Critical slowing down of the topological modes in numerical simulations of LQCD

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CERN

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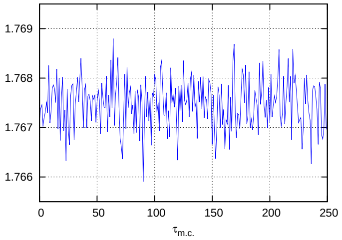

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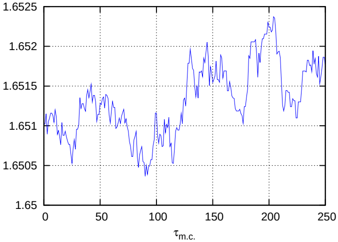

- the case of the topological charge

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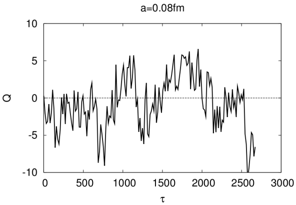

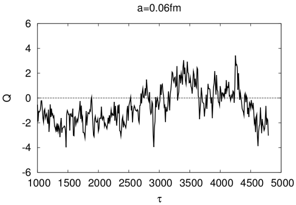

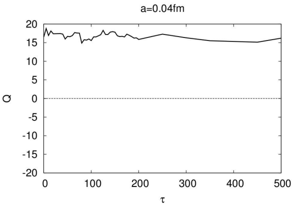

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practical:

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topology freezing is a show-stop problem

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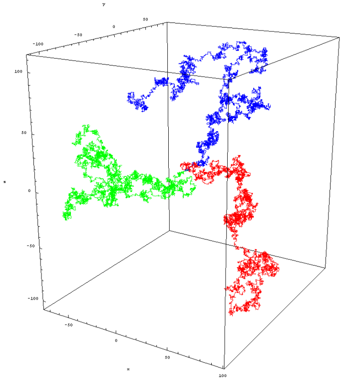

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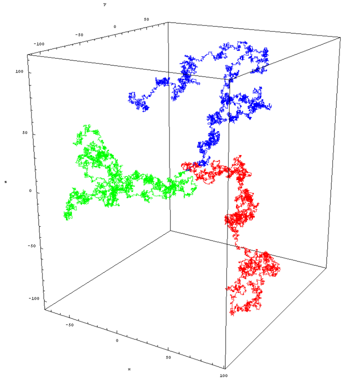

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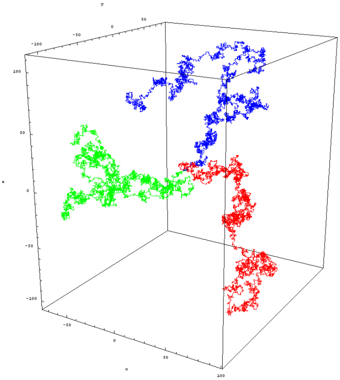

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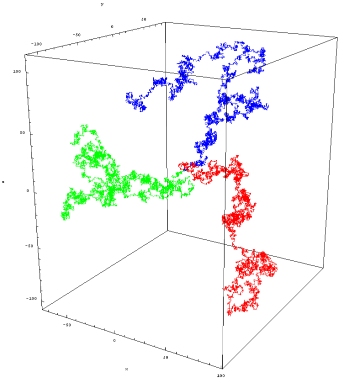

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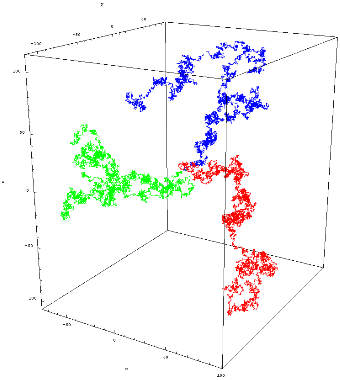

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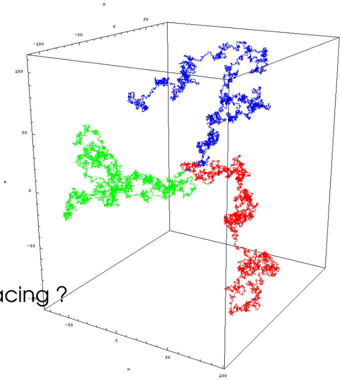
how do barriers (if) depend upon the lattice spacing ?



