The Description of Multi-Jet Events at the LHC

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History of Scales in a pp Collision

The description of a complete LHC collision is rather messy. . .
pp $\longrightarrow$ cascades of hadrons, leptons, photons

. . . but the work is nicely divided up into (largely) independent pieces

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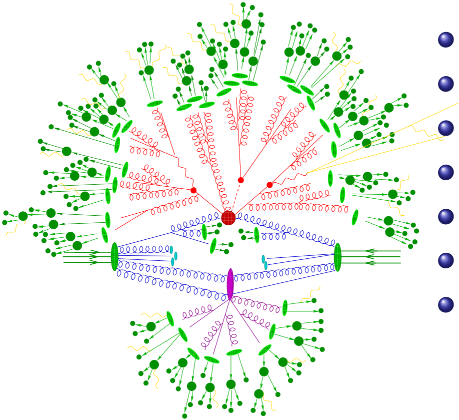
Inner Workings of a pp Collision



- Factorisation (PDF)
- Hard Scattering
- Parton Showering
- Hadronisation
- Multiple Interactions
- Underlying Event
- ...

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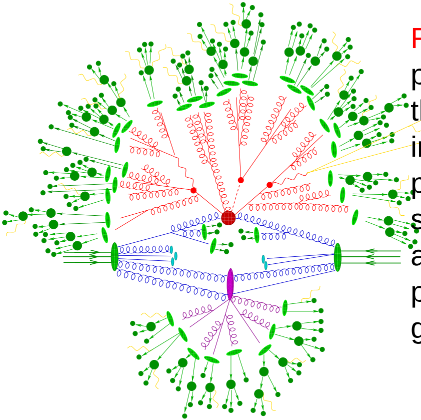

Factorisation allows us to parametrise our ignorance of the dynamics of confinement in functions which are independent of the process under study. Loosely interpreted as the probability to extract a parton from the proton with a given energy.

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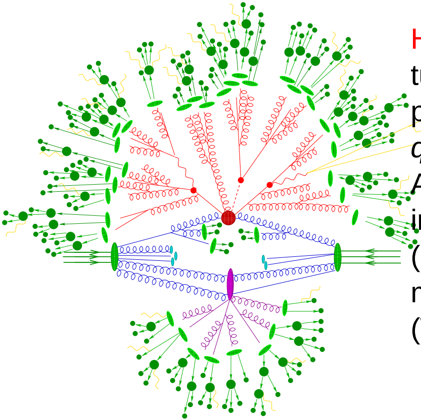

Hard Scattering is the perturbative study of partonic processes, e.g. $gg \to gg$, $qg \to qgg(W \to)e\nu, \ldots$

Applied to processes producing particles with larger than (say) 20-40GeV transverse momentum, or large mass (W,Z,H,...).

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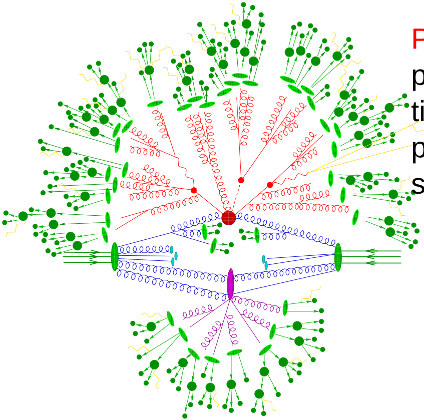

Parton Showering takes the few parton state of the hard interaction and evolves it to a many-parton state (simplifying assumptions for the radiation)…

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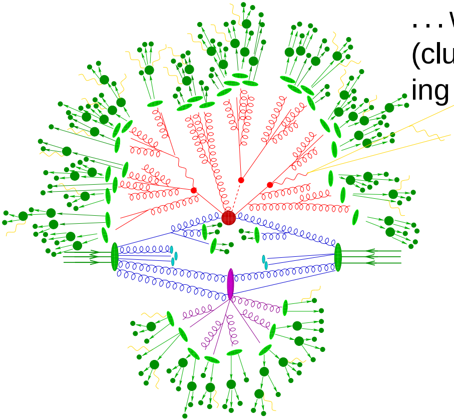

