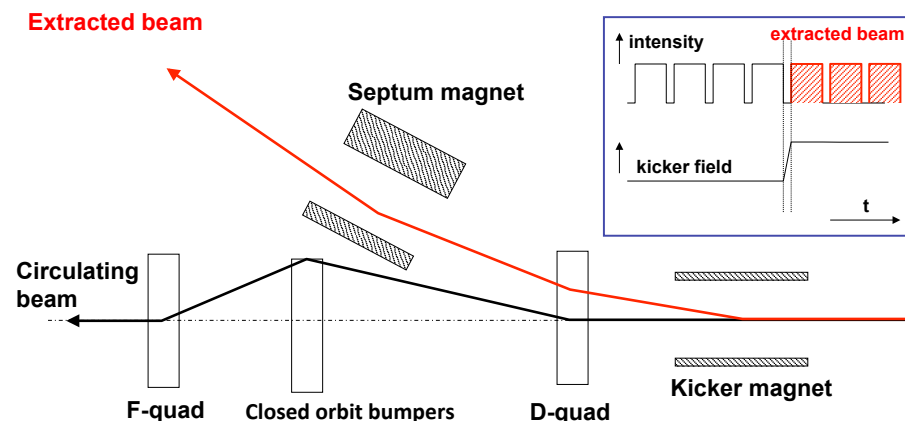


Kickers, septa and beam transfer lines

- Beam transfer devices
 - Kickers
 - Septa
 - Protection devices
- Beam transfer lines
 - Distinctions between transfer lines and circular machines
 - Linking machines/experiments together
 - Emittance blow-up from mismatch
 - Measure beam parameters (measurement lines)

Matthew Fraser, CERN (TE-ABT-BTP) based on lectures by M.J. Barnes, W. Bartmann, J. Borburgh, B. Goddard, V. Kain and M. Meddahi

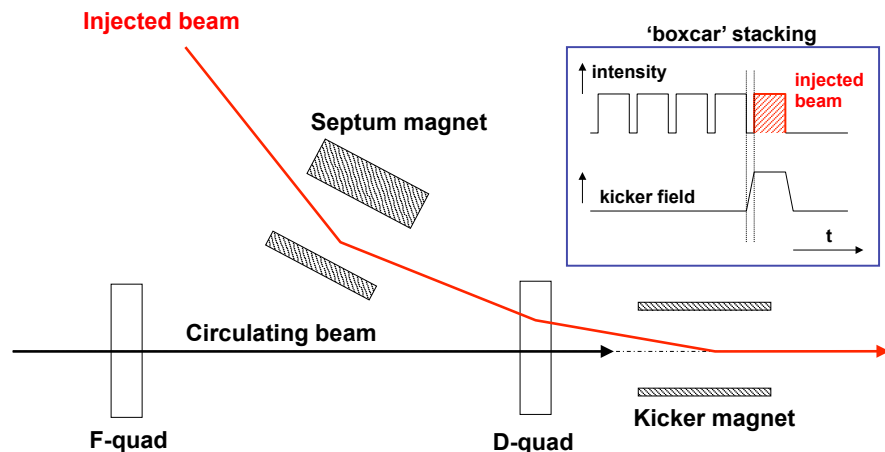
Reminder: extraction, septum and kicker



- Kickers produce fast pulses, rising their field within the particle-free gap in the circulating beam (**temporal separation**)
- Septa compensate for the relatively low kicker strength, and approach closely the circulating beam (**spatial separation**)

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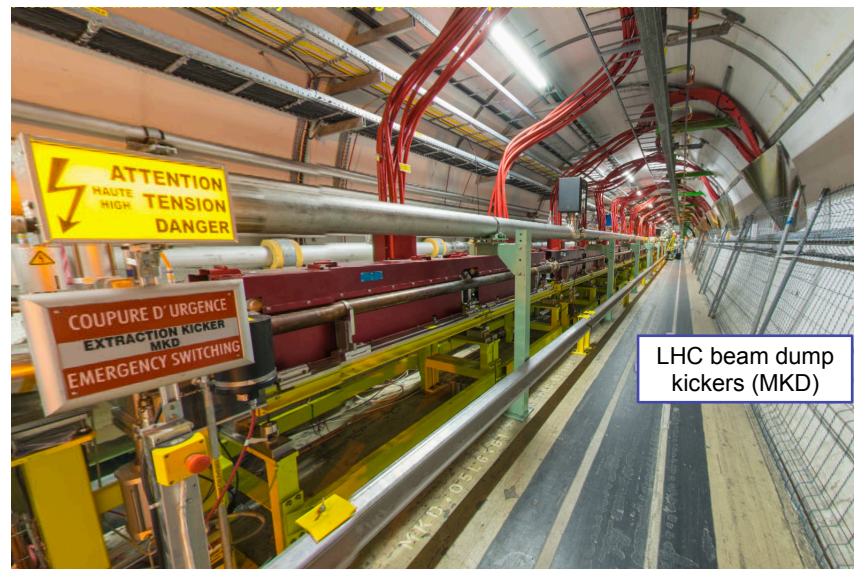
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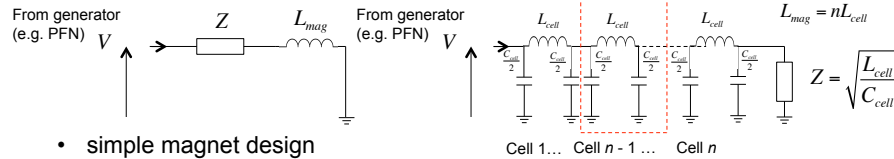
Kickers



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Magnets – design options

- Type: “lumped inductance” or “distributed inductance” (**transmission line**)

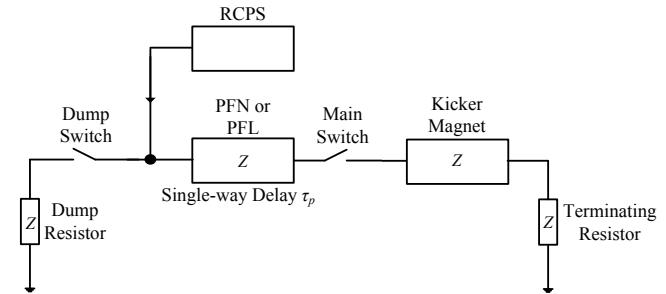


- simple magnet design
- magnet must be nearby the generator to minimise inductance
- exponential field rise-time:

$$I = \frac{V}{Z}(1 - e^{-t/\tau}) \quad \tau = \frac{L_{mag}}{Z}$$
- slow: rise-times $\sim 1 \mu\text{s}$
- Other considerations:
 - **Machine vacuum:** kicker in-vacuum or external
 - **Aperture:** geometry of ferrite core
 - **Termination:** matched impedance or short-circuit
- complicated magnet design
- impedance matching important
- field rise-time depends on propagation time of pulse through magnet:

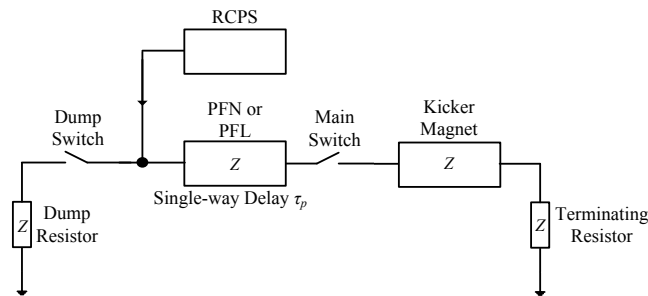
$$\tau = n \sqrt{L_{cell} \cdot C_{cell}} = n \frac{L_{cell}}{Z} = \frac{L_{mag}}{Z}$$
- fast: rise-times $\ll 1 \mu\text{s}$**

Simplified kicker system schematic



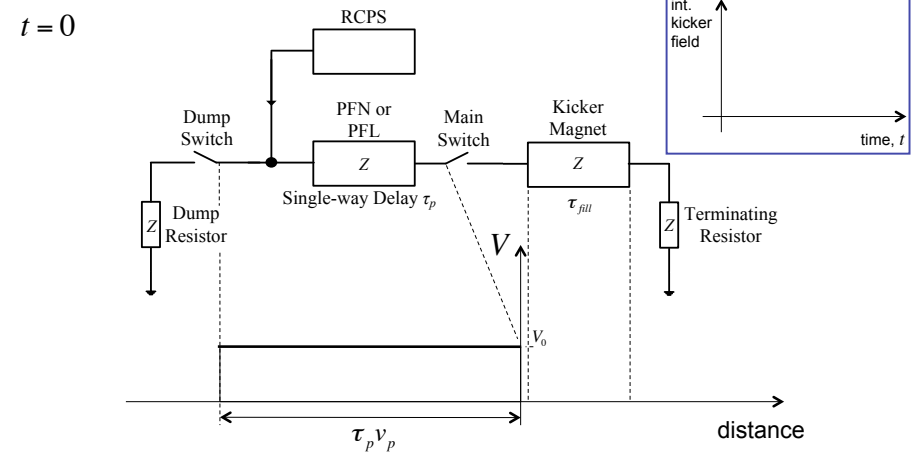
- PFL/PFN charged to voltage V_0 by the RCPS
- Main switch is closed...
 - ...voltage pulse of $V_0/2$ flows through kicker
- Once the pulse reaches the (matched) terminating resistor full-field has been established in the kicker magnet
- Pulse length controlled between $t = 0$ and $2\tau_p$ with dump switch

Simplified kicker system schematic



- Main sub-systems (“components”) of kicker system;
 - **RCPS** = Resonant Charging Power Supply
 - **PFL** = Pulse Forming Line (coaxial cable) or **PFN** = Pulse Forming Network (lumped elements)
 - Fast high power **switch(es)**
 - **Transmission line(s):** coaxial cable(s)
 - **Kicker Magnet**
 - **Terminators** (resistive)

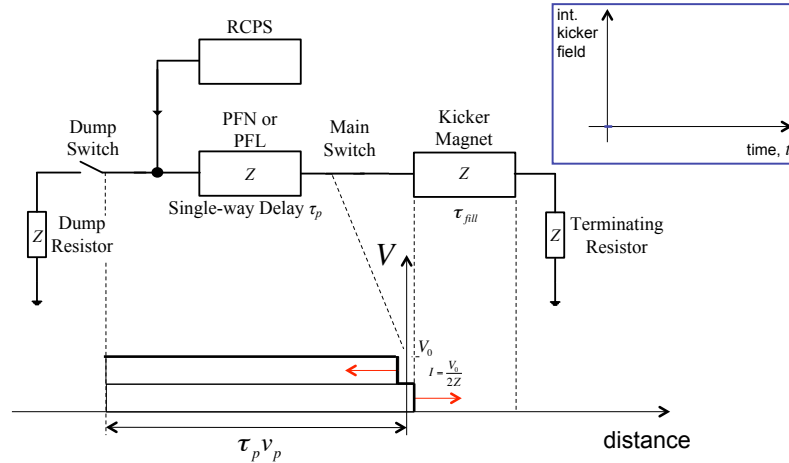
Simplified kicker system schematic



- Pulse forming network or line (PFL/PFN) charged to voltage V_0 by the resonant charging power supply (RCPS)
 - RCPS is de-coupled from the system through a diode stack

Simplified kicker system schematic

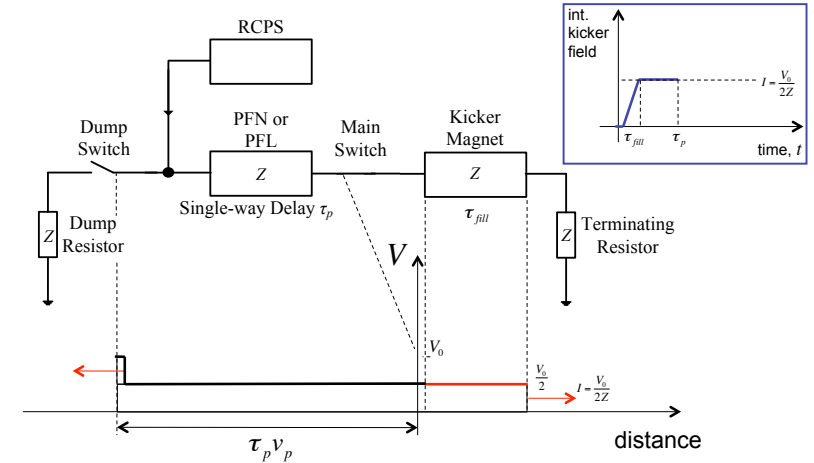
$t \approx 0$



- Pulse forming network or line (PFN/PFL) charged to voltage V_0 by the resonant charging power supply (RCPS)
 - RCPS is de-coupled from the system through a diode stack
- At $t = 0$, main switch is closed and current starts to flow into the kicker

