Electroweak (and QCD) corrections at the LHC

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NLO QCD calculations for the LHC: Les Houches priority list

Six-particle processes of priority list (2005/2007 Les Houches workshops)

$$\mathrm{pp} \to \quad \mathrm{t\bar{t}b\bar{b}}, \qquad \mathrm{t\bar{t}}jj, \qquad VV\mathrm{b\bar{b}}, \qquad VVjj, \qquad Vjjj, \qquad \mathrm{b\bar{b}b\bar{b}}$$

Importance of NLO QCD for the LHC

- heavy SM particles + jets $\Rightarrow$ large backgrounds to many Higgs and BSM signals
- large powers of α_S $\Rightarrow$ huge QCD scale uncertainties at LO

Technical challenges

- computer codes slower than sec/point $\Rightarrow$ CPU-months for precise distributions
- spurious singularities (Gram determinants) $\Rightarrow$ serious numerical instabilities

The optimal NLO method(s) for $n = 6, 7$ particle processes at the LHC?

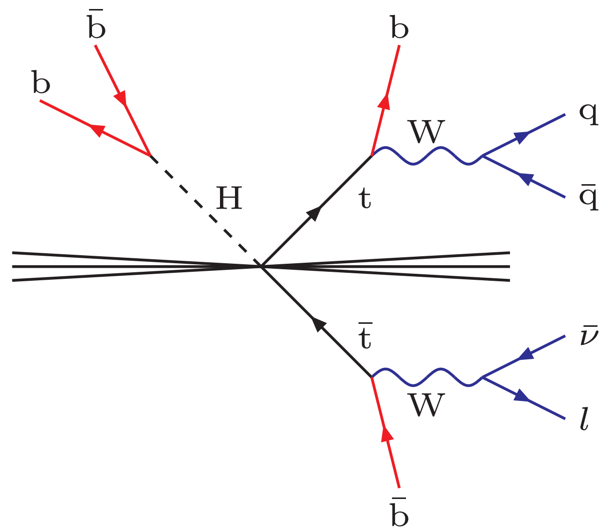
- Feynman diagrams and tensor reduction: very successful up to $n = 5$ but **complexity increases faster than factorially for** $n \gg 1$
- Methods of on-shell type: less practical experience but **complexity increases only polynomially for** $n \gg 1$

Completion of the first $2 \to 4$ calculations of the priority list

Within the last few months—four years after Les Houches wish list—four groups, using different methods, have completed two wish-list processes

- **Two calculations for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with permille agreement**
  - arXiv:0905.0110 by Bredenstein, Denner, Dittmaier and S. P. based on Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau and Worek based on OPP reduction and HELAC
- **Two calculations for $\mathrm{pp} \to \mathrm{W}jjj$ (leading-colour and full results)**
  - arXiv:0906.1445 by Ellis, Melnikov and Zanderighi based on D-dimensional unitarity (leading-colour approximation)
  - arXiv:0907.1984 by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre based on generalized unitarity (full colour)

Phenomenological motivation for $t\bar{t}b\bar{b}$: irreducible background to $t\bar{t}H(H \to b\bar{b})$



Associated $t\bar{t}H(H \to b\bar{b})$ production at LHC

- for $M_H < 135\,\text{GeV}$: exploit large $H \to b\bar{b}$ BR and measure top Yukawa coupling
- 5σ discovery potential of ATLAS TDR disappeared because of background

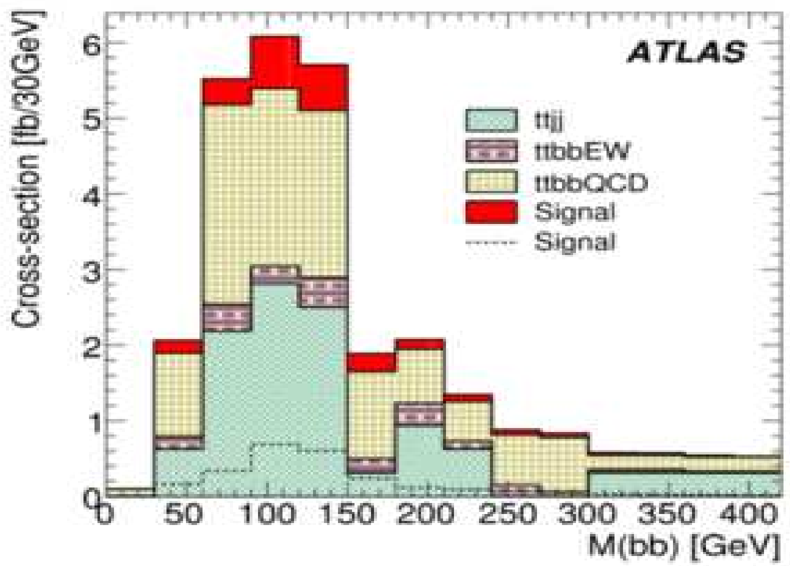
Statistics and systematics (30 fb^{-1})

- $S/\sqrt{B} \simeq 2$ sufficient for measurement
- $S/B \simeq 0.1$ implies that $\Delta B/B$ systematic uncertainty of $\mathcal{O}(10\%)$ kills measurement!

Main backgrounds (ATLAS analysis)

- $t\bar{t}b\bar{b}$ (`AcerMC`, $\mu_{QCD} = m_t + m_{\bar{b}b}/2$)
- $t\bar{t}jj$ (`MC@NLO`, $\mu^2_{QCD} = m^2_t + < p^2_{T,t} >$)

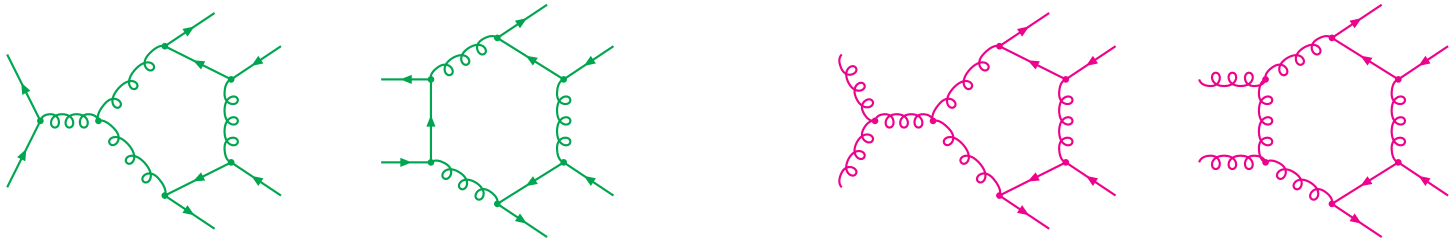
require NLO predictions!



