Electron dynamics with Synchrotron Radiation

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Useful books and references

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

H. Wiedemann, *Particle Accelerator Physics I and II*
Springer Study Edition, 2003

A.Hofmann, *The Physics of Synchrotron Radiation*
Cambridge University Press 2004

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

CERN Accelerator School Proceedings

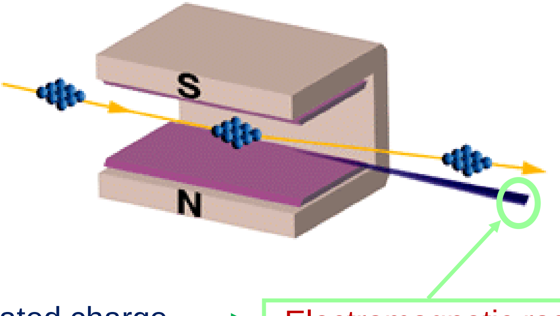
Synchrotron Radiation and Free Electron Lasers

Grenoble, France, 22 - 27 April 1996
(A. Hofmann's lectures on synchrotron radiation)
CERN Yellow Report 98-04

Brunnen, Switzerland, 2 – 9 July 2003
CERN Yellow Report 2005-012

Previous CAS Schools Proceedings

Curved orbit of electrons in magnet field

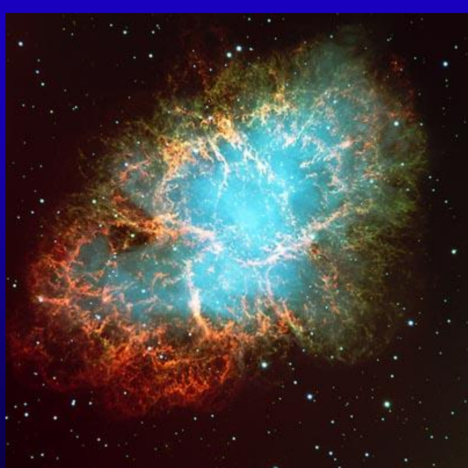


Accelerated charge → Electromagnetic radiation

Electron Beam Dynamics, L. Rivkin, Introduction to Accelerator Physics, Budapest

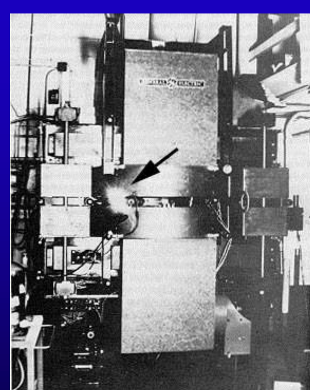
Electromagnetic waves

Crab Nebula
6000 light years away

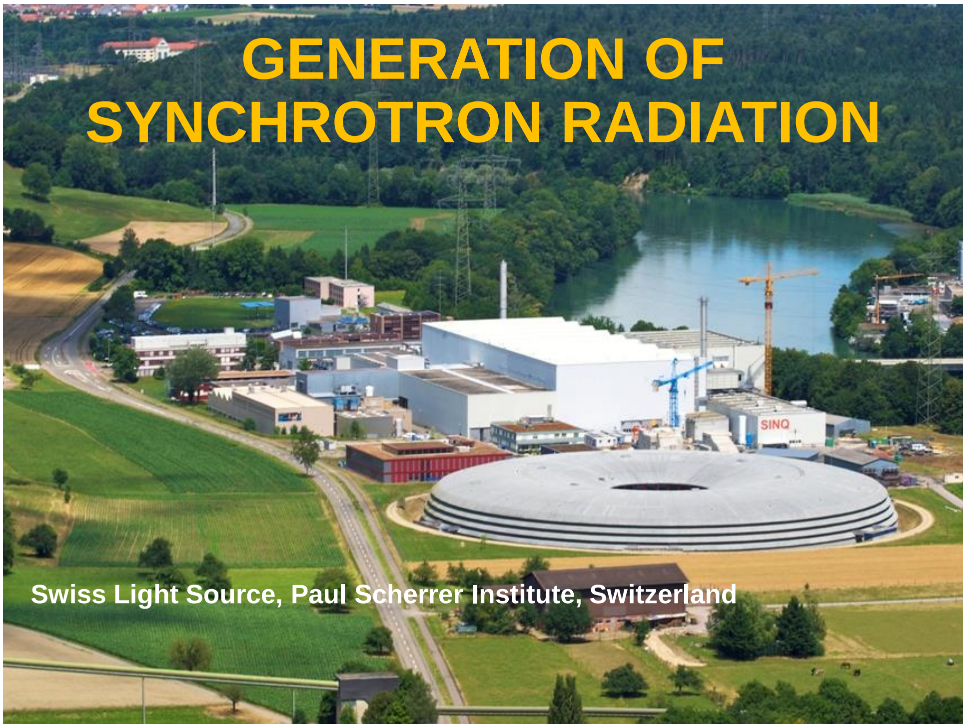


