Toy Model of AdS/CFT: Matrix Models

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My Motivation for Physics

- I am curious physical law on ultimate situations which are happening in astrophysics and cosmology scale
- Therefore my interests aim at understanding astro-scale events, for example, big bang (why and how universe was born?), cosmology, black holes (what is inside of it?), neutron stars (extreme nuclear physics) etc…
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Focus on today's talk

New development of Non-pert. Quantum Gravity

- Discovery of gauge/gravity dual (AdS/CFT)
- This duality says that quantum non-pert. quantum gravity in asymptotic AdS space is equivalent to the physics of some gauge theory with less dimension
- For example, the S-matrix of black hole formation/evaporation must be unitary (for Hawking's famous information paradox), since we can map this process by putting it in asymptotic AdS and consider it from dual gauge theory viewpoint, which is always ***unitary***

Now can AdS/CFT answer the physical questions we would like to know ?

=> I believe in principle this is yes, but not yet in practice.

AdS/CFT should be better understood, since we don't understand many nature of black holes

- What is wrong with the original Hawking's argument?
- If information is back, how will it be back?
- How do we see the quasi-local gravity from gauge theory?
- How do we see the non-local effects for black holes from gauge theory?
- How do we see the black hole complementarity?
- etc…

- In this talk, we will concentrate on unitarity issues & information problem of quantum gravity
- Hawking's original argument is based on semi-classical approximation. And he showed that black hole radiate thermally, so information is lost as far as black hole horizon is formed, due to Bogoliubov transformation.
- This contradicts with gauge/gravity duality, since black hole radiation is dual to unitary gauge theory evolution.
- So how things can be consistent?

Gauge/gravity correspondence

- AdS/CFT correspondence; $G_N \sim 1/N^2$, $l_s^2 R_{AdS} \sim \lambda^{-1/2}$
- Semiclassical approx. is $G_N \to 0$, $l_s^2 R_{AdS} \to 0$ with leading G_N correction *only for matter, but not for geometry*
- This means, Hawking's argument is at $N \to \infty$ theory in the dual gauge theory (with infinite 't Hooft)
- But note that in $N = \infty$, *information* "can" be lost
- This is because in this limit, we have infinite number of states for the system. System can absorb arbitrary amount of information as heat bath
- Also note that the number of states are infinity, Poincare recurrence time also becomes infinity as

  (recurrence time scale) ~ exp(S) ~ exp(N^2)
- On the contrary, if N is finite, then the field theory spectrum is discrete (on finite volume), and it evolves as QM system, so information is never lost

- So the question we would like to understand;

  Can we see the non-unitary black hole physics from unitary (at finite N) gauge theory, by taking $N=\infty$?

- Black hole is characterized by its horizon, where classically all information is incoming, and lost
- BH horizon makes all information (ie, correlation functions) decay exponentially at later time since information is absorbed inside the horizon
- Can we see this exponential damping/decay of correlation functions from unitary gauge theory at $N=\infty$?

- We can show this very simplified toy matrix models, where theory is manifestly unitary at finite *N*. (work with J. Polchinski)
- The reason why we work on toy models is that gravity-dual gauge theory are too complicated to solve analytically.
- To get the essence of physics, we simplify gravity-dual gauge theory and analyze completely so that we get general lesson. This is our philosophy.

- One of the model we studied is

(N.I. and J. Polchinski)

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a + \bar{a}^\dagger \bar{a}) + g(a^\dagger X a + \bar{a}^\dagger X^T \bar{a}) \ .$$

- We look up retarded Green function since we care only the late time behavior and found it decays.

$$G(t) = \int d\omega \tilde{G}(\omega) e^{-i\omega t}$$

- This is reflected as that infinite N spectrum is continuous, but at finite N, it is always discrete.