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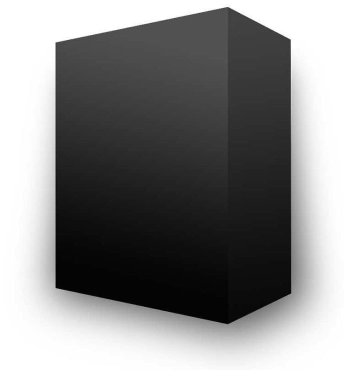

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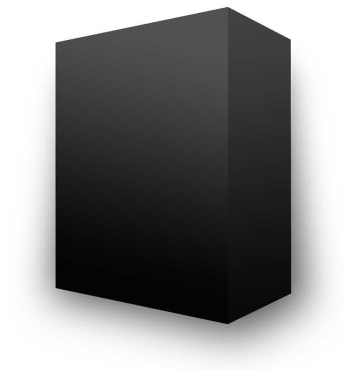

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...with at least some pin jacks on it!

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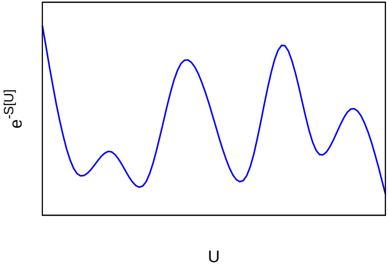
the theory is mapped onto its strong coupling limit and becomes trivial

M.Lüescher (arXiv:0907.5491)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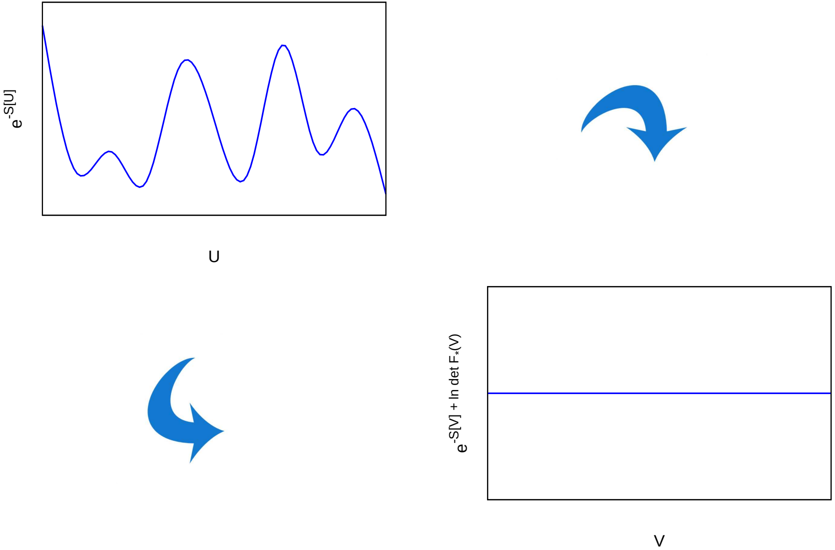
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e^-S[V] + ln det F*(V)
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original algorithm

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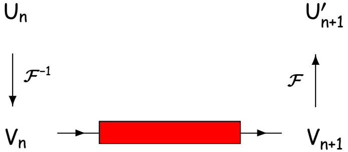
new algorithm



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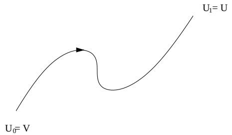

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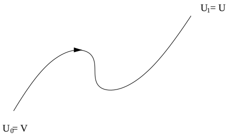

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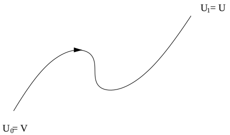

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- ...we are currently implementing the new proposal.