Non-relativistic Quantum Gravity

Oriol Pujolàs

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Les Houches
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Motivation

A fundamental problem in particle physics: Quantum Gravity

- String Theory is perhaps already close enough
  \[ \rightarrow \text{QFT framework} \]
A fundamental problem in particle physics: Quantum Gravity

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  -> QFT framework

Is it possible to have QG within QFT?

- Asymptotic safety (Weinberg ’79)?
- ‘Hořava Gravity’ (Hořava ’09)
  -> Lorentz invariance

A non-relativistic QG field theory which is (power-counting-) renormalizable
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a non-relativistic QG field theory which is (power-counting-) renormalizable

Really?
Motivation

Early attempt: ‘Higher order gravity’  (Stelle ‘$78$)

\[ \mathcal{L} = \sqrt{-g} \left\{ R + (Riem)^2 \right\} \]

\[ G \approx \frac{1}{p^2 + a p^4} \]

=>  Loops are less divergent

Renormalizable, yes.

But Ghosts!
Hořava’s proposal: Anisotropic Scaling

in the UV, \( w^2 \sim k^{2z} \quad z > 1 \)

\[ G = \frac{1}{w^2 - k^2 - a k^{2z}} \]

=> Loops are less divergent (& no ghosts)
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in the UV, \( w^2 \sim k^{2z} \quad z > 1 \)

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G = \frac{1}{w^2 - k^2 - a k^{2z}} \quad \Rightarrow \quad \text{Loops are less divergent (& no ghosts)}
\]

Eg: \[
L = (\dot{\phi})^2 + \phi \Delta \phi + \frac{\phi \Delta^2 \phi}{M^2} + \frac{\phi^{10}}{\Lambda^6}
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L = (\dot{\phi})^2 + \phi \Delta \phi + \frac{\phi \Delta^3 \phi}{M^4} + \frac{\phi^n}{\Lambda^{n-4}}
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\{ \text{are renormalizable!} \}
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\[
\Rightarrow \text{are renormalizable!}
\]

\[
\Rightarrow \text{Lifshitz exponent } z > 1 \text{ assists renormalizability}
\]

\[
\Rightarrow \text{Lorentz Invariance ‘emerges’ } \text{@ low energies}
\]
Does the ‘trick’ work for gravity?

Lorentz Invariance => (part of the) gauge group broken

=> additional degrees of freedom

One needs to be extra-careful, or else extra d.o.f.s pathological
Non-Relativistic Gravity

Hořava ‘09

3+1 split: (ADM)

dS^2 = (N^2 - N^i N_i) dt^2 - \gamma_{ij} (dx^i + N^i dt)(dx^j + N^j dt)

S = \int d^3x dt N \sqrt{\gamma} \left[ K_{ij} K^{ij} - \lambda (K_i^i)^2 - R_{(3)} - \delta V(\gamma_{ij}, R) \right]

\delta V(\gamma_{ij}, R) = \frac{R_{(3)^2}}{M_P^2} + ... + \frac{R_{(3)^3}}{M_P^4} + ...

t \mapsto \hat{t}(t)

x \mapsto \hat{x}(t, x)

Foliation-preserving diffs

K_{ij} \equiv \frac{1}{2N} \left( \gamma_{ij} - \nabla_i N_j - \nabla_j N_i \right)
Non-Relativistic Gravity

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\[ \delta V (\gamma_{ij}, R) = \frac{R^2_{(3)}}{M_P^2} + ... + \frac{R^3_{(3)}}{M_P^4} + ... + \alpha \left( \frac{\partial_i N}{N} \right)^2 + ... \]

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Non-Relativistic Gravity

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eg, sound speed for scalar graviton

\[ c_0^2 = \left( \frac{2}{\alpha} - 1 \right) \frac{\lambda - 1}{3\lambda - 1} \]

\[ \Rightarrow 0 < \alpha < 2 \]

<table>
<thead>
<tr>
<th>Original proposals</th>
<th>[ \alpha = 0 ]</th>
<th>[ \alpha \to \infty ] (Projectable, ( N(t) ))</th>
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Non-Relativistic Gravity

Isolating the new scalar mode (Stückelbergization)

\[ S = S_{GR} + S[\phi; \alpha, \lambda...] \quad \langle \phi \rangle = t \]

\[ S[\phi] = (\lambda - 1) M_P^2 \int d^4x \frac{1}{(\partial \phi)^2} \left( \Box \phi - \frac{\partial^\mu \phi \partial^\nu \phi}{(\partial \phi)^2} \nabla_\mu \nabla_\nu \phi \right)^2 + ... \]

\[ \phi = t + \chi(t,x) \]

\[ S[\chi] = M_P^2 \int d^4x \left[ \alpha (\partial_i \dot{\chi})^2 - (\lambda - 1) (\Delta \chi)^2 + \dot{\chi}(\Delta \chi)^2 + ... \right] \]

Strong coupling scale \[ \Lambda \approx \frac{\sqrt{|\lambda - 1|}}{M_P} \]
Non-Relativistic Gravity

Present status:

\[ \exists 1 \text{ formulation that is free from instabilities, strong coupling} \ldots \]

(“Projectable” version supplemented with all the operators allowed by symmetries)

Blas, OP & Sibiryakov 0909.3525

@ low energies: Lorentz scalar-tensor theory

deviations from GR (parameterized by \( \alpha, \lambda-1 \ldots \)) can be small

first observational tests place mild bounds \( \alpha, \lambda-1 \approx 10^{-1} - 10^{-2} \)

\[ \left( G_N^{\text{local}} \neq G_N^{\text{cosmo}} \right) \]
QG at a Lifshitz point?

problems / questions:

- is it really renormalizable?

- is it consistent with observations?

  probably stronger bounds from solar system tests

- recovery of Lorentz Inv in matter sector: fine tuning

\[ L = (\dot{\phi})^2 + c_\phi^2 \phi \Delta \phi + \frac{\phi \Delta^3 \phi}{M^4} + ... \]

\( c_\phi^2 \) are (running) coupling constants; generically different in the IR
QG at a Lifshitz point?

- BH physics
- cosmology
  - bouncing cosmologies, generation of scale-inv perturb
- (Dark matter)
- preferred frame effects
  - Solar system anomalies?
Thanks
&
Bon appétit !