...which can be hadronised (clustered into hadrons) according to various models

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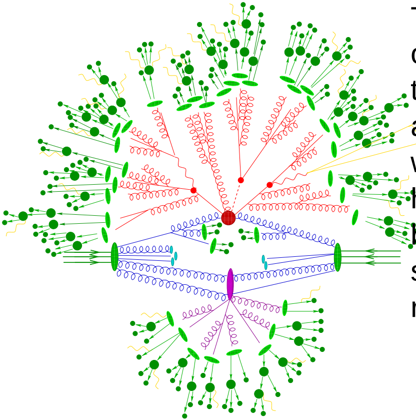


The notion of jets (algorithms for organising events according to the "number of hard objects"), allows one to define questions which are mostly insensitive to hadronisation etc. I.e. possibility of relating the few-parton state of the Hard Scattering directly to experimental data.

Describing the Hard Scattering

The Hard Scattering is often described using Fixed Order perturbation theory.

LO known for all relevant processes (given sufficient time, any process can by calculated)

NLO known for some processes (great recent progress in calculating the virtual (loop) corrections and automating subtraction)

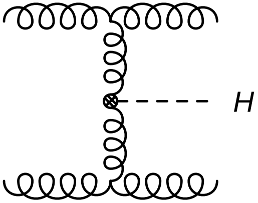
NNLO known only for a few, very simple processes (colour singlet final states at LO: $gg \to H/Z$)

The Born approximation (LO) may not always be sufficient. Not just because of large corrections to the total cross section (important for signal & background estimates). But also because HO may help in discriminating between background and signal.

All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate
Higgs Boson plus n jets at the LHC at leading order



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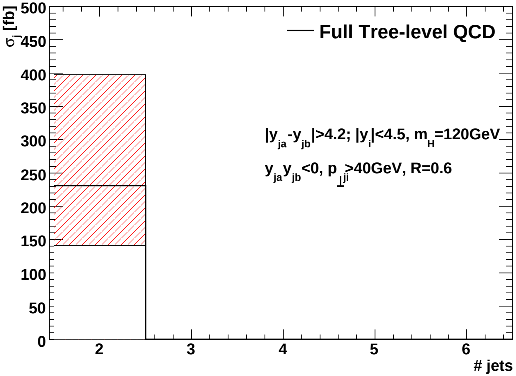

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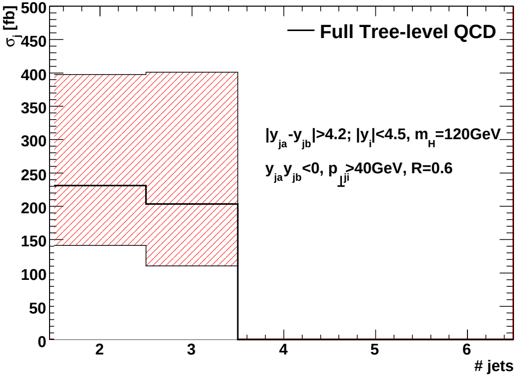

Indication that we need to go further! However, fixed order tools **exhausted** (full $2 \to 3$ with a massive leg at two loops **untenable!**).

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- The calculation of many perturbative corrections giving rise to hard Multi-Jet Emission is not a problem we can just choose to ignore
- It is not a problem which is already solved
- ...or can be solved with current techniques [a]

[a] Well, actually, we try

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of $2, 3, 4, \ldots$ parton rates) and in a flexible way (jets, W+jets, Z+jets, Higgs+jets,...)

Why?

$(n+1)$-jet rate not necessarily small compared to n-jet rate
Inclusive (hard) perturbative corrections important for e.g. hard end of W $p_\perp$-spectrum.

How?

Establish universal behaviour of radiative corrections (in the so-called High Energy Limit)

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What, Why, How?

Goal

- Sufficiently *simple* model for radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)
- Sufficiently *accurate* that the description is relevant

Factorisation of Matrix Elements

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\rightarrow$ **eikonal approximation** $\rightarrow$ enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

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Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\forall i \in \{2, \dots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\, s^2}{N_C^2 - 1}\, \frac{g^2\, C_A}{|p_{1\perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4\, g^2 C_A}{|p_{i\perp}|^2}\right) \frac{g^2\, C_A}{|p_{n\perp}|^2}.$$