ATLAS CSC-note,CERN-OPEN-2008-020

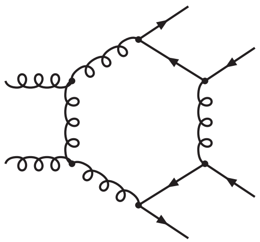
Partonic subprocesses and loop diagrams for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Sample one-loop diagrams in the $\mathrm{q\bar{q}}$ (188) and gg (1008) channels



24 pentagons | 8 hexagons | 114 pentagons | 40 hexagons

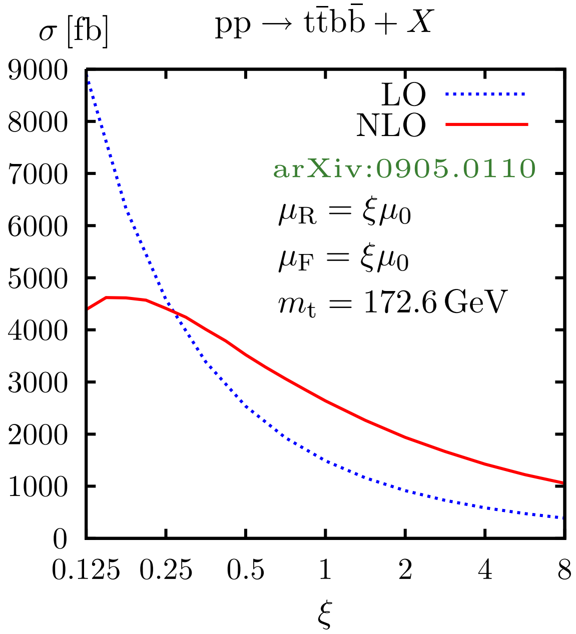
Tensor integrals $T^{(N)}_{i_1 \dots i_P}$ are reduced numerically. **Their coefficients $\Gamma_{\mu_1 \dots \mu_P} \{g \dots p\}^{\mu_1 \dots \mu_P}_{i_1 \dots i_P}$ undergo heavy** algebraic manipulations

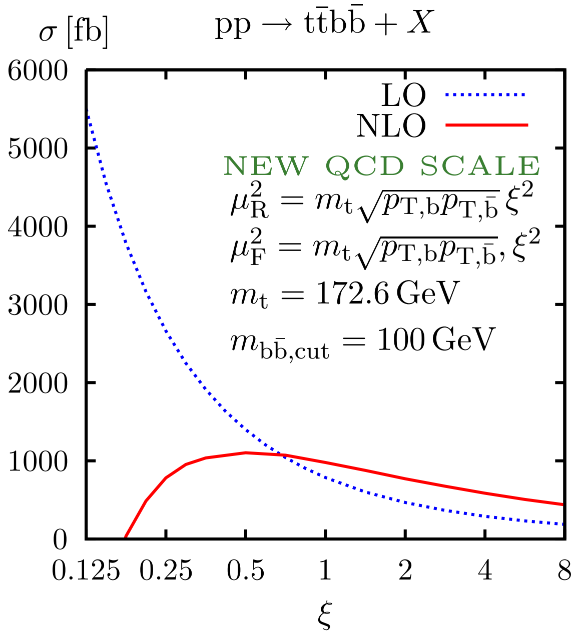


$$\Rightarrow \Gamma_{\mu_1 \dots \mu_P} \int \frac{q^{\mu_1} \dots q^{\mu_P}}{\prod_{i=0}^{N-1} [(q + p_i)^2 - m_i^2]} \Rightarrow \Gamma_{\mu_1 \dots \mu_P} \sum_{i_1 \leq \dots \leq i_P = 0}^{N-1} \{g \dots p\}^{\mu_1 \dots \mu_P}_{i_1 \dots i_P} \, T^{(N)}_{i_1 \dots i_P}$$

Automatic tools for algebraic manipulation and numerics

- Code size (Bible $\sim$ 4 MB): one Hexagons $\sim$ 1 MB, full executable $\sim$ 100 MB
- `FeynArts` and `FormCalc` plus in-house `MATHEMATICA` and `Fortran77` programs
- Code development took about two years but its *highly process-independent* character renders it *applicable to many other processes*





LO and NLO scale dependence of σ_{tot} at the LHC

Very high sensitivity to scale choice

- LO proportional to $\alpha_S(\mu_R)^4 \Rightarrow 78\%$ uncertainty

ATLAS scale choice: $\mu_0 = m_t + m_{b\bar{b}}/2 = E_{\text{thr}}/2$

- motivated by small NLO effects observed in similar processes: $t\bar{t}H$ ($K \simeq 1.2$), $t\bar{t}j$ ($K \simeq 1.1$), $t\bar{t}Z$ ($K \simeq 1.35$)
- but for $t\bar{t}b\bar{b}$ we found very large NLO corrections ($K \simeq 1.8 \pm 34\%$)

ATLAS scale is wrong and must be replaced by new (dynamical) QCD scale: $\mu_0^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$

- combines typical scales observed in $t\bar{t}b\bar{b}$ distributions
- reduces correction and uncertainty ($K = 1.25 \pm 21\%$)
- increases σ_{tot} by factor two wrt ATLAS simulations!

Statistical precision and speed of the calculation

Single 3GHz Intel Xeon processor & `pgf77` Portland compiler

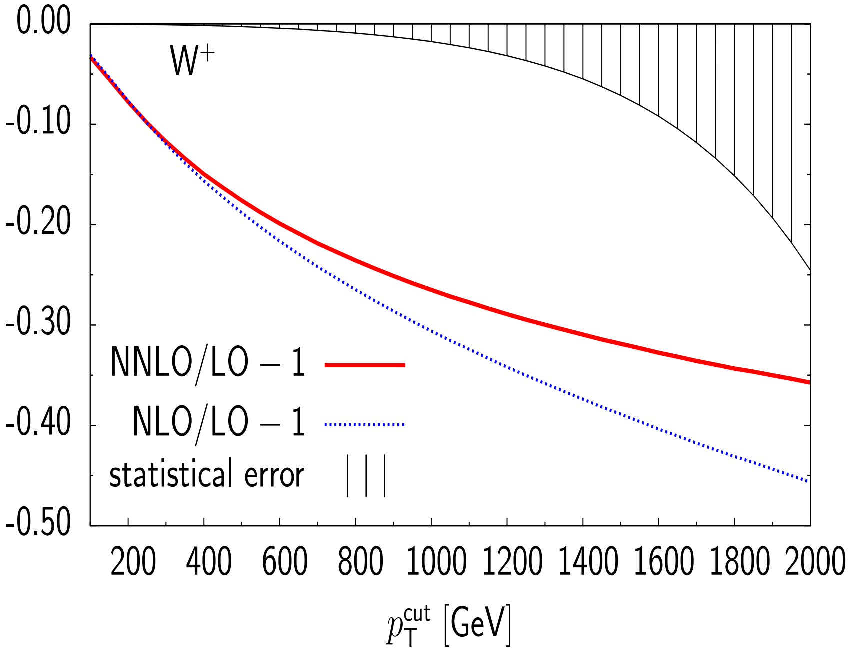
	$\sigma/\sigma_{\mathrm{LO}}$	# events (after cuts)	$(\Delta\sigma)_{\mathrm{stat}}/\sigma$	runtime	time/event
NLOtree (gg)	85%	5.8×10^6	0.4×10^{-3}	2h	< 1.4ms
virtual (**gg**)	10%	0.46×10^6	0.7×10^{-3}	20h	160ms
real + dipoles (gg/qg)	87%	16.5×10^6	2.6×10^{-3}	47h	10ms

- **2–3 CPU-days** $\Rightarrow$ $\mathcal{O}(10^7)$ events and $\mathcal{O}(10^{-3})$ **stat. accuracy** for σ_{tot} (distributions obtained with $\sim 5 \times 10^8$ events after cuts)
- **speed of virtual corrections** is remarkably high: 160 ms/event (including colour and polarization sums!)
- this is based on **completely process-independent techniques** and opens excellent perspectives to study many other multi-particle processes at the LHC!