First light observed
1054 AD

GE Synchrotron
New York State



First light observed
1947



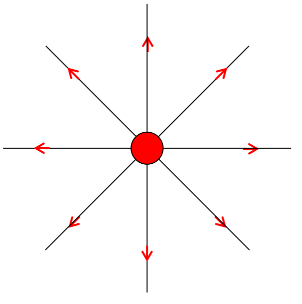
60‘000 SR users world-wide

Why do they radiate?

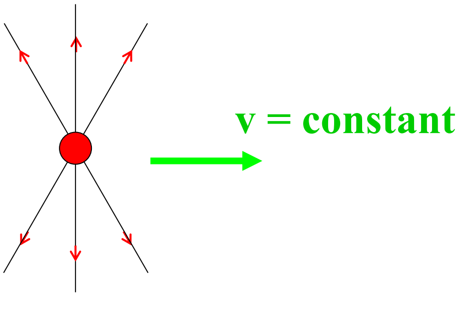
Synchrotron Radiation is not as simple as it seems

… I will try to show
that it is much simpler

Charge at rest
Coulomb field, no radiation



Uniformly moving charge does not radiate



But! Cerenkov!

Free isolated electron cannot emit a photon

Easy proof using 4-vectors and relativity

- momentum conservation if a photon is emitted

$$\boldsymbol{P}_i = \boldsymbol{P}_f + \boldsymbol{P}_\gamma$$

- square both sides

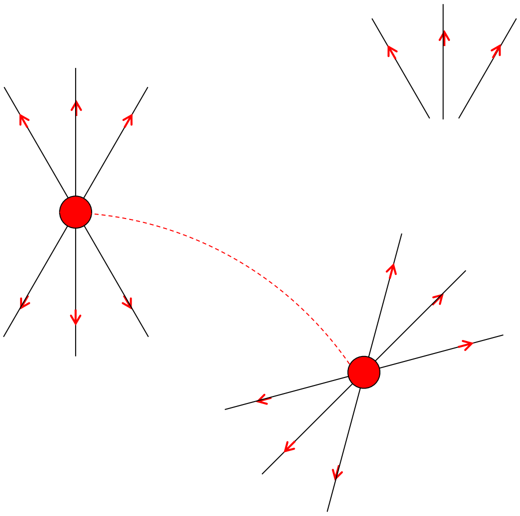
$$m^2 = m^2 + 2\boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma + 0 \quad \Rightarrow \quad \boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma = 0$$

- in the rest frame of the electron

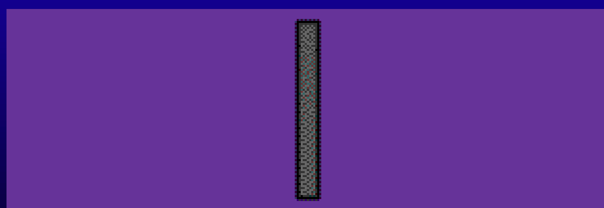
$$\boldsymbol{P}_f = (m, 0) \qquad \boldsymbol{P}_\gamma = (E_\gamma, p_\gamma)$$

this means that the photon energy must be zero.

