Non-relativistic Quantum Gravity

Oriol Pujolàs

CERN Theory Retreat

Les Houches

November 4-6, 09

Motivation

A fundamental problem in particle physics: Quantum Gravity

- String Theory is perhaps already close enough

-> QFT framework

Motivation

A fundamental problem in particle physics: Quantum Gravity

- String Theory is perhaps already close enough

-> ~~QFT framework~~

Is it possible to have QG within QFT ?

- Asymptotic safety (Weinberg '79) ?
- 'Hořava Gravity' (Hořava '09)

-> ~~Lorentz Invariance~~

a non-relativistic QG *field* theory which
is (power-counting-) renormalizable

Motivation

A fundamental problem in particle physics: Quantum Gravity

- String Theory is perhaps already close enough

  -> ~~QFT framework~~

Is it possible to have QG within QFT ?

- Asymptotic safety (Weinberg '79) ?
- 'Hořava Gravity' (Hořava '09)

  -> ~~Lorentz Invariance~~

  a non-relativistic QG *field* theory which is (power-counting-) renormalizable

Really?

Motivation

Early attempt: 'Higher order gravity' (Stelle '78)

$$\mathcal{L} = \sqrt{-g}\,\{R + (Riem)^2\}$$

$$G \approx \frac{1}{p^2 + a\,p^4}$$ => Loops are less divergent

Renormalizable, yes.

But Ghosts!

Motivation

Hořava's proposal: Anisotropic Scaling

in the UV, $w^2 \sim k^{2z}$ $z > 1$

$$G = \frac{1}{w^2 - k^2 - a\,k^{2z}}$$ => Loops are less divergent (& no ghosts)

Motivation

Hořava's proposal: Anisotropic Scaling

in the UV, $w^2 \sim k^{2z}$ $z > 1$

$G = \dfrac{1}{w^2 - k^2 - a\,k^{2z}}$ => Loops are less divergent (& no ghosts)

Eg: $L = (\dot{\phi})^2 + \phi\Delta\phi + \dfrac{\phi\Delta^2\phi}{M^2} + \dfrac{\phi^{10}}{\Lambda^6}$

$L = (\dot{\phi})^2 + \phi\Delta\phi + \dfrac{\phi\Delta^3\phi}{M^4} + \dfrac{\phi^n}{\Lambda^{n-4}}$

} are renormalizable!

Motivation

Hořava's proposal: Anisotropic Scaling

in the UV, $w^2 \sim k^{2z}$ $z > 1$

$$G = \frac{1}{w^2 - k^2 - a\,k^{2z}}$$ => Loops are less divergent (& no ghosts)

Eg:

$$L = (\dot{\phi})^2 + \phi\Delta\phi + \frac{\phi\Delta^2\phi}{M^2} + \frac{\phi^{10}}{\Lambda^6}$$

$$L = (\dot{\phi})^2 + \phi\Delta\phi + \frac{\phi\Delta^3\phi}{M^4} + \frac{\phi^n}{\Lambda^{n-4}}$$

are renormalizable!

=> Lifshitz exponent z > 1 assists renormalizability

=> Lorentz Invariance 'emerges' @ low energies

Does the 'trick' work for gravity?

~~Lorentz Invariance~~ => (part of the) gauge group broken

=> additional degrees of freedom

One needs to be extra-careful, or else extra d.o.f.s pathological

Non-Relativistic Gravity

Hořava '09

3+1 split: (ADM)

$$ds^2 = (N^2 - N^i N_i)dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \int d^3x\, dt\, N\sqrt{\gamma}\left[K_{ij}K^{ij} - \lambda (K^i_i)^2 - R_{(3)} - \delta V\left(\gamma_{ij}, R\right)\right]$$

$$\delta V\left(\gamma_{ij}, R\right) = \frac{R^2_{(3)}}{M_P^2} + \ldots + \frac{R^3_{(3)}}{M_P^4} + \ldots$$

$t \mapsto \hat{t}(t)$

$x \mapsto \hat{x}(t, x)$

$$K_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

Foliation-preserving diffs

Non-Relativistic Gravity

Hořava '09

3+1 split:
(ADM)

$$ds^2 = (N^2 - N^i N_i)dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \int d^3x\,dt\, N\sqrt{\gamma}\left[K_{ij}K^{ij} - \lambda(K_i^i)^2 - R_{(3)} - \delta V\left(\gamma_{ij}, R\right)\right]$$

$$\delta V\left(\gamma_{ij}, R\right) = \frac{R_{(3)}^2}{M_P^2} + \ldots + \frac{R_{(3)}^3}{M_P^4} + \ldots \quad + \alpha\left(\frac{\partial_i N}{N}\right)^2 + \ldots$$