Simplified kicker system schematic

$t \approx \tau_p$

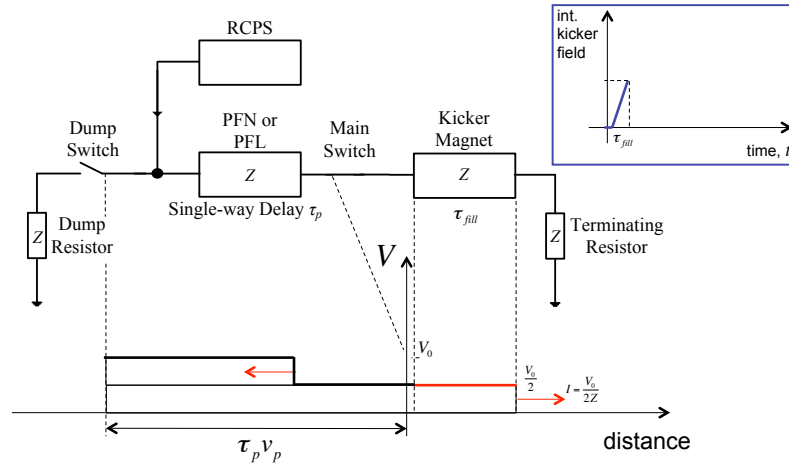


- PFN continues to discharge energy into kicker magnet and matched terminating resistor

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Simplified kicker system schematic

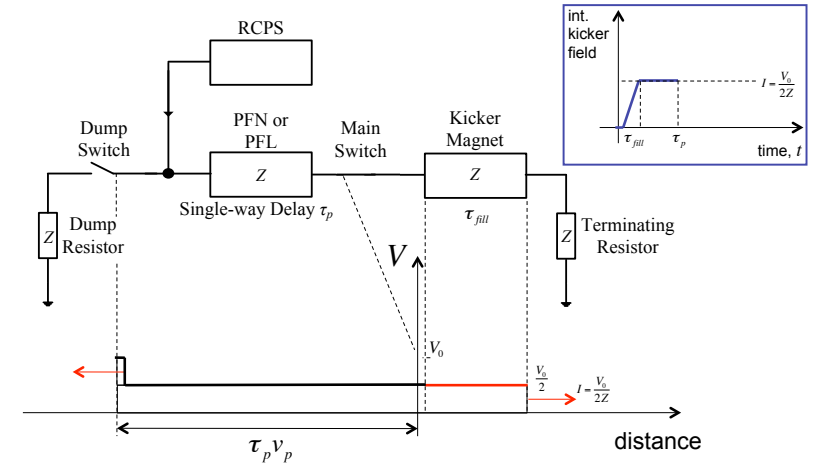
$t \approx \tau_{fill}$



- At $t = \tau_{fill}$, the voltage pulse of magnitude $V_0/2$ has propagated through the kicker and nominal field achieved with a current $V_0/2Z$
 - typically $\tau_p \gg \tau_{fill}$ (schematic for illustration purposes)

Simplified kicker system schematic

$t \approx \tau_p$

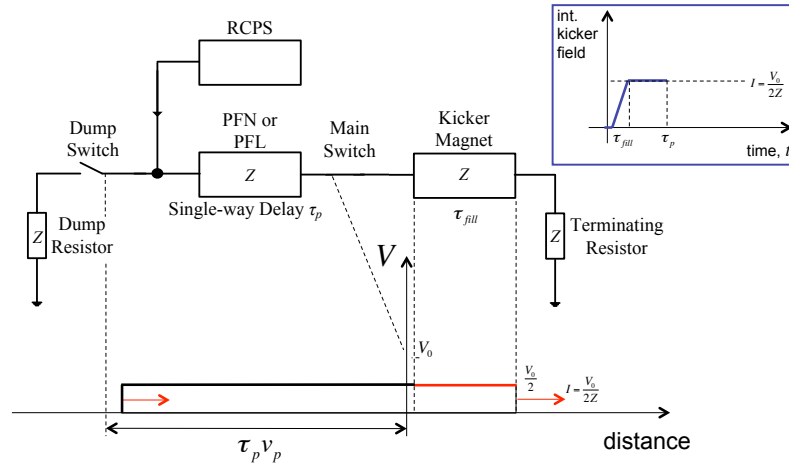


- PFN continues to discharge energy into kicker magnet and matched terminating resistor
- At $t \approx \tau_p$ the negative pulse reflects off the open end of the circuit (dump switch) and back towards the kicker

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Simplified kicker system schematic

$$t \approx \tau_p$$

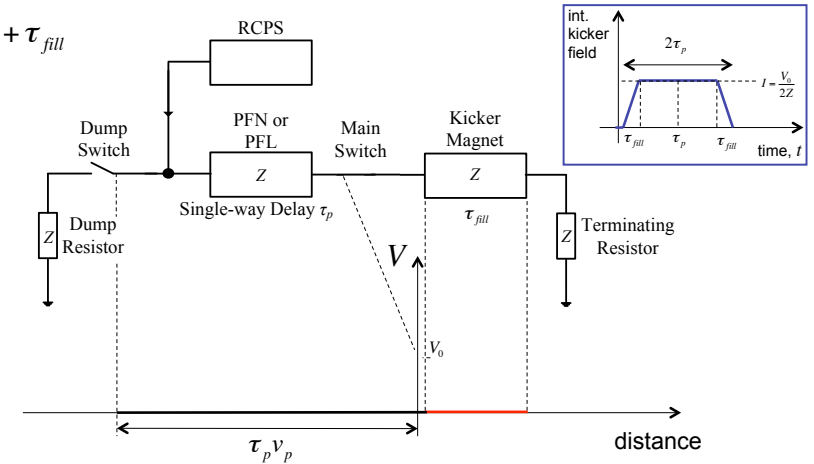


- PFN continues to discharge energy into matched terminating resistor
- At $t \approx \tau_p$ the negative pulse reflects off the open end of the circuit and back towards the kicker

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Simplified kicker system schematic

$$t = 2\tau_p + \tau_{fill}$$

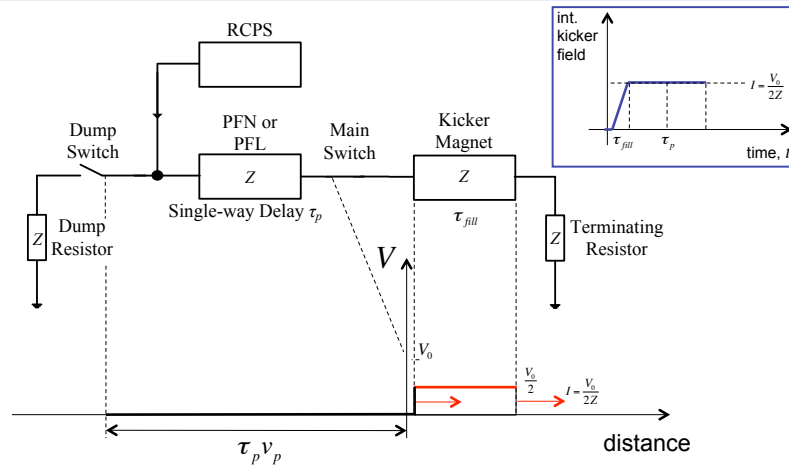


- A kicker pulse of approximately $2\tau_p$ is imparted on the beam and all energy has been emptied into the terminating resistor

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Simplified kicker system schematic

$$t \approx 2\tau_p$$

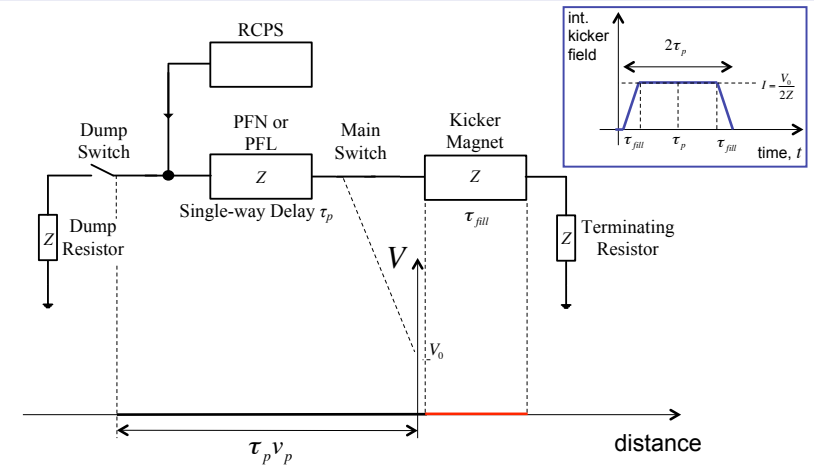


- At $t \approx 2\tau_p$ the pulse arrives at the kicker and field starts to decay

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Simplified kicker system schematic

$$t \approx 2\tau_p$$

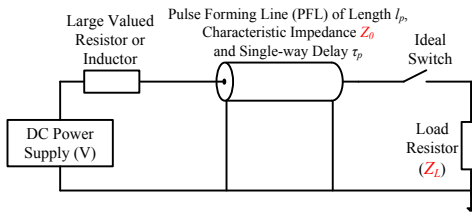


- Kicker pulse length can be changed by adjusting the relative timing of dump and main switches:
 - e.g. if the dump and main switches are fired simultaneously the pulse length will be halved and energy shared on dump and terminating resistors

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Reflections

- A simplified pulse forming circuit:



Match impedances to avoid reflections!

- When the switch is fired the voltage is divided as:

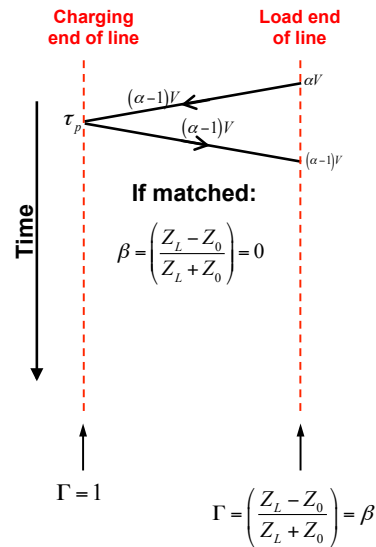
$$V_L = V \cdot \left(\frac{Z_L}{Z_0 + Z_L} \right) = \alpha V$$

- In the matched case:

$$Z_0 = Z_L \quad \alpha = \frac{1}{2}, \beta = 0$$

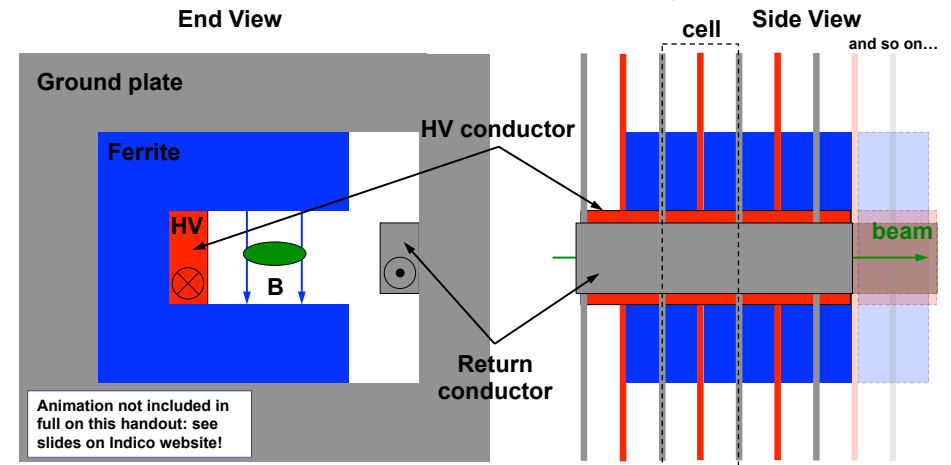
- Mismatches will ring in the circuit causing ripples on the pulse, or post-pulse.

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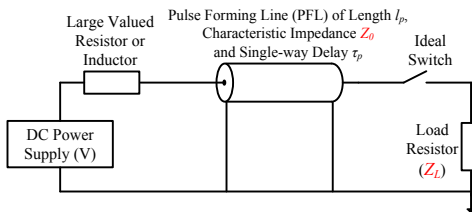
Magnets – transmission line

- Today's fast (rise-times of < few hundred ns) kicker magnets are generally **ferrite loaded** transmission lines:
 - Ferrite C-cores are sandwiched between HV plates
 - Grounded plates are interleaved to form a capacitor to ground



Reflections

- A simplified pulse forming circuit:



Match impedances to avoid reflections!

