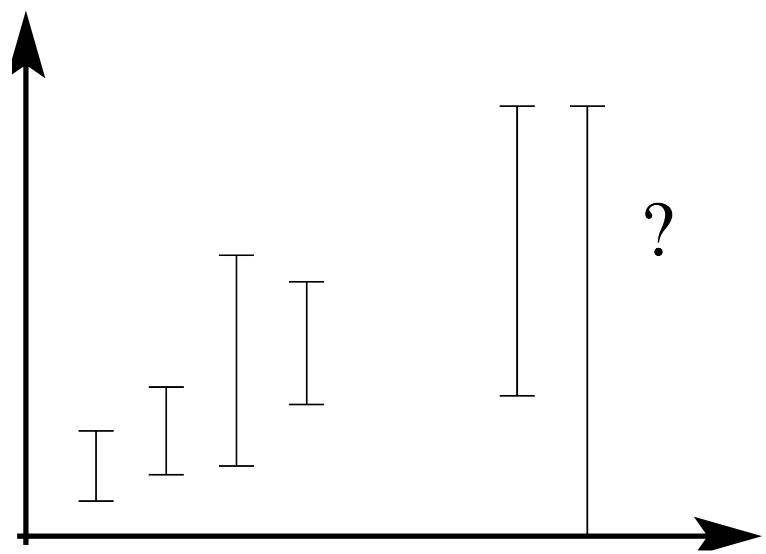

Formal String Theory: Introduction

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People in TH: Luis Alvarez-Gaume, Ignatios Antoniadis, Per Berglund, Matthew Buican, Sergio Ferrara, Steve Giddings, Marta Gomez-Reino, Norihiro Iizuka, Can Kozcaz, Wolfgang Lerche, Nicolas Orantin, Ioannis Papadimitriou, Sara Pasquetti, Chris Petersson, Oriol Pujolas, Samson Shatashvili, Tom Taylor, Angel Uranga, Johannes Walcher

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- Foundational questions in string theory, quantum gravity
- Supergravity
- Topological strings
- Supersymmetric/topological gauge theories
- AdS/CFT and Holography
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- No particular applications but many spinoffs
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What it is good for

- No particular applications but many spinoffs
- mathematics
- framework and ideas for particle physics and cosmology model building
- black hole entropy computations
- computation of perturbative amplitudes in gauge theory, integrability
- applied AdS/CFT: AdS/QCD, RHIC, condensed matter
- . . .

How does the topological string fit in?

