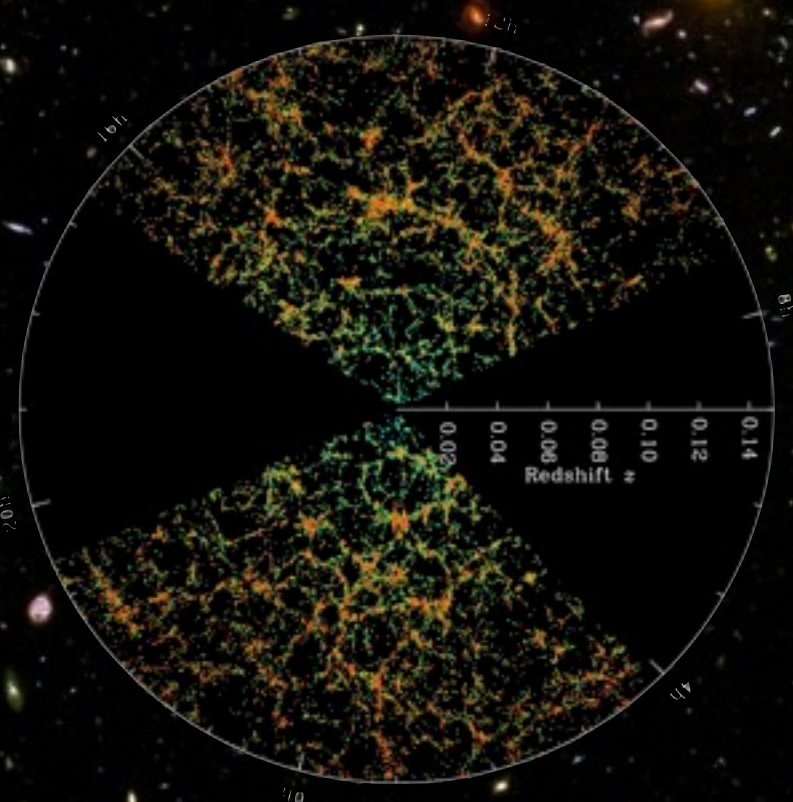
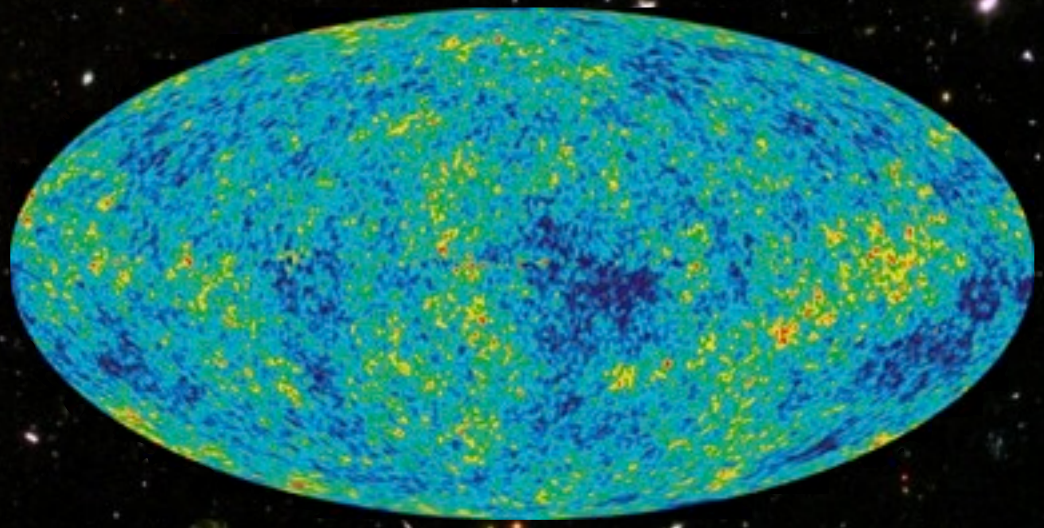
Motivation:

Everything originates from quantum fluctuations of the inflaton





Slow-roll Inflation

- Overdamped scalar field

$$\mathcal{L} = a^3 \left(\frac{1}{2}\dot{\phi}^2 - V(\phi) \right)$$

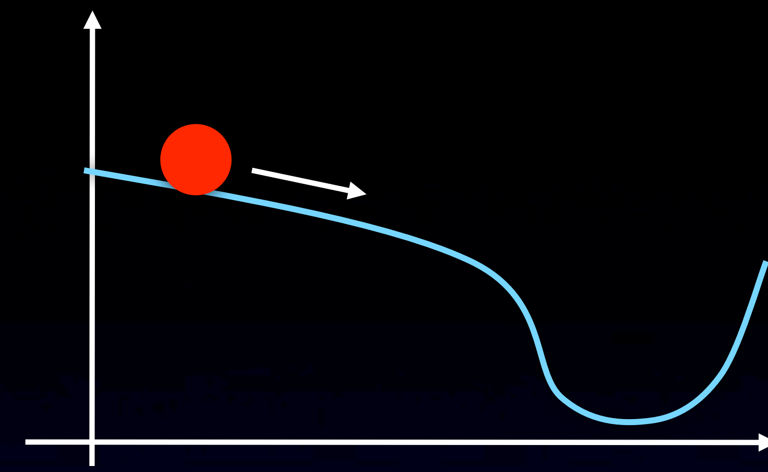
$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V' = 0$$

$$3H^2 = \rho \approx V$$

Slow-roll conditions

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2 << 1$$

$$\eta = m_p^2 \frac{V''}{V} << 1$$



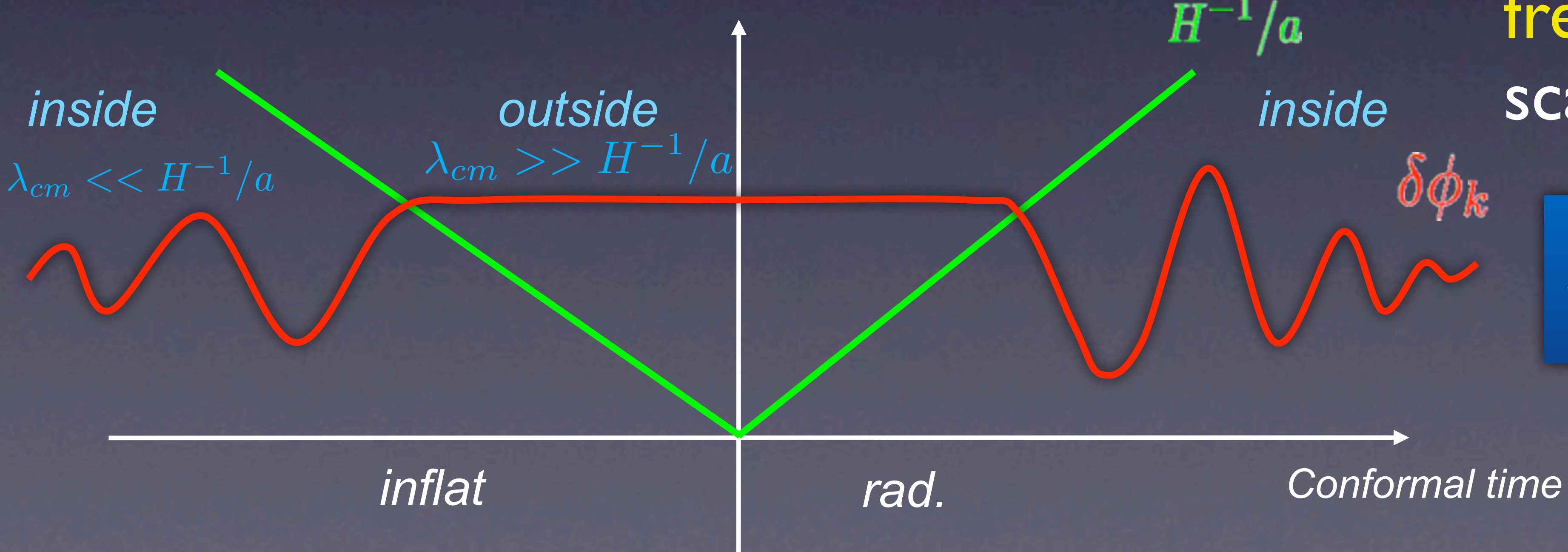
$$ds^2 = -dt^2 + a^2(t)dx^2$$

$$H^2 = \frac{\dot{a}}{a}$$

Quantum fluctuations of the inflaton

- For any light scalar field $V'' << H^2$

quantum fluctuations gets amplified, stretched and freeze on super-horizon scales during inflation



$$\langle \delta\phi_k \delta\phi_{k'} \rangle \approx \frac{H^2}{2k^3}\delta(\vec{k} + \vec{k'})$$