- When the switch is fired the voltage is divided as:

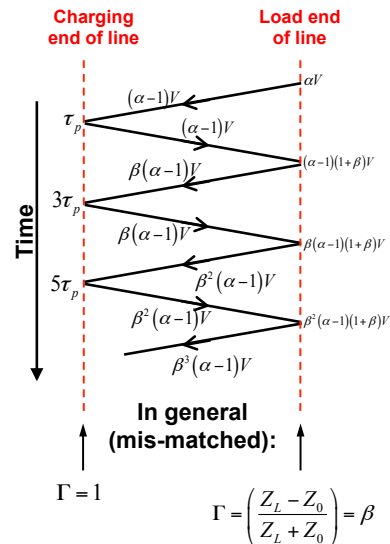
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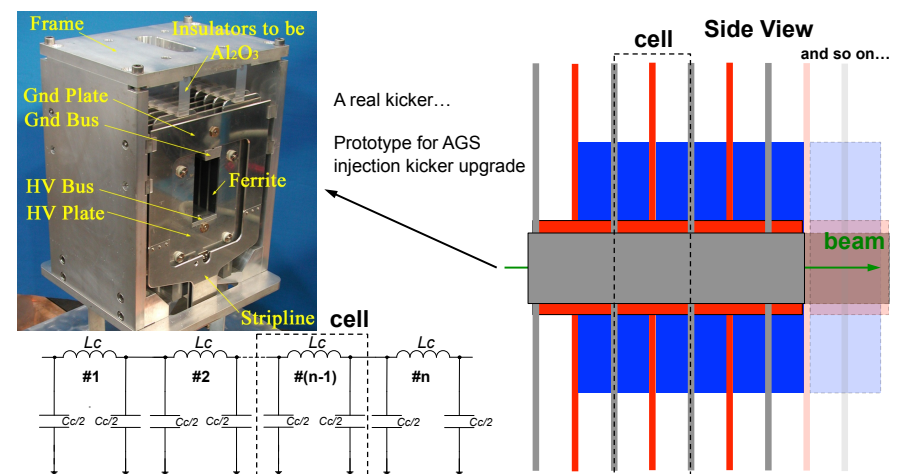
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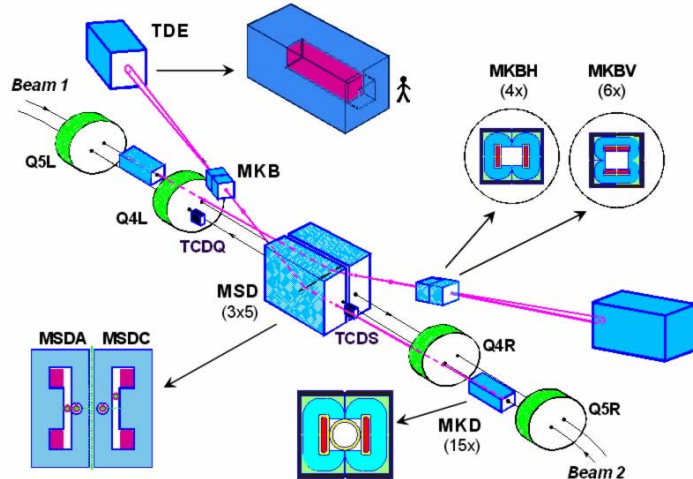
Magnets – transmission line

- Today's fast (rise-times of < few hundred ns) kicker magnets are generally **ferrite loaded** transmission lines:
 - Kicker magnets consists of many, relatively short, cells to approximate a broadband coaxial cable



Magnets – lumped inductance

- Lumped inductance kicker magnets are robust and reliable, and suitable for applications where the rise-time is typically $> 1 \mu\text{s}$:
 - e.g. LHC beam dump extraction and dilution kicker magnets



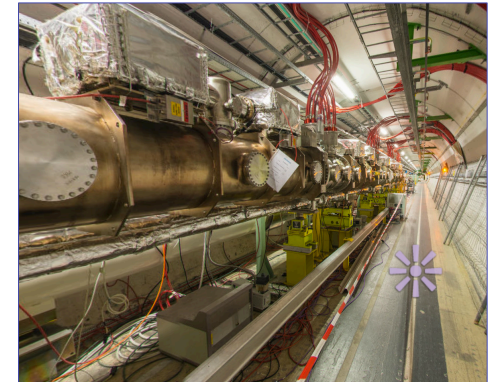
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Magnets – lumped inductance

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Generators nearby in gallery next to LHC tunnel

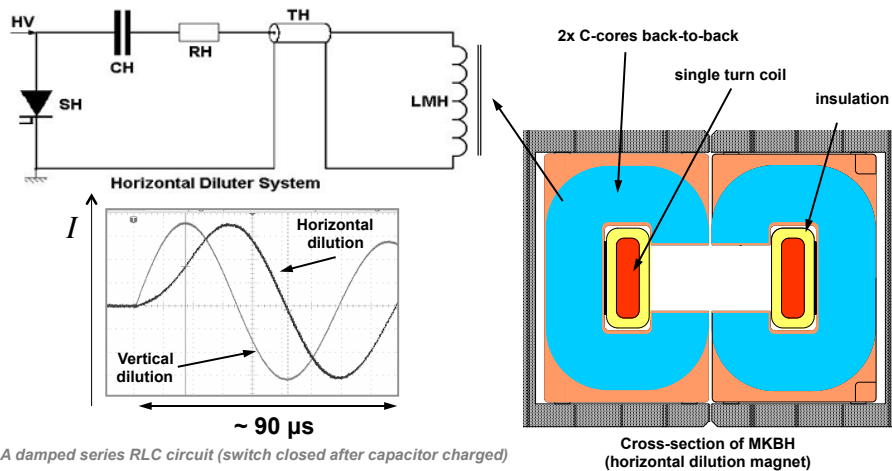


MKB dilution magnets in the LHC tunnel

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Magnets – lumped inductance

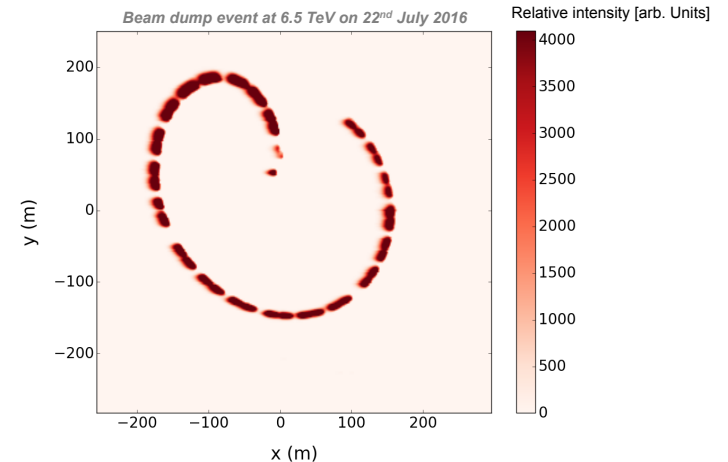
- Lumped inductance kicker magnets are robust and reliable, and suitable for applications where the rise-time is typically $> 1 \mu\text{s}$:
 - e.g. LHC beam dump extraction and dilution kicker magnets



A damped series RLC circuit (switch closed after capacitor charged)
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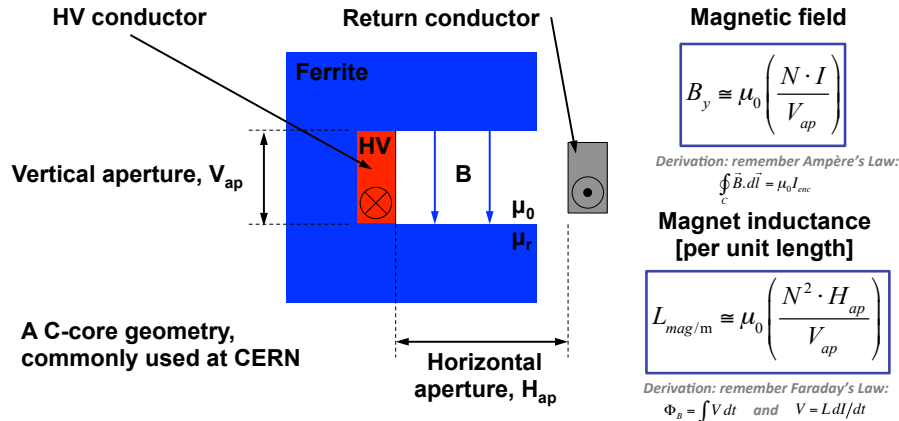
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Magnetic parameters



- Dimensions H_{ap} and V_{ap} specified by beam parameters at kicker location
- Ferrite ($\mu_r \approx 1000$) reinforces magnetic circuit and uniformity of the field in the gap
- For fast rise-times the inductance must be minimised: typically the number of turns, $N = 1$.
- Kickers are often split into several magnet units, powered independently.

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Magnetic field

$$B_y \approx \mu_0 \left(\frac{N \cdot I}{V_{ap}} \right)$$

Derivation: remember Ampère's Law:
 $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Magnet inductance [per unit length]

$$L_{mag/m} \approx \mu_0 \left(\frac{N^2 \cdot H_{ap}}{V_{ap}} \right)$$

Derivation: remember Faraday's Law:
 $\Phi_m = \int V dt$ and $V = L dI/dt$

Switches

Thyratrons

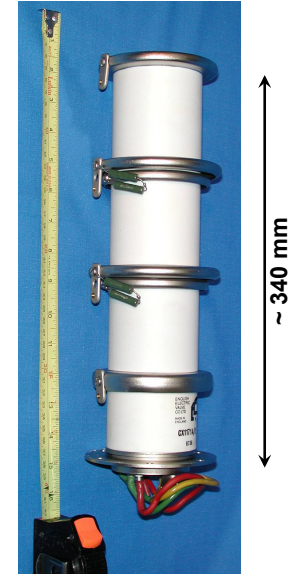
- Deuterium gas thyratrons are commonly used
- Hold off 80 kV and switch up to 6 kA
- **Fast switching** ~ 30 ns (~150 kA/ μ s)
- Erratic turn-on: use with RCPS to reduce hold-off time

Power semiconductor switches

- Suitable for scenarios where erratic turn-on is not allowed
- For example, LHC beam dump kickers held at nominal voltage throughout operation (>10h) ready to fire and safely abort at any moment.
- Hold off up to 30 kV and switch up to 18 kA
- **Slower switching** > 1 μ s (~18kA/ μ s)
- Low maintenance



Stack of high-power semiconductor switches (GTOs)



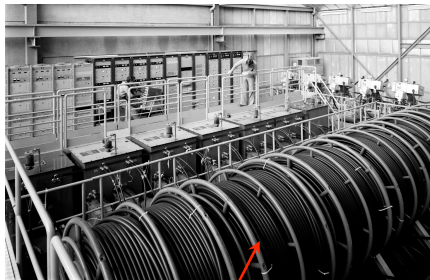
Thyratron

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PFL/PFN

Pulse Forming Line (PFL)

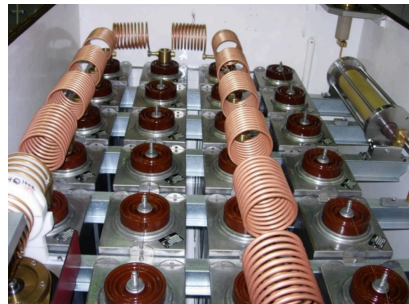
- Low-loss coaxial cable
- Fast and ripple-free pulses
- Attenuation (droop ~1%) becomes problematic for pulses > 3 μ s
- Above 50 kV SF6 pressurized PE tape cables are used
- Bulky: 3 μ s pulse ~ 300 m of cable



Reels of PFL used at the PS complex (as old as the photograph!)
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Pulse Forming Network (PFN)

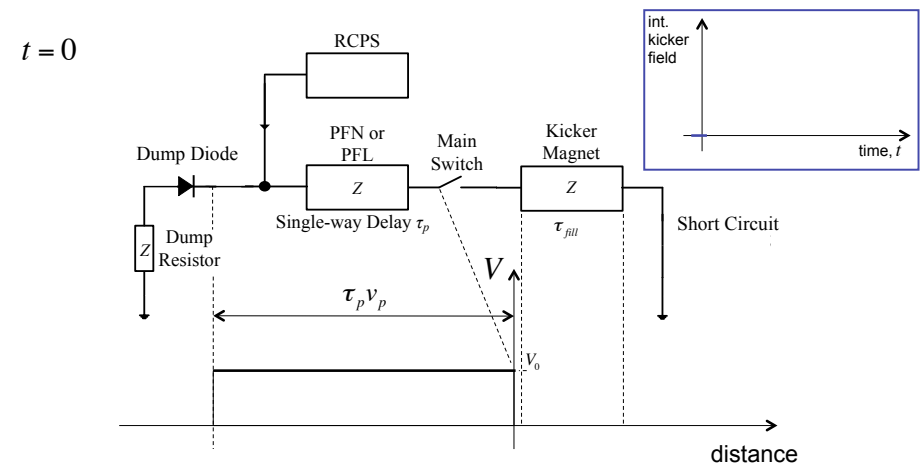
- Artificial coaxial cable made of lumped elements
- For low droop and long pulses > 3 μ s
- Each cell individually adjustable: adjustment of pulse flat-top difficult and time consuming.