Electroweak loop corrections at the TeV scale

Example: electroweak corrections to $pp \to W + \text{jet}$ at the LHC

$(\sigma^{\mathrm{NLO}} - \sigma^{\mathrm{LO}})/\sigma^{\mathrm{LO}}$



Kühn, Kulesza, S.P., Schulze (2007)

At small p_T

- Corrections of $\mathcal{O}(\alpha) \sim 1\%$

At $p_T > 100$ GeV

- **large negative corrections $\gg$ 1%**
- **increase with p_T**
- 30–40% at $p_T \sim$ 1–2 TeV!

Questions

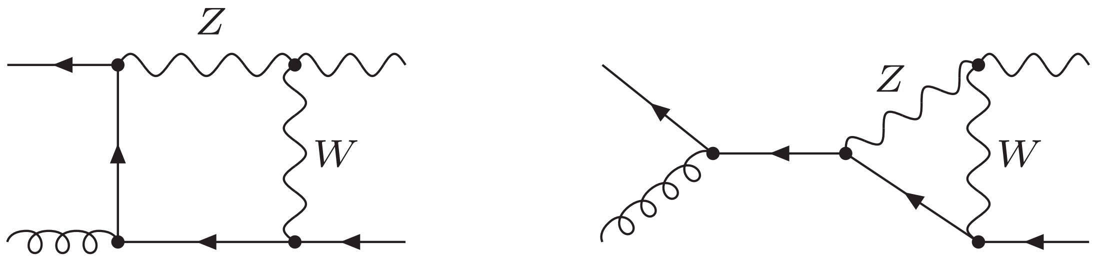
- **origin?**
- **large effects beyond one-loop?**

Origin: scattering energy $\gg$ characteristic scale of EW corrections

Large double logarithms

$$\frac{\delta\sigma}{\sigma} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{W,Z}^2}\right) \simeq -26\% \qquad \text{at} \quad \sqrt{s} \sim 1\,\mathrm{TeV}$$

from vertex and box diagrams involving virtual W and Z bosons



Kuroda, Moultaka, Schildknecht (1991); Degrassi, Sirlin (1992); Beenakker, Denner, Dittmaier, Mertig, Sack (1993); Denner , Dittmaier, Schuster (1995); Denner, Dittmaier, Hahn (1997), Beccaria, Montagna, Piccinini, Renard, Verzegnassi (1998); Ciafaloni, Comelli (1999)

Affect all hard scattering processes at LHC, ILC, CLIC!

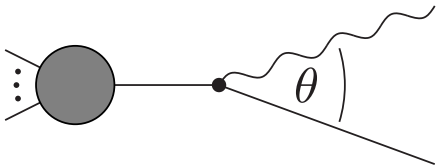
Asymptotic expansion of 1-loop EW corrections

General form of $M_W^2/s \to 0$ asymptotic limit

$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + C_1 \underbrace{\ln\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + \tilde{C}_1 \underbrace{\ln\left(\frac{s}{\mu_R^2}\right)}_{\text{UV}} + C_0 + \mathcal{O}\left(\frac{M_W^2}{s}\right)\right]$$

Origin of large logarithms

- Remnants of UV singularities can be absorbed into running couplings
- Mass singularities from soft/collinear gauge bosons coupling to external lines

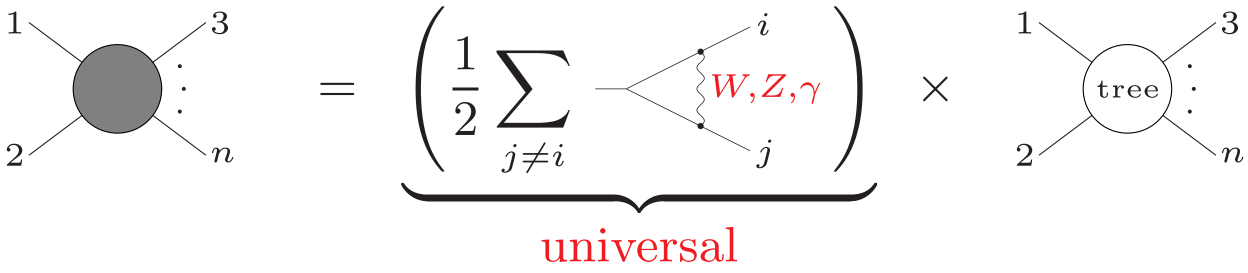


$$\Longrightarrow \int \frac{\mathrm{d}E}{E} \int \frac{\mathrm{d}\cos\theta}{(1-\cos\theta)}$$

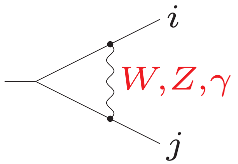
Analogies with QED and QCD? **Factorization and universality?**

Factorization and universality of one-loop EW logarithms [Denner, S.P. (2001)]

For **arbitrary processes** $(e, \nu, u, d, t, b, \gamma, Z, W^{\pm}, H, g)$



proven with **collinear Ward identities** for spontaneously broken YM theories



$$= \frac{\alpha}{4\pi}\left\{\sum_{V=\gamma,Z,W} I_i^V I_j^V \ln^2\frac{r_{ij}}{M_W^2} + 2 I_i^Z I_j^Z \ln\frac{r_{ij}}{M_W^2}\ln\frac{M_W^2}{M_Z^2} + \gamma_{ij}^{\mathrm{ew}} \ln\frac{s}{M_W^2}\right.$$

$$\left. + Q_i Q_j \sum_{k=i,j}\left[\ln\frac{r_{ij}}{m_k^2}\ln\frac{M_W^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{M_W^2}{m_k^2} - \ln\frac{M_W^2}{\lambda^2} - \frac{1}{2}\ln\frac{M_W^2}{m_k^2}\right]\right\}$$

Simple and general recipe for LL and NLL at one loop . . .

maybe too simple?!

Precision of NLL/NNLL approx. for pp $\rightarrow$ Zj [Kühn, Kulesza, S.P., Schulze (2005)]

Asymptotic expansion $(|\hat{s}|, |\hat{t}|, |\hat{u}| \gg M_W^2)$ for $q\bar{q} \rightarrow Zg$ amplitude