Curvature perturbation

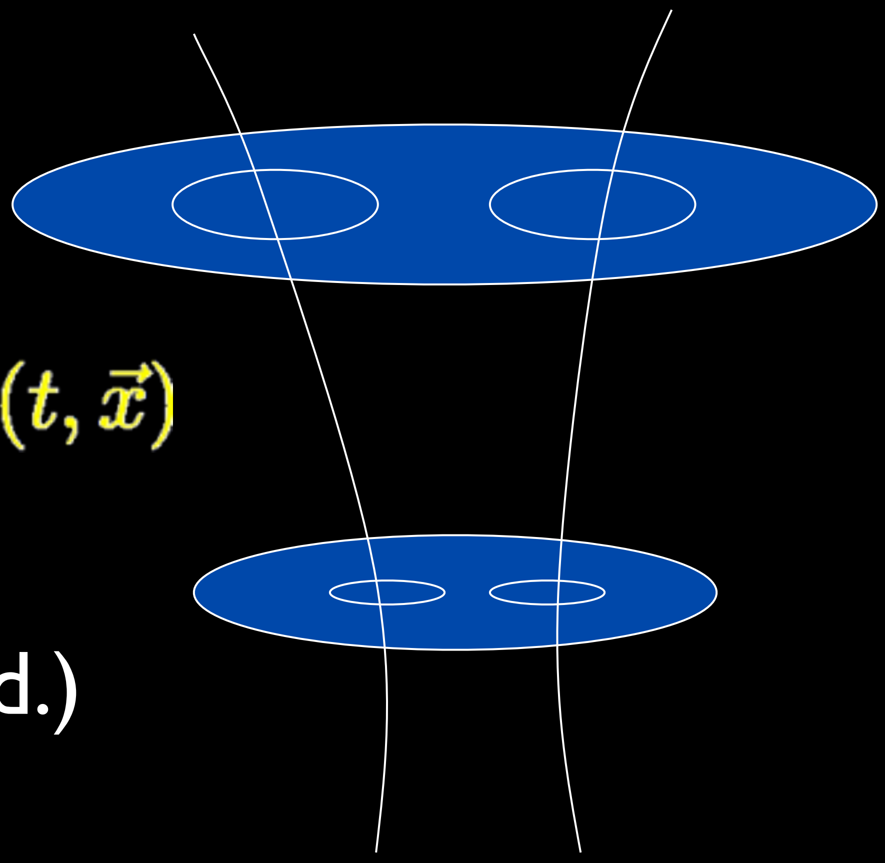
- Single field inflation (adiabatic pert.): only one clock

$\Rightarrow$ all horizon sized patches goes through same unpert. histories

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(\vec{x})} d\vec{x}^2 \ , \qquad \phi(t)$$

or

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \ , \qquad \phi(t) + \delta\phi(t, \vec{x})$$



where ζ is conserved on super-horizon scales (just a rescaling of coord.) and the linear gauge-transf. is