SPS extraction kicker (MKE) PFN (17 cells)

Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
 - Fill time of kicker magnet is doubled
 - Diode as dump switch provides solution for fixed pulse length

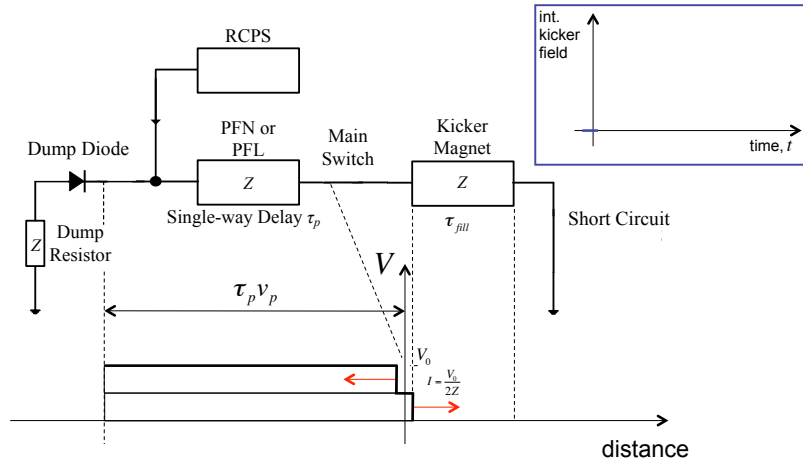


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$$t \approx 0$$

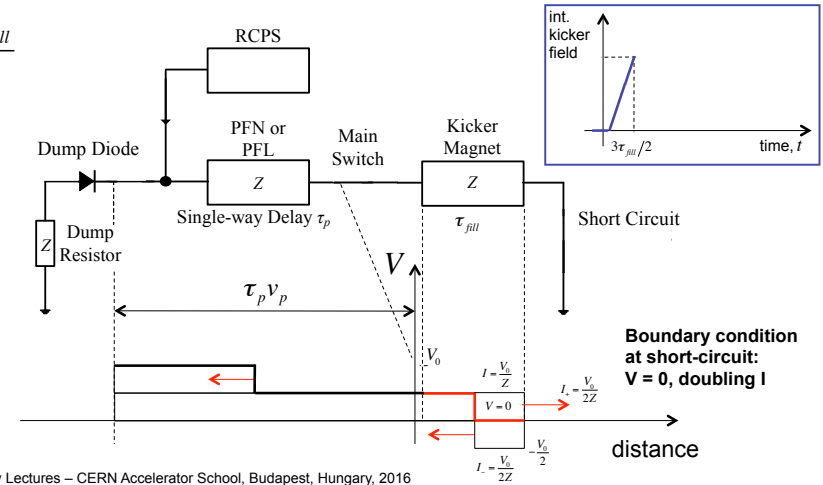


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$$t \approx \frac{3\tau_{fill}}{2}$$

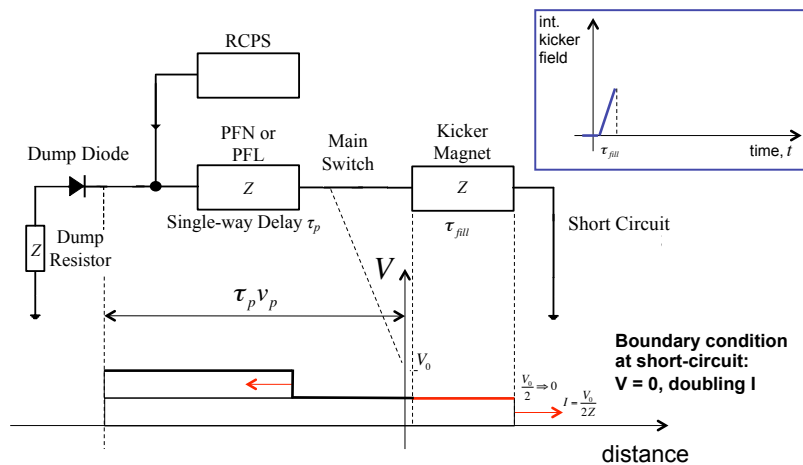


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$$t \approx \tau_{fill}$$

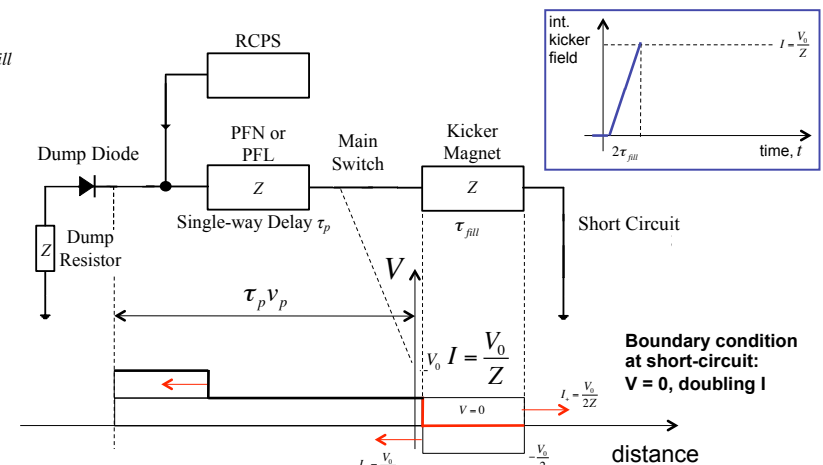


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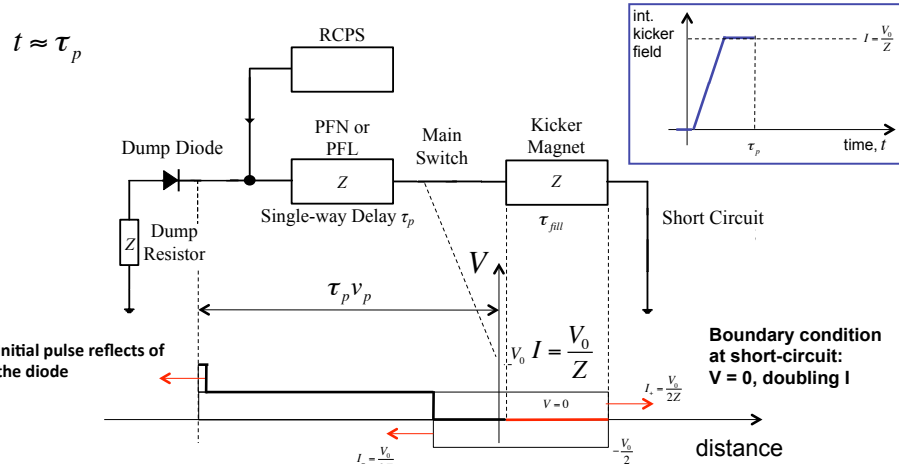
$$t \approx 2\tau_{fill}$$



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Terminated vs. short circuit

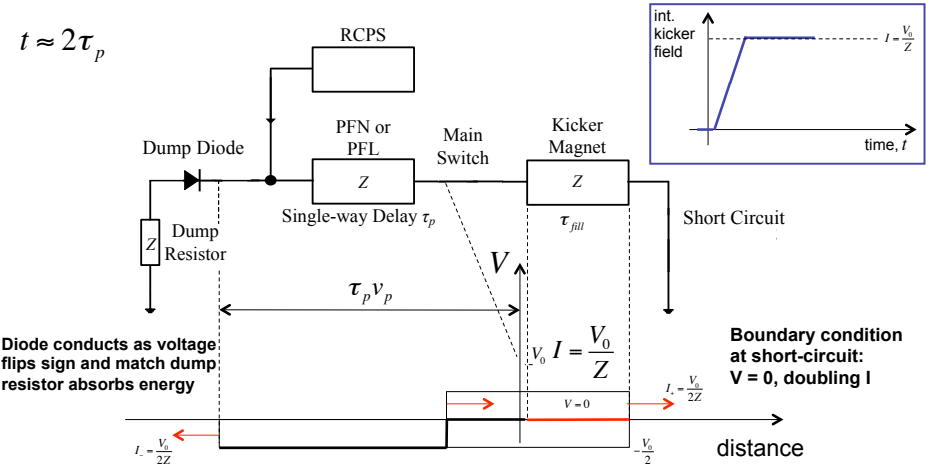
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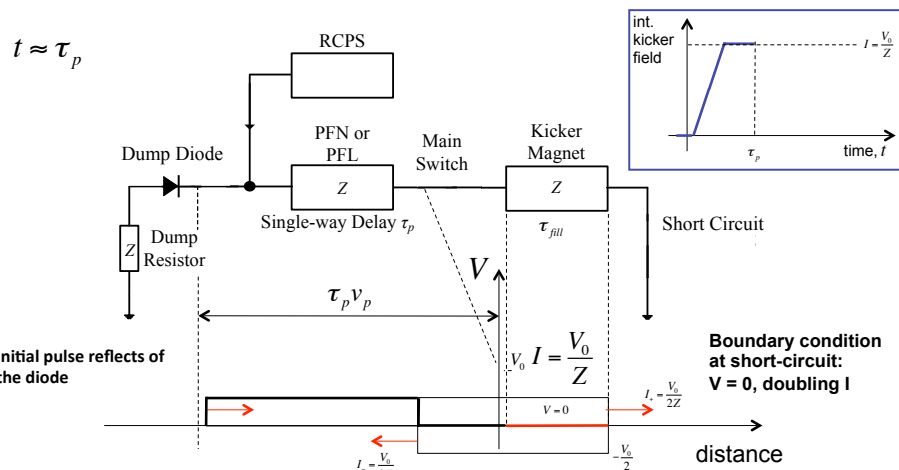
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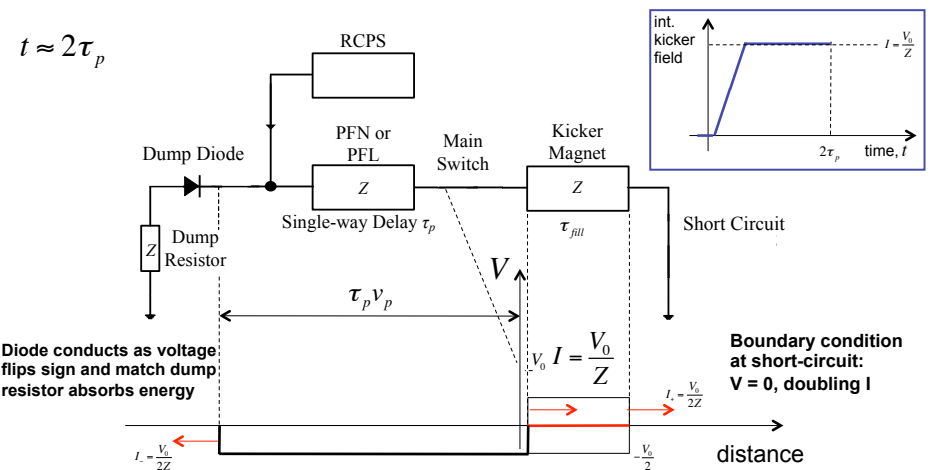
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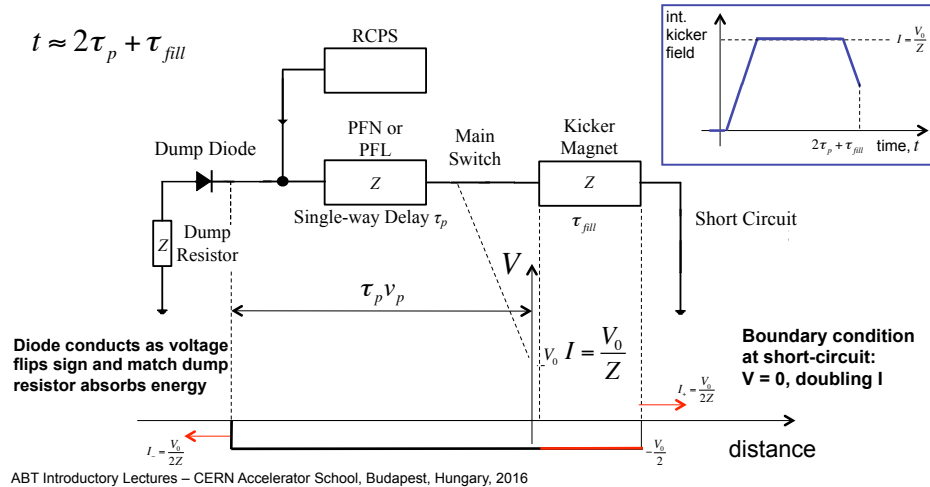
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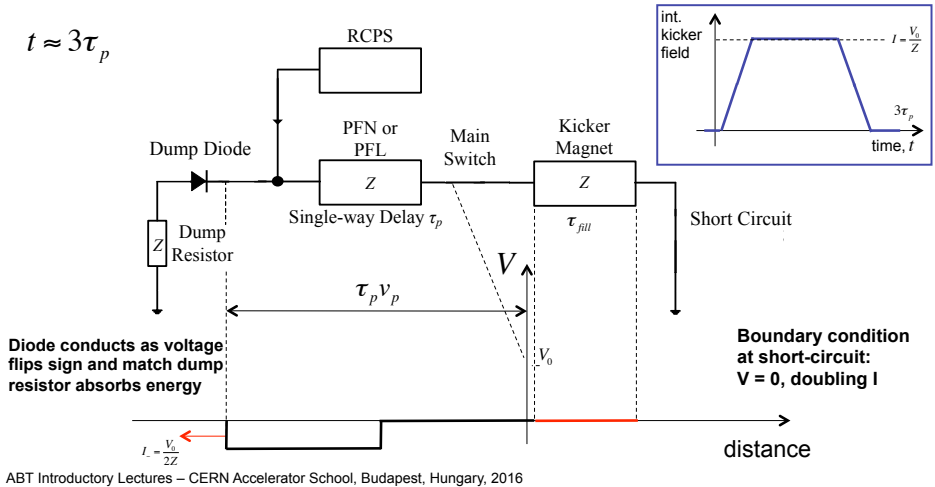
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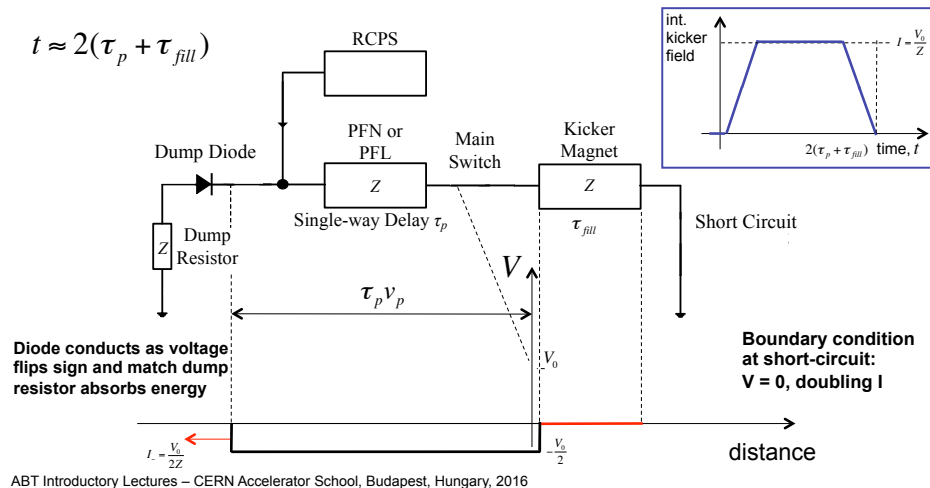
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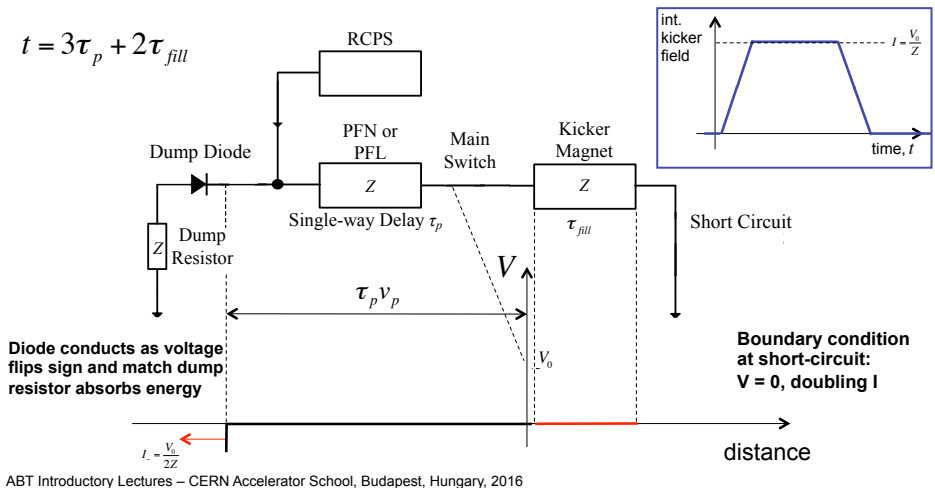
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Terminated vs. short circuit