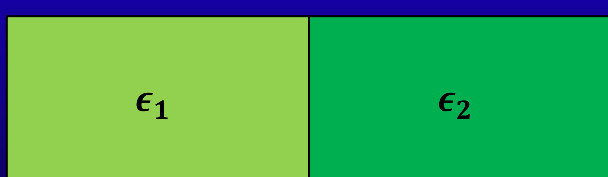
We need to separate the field from charge



Bremsstrahlung
or
“braking” radiation



Transition Radiation



$$c_1 = \frac{1}{\sqrt{\epsilon_1 \mu_1}} \qquad c_2 = \frac{1}{\sqrt{\epsilon_2 \mu_2}}$$

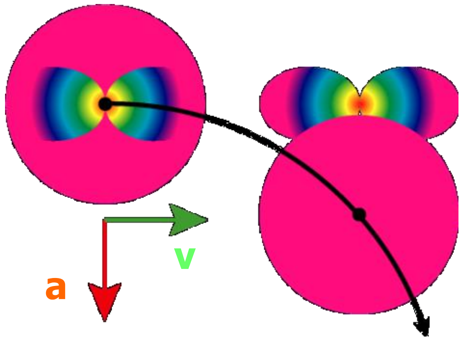
Fields of a moving charge

$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}}{\left(1 - \vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)^3\gamma^2}\cdot\frac{\mathbf{1}}{\mathbf{r}^2}\right]_{ret} +$$

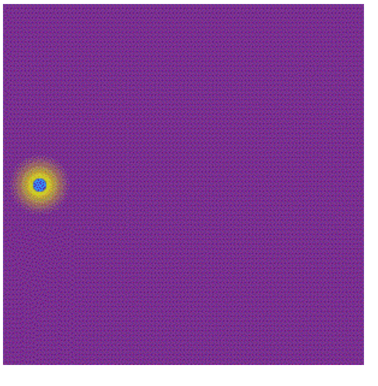
$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}} \times \left[\left(\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}\right) \times \dot{\vec{\boldsymbol{\beta}}}\right]}{\left(1 - \vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)^3\gamma^2}\cdot\frac{\mathbf{1}}{\mathbf{r}}\right]_{ret}$$

$$\vec{\mathbf{B}}(t) = \frac{1}{c}\left[\vec{\mathbf{n}} \times \vec{\mathbf{E}}\right]$$