$$\zeta = \frac{H}{\dot{\phi}} \delta\phi$$

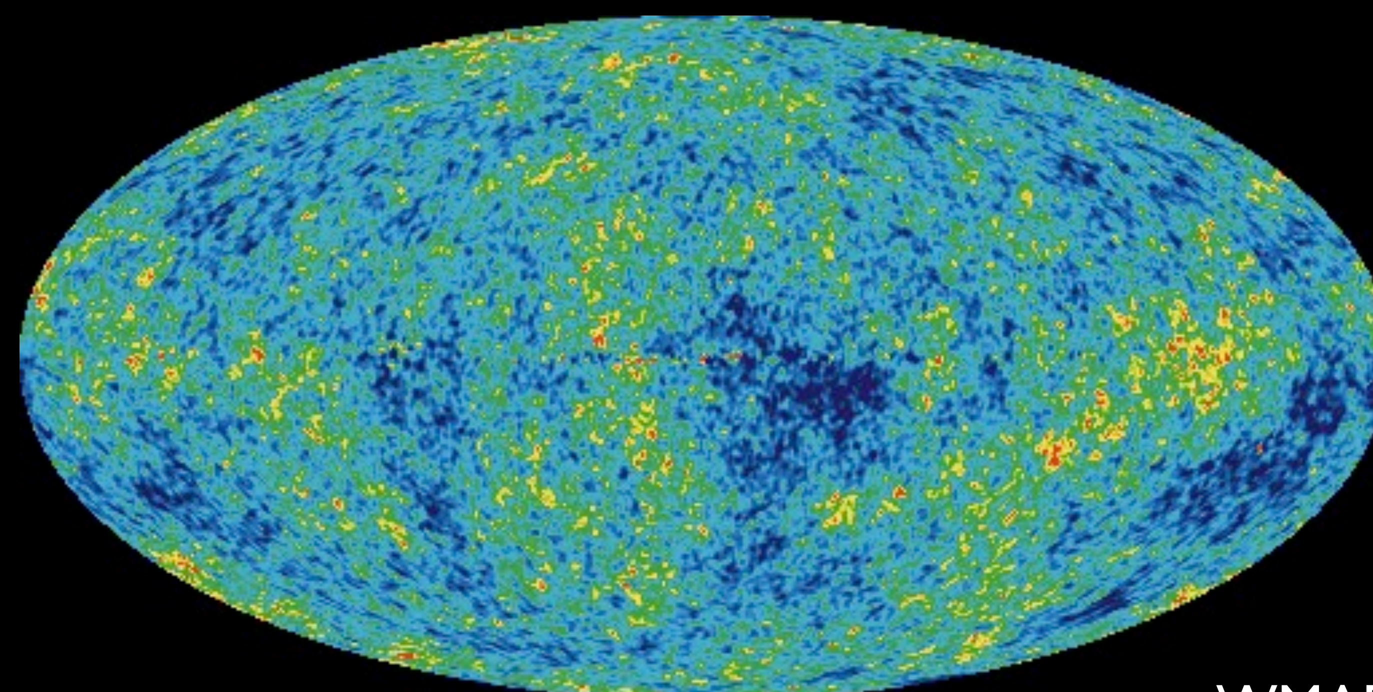
[Salopek,Bond '91]

CMB

- The curvature perturbations leads temperature fluctuations in the CMB photons

$$\frac{\delta T}{T} = -\frac{1}{5}\zeta$$

- Allows us to fit the spectrum of primordial curvature perturbations to data



WMAP5

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) \mathcal{P}_\zeta$$

(Non) Gaussianity

Fluctuations are practically Gaussian assuming:

- Single field inflation
- Canonical kinetic term
- Sufficiently smooth potential (slow-roll)

(Chaotic inflation, Hybrid inflation, SUGRA inflation, F-term inflation, D-term inflation, A-term inflation, Brane inflation, natural inflation, super-natural inflation, Hilltop inflation, etc...)

Detection of NG could rule out the **largest** class of inflationary models
and
shed light on other **non-standard** models

Major breakthrough in cosmology

Shapes of Non-Gaussianity

- To leading order, the perturbations are encoded in the two-point fuction

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_\zeta(k)$$

- A non vanishing three point function

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$$

is a signal of non-Gaussianity

- Introduce dimensionless f_{NL} :

$$f_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) \ + \ perm.$$

as a measure of non-Gaussinity

- Similarly

$$\tau_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle / P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_{14}) \ + \ perm.$$

Non-Gaussianity: Single field slow-roll

- Perturbations conserved on super-horizon scales: NG is computed at hoizon crossing

- Bispectrum from 3-point interaction

$$f_{NL} \approx \epsilon \qquad (\epsilon \approx 0.2)$$

[Maldacena '02,
Acquaviva, Bartolo, Matarrese, Riotto '02]

- Trispectrum from connected 4-point interaction and graviton exchange

$$\tau_{NL} \approx \epsilon$$

[Seery, Lidsey, Sloth '06,
Seery, Sloth, Vernizzi '08]

Semiclassical estimates

Squeezed limit:

- Consider

  $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ for $k_1 << k_2,\ k_3$

- The long wavelength mode will rescale the spatial coord. of the backgound of the two other modes

$$ds^2 = -dt^2 + e^{2\zeta_1} a^2(t) dx^2$$

- Taylor expanding the two point function in the background of ζ_1

$$\langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} = \langle \zeta_{k_2} \zeta_{k_3} \rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle + \dots$$

$$\Rightarrow \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1} \zeta_{k_1} \rangle\, k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$\Rightarrow \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

[Maldacena ’02]

- Also

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \sim (n_s - 1)^2 \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle$$

- So

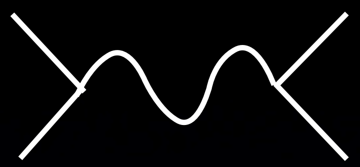
$$f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim \epsilon \qquad \tau_{NL} \sim \frac{\langle \zeta^4 \rangle}{\langle \zeta^2 \rangle^3} \sim \epsilon^2$$

!!!