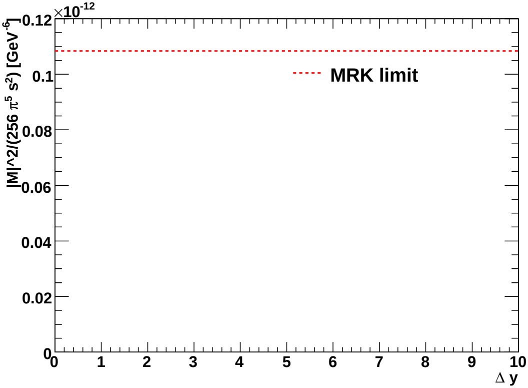
$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\, s^2}{N_C^2 - 1}\, \frac{g^2\, C_F}{|p_{1\perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4\, g^2 C_A}{|p_{i\perp}|^2}\right) \frac{g^2\, C_A}{|p_{n\perp}|^2},$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\, s^2}{N_C^2 - 1}\, \frac{g^2\, C_F}{|p_{1\perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4\, g^2 C_A}{|p_{i\perp}|^2}\right) \frac{g^2\, C_F}{|p_{n\perp}|^2},$$

However, how well does this actually approximate the amplitude?

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:



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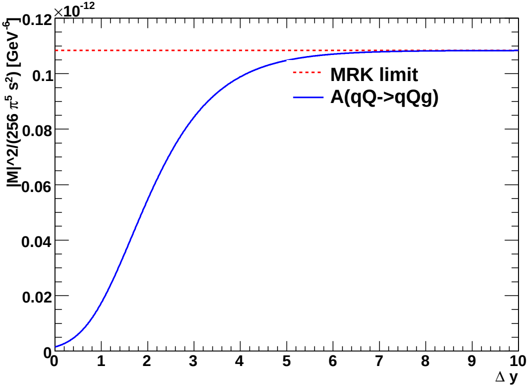

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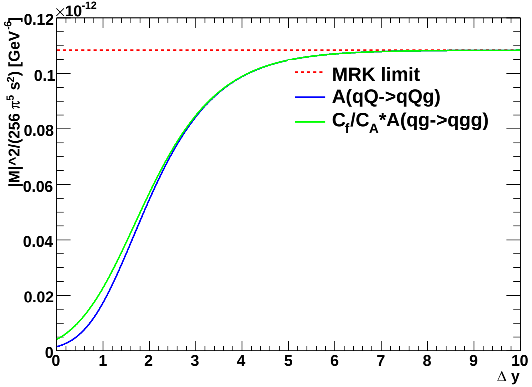

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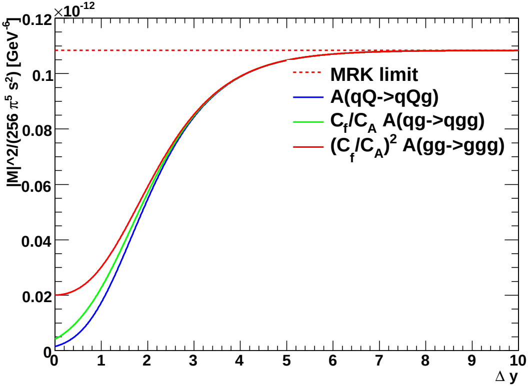

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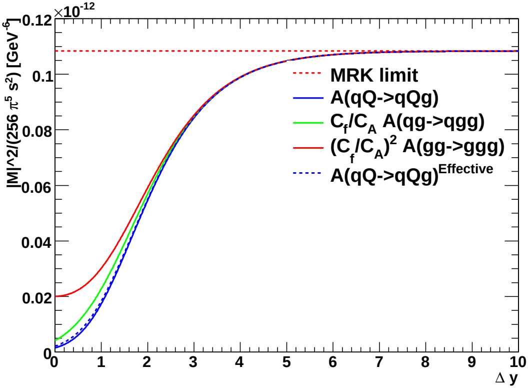
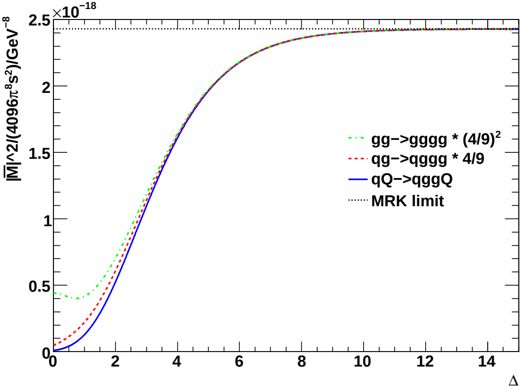