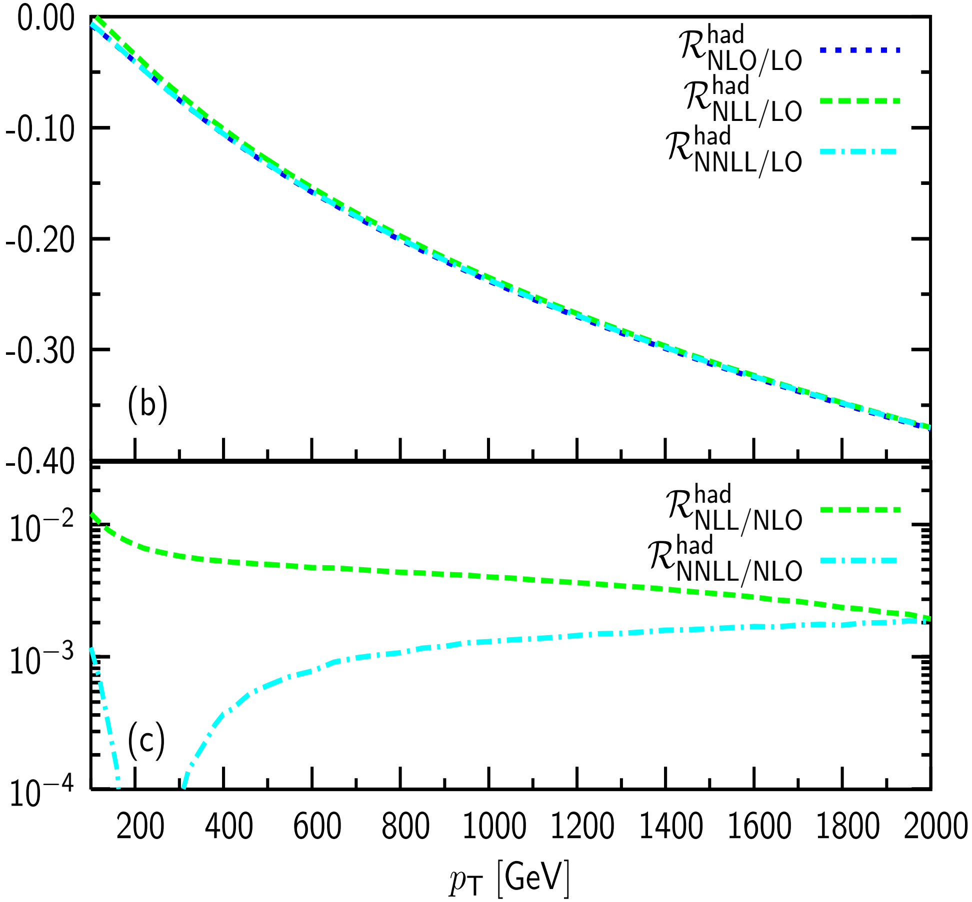
$$\overline{|\mathcal{M}_1|}^2 = 32\pi^2\alpha^2\alpha_S \sum_{\lambda=R,L} \Bigg\{ \left(I^Z_{q_\lambda}\right)^2 \sum_{V=Z,W^\pm} \left(I^V I^{\bar{V}}\right)_{q_\lambda} \Bigg\{ \Bigg[-\ln^2\left(\frac{-\hat{s}}{M_V^2}\right) + 3\ln\left(\frac{-\hat{s}}{M_V^2}\right) + \frac{3}{2}[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right)$$
$$+\ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right)] + \frac{7\pi^2}{3} - \frac{5}{2}\Bigg] \left(\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\right) + \frac{1}{2}\Bigg[3\ln\left(\frac{\hat{u}}{\hat{s}}\right) - 3\ln\left(\frac{\hat{t}}{\hat{s}}\right) - \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)\Bigg]$$
$$\times\left(\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}}\right) + 2\Bigg[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right) + 2\pi^2\Bigg]\Bigg\} + \frac{c_W}{s_W^3} T^3_{q_\lambda} I^Z_{q_\lambda} \Bigg\{\Bigg[\frac{4}{4-D} - 2\gamma_E$$
$$+2\ln\left(\frac{4\pi\mu^2}{M_Z^2}\right) + \ln^2\left(\frac{-\hat{s}}{M_W^2}\right) - \ln^2\left(\frac{-\hat{t}}{M_W^2}\right) - \ln^2\left(\frac{-\hat{u}}{M_W^2}\right) + \ln^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2}[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right)] - \frac{20\pi^2}{9} - \frac{\pi}{\sqrt{3}}$$
$$+2\Bigg]\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\right) + \frac{1}{2}\Bigg[\ln^2\left(\frac{\hat{u}}{\hat{s}}\right) - \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)\Bigg]\left(\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}}\right) - 2\Bigg[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right) + 2\pi^2\Bigg]\Bigg\}\Bigg\}$$

Very compact expressions

- NLL predicted by process-independent formula [Denner, S.P. (2001)]
- **NNLL** consist of $\pi^2, \pi/\sqrt{3}, \ln(\hat{t}/\hat{u}), \ldots$ not growing with energy

NLL/NNLL approximations vs exact calculation

$\mathrm{d}\sigma/\mathrm{d}p_T$ **for pp→ Zj**



Large negative corrections

- -25% at $p_T \sim 1$ TeV
- NLO, NLL, NNLL overlap!

Precision of NNLL approximation

- better than 0.2%

Precision of NLL approximation

- better than 1% ! (process-dependent)

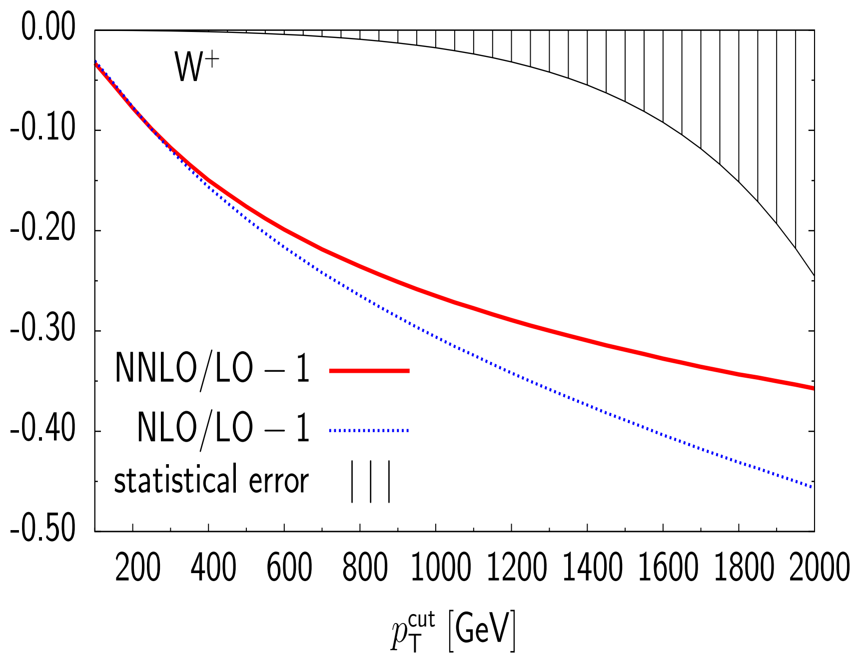
⇒ use asymptotic expansions for two-loop EW corrections at high energies

Two-loop LL and NLL corrections for pp→Wj [Kühn, Kulesza,S.P.,Schulze (2007)]

Compact formula for partonic scattering amplitudes ($\bar{q}q' \to Wg$)

$$\overline{\sum}|\mathcal{M}_2|^2 = 4\frac{\alpha^3\alpha_S}{s_W^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\left\{\left(C^{\mathrm{ew}}_{q_L}+\frac{C_A}{2s_W^2}\right)\left[\frac{C_A}{2s_W^2}\left(\ln^4\left(\frac{\hat{t}}{M_W^2}\right)+\ln^4\left(\frac{\hat{u}}{M_W^2}\right)-\ln^4\left(\frac{\hat{s}}{M_W^2}\right)\right)\right.\right.$$

$$\left.\left.+C^{\mathrm{ew}}_{q_L}\left(\ln^4\left(\frac{\hat{s}}{M_W^2}\right)-6\ln^3\left(\frac{\hat{s}}{M_W^2}\right)\right)\right]+\frac{1}{3}\left[\frac{b_1}{c_W^2}\left(\frac{Y_{q_L}}{2}\right)^2+\frac{b_2}{s_W^2}\left(C_F+\frac{C_A}{2}\right)\right]\ln^3\left(\frac{\hat{s}}{M_W^2}\right)\right\}$$



Size of one-loop and two-loop corrections

- $-27\% + 3\% = -24\%$ at $p_T \sim 1$ TeV
- $-42\% + 9\% = -33\%$ at $p_T \sim 2$ TeV

Two-loop $\simeq$ 1-2 σ_{stat}!