$k_1 << k_2,\ k_3,\ k_4$

Counter collinear limit:

- Exchanged graviton momentum goes to zero



- The effect of the long wave graviton is to rescale the background

$$dx^2 \to dx^2 + \gamma_{ij} dx^i dx^j \;\; \Rightarrow \;\; k^2 \to k^2 - \gamma_{ij} k^i k^j$$

- Taylor expanding in the new background

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_{\gamma_B} \sim \langle \zeta_{k_1} \zeta_{k_2} \rangle + \gamma_{ij\,B} \frac{\partial}{\partial \gamma_{ij\,B}} \langle \zeta_{k_1} \zeta_{k_2} \rangle + \ldots$$

$$\sim \langle \zeta_{k_1} \zeta_{k_2} \rangle + \gamma_{ij\,B} k^i k^j \frac{\partial}{\partial k^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle + \ldots$$

we get semiclassical contribution to the bispectrum from correlation between a pair of two-point functions due to the long wave graviton

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \approx \left\langle \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\gamma_B} \langle \zeta_{k_3} \zeta_{k_4} \rangle_{\gamma_B} \right\rangle$$

$$\Rightarrow \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle \approx \epsilon_{ij} \frac{k^i k^j}{k^2} \epsilon_{ij} \frac{k'^i k'^j}{k'^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle \langle \gamma_B \gamma_B \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle$$

$$\approx \text{Polarization} \times \epsilon \left\langle \zeta^2 \right\rangle^3$$

[Seery, Sloth, Vernizzi ’08]

$$\Rightarrow \quad \tau_{NL} \sim \frac{\left\langle \zeta^4 \right\rangle}{\left\langle \zeta^2 \right\rangle^3} \sim \epsilon$$

Obsevational bounds

Present:

$$f_{NL} \leq 100 \qquad \tau_{NL} \leq 10^8$$

In foreseeable future we can probe:

$$f_{NL} \sim 3 \qquad \tau_{NL} \sim 560$$

[Komatsu,Spergel '00] [Kogo,Komatsu '06]

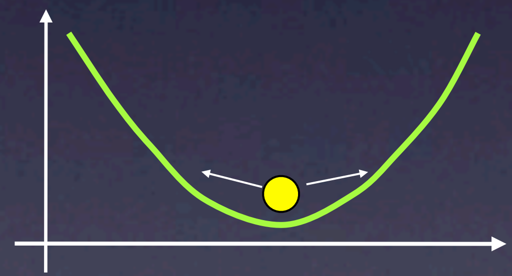
Models of large non-Gaussianity

Simplest model: Curvaton model

[Enqvist, Sloth '01, Wands, Lyth '01, Moroi, Takahashi '01]

- Light scalar field behaving as a test field during inflation
- Dominates the universe sometime after inflation and then decays

$$V(\sigma) = \frac{1}{2} m^2 \sigma^2$$



$$\frac{1}{2}\langle \dot{\sigma}^2 \rangle \approx \frac{1}{2} m^2 \langle \sigma^2 \rangle \quad \Rightarrow \quad P \approx 0$$

$$\rho_\sigma \propto 1/a^3 \;, \quad \rho_{rad} \propto 1/a^4$$

- Non-linear relation between $\delta\sigma$ and ζ

$$\zeta = \left(\frac{\rho_\sigma}{\rho}\right)_{dec} \left[\frac{\delta\sigma}{\sigma} + \frac{1}{2}\left(\frac{\delta\sigma}{\sigma}\right)^2\right] . \quad \Rightarrow \quad f_{NL} = \frac{5}{4}\left(\frac{\rho}{\rho_\sigma}\right)_{dec}$$

[Lyth, Ugarelli, Wands '02]

$$\tau_{NL} = \frac{36}{25} f_{NL}^2$$

[Byrnes, Sasaki, Wands '06]

More complicated models:

- Since we do not know the microscopic origin of inflation, from an Occam's razor point of view single field slow-roll is preferred - we expect small non-Gaussianity
- On the other hand, when inflation is embedded into a fundamental theory like string theory, it typically carries with it more complicated structures - typically leading to large non-Gaussianity

Example: DBI inflation

$$S = \int d^4x\sqrt{-g}\left[f^{-1}(\phi)\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi)\right]$$