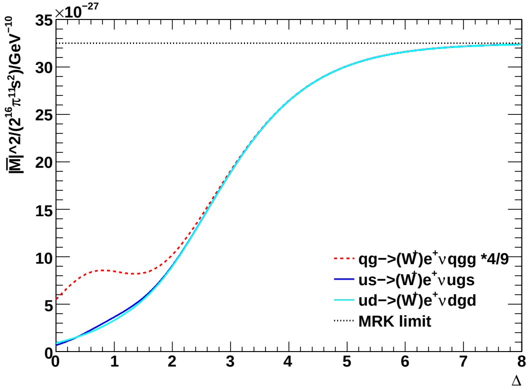
Comparison of 4-jet scattering amplitudes

Study just a slice in phase space:



Comparison of W+3-jet scattering amplitudes

Study just a slice in phase space:



Comparison of W+4-jet scattering amplitudes

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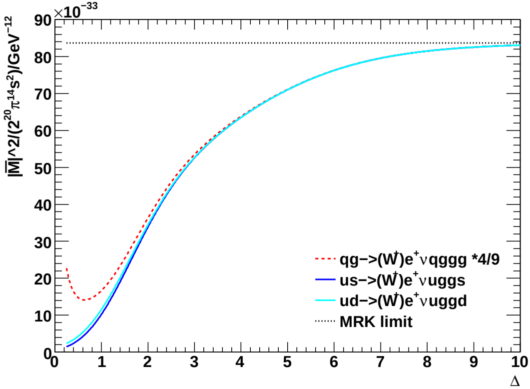
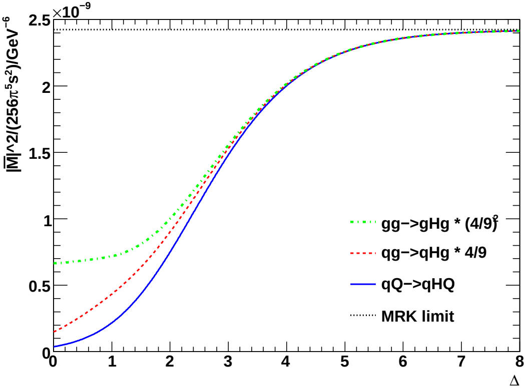


Comparison of H+2-jet scattering amplitudes

Study just a slice in phase space:



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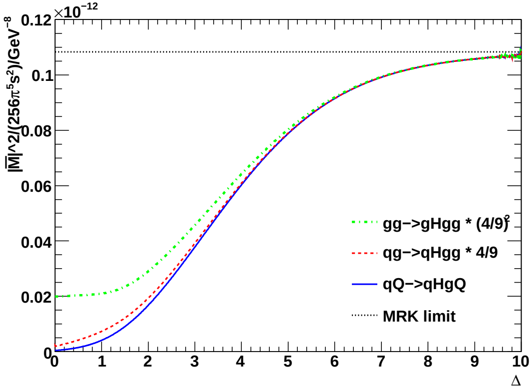


Conclusion from Study of Partonic Cross Sections

- Correct limit is obtained - but outside LHC phase space. Limit alone irrelevant.
- Universality obtained before limit is reached.

Will build frame-work which has the right MRK limit but also retains correct behaviour at smaller rapidities

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across *t*-channel pole

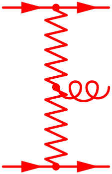
$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)} \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\, C_F\, \frac{1}{t_1}\right) \cdot \left(g^2\, C_F\, \frac{1}{t_2}\right).$$

Extend to $2 \to n$ ...

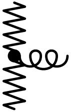
J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

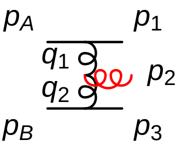


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

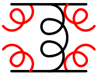


$$V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$V^\rho(q_1,q_2) = -(q_1+q_2)^\rho + \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A} + \frac{p_2\cdot p_B}{p_A\cdot p_B} + \frac{p_2\cdot p_n}{p_A\cdot p_n}\right) + p_A \leftrightarrow p_1 - \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B} + \frac{p_2\cdot p_A}{p_B\cdot p_A} + \frac{p_2\cdot p_1}{p_A\cdot p_1}\right) - p_B \leftrightarrow p_3.$$

