

# Non-perturbative Topological Strings

Sara Pasquetti

November 6, 2009

# Outline

- 1 Introduction
- 2 Non-perturbative in  $\alpha'$
- 3 Non-perturbative in  $g_s$

# The nature of the topological string series

The **A-model** topological string large radius expansion counts maps  $\phi : \Sigma_g \rightarrow X$ :

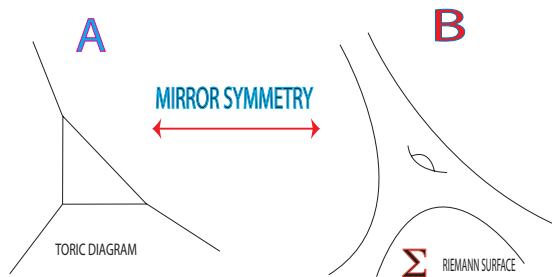
$$F(g_s; t) = \sum_{g=0}^{+\infty} g_s^{2g-2} F_g(t), \quad F_g(t) = \sum_{n \geq 0} N_{g,n} e^{-nt}.$$

- $t$  is the Kähler parameter measuring sizes in the target in units of  $\alpha'$ .  
The series in  $e^{-t}$  has finite convergence radius  $t = t_c$ .  
For  $t < t_c$  non-geometric phases (orbifolds)  $\Rightarrow$  **mirror B-model description**, exact in  $\alpha'$ .  
The problem then is “just” to compute. E.g. holomorphic anomaly equation method, **[BCOV]** (closed), **[Walcher]** (open), **[EMO]** (open local).
- The expansion in  $g_s$  is asymptotic  $F_g \sim (2g)! \Rightarrow e^{-1/g_s}$  effects, need D-branes.  
The problem here is to compute and find sources of non-perturbative effects.

# Toric Calabi–Yau's

- interesting in enumerative geometry
- nontrivial moduli spaces (orbifold points)
- can be used to engineer  $N = 2$  gauge theories
- large  $N$  gauge theory dual (Chern–Simons theories)

For toric CY's **the mirror geometry is encoded in a Riemann surface  $\Sigma$** :



Since from [SW,DV] it is expected that the mirror curve  $\Sigma_{mc}$  encodes all the relevant information  $\rightarrow$  we need a clever way to extract it...

# Matrix Model

Matrix models are integrals of Hermitian  $N \times N$  matrices. Free energies and correlation functions have a genus expansion:

$$F = \log \int dM e^{-\frac{\text{Tr}V(M)}{g_s}} = \sum_{g=0}^{\infty} g_s^{2g-2} F_g(t),$$

$$W(t; p_1, \dots, p_h) = \left\langle \text{tr} \frac{1}{p_1 - M} \cdots \text{tr} \frac{1}{p_h - M} \right\rangle^{(c)} = \sum_{g=0}^{\infty} g_s^{2g-2+h} W_h^{(g)}(t; p_1, \dots, p_h)$$

$t = g_s N$  is the 't Hooft coupling.

In the large  $N$  limit the eigenvalues of  $M$  condense along the branch cuts of the **spectral curve**  $\Sigma_{\text{mm}}$ .

# Matrix Models & the Remodelling Approach

[Eynard–Orantin] developed a recursive method to solve matrix models **in terms of the spectral curve  $\Sigma_{mm}$** :

Input a curve  $\Sigma$   $\Rightarrow$  **Topological Recursion**  $\Rightarrow$

Open and closed correlators:

$$W_h^{(g)}(t; p_1, \dots, p_h), F_g(t)$$

[Mariño, Bouchard–Klemm–Mariño–S.P.] conjectured and tested the the **Remodelling approach**:

Input the mirror curve  $\Sigma_{mc}$   $\Rightarrow$  **Topological Recursion**  $\Rightarrow$

Open and closed topological string amplitudes:

$$W_h^{(g)}(t; p_1, \dots, p_h), F_g(t)$$

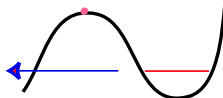
This is the **exact and non-ambiguous solution in  $\alpha'$** .

Better than the topological vertex: amplitudes are defined all over the moduli space.

Better than holomorphic anomaly equation: no ambiguities in the open case.