Factorization and Exponentiation of IR logarithms: QCD vs EW

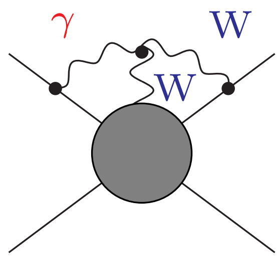
Logarithmic dependence of QCD scattering amplitudes on IR cut-off

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_{\mathrm{T}})} = K(\mu_{\mathrm{T}})\mathcal{M} \qquad \Rightarrow \qquad \mathcal{M}(\mu_{\mathrm{T}}) = \exp\left[-\int_{\mu_{\mathrm{T}}}^{Q} \frac{\mathrm{d}\mu}{\mu} K(\mu)\right] \mathcal{M}(Q)$$

Powerful formalism to describe higher-order logarithms in QCD: can be used to derive higher-order EW logarithms?

How to deal with mass gap in the electroweak gauge sector?

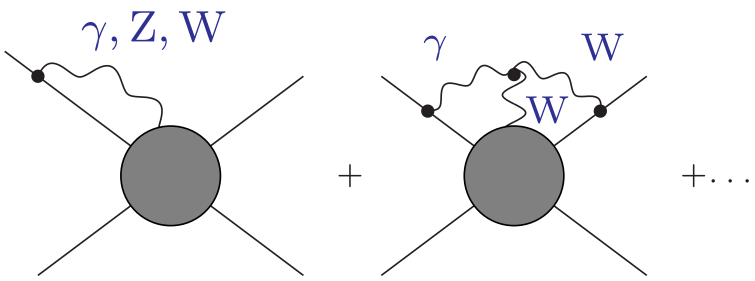
$M_\gamma = 0 \ll M_{\mathrm{Z}} \sim M_{\mathrm{W}}$:



$$\Rightarrow \quad \alpha^2 \frac{1}{\varepsilon} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right)$$

Symmetry-breaking problem reduced to two problems with unbroken symmetry

SU(2)×U(1) regime: $\mu_{\mathrm{T}} > M_{\mathrm{W,Z}}$

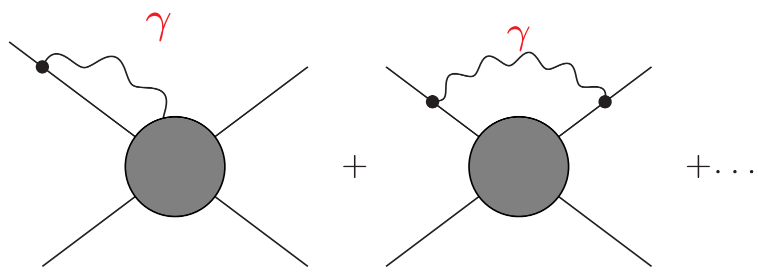


mass gap irrelevant $(M_\gamma = M_{\mathrm{Z}} = M_{\mathrm{W}})$

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_{\mathrm{T}})} = K_{\mathrm{EW}}(\mu_{\mathrm{T}})\mathcal{M}$$

as in symmetric $\mathrm{SU}(2) \times \mathrm{U}(1)$ theory

$\mathrm{U}(1)_{\mathrm{em}}$ regime: $\mu_{\mathrm{T}} < M_{\mathrm{W,Z}}$



weak boson frozen $(M_{\mathrm{Z}}, M_{\mathrm{W}} = \infty)$

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_{\mathrm{T}})} = K_{\mathrm{QED}}(\mu_{\mathrm{T}})\mathcal{M}$$

as in QED

Prediction: double factorization and exponentiation

$$\mathcal{M}(\mu_{\mathrm{T}}) = \exp\left\{-\int_{\mu_{\mathrm{T}}}^{M_{\mathrm{W}}} \frac{\mathrm{d}\mu}{\mu} K_{\mathrm{QED}}(\mu)\right\} \exp\left\{-\int_{M_{\mathrm{W}}}^{\sqrt{s}} \frac{\mathrm{d}\mu}{\mu} K_{\mathrm{EW}}(\mu)\right\} \mathcal{M}_{\mathrm{Born}}$$

Fadin, Lipatov, Martin, Melles (2000)

Two-loop calculations based on EW Feynman rules

QCD-inspired resummations rely on strong theoretical assumptions

- EWSB completely neglected! (apart from two-regime splitting)

Can be checked against explicit 2-loop calculations

- select relevant loop diagrams generated from SB EW Lagrangian
- extract logarithms arizing in IR and UV regions

The (few) existing results agree with the QCD-inspired resummations

$$\alpha^2 \Bigg[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right)} + \underbrace{C_3 \ln^3\left(\frac{s}{M_W^2}\right)} \Bigg]$$

Melles; Hori,Kawamura, Kodaira (2000)
Beenakker, Werthenbach (2000,2002)

S.P. (2004)

Denner, Melles, S. P. (2003)

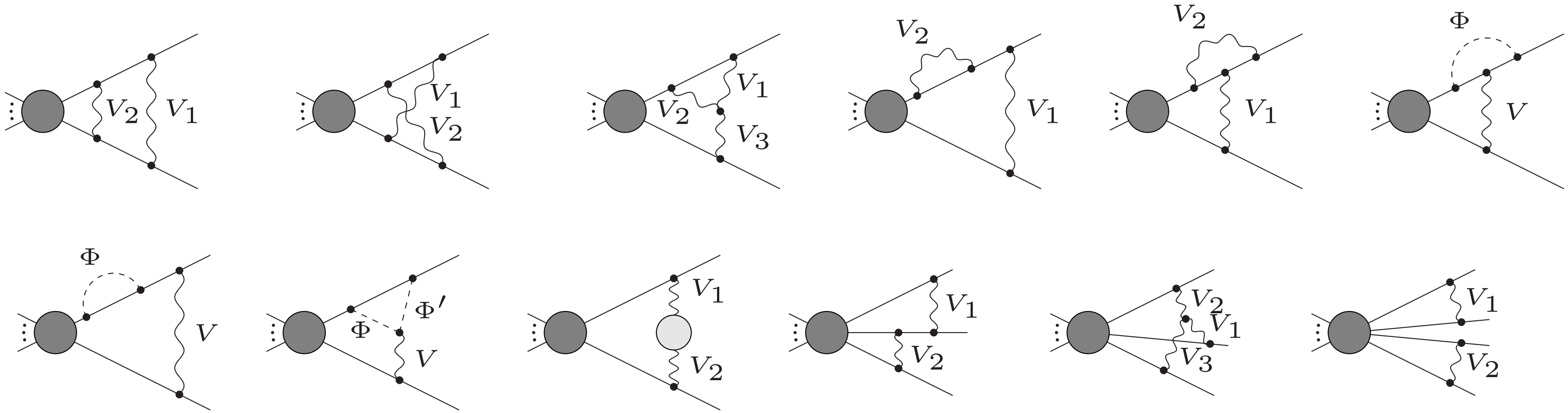
Denner, Jantzen, S.P. (2006,2008)

