Chiral symmetry breaking on the lattice

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Introduction

QCD with N_f light flavors: spontaneous chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$$

- $N_f^2 - 1$ pseudo Nambu-Goldstone bosons
- Chiral condensate:

$$\lim_{m\to 0} \lim_{V\to\infty} \frac{\langle 0|\bar{\psi}\psi|0\rangle}{N_f} = -\Sigma \neq 0$$

- Coupling pions - axial current:

$$\langle 0|A_\mu^a(0)|\pi^b(p)\rangle = ip_\mu F \delta^{ab}$$

In presence of SSB:
close to the chiral limit, the dynamics of the pseudo Nambu-Goldstone bosons is governed by Σ, F only

- GMOR relation:

$$M^2 = \frac{2m\Sigma}{F^2}$$

Spontaneous chiral symmetry breaking + QCD symmetries: $\rightarrow$ Chiral Effective Theory

Weinberg ('79), Gasser and Leutwyler ('84,'85)

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\mathbf{Tr}(\partial_\mu U \partial_\mu U^\dagger) - \frac{\Sigma}{2}\mathbf{Tr}(\mathcal{M}U^\dagger + \mathcal{M}^\dagger U),$$

$$\mathcal{L}_\chi^{(4)} = \sum_i \Lambda_i \mathcal{O}_i$$

- $U \in SU(N_f)$; $\mathcal{M}$ mass matrix
- F, Σ, Λ_i parametrize low-energy dynamics (Low Energy Couplings)
- NLO formulae, $N_f = 2$:

$$M_\pi^2 = M^2 + \frac{M^4}{32\pi^2 F^2} \ln\left(M^2/{\Lambda_3}^2\right) + \ldots$$

$$F_\pi = F - \frac{M^2}{16\pi^2 F} \ln\left(M^2/{\Lambda_4}^2\right) + \ldots$$

Lattice QCD in the chiral regime

Non-perturbative regularization of QCD $\rightarrow$ UV cut-off $\propto \frac{1}{a}$

Study from first principles how chiral symmetry is realized in low-energy QCD

Scales: a, m, $V = L^3 T$

- Volume: $m \rightarrow 0$, $V \rightarrow \infty$;

  no spontaneous symmetry breaking at finite volume;

  corrections to physical observables can be computed in χPT

  - $M_\pi L \gg 1$: like $V = \infty \rightarrow$ dominant corrections $\propto e^{-M_\pi L}$ (p-regime)
  - $m\Sigma V \sim O(1) \rightarrow M_\pi L \lesssim 1$: corrections are polynomial $\propto 1/(FL)^2$

    $\rightarrow$ extract informations from finite-size scaling (ϵ- regime)

Lattice QCD in the chiral regime

Non-perturbative regularization of QCD $\rightarrow$ UV cut-off $\propto \frac{1}{a}$

Study from first principles how chiral symmetry is realized in low-energy QCD

Scales: a, m, $V = L^3 T$

- Finite lattice spacing a: chiral properties at $a \neq 0$
  - Wilson fermions: explicit breaking of chiral symmetry at $a \neq 0$
    -additive mass renormalization
    -lattice artefacts $\propto O(a)$: improvement program $\rightarrow O(a^2)$
    -dynamical simulations feasible
  - Ginsparg-Wilson fermions:

$$\gamma_5 D + D\gamma_5 = \frac{a}{\rho} D\gamma_5 D$$

Ginsparg, Wilson ('82)

exact chiral symmetry at finite lattice spacing

Lüscher ('98)

-no additive mass renormalization, operator mixing continuum-like
-discretization errors $O(a^2)$
-numerically very expensive

  - ...