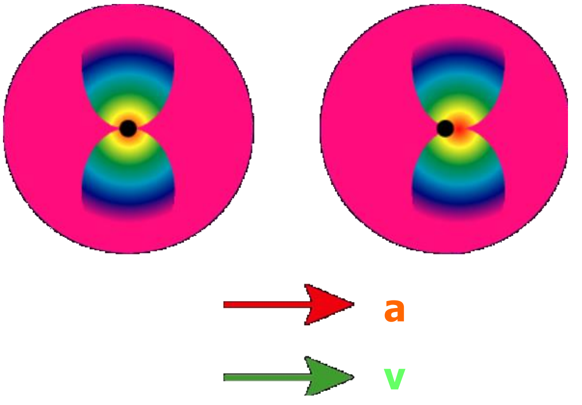
Transverse acceleration



Radiation field quickly separates itself from the Coulomb field



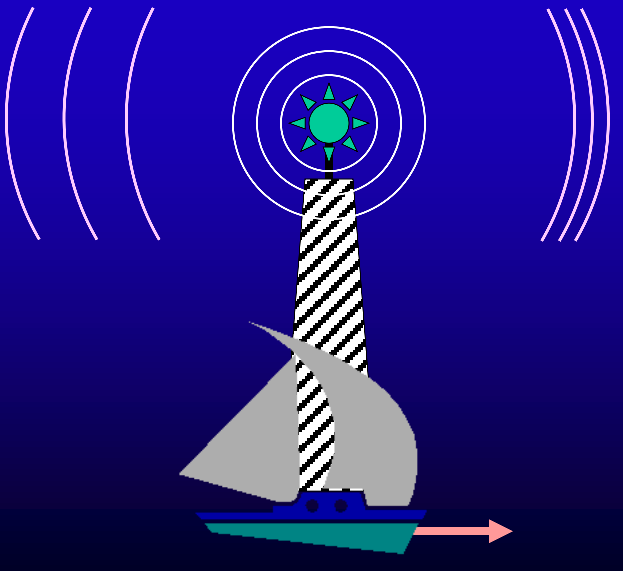
Longitudinal acceleration



Radiation field cannot separate itself from the Coulomb field

Synchrotron Radiation Basic Properties

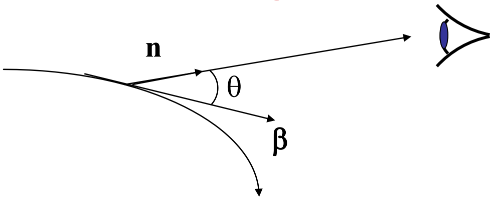
Moving Source of Waves



Cape Hatteras, 1999

Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n}\cdot\boldsymbol{\beta})\, T_{emit}$$

The wavelength is shortened by the same factor

$$\lambda_{obs} = (1 - \beta\cos\theta)\,\lambda_{emit}$$

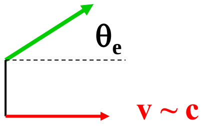
in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{obs} = \frac{1}{2\gamma^2}\lambda_{emit}$$

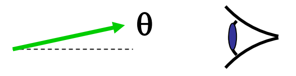
since

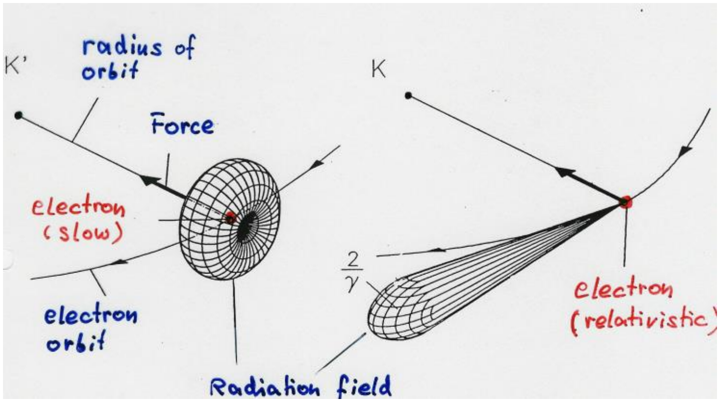
$$1 - \beta = \frac{1-\beta^2}{1+\beta} \cong \frac{1}{2\gamma^2}$$

Radiation is emitted into a narrow cone



$$\theta = \frac{1}{\gamma} \cdot \theta_e$$

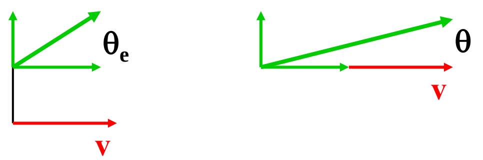


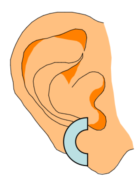


v << c | **v ≈ c**

Sound waves (non-relativistic)

Angular collimation





$$\theta = \frac{v_{s\perp}}{v_{s\parallel} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted}\left(1 - \frac{\mathbf{v}}{v_s}\right)$$