arbitrary processes involving $Z, W, H, b, t, \ldots$

$f_1 f_2 \to f_3 \ldots f_n$

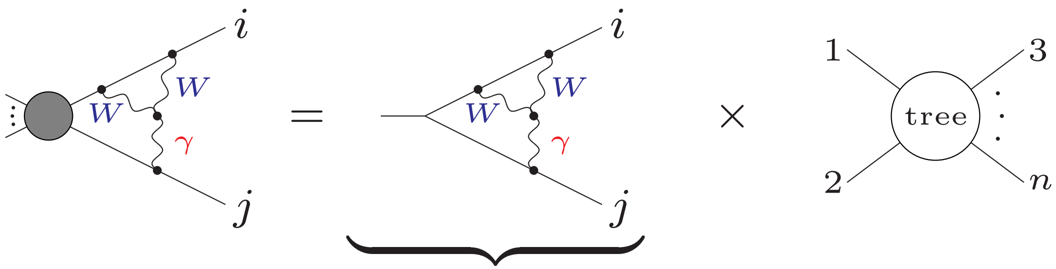
(A) Factorizable two-loop diagrams

Soft/collinear gauge bosons and Higgs/Goldstone bosons coupling only to ext. lines



Factorization and explicit calculation using sector decomposition [Denner, S.P. (2004)]

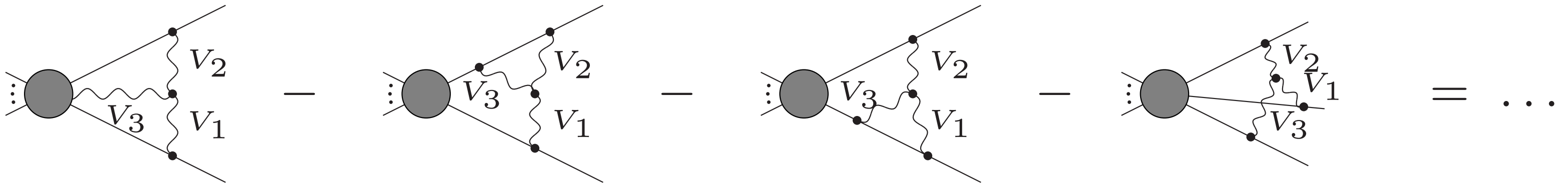
$\Rightarrow L = \ln(Q^2/M^2) \gg 1$ and poles in $D = 4 - 2\epsilon$ from massless γ and fermions



$$\frac{\mathrm{i}e^4 \epsilon^{W\bar{W}\gamma} I_i^W I_i^{\bar{W}} I_j^\gamma}{s_\mathrm{W}} \left(\frac{s}{Q^2}\right)^{2\epsilon} \left[-\frac{1}{3} L^3 \epsilon^{-1} - 5L^4 - 6\epsilon^{-3} - 6L\epsilon^{-2} - 2L^2\epsilon^{-1} + \frac{2}{3} L^3\right]$$

(B) Cancellation of non-factorizable two-loop diagrams

Collinear gauge bosons coupling to external and internal lines



Collinear Ward identities for SB non-abelian theories [Denner, Jantzen, S.P. (2001,2006)]

$$\dots = \mu_0^{4\epsilon} \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \int \frac{\mathrm{d}^D q_2}{(2\pi)^D} \frac{4\mathrm{i}e^2 g_2 \varepsilon^{V_1 V_2 V_3}}{(q_1^2 - M_{V_1}^2)(q_2^2 - M_{V_2}^2)(q_3^2 - M_{V_3}^2)(p_i - q_2)^2 (p_j - q_1)^2}$$

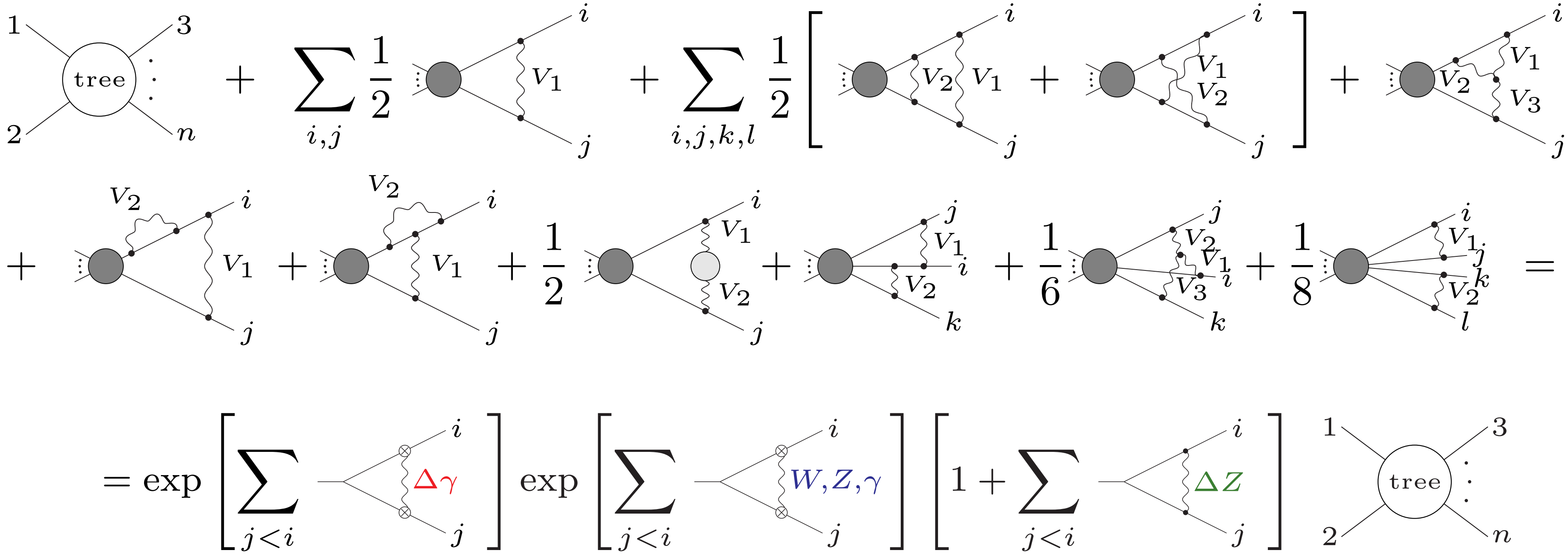
$$\times \lim_{q_1^\mu \to 0} \lim_{q_2^\mu \to x p_i^\mu} (p_i - q_2)^{\mu_2} (p_j - q_1)^{\mu_1} \left[g_{\mu_1 \mu_2} (q_1 - q_2)^{\mu_3} + g_{\mu_2}^{\mu_3} (q_2 + q_3)_{\mu_1} - g_{\mu_1}^{\mu_3} (q_3 + q_1)_{\mu_2} \right]$$