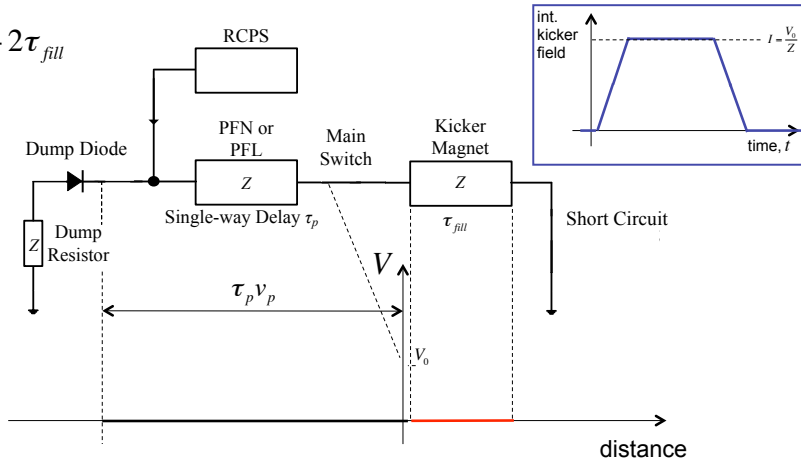
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$$t = 3\tau_p + 2\tau_{fill}$$

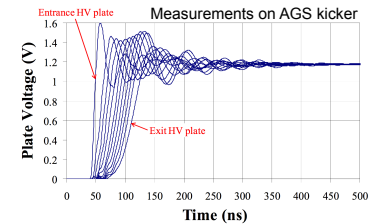


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Other topics and considerations

- Ripple:** cells of a transmission line kicker have a cut-off frequency that introduces dispersion in pulse:
 - Cut-off frequency:

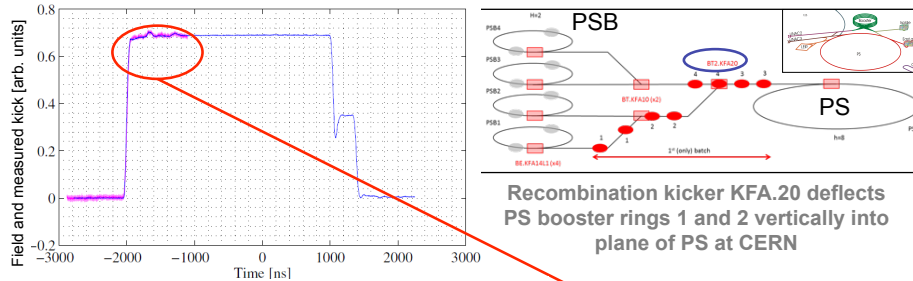
$$\omega_c = \frac{2}{\sqrt{L_{cell} C_{cell}}} = \frac{Z}{L_{cell}}$$



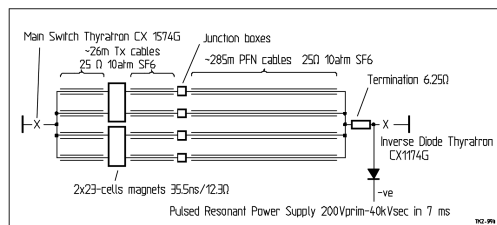
- In vacuum:** aperture dimensions (H_{ap} and V_{ap}) minimised if in vacuum:
 - For given B, lower I and L can be achieved with smaller H_{ap} and V_{ap}
 - Machine vacuum is a reliable dielectric, recovers after flashover
 - Costly and time consuming to construct/maintain (cleanliness, bake-out)
- Beam coupling impedance:** kickers are a source of beam impedance in accelerators (wakefields and beam instabilities)
 - Ferrite is shielded from beam with beam screens or serigraphy by permitting a smooth conducting path for beam induced image charges
 - Beam induced heating of ferrite yoke can heat it above the Curie temp.

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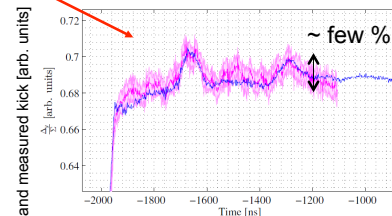
An example of reflections



Recombination kicker KFA.20 deflects PS booster rings 1 and 2 vertically into plane of PS at CERN



BT.KFA20 circuit schematic: operates in "virtual" short-circuit

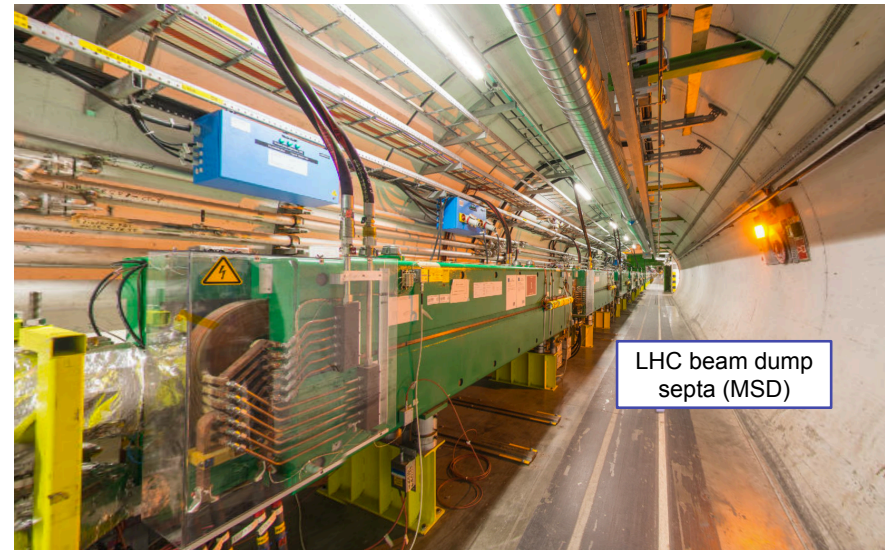


Current loop measurement (L. Sermeus) Beam-based measurement (I=200e10 ppb)

Beam-based kicker measurements at higher intensities, V. Forte BT + PS injection kicker meeting, CERN (15th August 2016)

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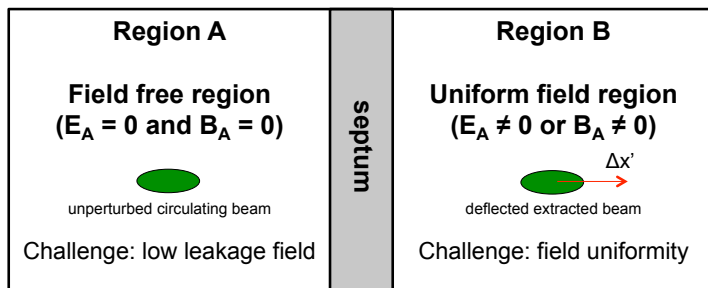
Septa



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Septa

- Two main types:
 - Electrostatic septa (DC)
 - Magnetic septa (DC and pulsed):
 - Direct drive septum
 - Eddy current septum (pulsed only)
 - Lambertson septum (deflection parallel to septum)



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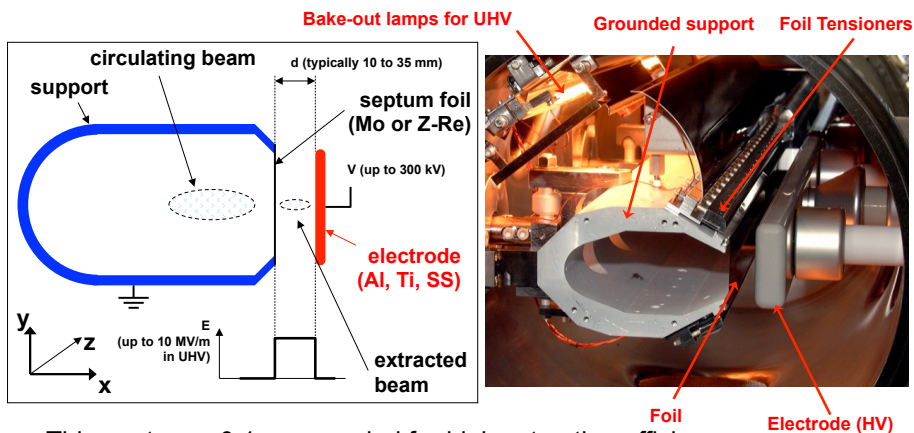
Electrostatic septum

- At SPS we slow-extract 400 GeV protons using approximately 15 m of septum split into 5 separate vacuum tanks each over 3 m long:
 - Alignment of the 60 - 100 μm wire array over 15 m is challenging!