$t \mapsto \hat{t}(t)$

$x \mapsto \hat{x}(t,x)$

$$K_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

Foliation-preserving diffs

Non-Relativistic Gravity

Hořava '09

3+1 split:
(ADM)

$$ds^2 = (N^2 - N^i N_i)dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \int d^3x\,dt\, N\sqrt{\gamma}\left[K_{ij}K^{ij} - \lambda(K_i^i)^2 - R_{(3)} - \delta V\left(\gamma_{ij}, R\right)\right]$$

$$\delta V\left(\gamma_{ij}, R\right) = \frac{R_{(3)}^2}{M_P^2} + \ldots + \frac{R_{(3)}^3}{M_P^4} + \ldots \quad + \alpha\left(\frac{\partial_i N}{N}\right)^2 + \ldots$$

Non-Relativistic Gravity

Hořava '09

3+1 split:
(ADM)

$$ds^2 = (N^2 - N^i N_i)dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \int d^3x\, dt\, N\sqrt{\gamma}\left[K_{ij}K^{ij} - \lambda (K^i_i)^2 - R_{(3)} - \delta V\left(\gamma_{ij}, R\right)\right]$$

$$\delta V\left(\gamma_{ij}, R\right) = \frac{R^2_{(3)}}{M_P^2} + \ldots + \frac{R^3_{(3)}}{M_P^4} + \ldots \quad + \alpha\left(\frac{\partial_i N}{N}\right)^2 + \ldots$$

eg, sound speed for scalar graviton

$$c_0^2 = \left(\frac{2}{\alpha} - 1\right)\frac{\lambda - 1}{3\lambda - 1}$$

$$\Rightarrow 0 < \alpha < 2$$

Original proposals	
$\alpha = 0$ 	$\alpha \to \infty$ (Projectable, $N(t)$) 

Non-Relativistic Gravity

Isolating the new scalar mode (Stückelbergization)

$$S = S_{GR} + S[\phi;\alpha,\lambda\ldots] \qquad \langle\phi\rangle = t$$

$$S[\phi] = (\lambda-1)\,M_P^2\int d^4x\,\frac{1}{(\partial\phi)^2}\left(\Box\phi - \frac{\partial^\mu\phi\partial^\nu\phi}{(\partial\phi)^2}\nabla_\mu\nabla_\nu\phi\right)^2 + \ldots$$

$$\phi = t + \chi(t,x)$$

$$S[\chi] = M_P^2\int d^4x\,\left[\alpha\,(\partial_i\dot{\chi})^2 - (\lambda-1)\,(\Delta\chi)^2 + \dot{\chi}(\Delta\chi)^2 + \ldots\right]$$

Strong coupling scale $\quad \Lambda \approx \dfrac{\sqrt{|\lambda-1|}}{M_P}$

Non-Relativistic Gravity

Present status:

$\exists$ 1 formulation that is free from instabilities, strong coupling ….

("Projectable" version supplemented with all the operators allowed by symmetries)

Blas, OP & Sibiryakov 0909.3525

@ low energies: ~~Lorentz~~ scalar-tensor theory

deviations from GR (parameterized by $\alpha,\ \lambda-1$ …) can be small

first observational tests place mild bounds $\alpha,\ \lambda-1 \approx 10^{-1}-10^{-2}$

$$\left(G_N^{\text{local}} \neq G_N^{\text{cosmo}} \right)$$

QG at a Lifshitz point?

problems / questions :

- is it really renormalizable?

- is it consistent with observations ?

  probably stronger bounds from solar system tests

- recovery of Lorentz Inv in matter sector: fine tuning

$$L = (\dot{\phi})^2 + c_\phi^2 \phi \Delta \phi + \frac{\phi \Delta^3 \phi}{M^4} + \ldots$$

c_ϕ^2 are (running) coupling constants; generically different in the IR

QG at a Lifshitz point?

applications / distinctive predictions

- BH physics

- cosmology

  bouncing cosmologies, generation of scale-inv perturb

- (Dark matter)

- preferred frame effects

  Solar system anomalies?

Thanks

&

Bon appétit !