Lattice phenomenology

CERN Theory Retreat, Les Houches, 04-06.11.2009

Andreas Jüttner

Lattice phenomenology

- simulations of lattice FT are a tool to study non-perturbative effects, in particular QCD
- theoretical developments (field theoretic, algorithmic) as well as increase in available computing resources allow a constant improvement in the control of systematics in simulations
- motivation apart from understanding QCD itself is to make predictions for SM observables from first principles
- here *first principles* means to start from $\mathcal{L}_{QCD}$ and to make predictions without tuning anything else than the bare parameters

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- Review talks at Lattice 2009, Beijing

  http://rchep.pku.edu.cn/workshop/lattice09/plenaryprogram.html
  (in the future also on POS)

C. Aubin	Lattice studies of hadrons with heavy flavors
V. Lubicz	Kaon Physics from Lattice QCD
R. Van de Water	The CKM matrix and flavor physics from lattice QCD
D. Renner	Status and prospects for the calculation of hadron structure from lattice QCD
M. Laine	Finite-temperature QCD

- Flavia Net Lattice Averaging Group (FLAG) - in preparation

Beware

- there are many systematic effects that have to be controlled and estimated correctly
  - algorithms → Filippo Palombi, next talk
  - quark mass → extrapolation in m_q (applicability of chiral perturbation theory)
  - cut-off effects → continuum extrapolation
  - finite volume effects → infinite volume limit
  - ...

  before QCD is *solved*

in the following I want to give two examples

Two examples

a) flavour physics in the SM: determination of the elements of the CKM matrix, in this talk first row unitarity
b) muon anomalous moment (leading hadronic contribution)

both computations provide constraints and tests of the SM by comparison of

experimental measurement $\leftrightarrow$ SM prediction

$$R(A \to B)|_{\rm exp.} \stackrel{?}{=} \left[(\rm EM) \times (\rm Weak) \times (\rm Strong) \times (\rm BSM)\right]|_{\rm theory}$$

Two examples

example (a): CKM first row unitarity

(a) Flavour changing processes: e.g. $K \to \pi$

CKM first row unitarity

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$

$|V_{ud}|$ super-allowed nuclear β-decays + theory

$|V_{us}|$ $K \to \pi$, $K \to \mathrm{vac}$ + lattice QCD

$|V_{ub}|$ negligible

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semi-leptonic kaon decay

$$\underbrace{\Gamma_{K\to\pi l\nu}}_{\text{experiment}} = \underbrace{C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\mathrm{EW}}[1 + \Delta_{SU(2)} + 2\Delta_{\mathrm{EM}}]}_{\text{well known or PT}} |V_{us}|^2 \underbrace{f_+^{K\pi}(0)^2}_{\text{non-PT}}$$

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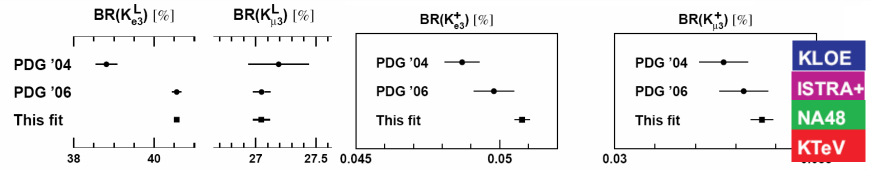

(FLAVIANet Kaon working group)

Exp. + PT: $|V_{us} f_+^{K\pi}(0)|^2 = 0.21661(47)$

arXiv:0801.1817

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$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle|_{\mathrm{QCD}} = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

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Also via leptonic Kaon decay

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{m_K(1 - m_\mu^2/m_K^2)}{m_\pi(1 - m_\mu^2/m_\pi^2)} \times 0.9930(35) \times \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2$$

Marciano PRL 93,2004

$$\langle 0|V_\mu(0)|PS(p)\rangle|_{\mathrm{QCD}} = ip_\mu f_{PS}$$

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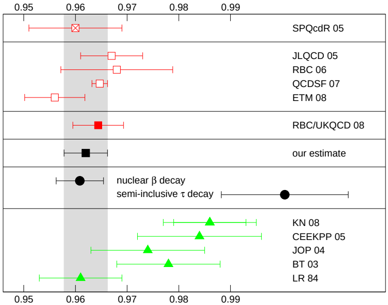
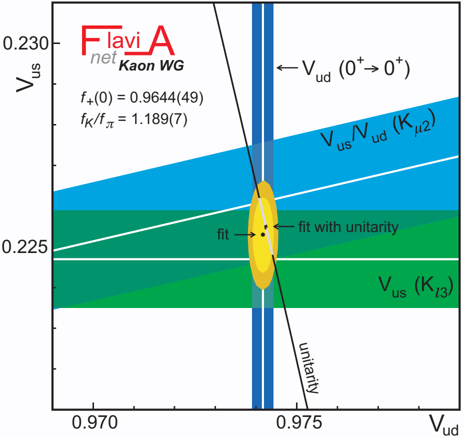