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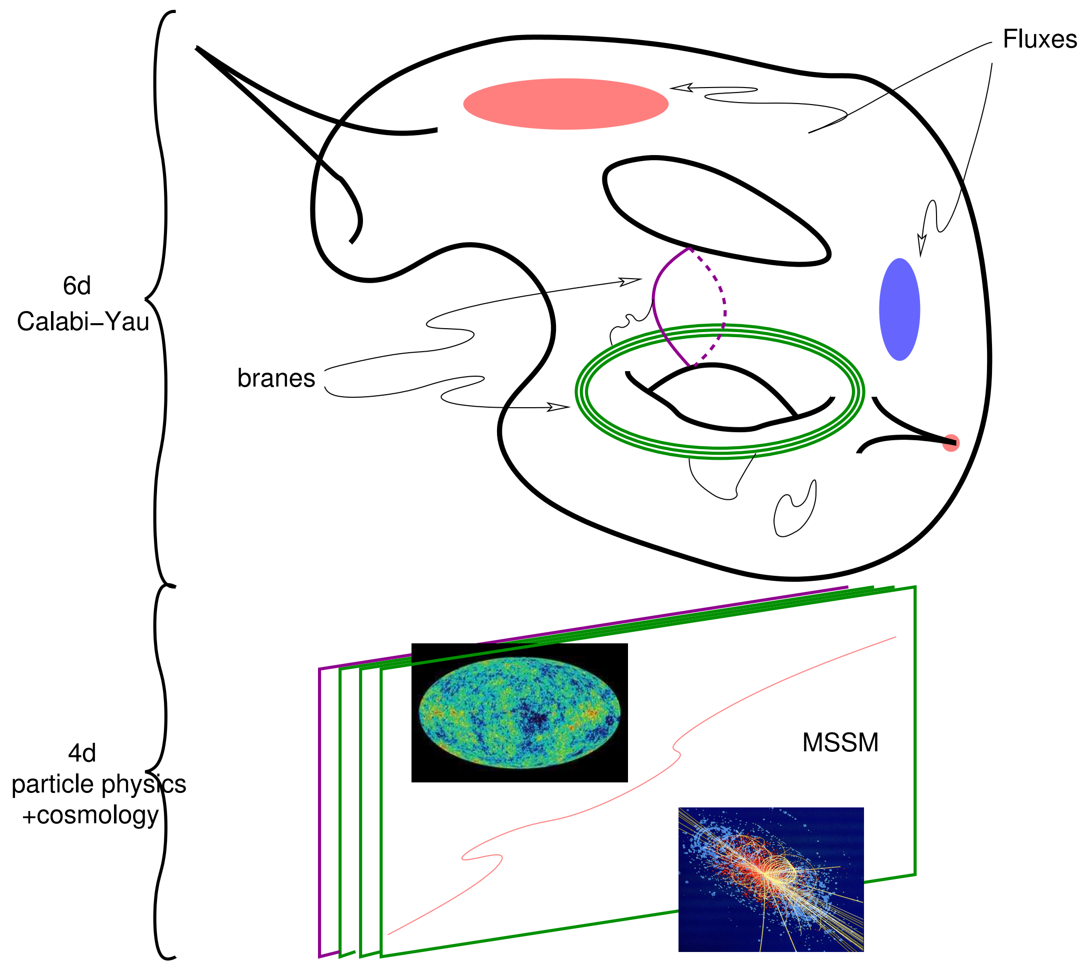
- crit. dim. of superstring $= 10$
- $4 = 10 - 6$; need 6-dimensional compactification manifold (Calabi-Yau)
- for most practical purposes: Only dynamics of low-lying ("topological") modes is important
- completely controlled by topological string. E.g., superpotential $\mathcal{W}$ of chiral fields.
- critical dimension of topological string $= 6 = 10 - 4$.



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- A solvable toy model of string dynamics

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- A solvable toy model of string dynamics

(Ordinary) string theory makes very strong claims about unification of theoretical physics.

Conceptually challenging is the absence (at the fundamental level) of freely adjustable parameters. Ultimate theory should not (be able to) rely on perturbation theory for its definition. It is hard to know where to begin. This is the issue of *background independence*.

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Even more remarkably, the topological string rather directly controls a physically most relevant subsector of the superstring, the vacuum (and BPS) sector. This makes it a very powerful calculational tool ($e.g.$, black hole entropy), and a teacher of valuable lessons about fundamental question in quantum gravity ($e.g.$, transitions changing the topology of spacetime).

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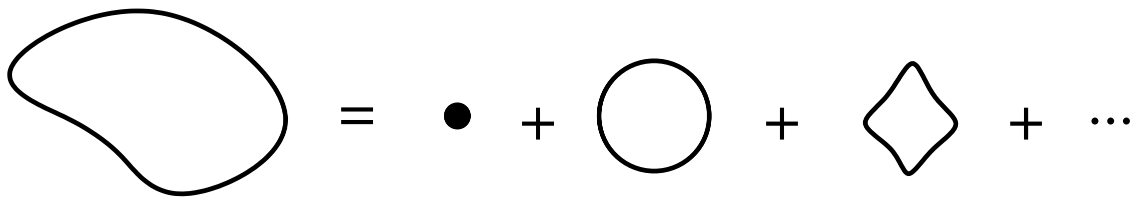
$*$ What follows is a brief sketch of how this is achieved, followed by one example.