$$\times \sum_{\varphi'_i, \varphi'_j} \Bigg\{ G_{\mu_3}^{[\underline{\bar{V}_3}\,\underline{\varphi'_i}]} (q_3, p_i - q_2) u(p_i, \kappa_i) + \frac{2(p_j + q_2)_{\mu_3}}{(p_j + q_2)^2} \sum_{\varphi''_j} e I^{\bar{V}_3}_{\varphi''_j \varphi'_j} \mathcal{M}_0^{\varphi_1 \cdots \varphi'_i \cdots \varphi''_j \cdots \varphi_n}$$

$$+ \sum_{\substack{k=1 \\ k \neq i,j}}^{n} \frac{2(p_k + q_3)_{\mu_3}}{(p_k + q_3)^2} \sum_{\varphi'_k} \mathcal{M}_0^{\varphi_1 \cdots \varphi'_i \cdots \varphi'_j \cdots \varphi'_k \cdots \varphi_n} e I^{\bar{V}_3}_{\varphi'_k \varphi_k} \Bigg\} I^{\bar{V}_1}_{\varphi'_j \varphi_j} I^{\bar{V}_2}_{\varphi'_i \varphi_i} = 0$$

This cancellation mechanism permits process-independent treatment

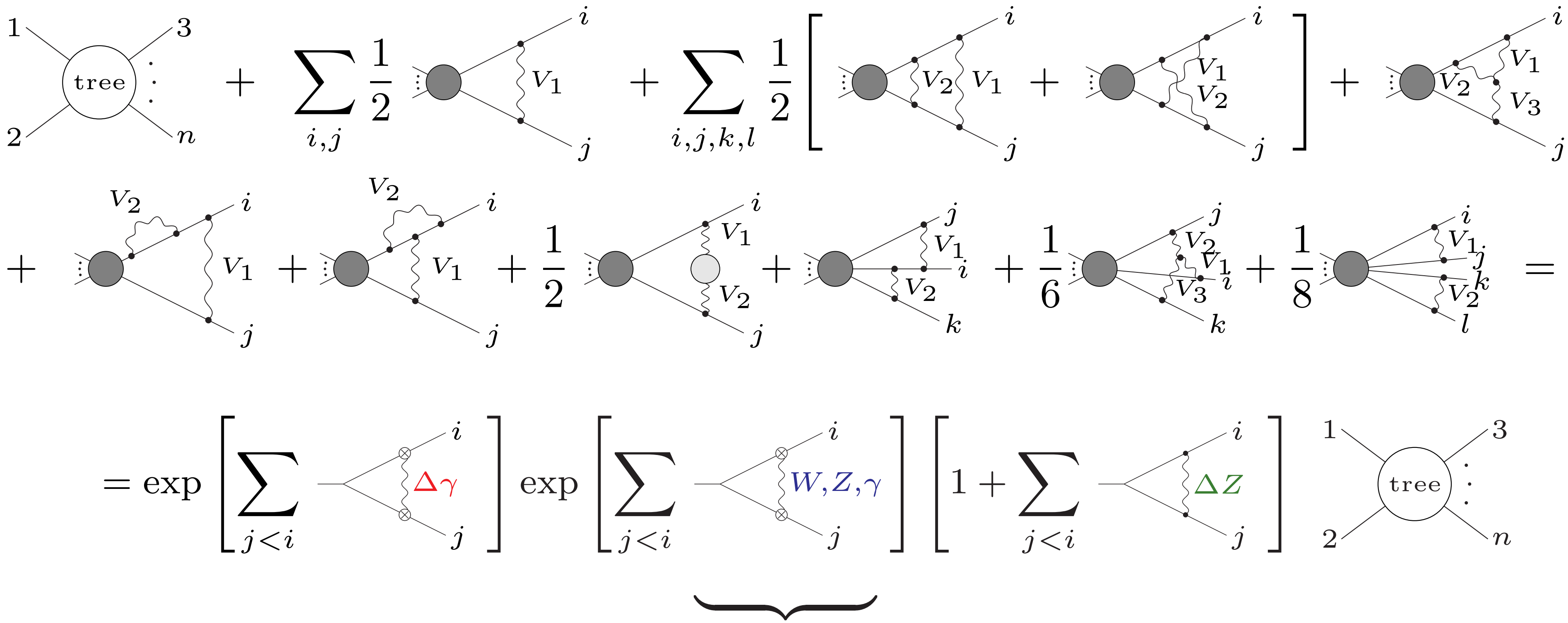
Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$



$\mathcal{O}(100)$ **inequivalent two-loop diagrams** $\Rightarrow$ **very simple result!**

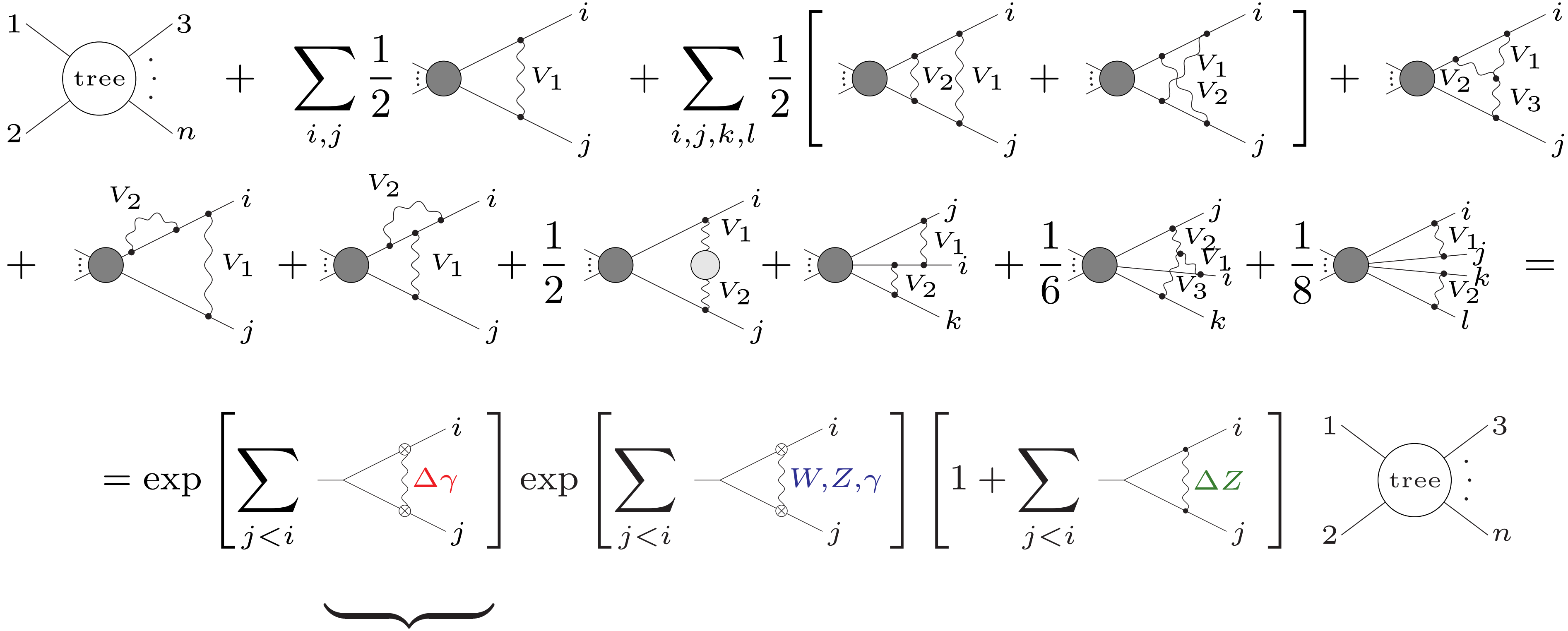
- Two-loop $\equiv$ exp(1-loop)$\times$ Born
- Confirms structure predicted by QCD-inspired resummations

Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$



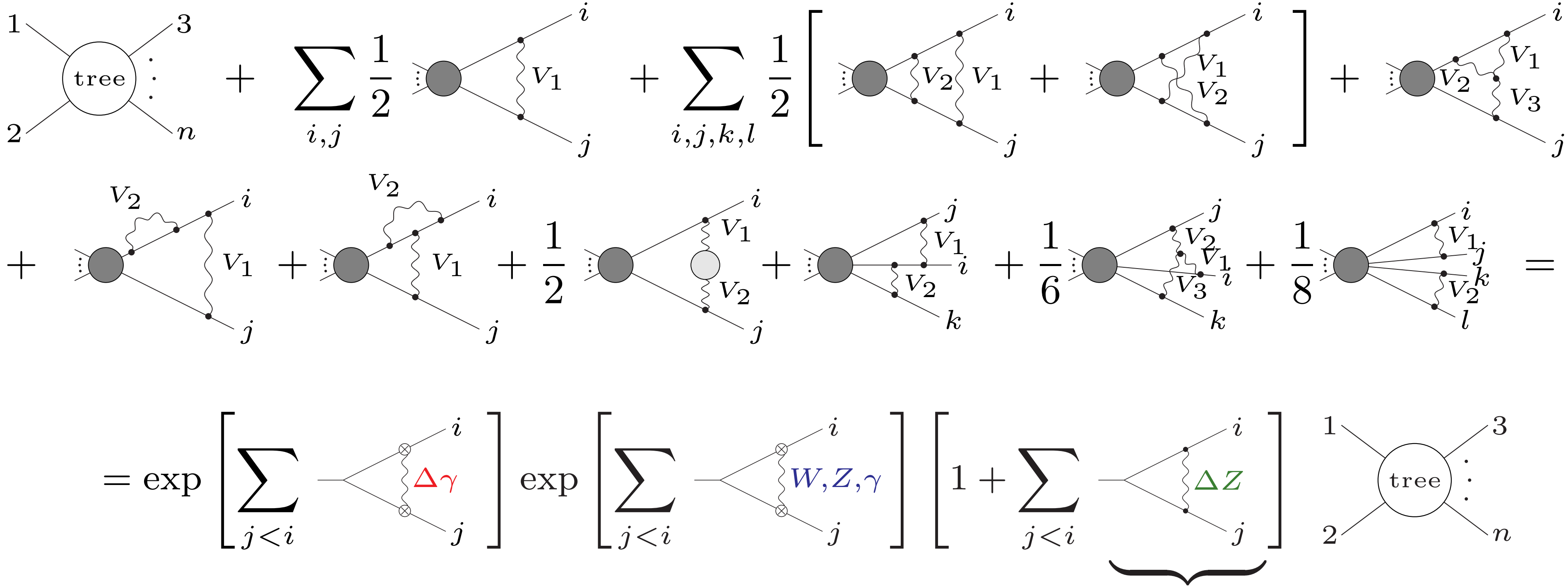
contains only $L = \ln(s/M_W^2)$ and behaves as in a symmetric SU(2)xU(1) theory with $M_W = M_Z = M_\gamma$

Two-loop NLL result for $f_1 f_2 \to f_3 \ldots f_n$



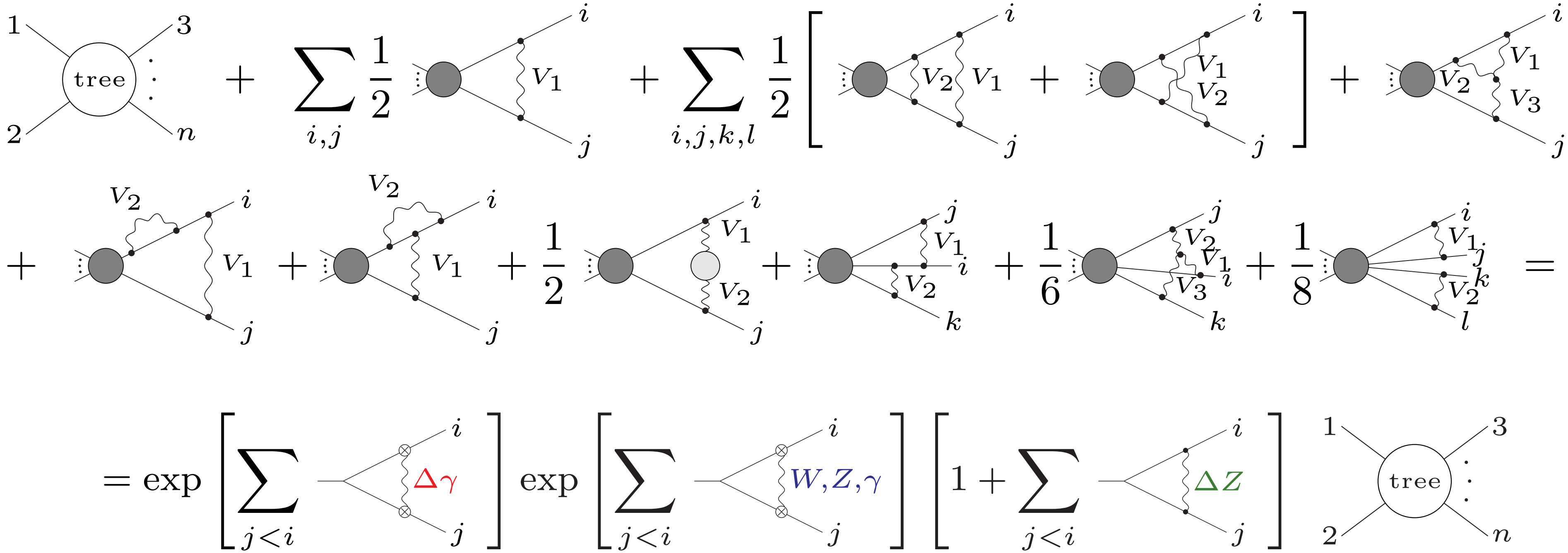
photonic $1/\epsilon$ singularities
factorize and behave as in QED

Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$



Additional mixing correction depending on Z-W mass difference $\Rightarrow \mathcal{O}(10^{-3})$ effect at two loops

Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$



- these results applicable to $q\bar{q} \to \mu^+\mu^-$, $u\bar{d} \to t\bar{b}$, $gg \to b\bar{b}$, ...
- our tools permit to extend this analysis to processes with γ, W, Z, H
- will start to play a (small) role only at 100 fb^{-1} or higher integ. luminosities