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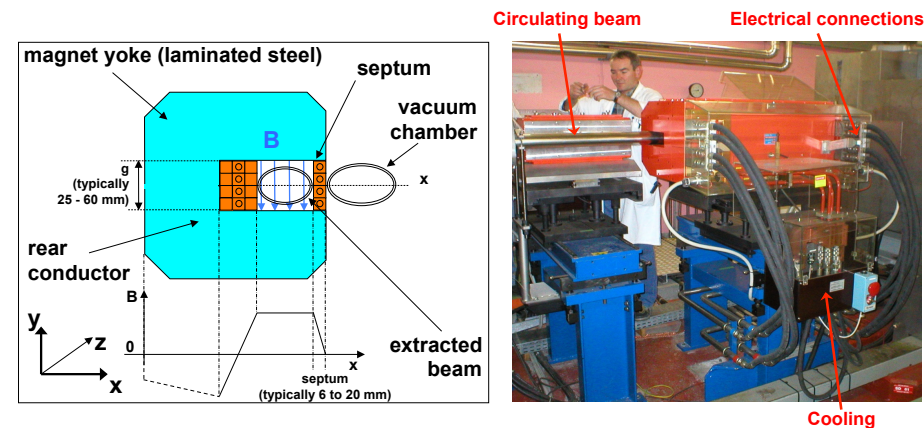
Electrostatic septum



- Thin septum ~ 0.1 mm needed for high extraction efficiency:
 - Foils typically used
 - Stretched wire arrays provide thinner septa and lower effective density
- Challenges include conditioning and preparation of HV surfaces, vacuum in range of 10^{-9} – 10^{-12} mbar and in-vacuum precision position alignment

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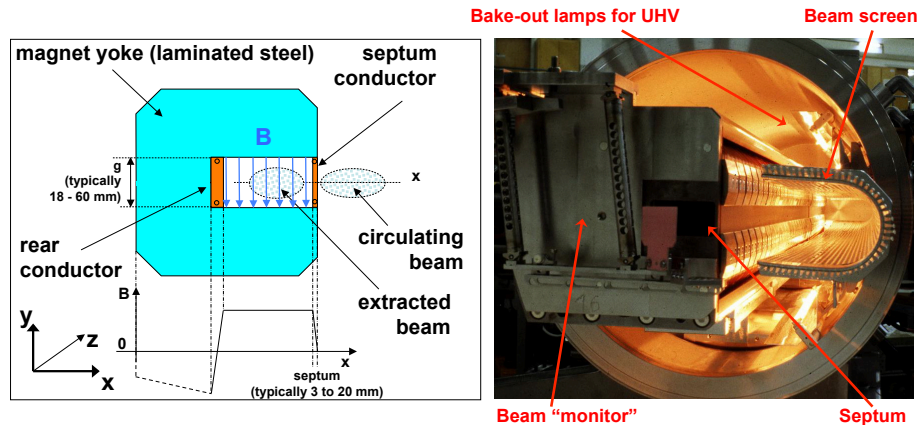
DC direct drive magnetic septum



- Continuously powered, rarely under vacuum
- Multi-turn coil to reduce current needed but cooling still an issue:
 - Cooling water circuits flow rate typically at 12 – 60 l/min
 - Current can range from 0.5 to 4 kA and power consumption up to 100 kW!

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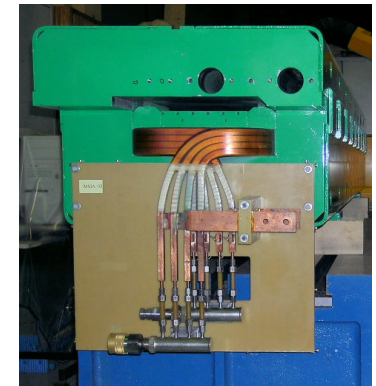
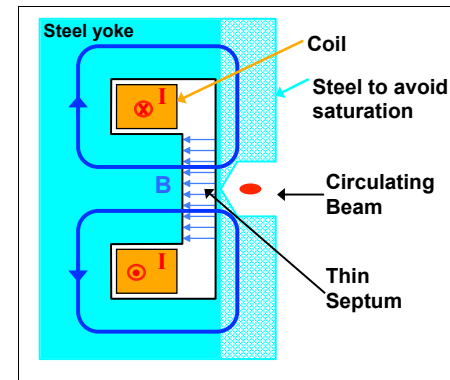
Direct drive pulsed magnetic septum



- In vacuum, to minimise distance between circulated and extracted beam
- Single-turn coil to minimise inductance, bake-out up to 200 °C (~10⁻⁹ mbar)
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
 - Current in range 7 – 40 kA with a few ms oscillation period
 - Cooling water circuits flow rate from 1 – 80 l/min

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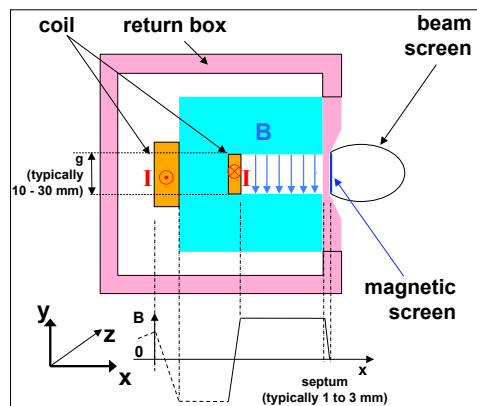
Lambertson septum



- Magnetic field in gap orthogonal to previous examples of septa:
 - Lambertson deflects beam orthogonal to kicker: dual plane injection/extraction
- Rugged design: conductors safely hidden away from the beam
- Thin steel yoke between aperture and circulating beam – however extra steel required to avoid saturation, magnetic shielding often added

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Eddy current septum

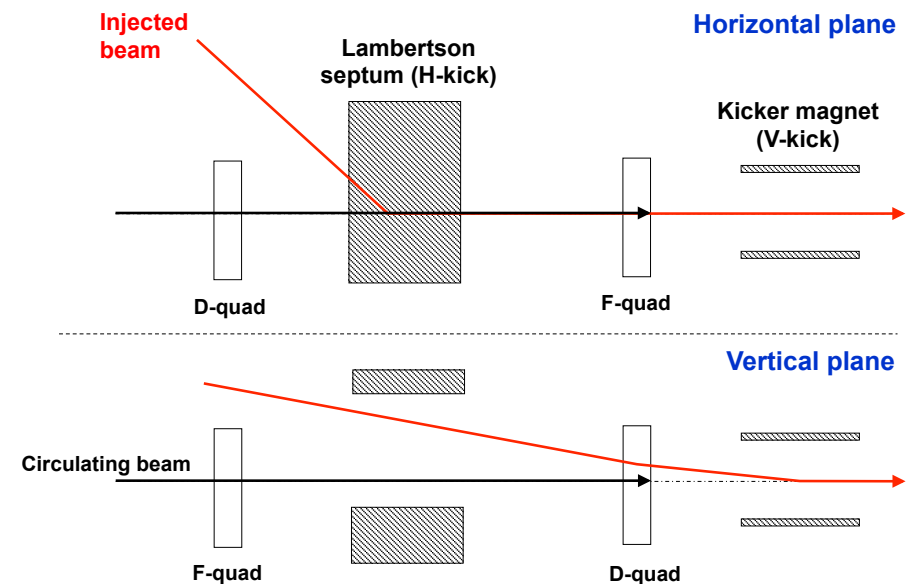


- Coil removed from septum and placed behind C-core yoke:
 - Coil dimension not critical
 - Very thin septum blade
- Magnetic field pulse induces eddy currents in septum blade
- Eddy currents shield the circulating beam from magnetic field
- Return box and magnetic screen reduce fringe field seen by circulating beam

- In or out of vacuum, single-turn coil
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
 - Current ~10 kA fast pulsed with ~ 50 μs oscillation period
 - Cooling water circuits flow rate from 1 – 10 l/min

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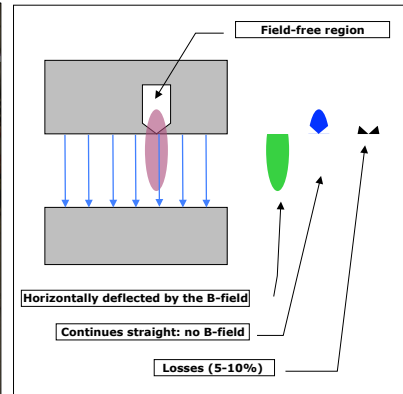
Two plane injection with Lambertson



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Lambertson septum

- At SPS we use Lambertson septa to split the 400 GeV slow-extracted proton spill (~ seconds) to different target stations simultaneously:
 - These devices are radioactive: critical that coils are located away from the septum



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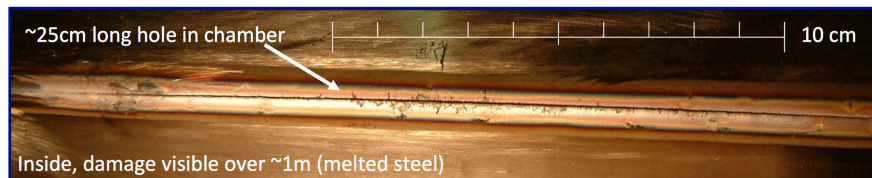
Protection devices

- When beam energy exceeds damage limit for machine equipment one has to design for certain failure scenarios
- Critical beam transfer systems have redundancy and multiple layers of protection:
 - Passive protection devices form the last layer of this security
- Protection devices are designed to dilute and absorb beam energy safely
- Failures associated with beam transfer equipment are typically very fast and difficult to catch, for example:
 - No turn-on of kicker: injection protection
 - Erratic turn-on of kicker: sweep circulating beam in the machine
 - Flash-over (short-circuit) in kicker: impart the wrong kicker angle
 - Transfer line magnet failure: steering beam onto aperture of downstream machine

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Protection devices

- When things go wrong...!
 - SPS extraction septum power supply tripped during setting-up of LHC beam, 25th October 2004:

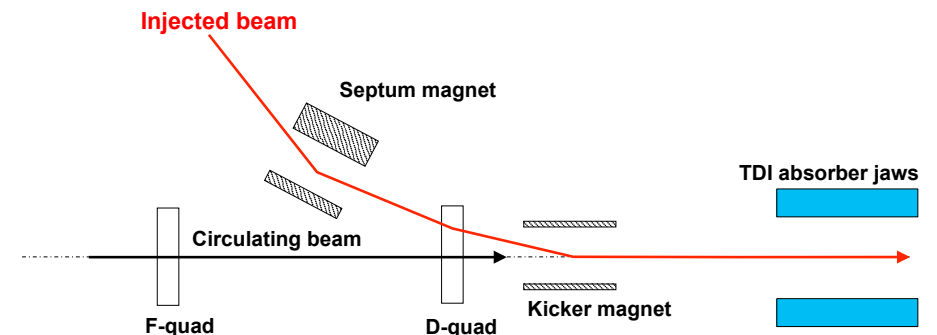


- Septum field dropped by 5% in 11 ms
- 3.4×10^{13} protons at 450 GeV, i.e. 2.5 MJ of beam energy dissipated on the aperture of the transfer line
- Vacuum chamber and quadrupole magnet damaged requiring replacement
- Upgraded fast interlock system was implemented to protect against such fast failures

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Injection protection: e.g. LHC injection

- LHC has a dedicated injection dump (TDI) to protect against fast failures on the injection kicker

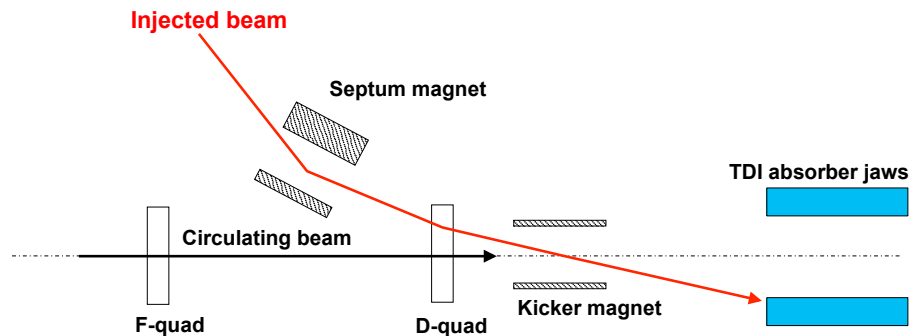


In reality the LHC injection is dual plane: Lambertson septum kick orthogonal to kicker