In string perturbation theory, we have two-parameter expansion

- α' expansion ($\alpha' = l_s^2$)



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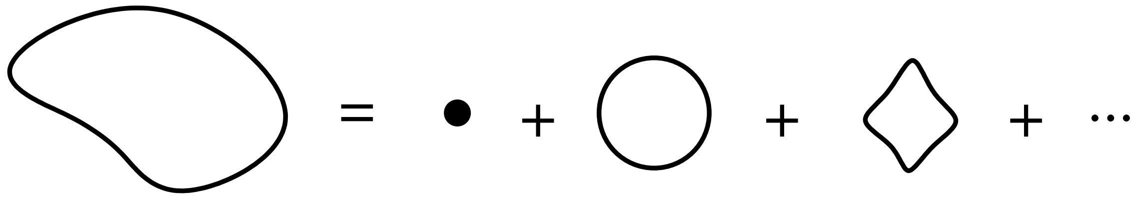

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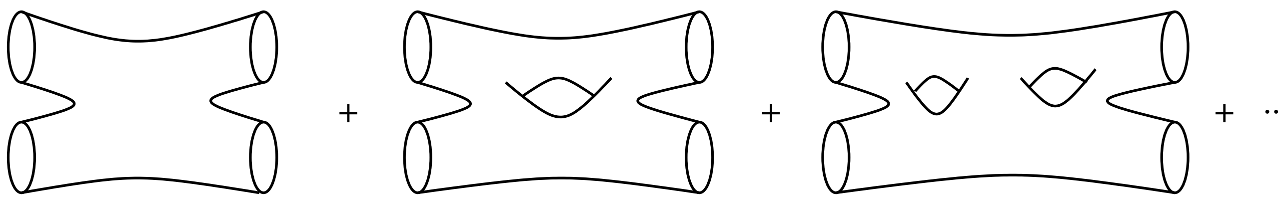
- Loop expansion:



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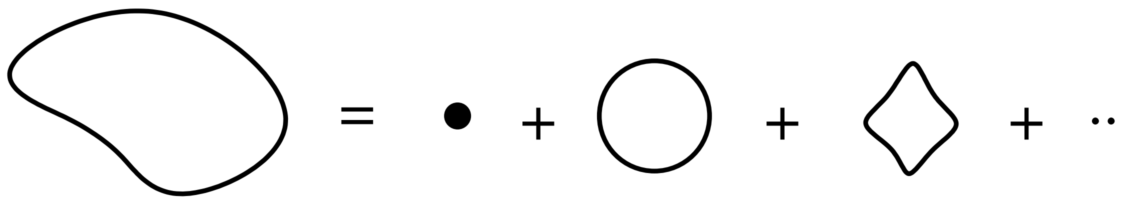

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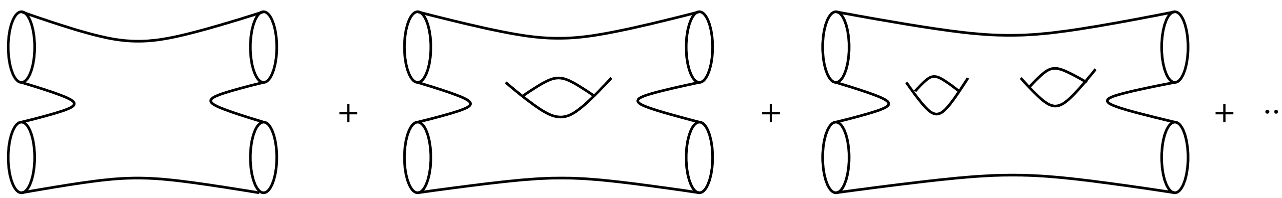