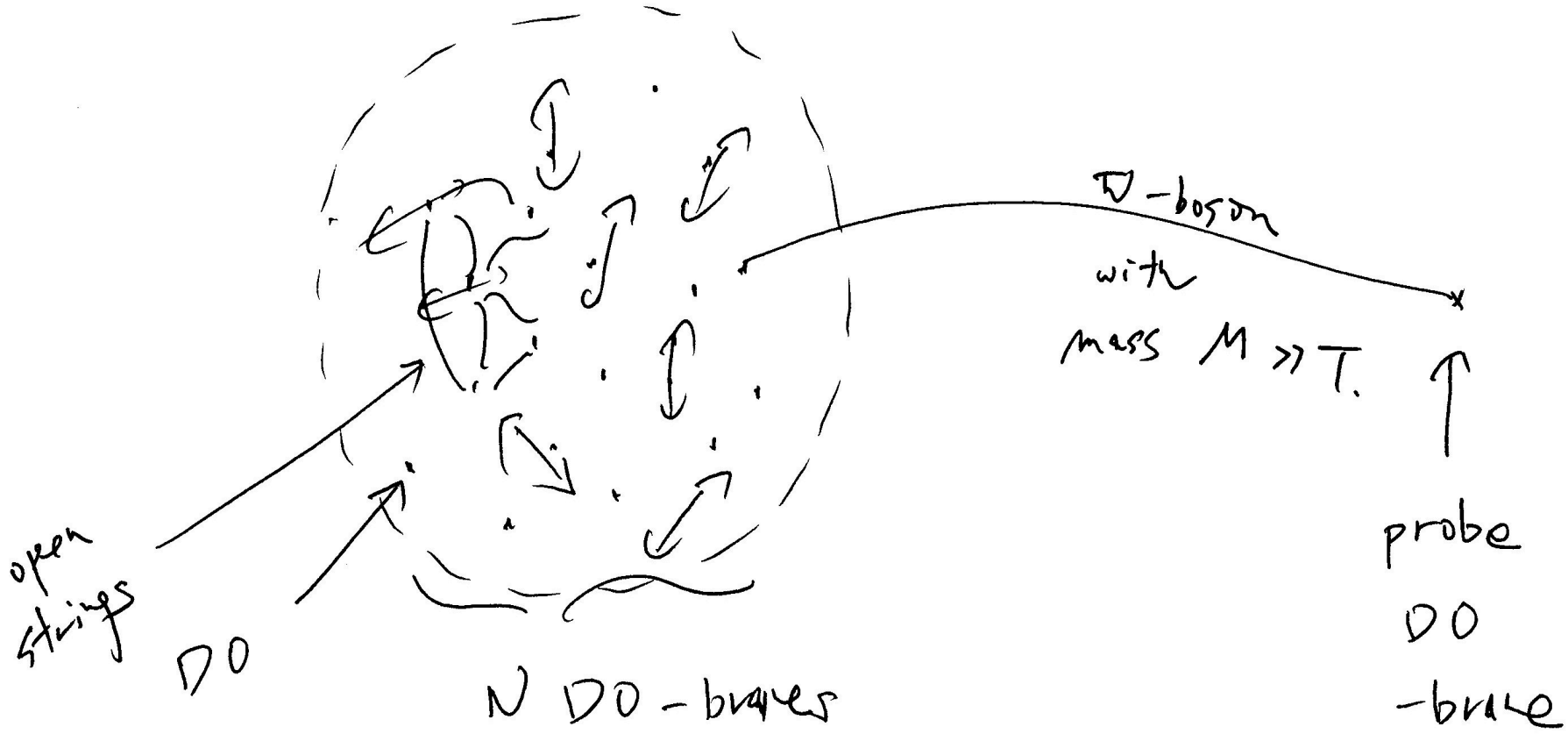
finite N vs. N $= \infty$

- For SU(N) field theory at $N = \infty$,
  is theory unitary?
- N plays the role of IR cut-off of the phase space volume. With finite N, phase space volume is finite
- The reason why we see exponential damping is the same as correlators decay exponentially at late time, for the system which has literary infinite volume