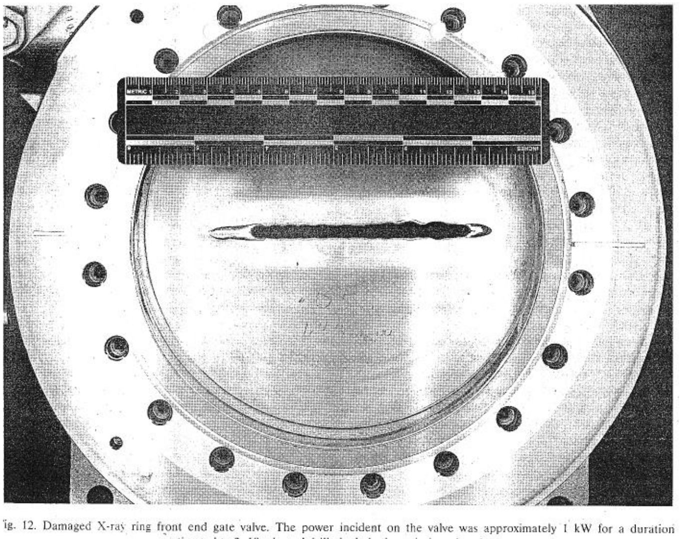
Synchrotron radiation power

Power emitted is proportional to: $P \propto E^2 B^2$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

The power is all too real!



ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Synchrotron radiation power

Power emitted is proportional to:

$$P \propto E^2 B^2$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3} \alpha \hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\text{m}}{\text{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197 \text{ Mev} \cdot \text{fm}$$

Energy loss per turn:

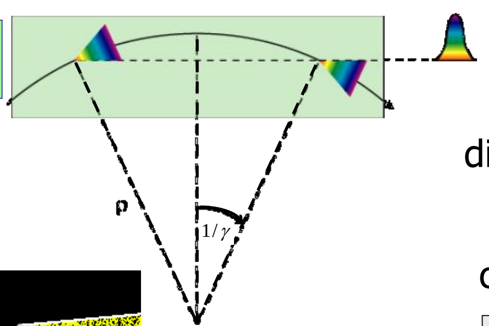
$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho}$$

Typical frequency of synchrotron light

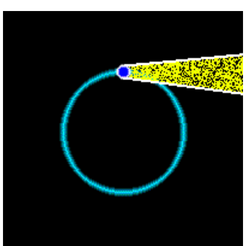
Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

$$l \sim \frac{2\rho}{\gamma}$$



Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1 - \beta)$$

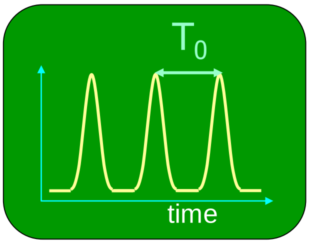


$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$

Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every T_0 (revolution period)



- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

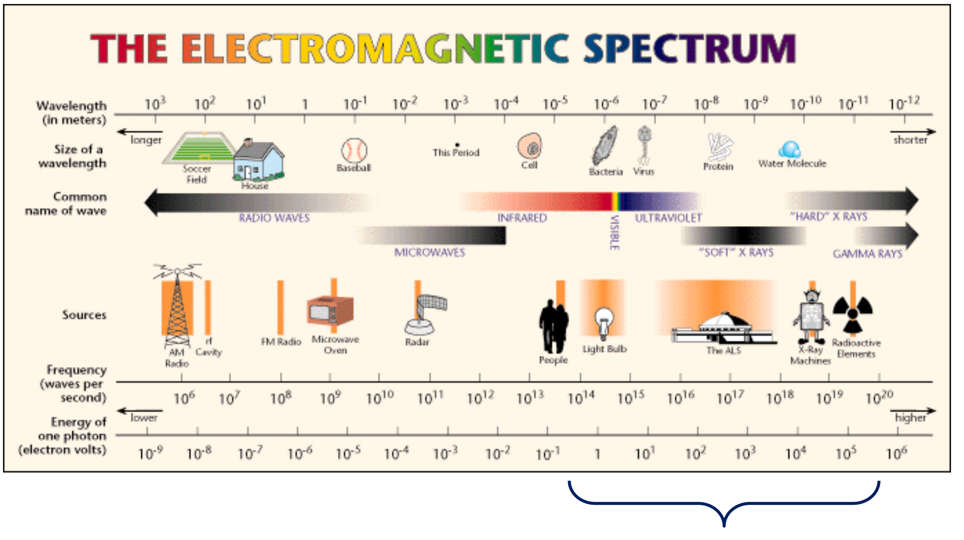
- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