$|f_{NL}| \sim 100$ for model to be consistent

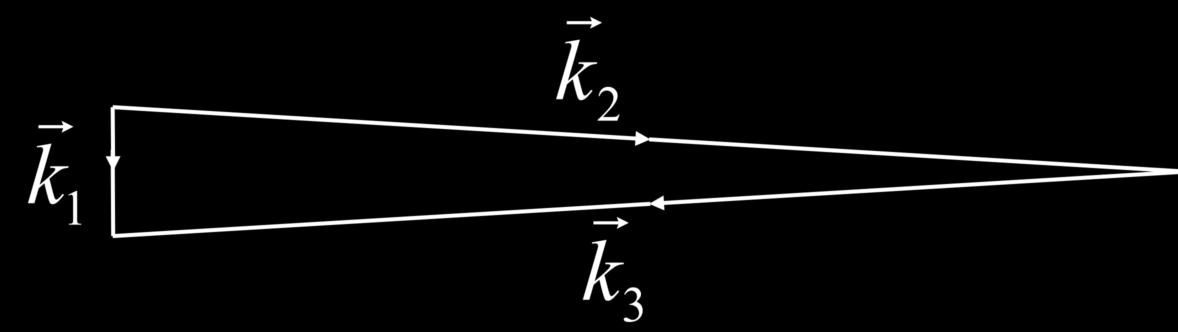
Conclusions:

- Primordial non-Gaussianity can provide an interesting consistency check for different models of inflation
- If primordial non-Gaussianity is discovered, it could be very powerfull to discriminate between different models of inflation, since they will each have their distinct shape of non-Gaussinity

Definition and shape of f_{NL}

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle \equiv (2\pi)^3\delta^{(3)}(\Sigma_i\vec{k}_i)\frac{3}{5}f_{NL}P_\zeta(\vec{k_1})P_\zeta(\vec{k_2}) + perm.$$

- Squeezed: $k_1 << k_2, k_3$

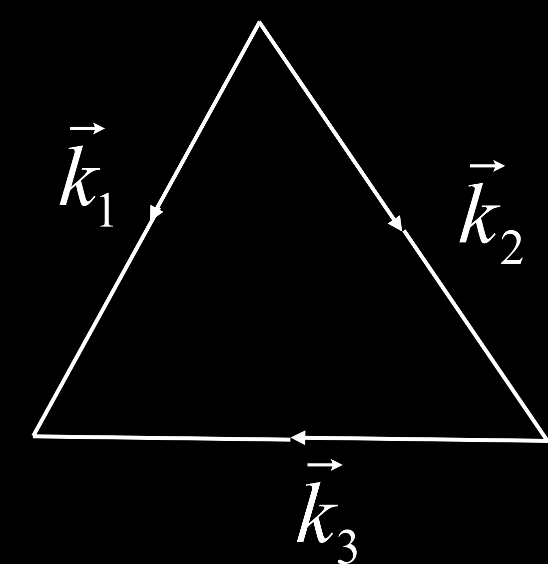


f_{NL}^{local} maximal in squeezed limit

$$\zeta(\vec{x}) = \zeta_g(\vec{x}) + \frac{3}{5}f_{NL}^{local}\zeta_g^2(\vec{x})$$

Multifield inflation

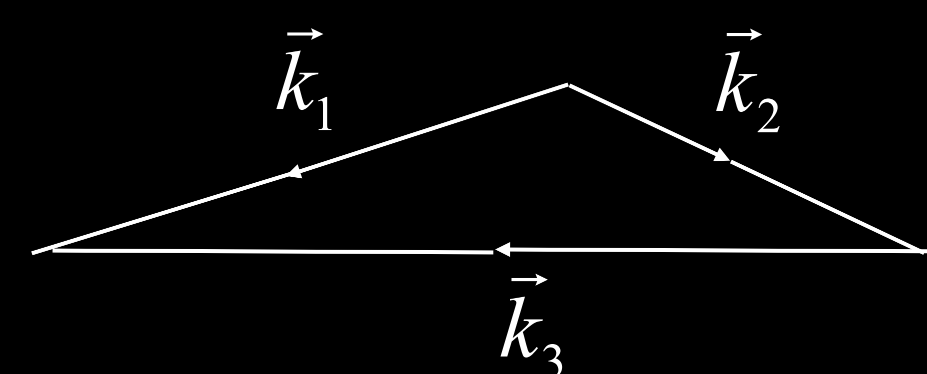
- Equilateral: $k_1 \sim k_2 \sim k_3$



f_{NL}^{equil} maximal in the equilateral limit

Non canonical kinetic term

- Folded/flattened: $k_3 \sim 2k_1 \sim 2k_2$



f_{NL}^{folded} maximal in the folded limit

Non vacuum initial cond.