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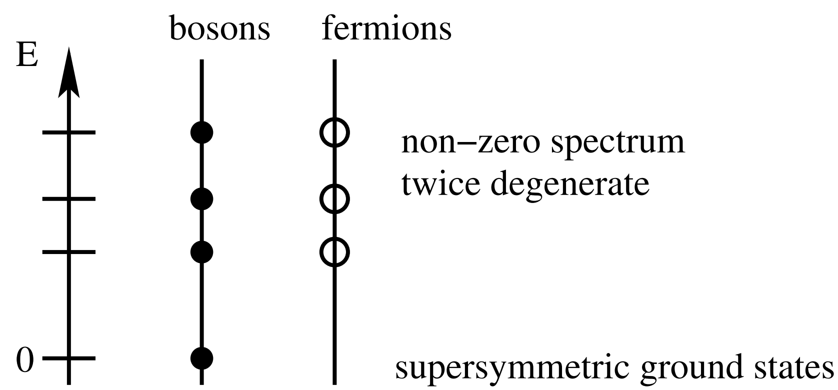
In the topological string, α'-expansion is reduced to non-perturbative contributions, and g_s-expansion is under better technical control.

Topological truncation of supersymmetric quantum theories

Supersymmetric quantum theories possess fermionic symmetries (supercharges) which square to the Hamiltonian. Say quantum mechanics:

$$Q^2 = 0, \qquad , (Q^\dagger)^2 = 0, \qquad \{Q, Q^\dagger\} = 2H$$

Main observation: The states of zero energy are in short (one-dimensional) representations and thereby protected by supersymmetry under deformations of H.



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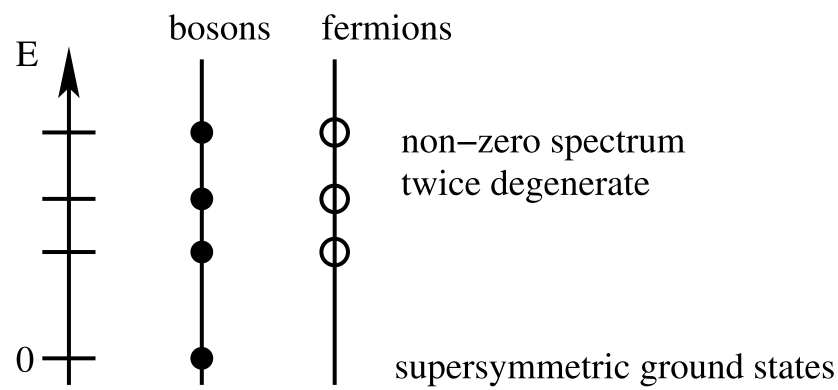

By truncating the theory to supersymmetric ground states, we obtain exact results that are independent of a large number of details of the full physical theory.

Starting from a supersymmetric quantum field theory, we may interpret (after a "twist", which makes supercharges scalars) supercharge Q as BRST charge resulting from "gauge-fixing of some unknown gauge symmetry". The energy-momentum tensor is Q-exact, implying that the theory is independent of worldsheet metric and location of operator insertions.

$$\frac{\delta}{\delta g_{\mu\nu}}\langle\cdots\rangle = \langle T^{\mu\nu}\cdots\rangle = \langle\{Q, G^{\mu\nu}\}\cdots\rangle = 0$$

if $\cdots$ and vacuum is Q-invariant. The topological quantum field theory is the truncation of the operator content to the Q-cohomology classes (kernel modulo image of Q). This is typically a finite-dimensional space. Moreover, the path-integral for computation of correlators of the Q-cohomology classes reduces to a finite-dimensional integral.