1. Direct computation of the quark condensate

- GW fermions (exact chiral symmetry)
  - In the chiral limit, infinite volume:

$$\langle\bar{\psi}\psi\rangle = \lim_{a\to 0} Z_S\langle\bar{\psi}\psi\rangle$$

Z_S renormalization of scalar density

  - For $m \neq 0$:

$$Z_s\langle\bar{\psi}\psi\rangle = b_1 m + b_2 m^3 + \{finite \;\; terms\}$$

UV divergences: $b_1 \propto 1/a^2$, $b_2 \propto \ln(a)$
need to be subtracted

- Wilson fermions: UV divergences persist in the chiral limit

Banks-Casher relation

$$\lim_{\lambda \to 0} \lim_{m \to 0} \lim_{V \to \infty} \rho(\lambda) = \frac{\Sigma}{\pi}$$

$$\rho(\lambda) = \frac{1}{V} \sum_{k=1}^{\infty} \langle \delta(\lambda - \lambda_k) \rangle$$

Banks, Casher ('80)

spectral density of Euclidean massless Dirac operator:

- well defined thermodynamic limit
- is renormalizable → universal continuum limit

Del Debbio et al ('06), Giusti and Lüscher ('09)

$$\langle \bar{\psi}\psi \rangle = -2m \int_0^{\infty} d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

UV divergences come from probe function

Giusti,SN ('07)

Mode number:

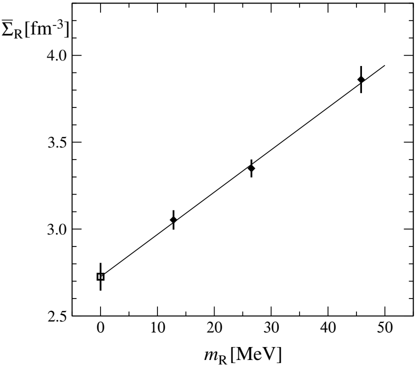
average number of eigenmodes of $D^\dagger D + m^2$ with eigenvalues $\alpha \leq M^2$

$$\nu(M, m) = V \int_{-\Lambda}^{\Lambda} d\lambda \, \rho(\lambda); \qquad \Lambda = \sqrt{M^2 - m^2}$$

is renormalizable

Giusti,Lüscher ('09)

$$\Sigma_{\text{eff}} = \frac{\pi}{2} \frac{\nu(M, m)}{\Lambda V}; \qquad \Sigma_{\text{eff}} = \Sigma + \ldots$$



$O(a)$ improved Wilson fermions, $N_f = 2$

$a \simeq 0.08$ fm, $L \simeq 1.9, 2.5$ fm, $T = 2L$

$$(\Sigma^{\overline{\text{MS}}}(2\ GeV))^{1/3} = 276(3)(4)(5)\ \text{MeV}$$

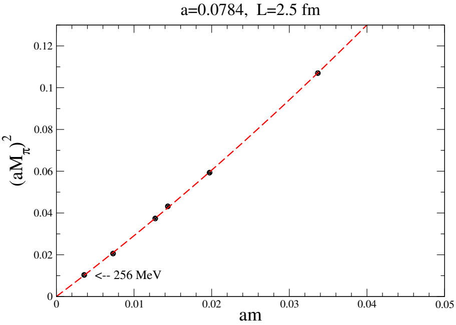
2. GMOR relation

$M^2 = \frac{2m\Sigma}{F^2}$

Pseudoscalar correlator: $P^{rs} = \bar{r}\gamma_5 s$

$$f_{PP}(x_0) = a^3 \sum_{\vec{x}} \langle P^{rs}(x) P^{rs}(0) \rangle = -\frac{G_{PS}^2}{M_{PS}} e^{-M_{PS} x_0} + \ldots$$

$$G_{PS} = \frac{M_{PS}^2 F_{PS}}{m_{rs}}$$



$O(a)$ improved Wilson fermions, $N_f = 2$

$M_{\pi,min} L \simeq 3.2$

Del Debbio et al ('06,'08)

M_π^2 is linear in m up to $\simeq m_s/2$
$\rightarrow$ pion mass dominated by GMOR