$$\omega_0 \sim 1\,\mathrm{MHz}$$
$$\gamma \sim 4000$$
$$\omega_{typ} \sim 10^{16}\,\mathrm{Hz}\,!$$

- At high frequencies the individual harmonics overlap

continuous spectrum !



Wavelength continuously tunable !

Electron Beam Dynamics, L. Rivkin, Introduction to Accelerator Physics, Budapest

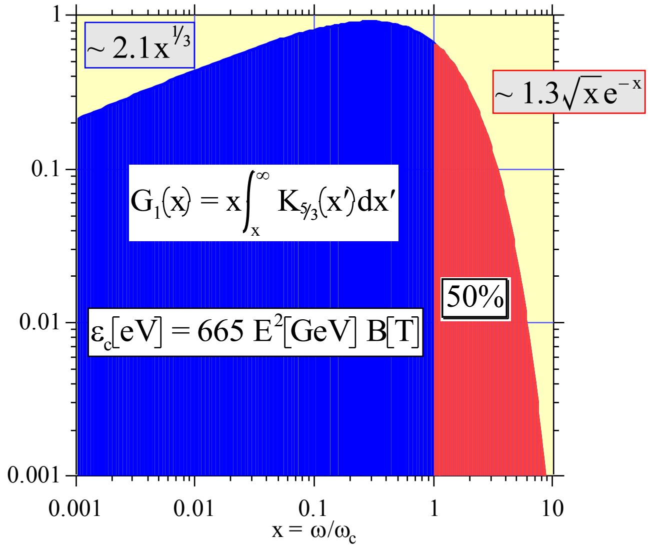
$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$S(x) = \frac{9\sqrt{3}}{8\pi}\, x \int_x^\infty K_{5/3}(x')dx'$$

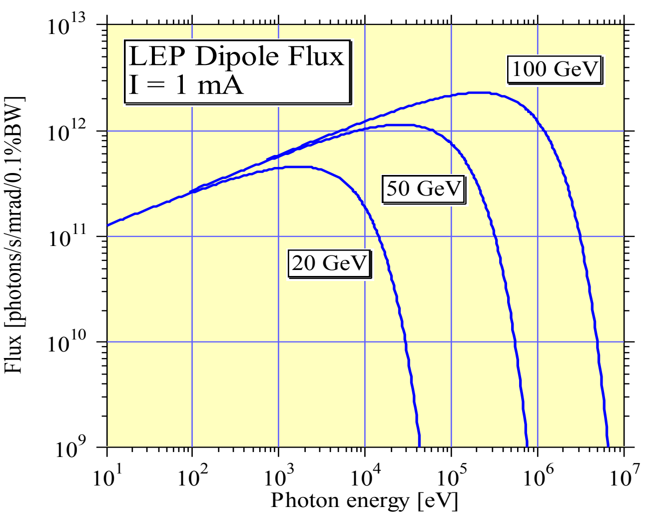
$$\int_0^\infty S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3}\hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$