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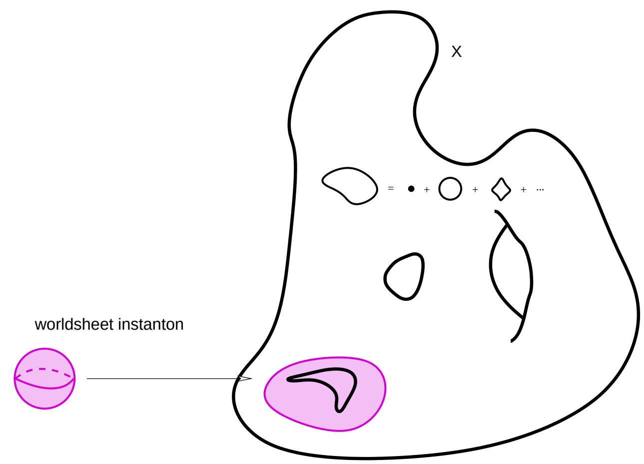
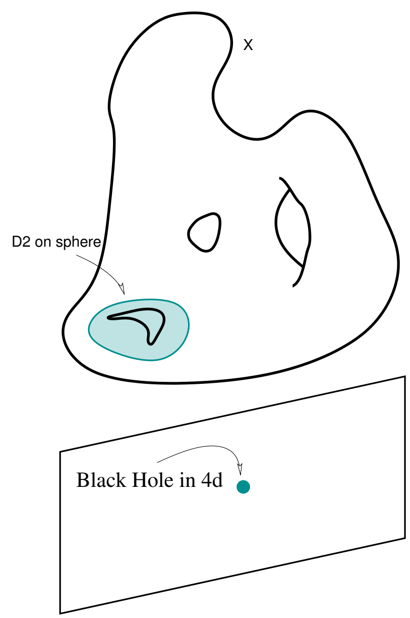
Key points:

- Deformation invariance
- Reduction to finite-dimensional problem

From worldsheet to target space

Topological A-model is, in perturbation theory, a theory of counting worldsheet instantons. This computes certain (supersymmetric) couplings in 4d effective theory of type II on Calabi-Yau (Antoniadis et al.)





via M-theory, this problem is related non-perturbatively to counting degeneracy of BPS states (susy black holes) in four dimensions (Gopakumar-Vafa duality)

An example: Topological quantum foam

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Besides black holes, one might expect a theory of quantum gravity to describe the structure of spacetime at very small scales ($< l_{\mathrm{Planck}} \approx 10^{-33} cm$). It is an old idea (Wheeler, Hawking) that the geometry and topology of spacetime should be subject to wild quantum fluctuations at this scale. "Quantum Foam".

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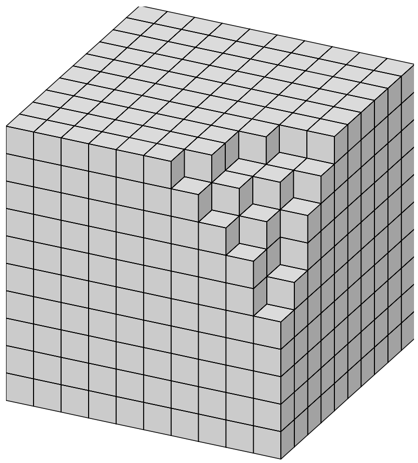
The simplest Calabi-Yau manifold is complex three-dimensional space $\mathbb{C}^3$ (Non-compact!)

$$\begin{aligned}\log Z^{\text{top}}(\mathbb{C}^3) &\sim \frac{1}{g_s^2}\zeta(3) - \frac{1}{12}\log g_s + \sum_{g=2}^{\infty} g_s^{2g-2}\frac{B_{2g}B_{2g-2}}{2g(2g-2)(2g-2)!} \\ Z^{\text{top}} &= \prod_n \frac{1}{(1-q^n)^n} = M(q)\end{aligned}$$

$q = e^{-g_s}$, $M(q)$: MacMahon function. ($asymptotic\ expansion$)

