Holography and Applications

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An unorthodox ‘derivation’ of AdS/CFT

- Let us imagine we are back in 1996 and know nothing about AdS/CFT.
- We can go a long way in ‘deriving’ the AdS/CFT correspondence by studying classical AdS gravity:

$$R_{\mu\nu} - \frac{1}{2}(R - 2\Lambda)g_{\mu\nu} = 0$$

- A mathematically interesting question to ask is what is the space of solutions, i.e. the classical phase space, of such an equation.
- Finding the space of non-singular solutions is an extremely hard problem in dimensions higher than 3. In 3 dimensions it is given by the Teichmüller space of Riemann surfaces [K. Krasnov, ...]
- The problem remains hard if one allows for singular solutions, but progress can be made...

The Fefferman-Graham theorem

Fefferman-Graham theorem

- In an open neighborhood of the conformal boundary at $r = \infty$ the most general solution of Einstein's equation with a negative cosmological constant takes the form

$$ds^2 = dr^2 + \gamma_{ij}(r,x)dx^i dx^j, \quad i,j = 1,\ldots,d$$

where

$$\gamma_{ij} = e^{2r}\left(g_{(0)ij} + e^{-2r}g_{(2)ij} + \cdots + g_{(d)ij}e^{-dr} + h_{(d)ij}e^{-dr}r + \cdots\right)$$

- The tensors $g_{(0)ij}(x)$ and the transverse traceless part of $g_{(d)ij}(x)$ can be independently prescribed and they *uniquely* determine the bulk metric in an open neighborhood of the conformal boundary.
- In particular, $g_{(2)},\ldots,g_{(d-2)}$ and $h_{(d)}$ are algebraically determined in terms of $g_{(0)}$ and its derivatives, while all higher order terms are determined in terms of $g_{(0)}$ and $g_{(d)}$.

Classical phase space

- The space parameterized by the tensors $g_{(0)ij}$ and $g_{(d)}$ is a symplectic manifold equipped with the symplectic form

$$\Omega = \int d^d x \delta g_{(0)ij} \wedge \delta {g_{(d)}}^{ij}$$

- This classical phase space contains singular solutions as a generic assignment of $g_{(0)}$ and $g_{(d)}$ leads to a singularity at some point far enough from the conformal boundary.
- There is nevertheless enough structure to allow us to 'derive' the kinematics of AdS/CFT...

Classical phase space symmetries and QFT kinematics

- Einstein's equation implies that $g_{(d)}$ must satisfy the constraints:

$$g_{(0)}{}^{ij} g_{(d)ij} = \mathcal{A}[g_{(0)}], \quad D_{(0)}{}^{i} \left(g_{(d)ij} - g_{(0)}{}^{kl} g_{(d)kl} g_{(0)ij} \right) = 0$$

  where $\mathcal{A}[g_{(0)}]$ is a local functional of $g_{(0)}$ that coincides with the conformal anomaly of a d-dimensional conformal field theory! (with certain values for the anomaly coefficients.)

- Given a conformal Killing vector of the metric $g_{(0)ij}$, the above identities imply that the following charges are conserved:

$$Q_{\xi} = \int d\sigma^{i} \left(g_{(d)ij} - g_{(0)}{}^{kl} g_{(d)kl} g_{(0)ij} \right) \xi^{j}$$

- The Poisson algebra of these charges provides a represantation of the asymptotic symmetry slgebra $so(2, d)$ on phase space. In three dimensions ($d = 2$), this turns out to be two copies of a centrally extended Virasoro algebra [Brown-Henneaux]!

Classical phase space II: dynamics

- We have seen that the quantity

$$\mathcal{T}_{ij}[g_{(0)}] = \left(g_{(d)ij} - g_{(0)}{}^{kl} g_{(d)kl} g_{(0)ij}\right)$$

has the kinematic properties required of the stress tensor of a conformal field theory coupled to the metric $g_{(0)ij}$. What about the dynamics?

- To compute the corresponding 'correlation functions' we need to evaluate the functional derivatives

$$\langle T(x_1) \ldots T(x_n) \rangle \sim \frac{\delta \mathcal{T}[g_{(0)}(x_1)]}{\delta g_{(0)}(x_2) \ldots \delta g_{(0)}(x_n)}$$

- Determining $g_{(d)}[g_{(0)}]$ in full generality corresponds to finding the subspace of the above phase space that includes only *non-singlular* solutions. We now see that it also corresponds to computing *all* correlation functions of the dual QFT!

- To compute the n-point function we need to compute $g_{(d)}[g_{(0)}]$ to order $n-1$ in $g_{(0)}$ around a background solution.
- Remarkably, the dynamics encoded in such correlation functions is consistent with that of a strongly coupled CFT!

Recent examples of CFT dynamics from AdS gravity

- 2-point function in the background of a black hole: quark-gluon plasma
- $g_{(d)}[g_{(0)}]$ in a derivative expansion around a black hole: relativistic Navier-Stokes equation [Bhattacharyya, Hubeny, Minwalla, Rangamani]

To summarize...

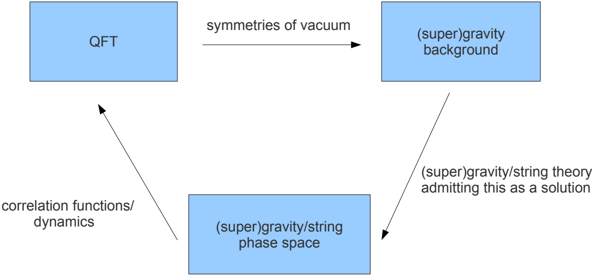
The classical phase space of AdS gravity has the structure of a CFT!

Holographic dualities

- Ultimately, an holographic duality must involve a quantum theory of gravity, such as string theory, on one side, and a non-gravitational quantum theory on the other.
- In systems that admit a semicalssical limit, such as the large N limit, one can expect that gravity captures some corner of the dynamics, around which systematic corrections can be computed.
- Within this class of systems, the systematic construction of the classical phase space of gravitational theories provides a method for ‘constructing’ holographic dualities in both directions:

From QFT to gravity and back

- There are many physically important strongly coupled QFTs that we would like to try to understand via holography.
- Prominent examples are QCD and non-relativistic CFTs that arise at quantum critical points in condensed matter physics.
- Starting with some basic knowledge, such as the symmetries of the vacuum, of such QFTs, one can try to construct a gravitational background as a starting point in finding the holographic dual.
- Constructing the gravitational phase space in such a background then will lead to dynamical information about the QFT...


QFT
symmetries of vacuum
(super)gravity background
(super)gravity/string theory admitting this as a solution
(super)gravity/string phase space
correlation functions/ dynamics

From gravity to exotic QFTs

- However, systematically constructing the classical phase space of gravity can lead to exotic QFTs, non-local quantum theories, or exotic UV completions.
- The Klebanov-Strassler and the Maldacena-Nùñez backgrounds provide such examples of exotic UV completions to versions of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory.
- Moreover, one may hope that such an approach may shed light on the holographic dual of gravity in asymptotically flat spacetimes.

Summary & Conclusions

Classical gravity contains QFT!