Synchrotron radiation flux for different electron energies



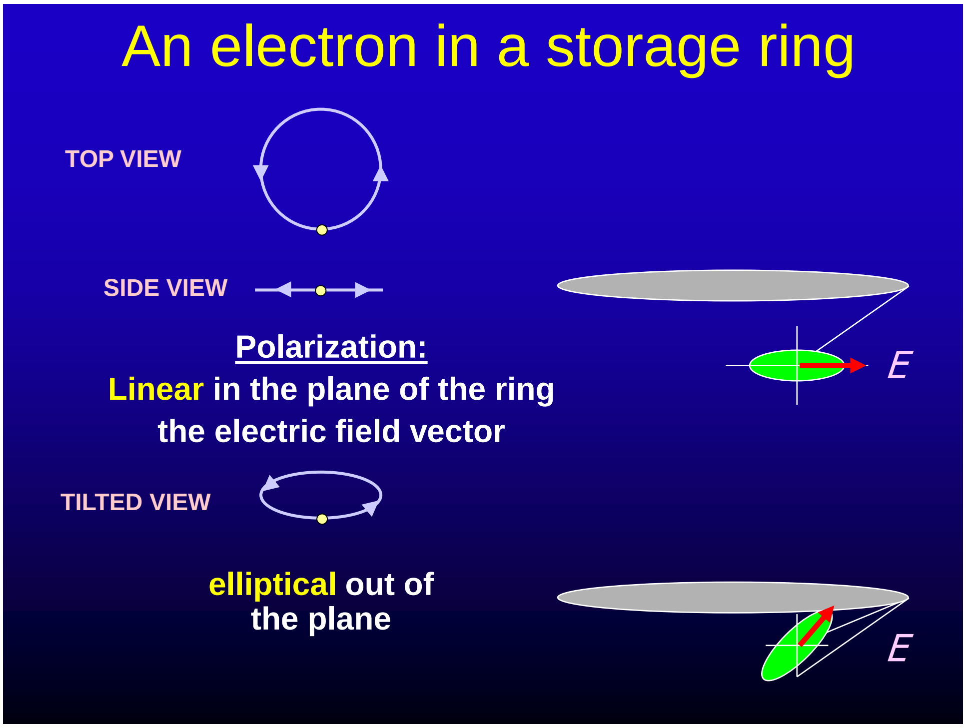
Angular divergence of radiation

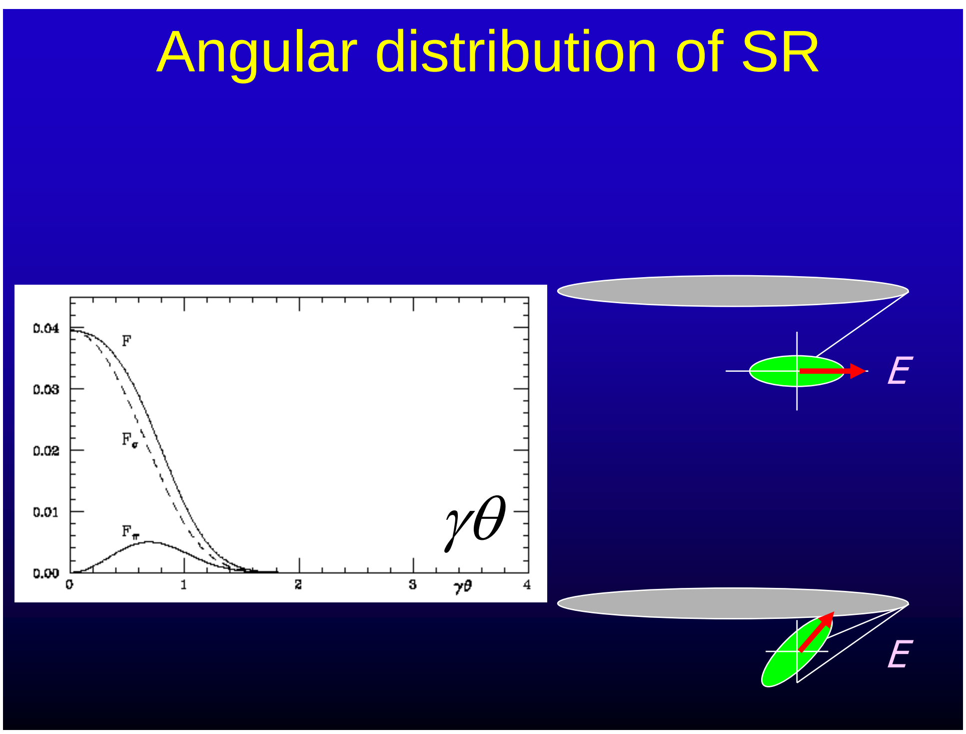
The rms opening angle R'