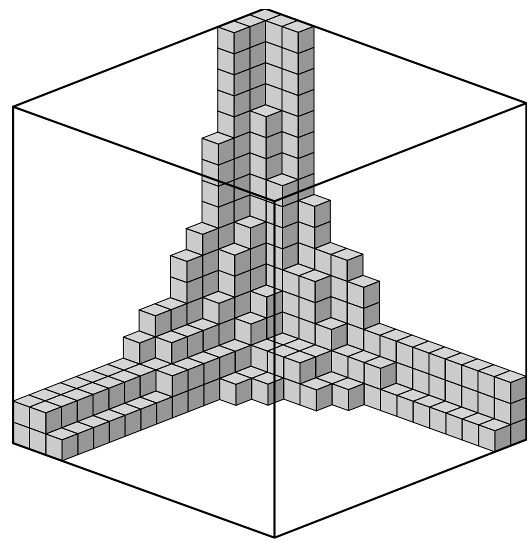
Topological string and melting crystal

MacMahon function is the generating function of three-dimensional partitions, or boxes in the corner of room, or configurations of a melting crystal corner.



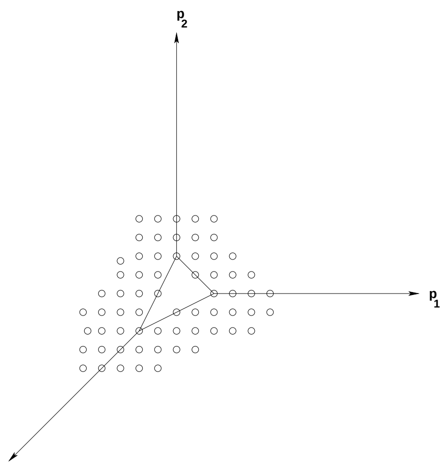
$$M(q)=\sum_{\text{box configs.}} q^{\#\text{boxes}}$$
$$= 1 + q + 3q^2+6q^3 + 13q^4 + 24q^5 + \cdots$$

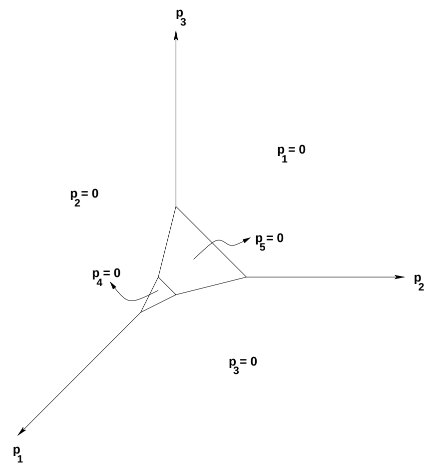
Topological vertex corresponds to imposing boundary conditions along the three axes (Okounkov, Reshetikhin, and Vafa, 2003)

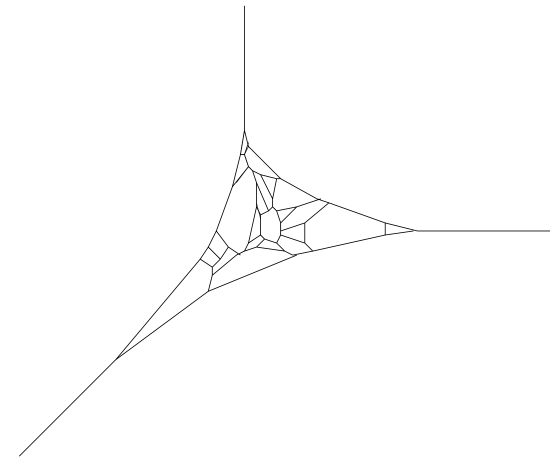


Topological quantum foam

The configurations of melting crystal can be identifed with certain modifications of the underlying geometry $\mathbb{C}^3$. (Iqbal, Okounkov, Nekrasov, Vafa, 2003)