preliminary compilation for $f_+^{K\pi}(0)$

$f_+^{K\pi}(0)$-compilation by FLAVIANet Lattice Averaging Group (preliminary 08/09 - watch out for update)

(a) First row unitarity



FLAVIANet Working Group on Kaons arXiv:0801.1817
watch out for update by FLAG (FLAVIANet Lattice Averaging Group)

- rather mature computation after recent theoretical developments that allow for better control of systematics
- theory error ≈ experimental error
- although confirmation of SM *not exciting* - can be used to constrain new physics

example (b): μ anomalous magnetic moment

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exp. $a_\mu = 11659208(6.3) \times 10^{-10}$ *[Muon g-2 Collab.], Phys. Rev. D 73 (2006) 072003*

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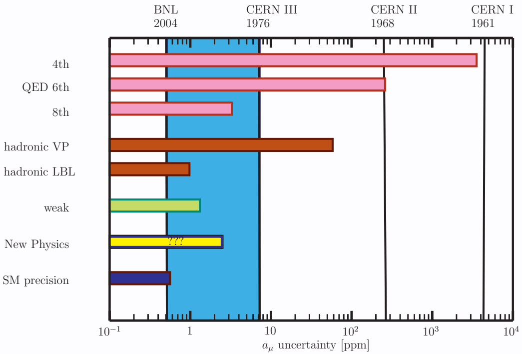
- remarkable in that all sectors of the SM contribute:



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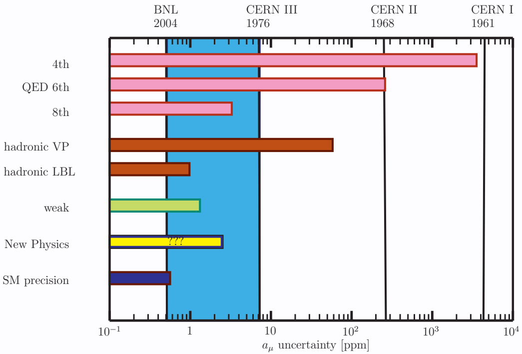
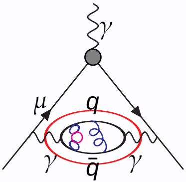


F. Jegerlehner, A. Nyffeler, arXiv:0902.3360

hadronic VP: $690.3(5.3)\times 10^{-10}$ semi-phenomenologically from e^+e^--anihilation

at least independent confirmation or even smaller error crucial!

Leading hadronic contribution to muon $g-2$



- vacuum polarisation:
  $$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$
  $$= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$
- $a_\mu^{\text{had. VP}} = \left(\frac{\alpha}{\pi}\right)^2 \int dQ^2 K(Q^2) \Big(\Pi(Q^2) - \Pi(0)\Big)$

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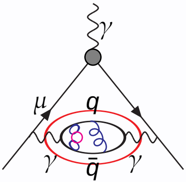

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- there were previous attempts but inconsistent and not satisfying
  T. Blum, C. Aubin, PRD 75, 114502 (2007) QCDSF Collaboration NPB 688 (2004) 135164 D. Renner and X. Feng, arXiv:0902.2796
- brute force doesn't help - one needs new ideas
- next to standard lattice systematics we identify three problems that need to be adressed:
  - momentum resolution in a finite box
  - vector meson
  - quark-disconnected diagrams
- Mainz lattice group working on these issues - I will briefly present progress with quark-disconnected diagrams *Michele Della Morte, A.J. arXiv:0910.3755*

What we compute

All we need to do is to compute a *simple* two-pt function

$$C_{\mu\nu}^{(N_f=2)}(q) = \sum_x e^{-iqx} \langle j_\mu^{(N_f=2)}(x) j_\nu^{(N_f=2)}(0) \rangle$$

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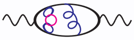