Final Comments

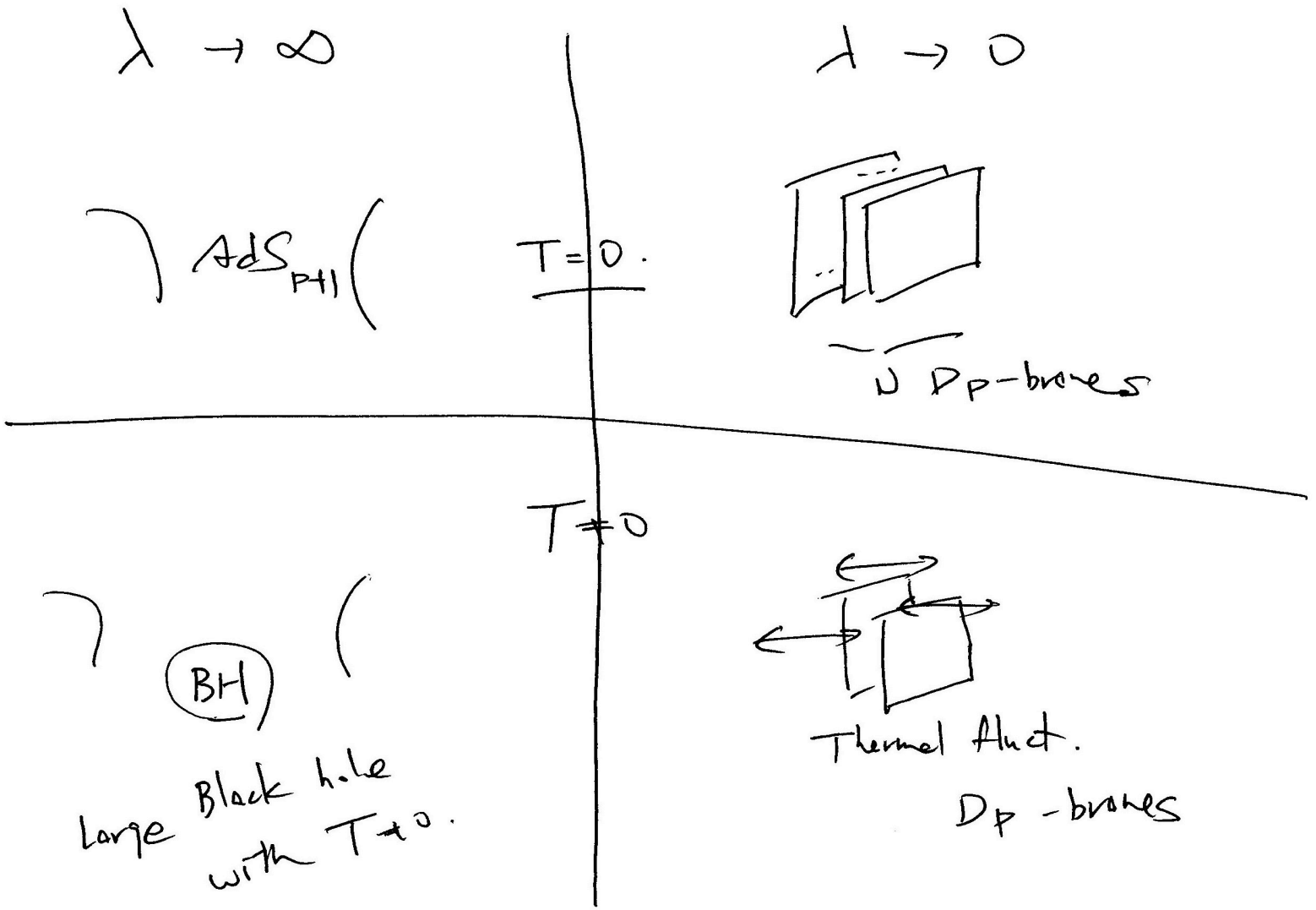
- In *our* toy model, $1/N^2$ corrections does not help to restore the information come back
- This is expected since non-pert. width of $e^{(-N^2)} << 1/N^2$ precision spectrum is crucial
- One goal is to obtain some solvable model where we can obtain *full $1/N^2$ expansion* and re-sum that in systematic way: so that we can obtain the finite N effect
- Systematic understanding of $1/N^2$ expansion is necessary
- More analysis should be done to understand better

Thank You



$$U(N+1) \to U(N) \times U(1)$$

with $0 << T \leq g^{2/3} N^{1/3}$



(Itzhaki-Maldacena-Sonnenschein-Yankielowicz)

$N = \infty$ $\lambda \neq 0$

- We would like to find simple enough toy model where resumming Feynman diagrams is systematic enough so that we can see the full planner physics non-perturbatively
- If we can resum all diagrams, unitarity is guaranteed at finite N
- Our toy model is kind of reduction of D0-brane black hole with a probe D0-brane. We have one U(N) adjoint and one U(N) fundamental representation
- Here, [adjoint field] = black hole degrees of freedom and [fundamental field] = open strings or W-bosons between the black hole and a probe
- Adjoint plays the role of thermal heat bath, whose correlator are thermal one with some mass m, and since probe is away from black hole, W-bosons masses M are heavy enough
- They couple by Yukawa interaction so that U(N) indices are contracted