# Instantons in matrix models

In the **one–instanton** configuration we integrate  $N - 1$  eigenvalues on the principal saddle and **one eigenvalue on the non–trivial saddle**, for instance a local maximum:


$$F = \sum_g g_s^{2g-2} F_g + e^{-A/g_s}$$

$A$  is the **instanton action** related to the **tunneling probability**, it controls the **large order** behavior of the theory:

$$F_g(t) \sim (2g)! A^{-2g} \quad \Rightarrow \text{TESTABLE!!}$$

- **[David]**: mm at criticality, in a double scaling limit  $\rightarrow$  minimal strings.
- **[Mariño et al.]**: mm away from criticality  $\rightarrow$  topological strings ( $c = 0$ ), toric branes interpretation.
- **[S.P.–Schiappa]**: mm  $\rightarrow$  topological strings ( $c = 1$ ) and toric branes interpretation, relation to Stokes phenomena and to the Gopakumar–Vafa formula.

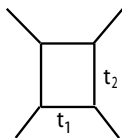
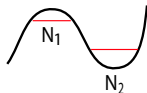
# Gauge–Strings Duality

[Gopakumar–Vafa, AKMV]

$U(N)$  Chern–Simons Gauge Theory  $\hat{k}$ ,  
on a 3-manifold  $\mathcal{M} \Rightarrow$  **Matrix Model**  
formulation [Mariño]

Topological String on toric CY's.

Chern-Simons  
matrix model  
on  
 $\mathcal{M} = S^3/Z_2$



top string on  
local  $F_0$

Choose a vacuum:

$U(N) \rightarrow U(N_1) \times U(N_2)$ , **perturbative saddle expansion** around  $N_1, N_2$ :

$$\log Z(\hat{k}, N; N_1, N_2) = \sum_g g_s^{2g-2} F_g(N_1, N_2)$$

$t_1 = g_s N_1, t_2 = g_s N_2$  are the Kähler parameters.

$F_g(t_1, t_2)$  are the perturbative **background dependent** free energies.

... but all this is **perturbative**  $F_g \sim (2g)!$

# Non–Perturbative Gauge–Strings Duality

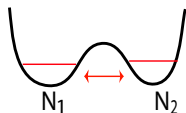
The **non–perturbative** partition function [Eynard–Mariño]:

$$Z_{\text{np}}(\hat{k}, N) = \sum_{N_1+N_2=N} Z(\hat{k}, N; N_1, N_2)$$

is clearly **background independent** (also modular invariant).

What is the dual geometry now? [Mariño–S.P.–Putrov]

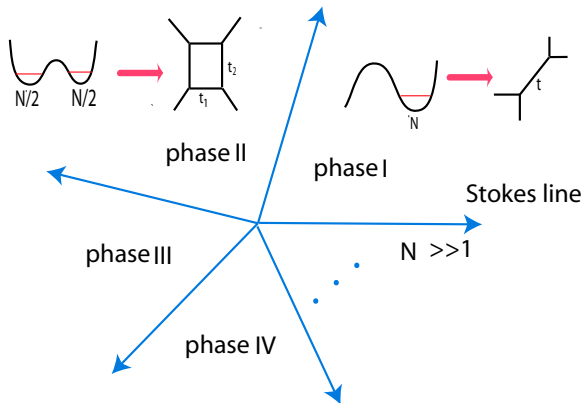
- $\hat{k}$  sets the shape of the effective potential  $\rightarrow$  **many phases**.
- In each phase, at large  $N$ , there is only one equilibrium configuration  $(N_1, N_2) \rightarrow (t_1, t_2)$  geometry, **background dependent perturbative expansion**.
- **Other configurations/geometries** contribute to the free energy with **non–perturbative corrections**. They are not always of the  $e^{-A/g_s}$  type!!



$$F = g_s^{-2} F_0 + F_1 + \log \Theta + \dots$$

- There are **large  $N$ , topology–changing phase transitions** when going from one region to the next one.

# Exact Gauge–Strings Duality



This simple toy model is in fact extremely rich and insightful, and we can test everything numerically!!

# More reasons to study Topological Strings

- Non-Geometric (Orbifolds) Phases of String Theory.
- Non-Perturbative Effects related to the Large Order Behavior.
- Non-Perturbative Definition & Background Independence.
- Gauge-String Dualities Beyond Genus Expansion.
- *Falsifiability*: results can often be proved mathematically, or checked via dual computations, or tested numerically
- *Aesthetics*