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using iso-spin and carrying out the Wick contraction

$$= \sum_x e^{-iqx} \left\langle \frac{5}{9}\mathrm{Tr}\left\{\bar{q}(x)\gamma_\nu q(x)\bar{q}(0)\gamma_\mu q(0)\right\} + \frac{1}{9}\mathrm{Tr}\left\{\gamma_\nu q(x)\bar{q}(x)\right\}\mathrm{Tr}\left\{\gamma_\mu q(0)\bar{q}(0)\right\} \right\rangle$$

quark-connected quark-disconnected

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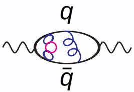

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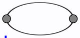

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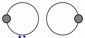

quark-disconnected

- quark-disconnected extremely expensive numerically
- unknown size (have been neglected so far)

Vacuum polarization in $SU(2)$ χPT

- so what about constructing in chiral effective theory expressions for the quark-connected and quark-disconnected pieces, respectively?
  Recall:

$$
\begin{aligned}
C^{(2),conn}_{\mu\nu}(t,\vec{q}) &\equiv \sum_{\vec{x}} e^{i\vec{q}\vec{x}} \left\langle \mathrm{Tr}\left\{\bar{u}(0)\gamma_\nu d(0)\bar{d}(x)\gamma_\mu u(x)\right\}\right\rangle \\
C^{(2),disc}_{\mu\nu}(t,\vec{q}) &\equiv \sum_{\vec{x}} e^{i\vec{q}\vec{x}} \left\langle \mathrm{Tr}\left\{\bar{u}(0)\gamma_\nu u(0)\right\}\mathrm{Tr}\left\{\bar{d}(x)\gamma_\mu d(x)\right\}\right\rangle
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\end{aligned}
$$

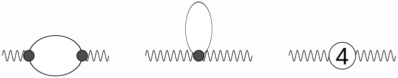
- these correlation functions are built from the flavour currents

$$
\begin{aligned}
j_\mu^{ud} &= \tfrac{1}{2}\bar{\psi}(\sigma_1+\sigma_2)\gamma_\mu\psi \\
j_\mu^{uu} &= \tfrac{1}{2}\bar{\psi}(\sigma_0+\sigma_2)\gamma_\mu\psi \\
j_\mu^{dd} &= \tfrac{1}{2}\bar{\psi}(\sigma_0-\sigma_3)\gamma_\mu\psi
\end{aligned}
$$

- can be described in $SU(2)$ chiral perturbation theory with scalar couplings added (only turn up in $L^{(4)}$)

Consequences

- with this Lagrangian we compute the quark-connected and quark-disconnected part independently:



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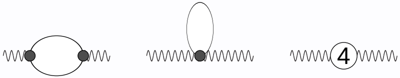

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- the contributions $\Pi_{\mu\nu}^{(a,a)}$ ($a = 0,1,2,3$) are the effective theory descriptions of 2pt-correlators constructed of the currents $c\bar{\psi}\gamma_{\mu/\nu}\sigma_a\psi$, where $c \in \mathbb{C}$ is a normalization
- it is possible to express one through the other:

$$\Pi^{(N_f=2)}(q)|_{\text{disc}} \quad = \quad \frac{1}{9} \quad \underbrace{\Pi^{(0,0)}}_{\substack{\text{due to scalar coupling} \\ \text{in } N_f=2 \text{ EM current}}} \quad -\frac{1}{10}\Pi^{(N_f=2)}(q^2)|_{\text{con}}$$

  - disconnected part (at NLO) has same qualitative q^2-dependence as connected part
  - compute $\Pi^{(0,0)}$ once and then predict q^2 and m_π^2 dependence

How large is the quark-disconnected contribution?

- it will be hard in practice to compute $\Pi^{(0,0)}$
- but recall one thing:

$$a_\mu^{\text{had.}} = \left(\frac{\alpha}{\pi}\right)^2 \int dQ^2 K(Q^2) \left(\Pi(Q^2) - \Pi(0)\right)$$

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$$\frac{\hat{\Pi}(q^2)|_{\text{disc}}}{\hat{\Pi}(q^2)|_{\text{con}}} = -\frac{1}{10} \qquad \text{(at NLO)}$$

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- we (Mainz) have started implementing relevant correlation functions in a C code and we will check this numerically
- applications to other observables (e.g. scalar form factor ?) would of course be desireable
- extension to $N_f = 2 + 1$ under way

Summary/Outlook

- two examples for lattice phenomenology which can provide crucial non-perturbative input to SM tests
- large scale simulations go hand in hand with new field theoretic and algorithmic developments
- topics related to $g-2$ not covered but currently working on:
    - vector meson dominance and chiral extrapolation of lattice data
    - momentum resolution for hadrons in finite volume