2. GMOR relation

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$$G_{PS} = \frac{M_{PS}^2 F_{PS}}{m_{rs}}$$

- Other studies of quark mass dependence of M_{PS}^2, F_{PS}:
  - $N_f = 2$: ETM Collaboration ('08), JLQCD+TWQCD ('08)
  - $N_f = 2 + 1$: RBC/UKQCD ('08), PACS-CS ('08)

3. Finite-size scaling in the ϵ-regime

For $\mu = m\Sigma V \lesssim 1$, $N_f > 1$

$$\Sigma(\mu) \propto \Sigma\mu \to_{(\mu\to 0)} 0$$

is a sign of SSB in infinite volume

Hasenfratz & Leutwyler ('90), Leutwyler & Smilga ('92)

If we assume SSB → determine F, Σ by matching lattice results with finite-size scaling predicted by the Chiral Effective Theory

$$\mathcal{Z}_{LO} = \int_{\mathrm{SU(N_f)}} dU_0 \exp\left[\mu \mathbf{ReTr} U_0\right]; \qquad U_0 \to \quad \textit{pion zero mode}$$

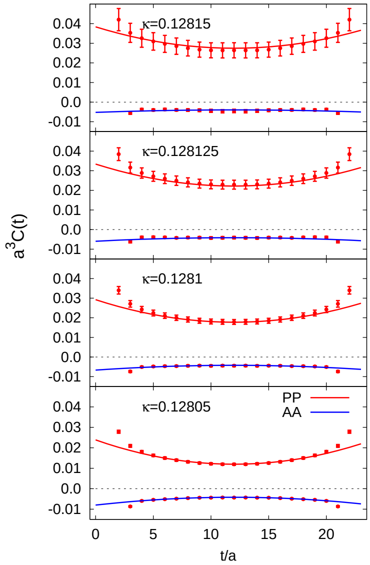
Gasser & Leutwyler ('87)

Pseudoscalar and axial correlators at NLO in the ϵ-expansion

$$C^{ab}_{PP,AA}(t) = \delta^{ab}\left\{a_{P,A} + b_{P,A} h_1\left(\frac{t}{T}\right)\right\}$$

- $h(\tau) = \frac{1}{2}\left[\left(\tau - \frac{1}{2}\right)^2 - \frac{1}{12}\right]$
- a_P, b_P, a_A, b_A functions of $F, \Sigma, L, T, \mu = m\Sigma V$

Hansen ('90)



Tree level $O(a)$ Wilson fermions, $N_f = 2$
$a \simeq 0.115$ fm, $L \simeq 1.84, 2.8$ fm,
$m\Sigma V \simeq 0.7 - 5$

A. Hasenfratz, Hoffmann & Schaefer (08)

- Continuum NLO fit: ($L = 2.8$ fm)

$F = 90(4)$ MeV,
$(\Sigma^{\overline{MS}}(2\ GeV))^{1/3} = 248(6)$ MeV

- Lattice artefacts can be studied in χPT $\rightarrow$ Wilson χPT
  ϵ-regime: lattice artefacts are suppressed for $m \sim a\Lambda^2_{QCD}$.
  Include leading $O(a^2)$ correction at NLO: *Bär, SN & Schaefer (08)*

$F = 88(3)$ MeV
$(\Sigma^{\overline{MS}}(2\ GeV))^{1/3} = 249(4)$ MeV
$c_2 = 0.02(8)$ GeV^4

See also JLQCD+TQWCD ('07,'08)

Conclusions

- Lattice QCD allows to study spontaneous symmetry breaking of chiral symmetry from first principles
- Lattice computations in the chiral regime are challenging
  - control over systematic uncertainties:
    finite volume effects, lattice artefacts, renormalization...
- Once SBB tested $\rightarrow$ test Chiral Effective Theory at higher orders
  $\rightarrow$ compute LECs
- different observables and methods
  $\rightarrow$ consistent picture of QCD at low energy is emerging