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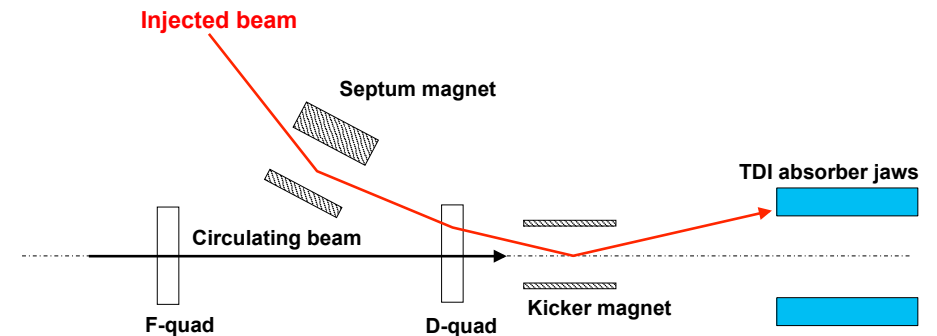


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Injection protection: e.g. LHC injection

- LHC has a dedicated injection dump (TDI) to protect against fast failures on the injection kicker
 - No turn-on of kicker: beam steered safely onto absorber
 - Erratic turn-on of kicker: circulating beam steered safely onto absorber
 - Flash-over (short-circuit) in kicker: “worst-case” gives twice deflection:

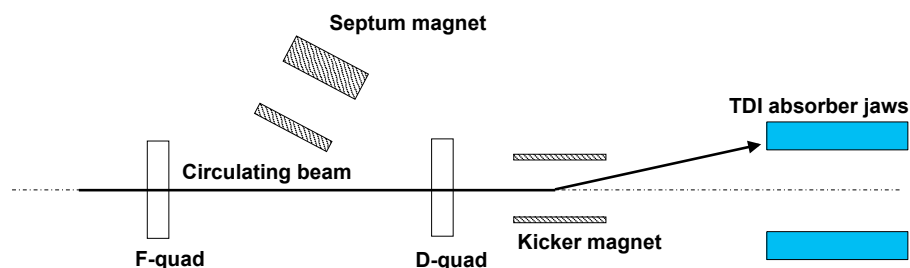


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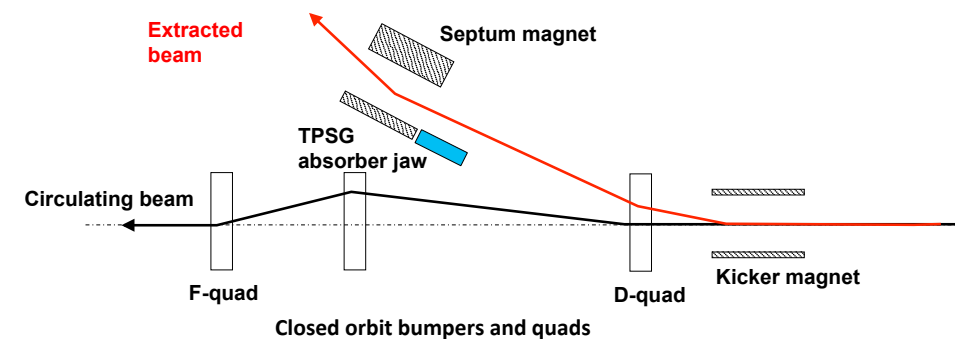


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Extraction protection: e.g. SPS extraction

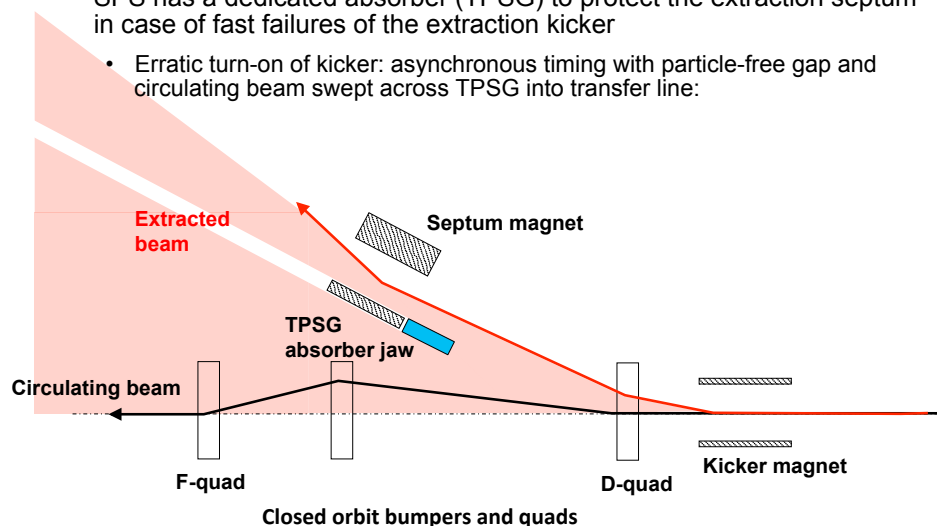
- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker



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Extraction protection: e.g. SPS extraction

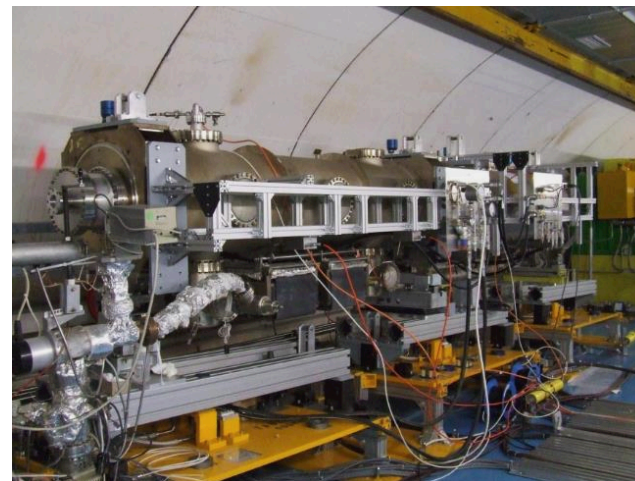
- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker
- Erratic turn-on of kicker: asynchronous timing with particle-free gap and circulating beam swept across TPSG into transfer line:



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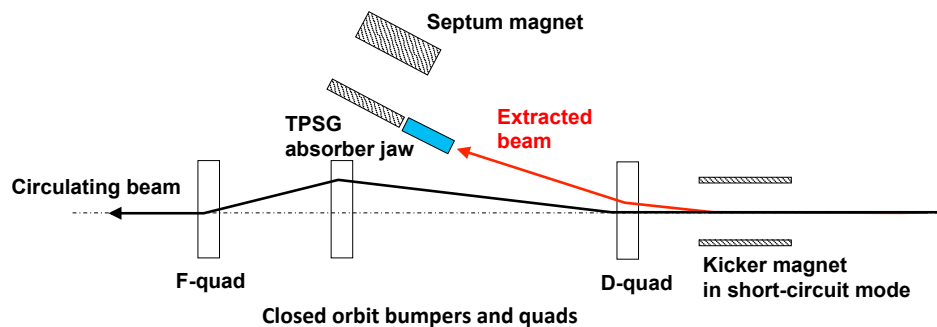


TPSG and MSE (magnetic septum) installed at HIRADMAT irradiation test facility in 2012: impacted with LHC nominal intensity (288b and 1.1×10^{11} p/b): both devices survived!

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Extraction protection: e.g. SPS extraction

- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker
- Erratic turn-on of kicker: asynchronous timing with particle-free gap and circulating beam swept across TPSG into transfer line
- Flash-over (short-circuit) in kicker: worst-case amplitude places the extracted beam onto the absorber jaw:

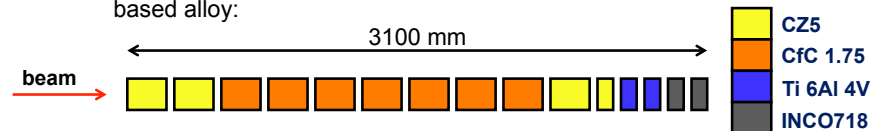


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Extraction protection: e.g. TPSG

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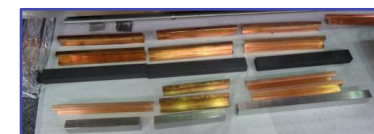
- Diluter made of graphite, 2D carbon composite, titanium alloy and nickel based alloy:



- Designed to protect downstream septum from direct impact of 450 GeV LHC ultimate beam (288 bunches at 1.7×10^{11} protons per bunch, 3.5 MJ)

Water cooling channel

Absorber blocks



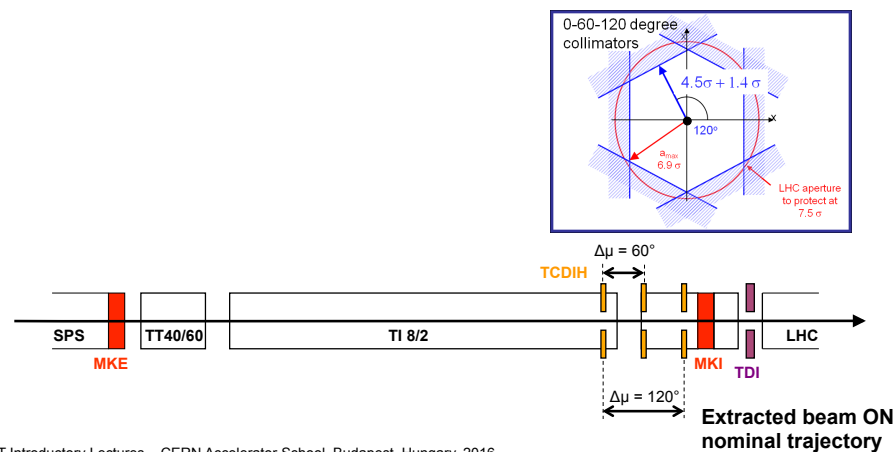
Absorber blocks inspected after impact of HIRADMAT test #6: survived and re-installed

Comment: small emittance (high beam brightness) can be just as much a concern as the total intensity for thermo-mechanical stresses during beam impact

TPSG assembly without vacuum tank
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Transfer protection: e.g. SPS-to-LHC

- SPS to LHC transfer lines have dedicated transfer line collimators (TCDIH and V) in case of fast failures to protect LHC aperture:



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Transfer protection: e.g. SPS-to-LHC

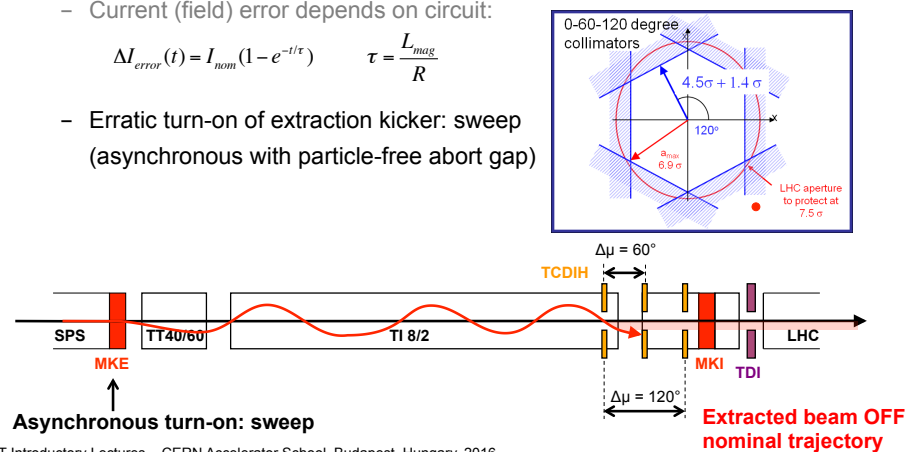
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- Magnet power supply trips at time t after the last extraction interlock check: beam steered onto collimator

- Current (field) error depends on circuit:

$$\Delta I_{error}(t) = I_{nom}(1 - e^{-t/\tau}) \quad \tau = \frac{L_{mag}}{R}$$

- Erratic turn-on of extraction kicker: sweep (asynchronous with particle-free abort gap)