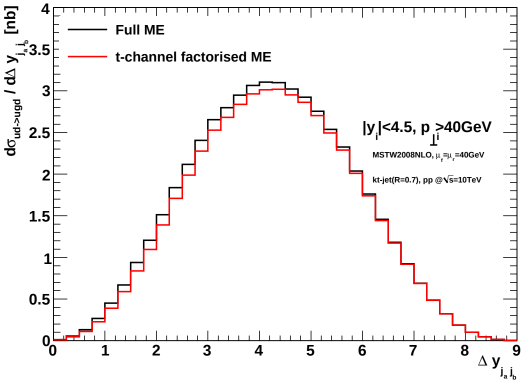


Building Blocks for an Amplitude

The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^{t}_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2$$
$$\cdot\ \left(g^2\ C_F\ \frac{1}{t_1}\right)\cdot\ \left(g^2\ C_F\ \frac{1}{t_2}\right)$$
$$\cdot\ \left(\frac{-g^2 C_A}{t_1 t_2}\ V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right).$$

3 Jets @ LHC



J.M.Smillie and JRA: arXiv:0908.2786

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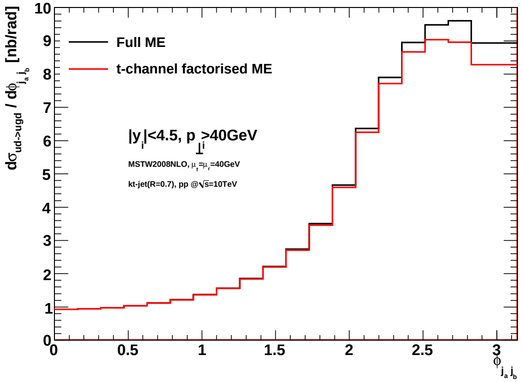

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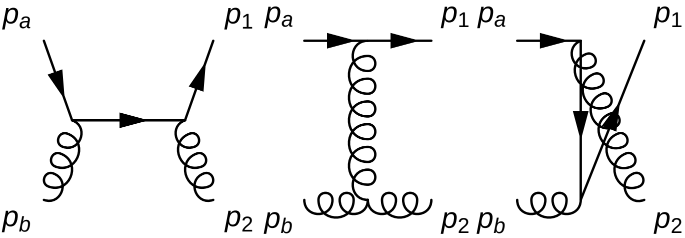
Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \; \times \langle 1|\sigma|a\rangle .$$

Complete t-channel factorisation!

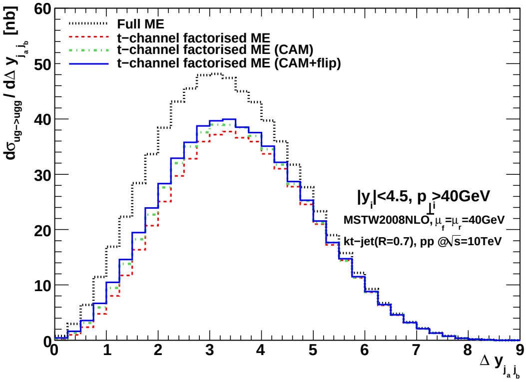
Quark-Gluon Scattering

The t-channel current generated by a helicity non-flipping gluon is that of a quark with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in MRK limit.

Quark-Gluon Scattering



J.M.Smillie and JRA

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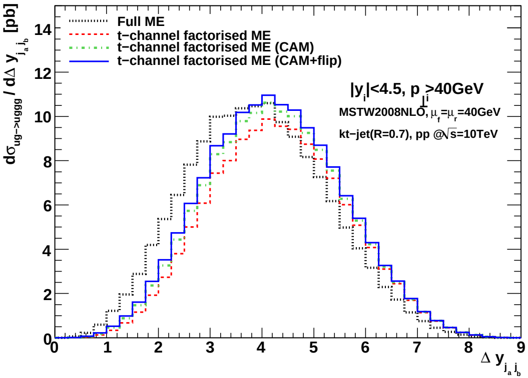