- **at the critical frequency:** $\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$
- **well below** $\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4\left(\frac{\lambda}{\rho}\right)^{1/3}$

  independent of γ !
- **well above** $\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$

Synchrotron light polarization

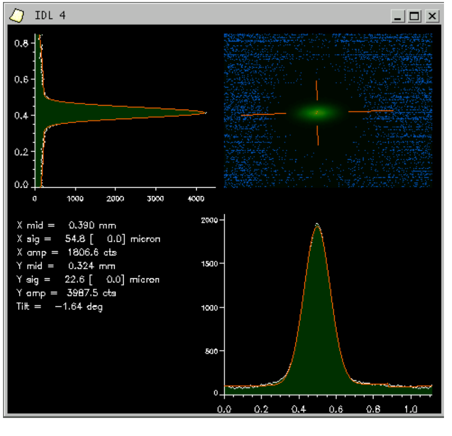

An electron in a storage ring
TOP VIEW
SIDE VIEW
Polarization:
Linear in the plane of the ring
the electric field vector
TILTED VIEW
elliptical out of
the plane
E
E

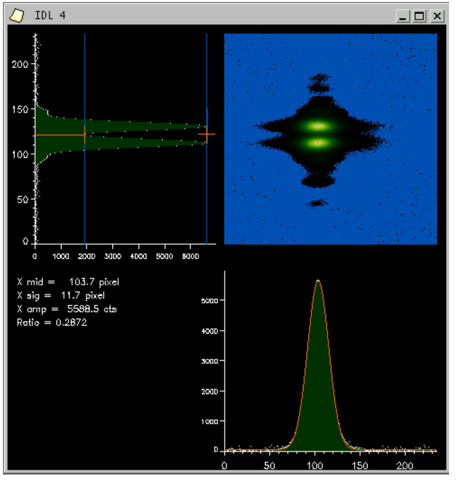

Angular distribution of SR
0.04
0.03
0.02
0.01
0.00
0
1
2
3
4
F
F_σ
F_π
$\gamma\theta$
$\gamma\theta$
E
E

Synchrotron light based electron beam diagnostics

Seeing the electron beam (**SLS**)

X rays | visible light, vertically polarised

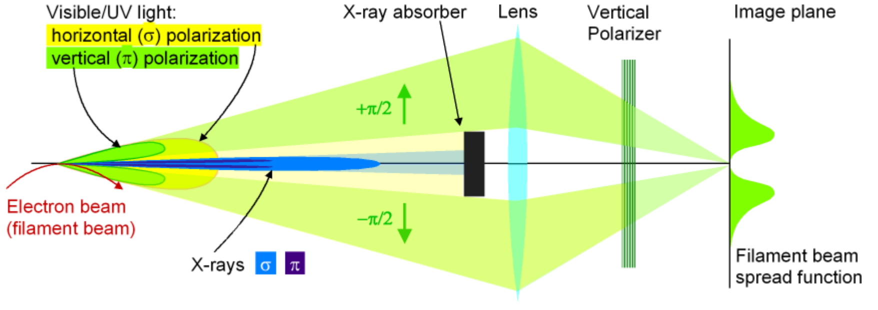




$\sigma_x \sim 55\mu m$

Seeing the electron beam (SLS)

Making an image of the electron beam using the vertically polarised synchrotron light



High resolution measurement

Wavelength used: 364 nm

For point-like source the intensity on axis is zero

Peak-to-valley intensity ratio is determined by the beam height

Present resolution: **3.5 µm**