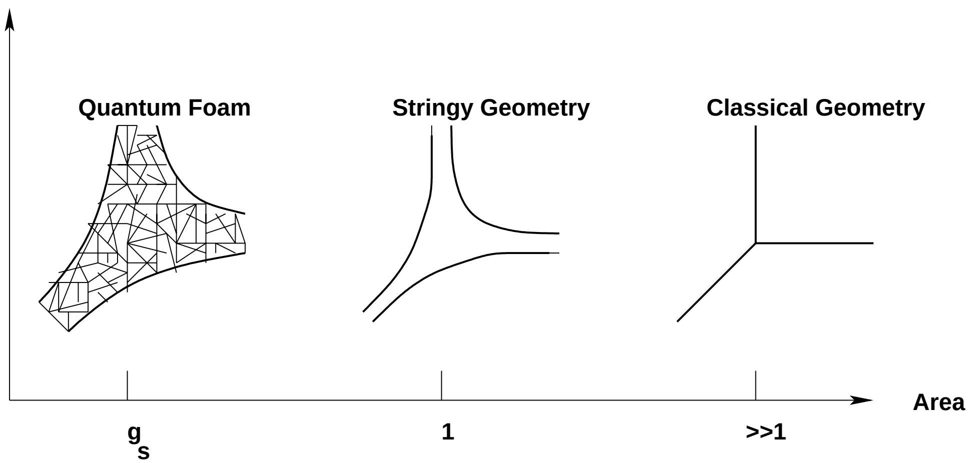


The solution of topological string via topological vertex/melting crystal identifies precisely the class of contributing topologies/geometries.

- Via mirror symmetry, we can make sense of length scales $R < l_s$ in topological string (and extract some lessons for physical string)

- Even more interesting is Planck scale, in 10-d physical string $l_{\rm Planck} = g_s^{1/4} l_s$.

- The length scale of quantum foam in topological string is $g_s^{1/2} l_s$.



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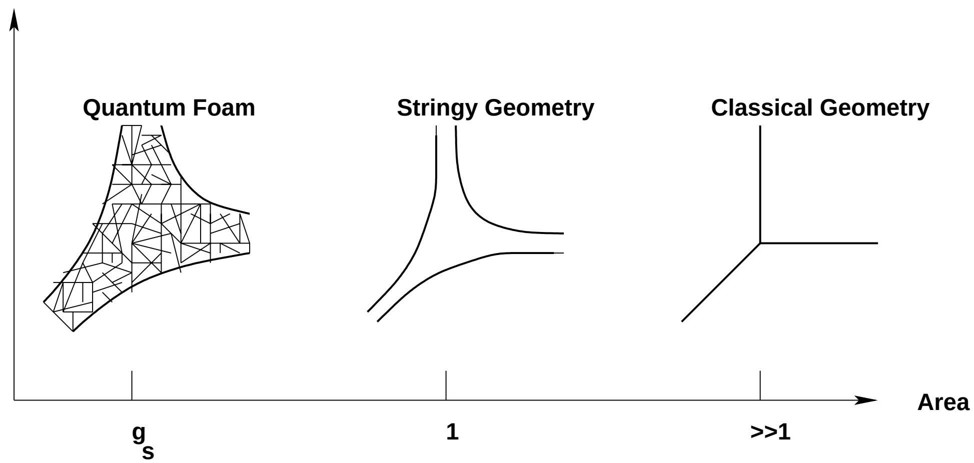

• Awaits realization in physical string

Unification and Duality in the topological world

- Gromov-Witten theory; A-model topological strings
- B-model topological string, modularity (and arithmetics?)
- BPS state counting
- (topologically twisted) $\mathcal{N} = 2$ gauge theory
- integrable systems
- matrix models
- Chern-Simons theory
- (elusive) topological M-theory
- topological quantum foam
- . . .

Some of these dualities can be checked to all orders in perturbation theory, and beyond. The equivalences can be checked completely, and not just for a subsector (what?).

Outlook

"Formal string theory" is a core subject of high-energy physics, and mathematical physics in general.

It will continue to generate results interesting to other subjects as well

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Coming up:

- Can Kozcaz: A-model and geometric engineering
- Sara Pasquetti: B-model and non-perturbative effects
- Ioannis Papadimitriou: Holography and implications
- Norihiro Iizuka: Toy models of CFT/AdS: Matrix models
- Oriol Pujolas: Non-relativistic quantum gravity