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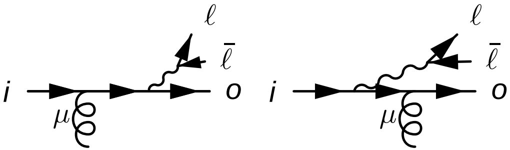
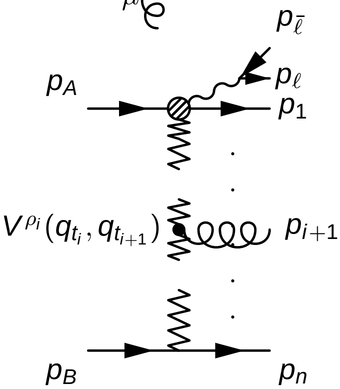
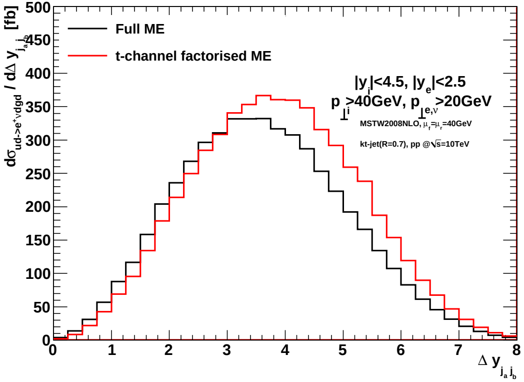
W+Jets

Two currents to calculate for $W + jets$:





W+ 3 Jets @ LHC



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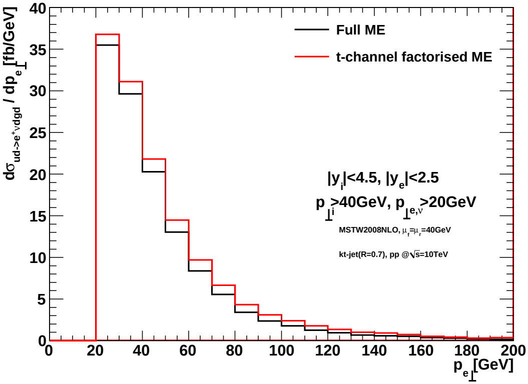

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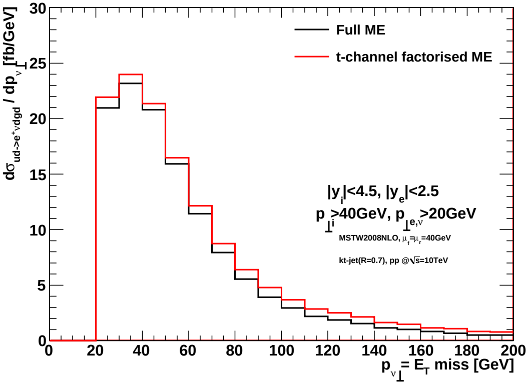


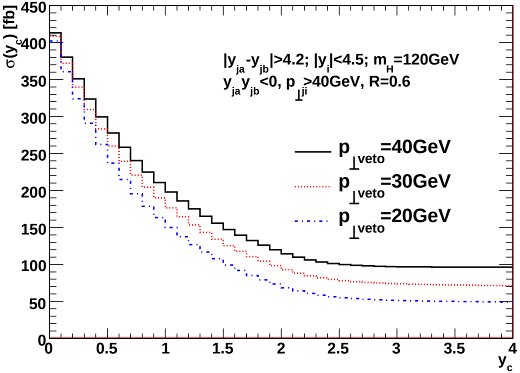
J.M.Smillie and JRA: arXiv:0908.2786

All-Orders and Regularisation

- Have prescription for $2 \to n$ matrix element, including virtual corrections
- Organisation of cancellation of IR (soft) divergences easy
- Can calculate the sum over the n-particle phase space explicitly ($n \sim 25$) to get the all-order corrections

V. Del Duca, C.D. White, JRA arXiv:0808.3696, J.M. Smillie, JRA arXiv:0908.2786

Effect of Central rapidity jet veto in H+diJets



$$\forall j \in \{\text{jets with } p_{j\perp} > p_{\perp,\text{veto}}\} \setminus \{a,b\} \ : \ \left| y_j - \frac{y_a + y_b}{2} \right| > y_c$$

Summary

Conclusions

- Emerging framework for the study of processes with multiple hard jets
- For each number of particles n, the approximation to the matrix element (real and virtual) is sufficiently simple to allow for the all-order summation to be constructed as an explicit sum over n-particle final states (exclusive studies possible)
- Resummation based on approximation which really does capture the behaviour of the scattering processes at the LHC
- Matching will correct the approximation where the full matrix element can be evaluated