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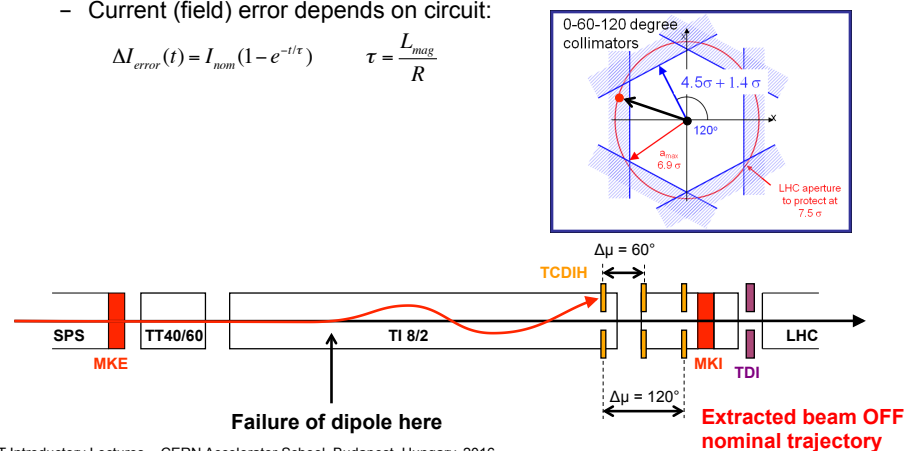
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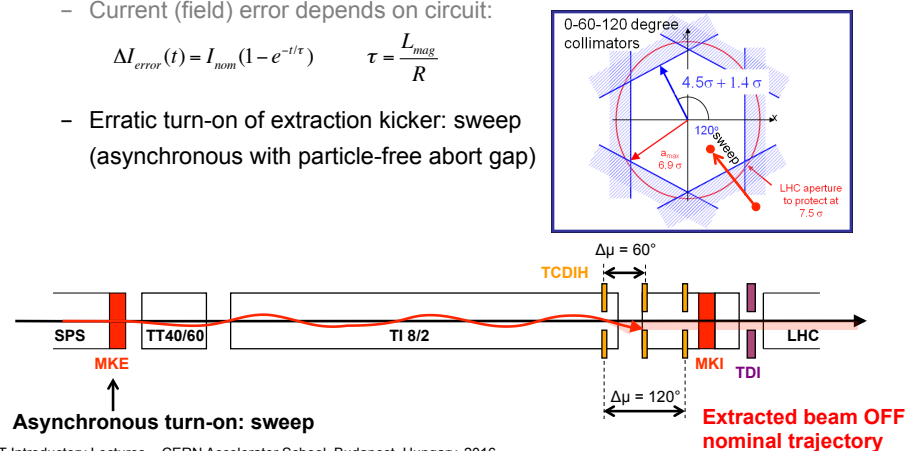
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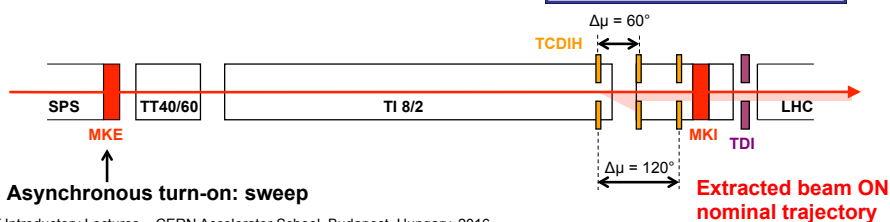
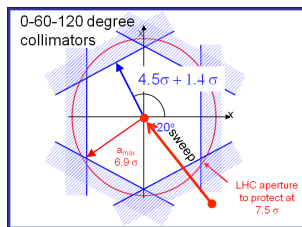
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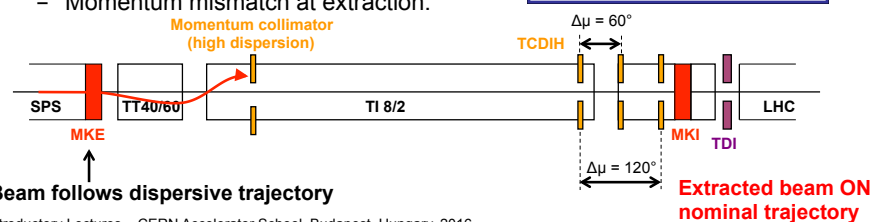
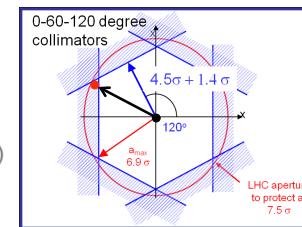
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- Erratic turn-on of extraction kicker: sweep (asynchronous with particle-free abort gap)

- Flash-over (short-circuit) in kicker

- Momentum mismatch at extraction:



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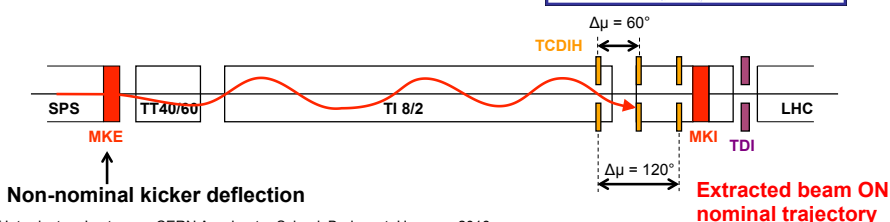
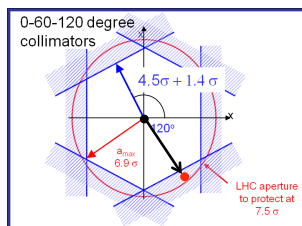
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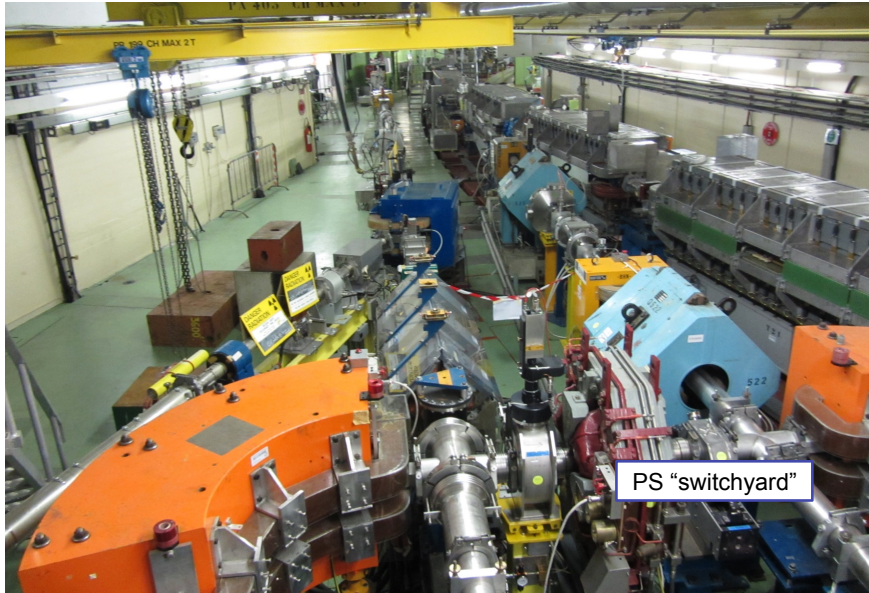
Beam transfer lines



LHC beam dump extraction line (TD62/68)

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Beam transfer lines



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Functional requirements

Transfer lines transport beams between accelerators (extraction of one to injection of the next) and on to experimental targets and beam dumps

- Requirements:
 - Geometric link between machines/experiment
 - Match optics between machines/experiment
 - Preserve emittance
 - Change particles' charge state (stripping foils)
 - Measure beam parameters (measurement lines)
 - Protect downstream machine/experiment

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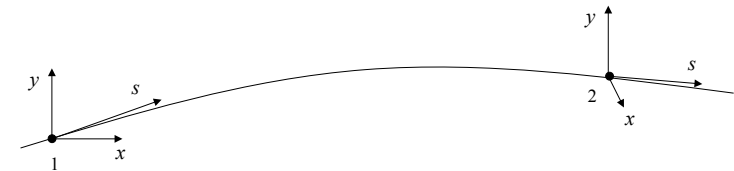
Beam transfer lines

- Beam transfer lines
 - Functional requirements
 - Distinctions between transfer lines and circular machines
 - Linking machines/experiments together
 - Emittance blow-up from mismatch
 - Measurement of beam parameters (measurement lines)

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General transport

Beam transport: moving from s_1 to s_2 through n elements, each with transfer matrix M_i



$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1 \rightarrow 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} \quad \mathbf{M}_{1 \rightarrow 2} = \prod_{i=1}^n \mathbf{M}_i$$

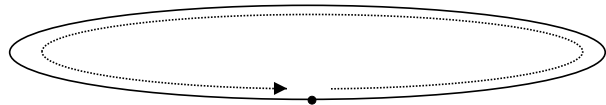
The transfer matrix (M_i) can be expressed using the Twiss formalism:

$$\mathbf{M}_{1 \rightarrow 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1} (\cos \Delta\mu + \alpha_1 \sin \Delta\mu) & \sqrt{\beta_1\beta_2} \sin \Delta\mu \\ \sqrt{1/\beta_1\beta_2} [(\alpha_1 - \alpha_2) \cos \Delta\mu - (1 + \alpha_1\alpha_2) \sin \Delta\mu] & \sqrt{\beta_1/\beta_2} (\cos \Delta\mu - \alpha_2 \sin \Delta\mu) \end{bmatrix}$$

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Circular Machine

Circumference = L



One turn:
 $\Delta\mu = 2\pi Q$

$$\mathbf{M}_{1 \rightarrow 2} = \mathbf{M}_{0 \rightarrow L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta} (1 + \alpha^2) \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1 = \alpha_2, \beta_1 = \beta_2, D_1 = D_2$
- This condition *uniquely* determines $\alpha(s), \beta(s), \mu(s), D(s)$ around the whole ring
 - i.e. a single matched ellipse exists for each given location, s

Transfer line

One pass:
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1 \rightarrow 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

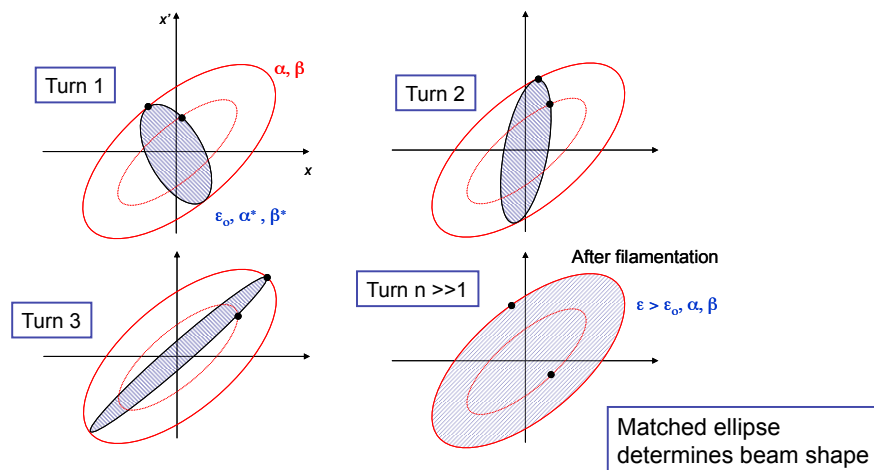


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- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, $\alpha(s) \beta(s)$ are functions of α_1 and β_1

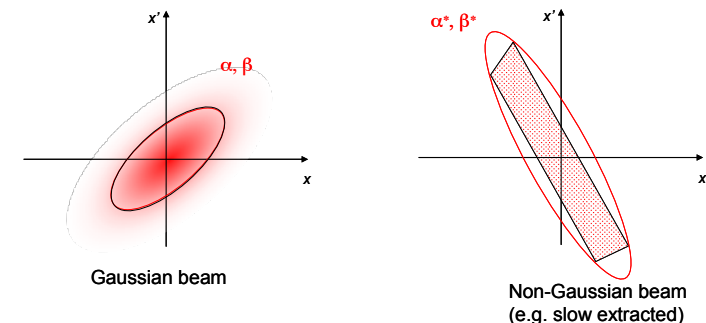
Circular Machine

- At a location with matched ellipse (α, β) a mismatched injected beam (α^*, β^*) with emittance ϵ_0 , generates (via filamentation) a larger ellipse with the matched α, β , but larger emittance: $\epsilon > \epsilon_0$



Transfer line

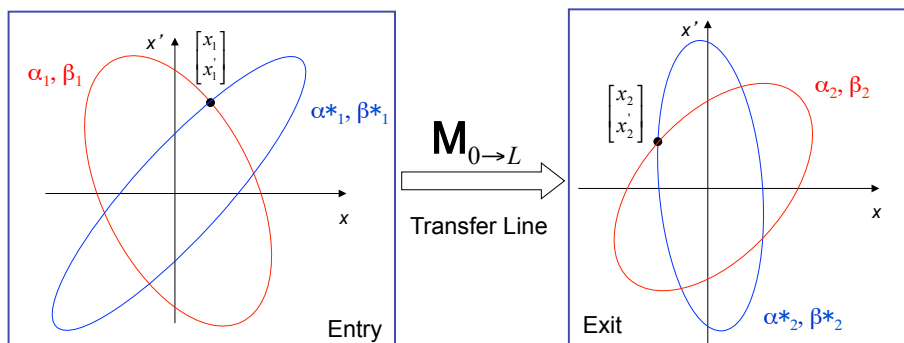
- Initial α, β are defined for a transfer line by the beam shape at the entrance



- Propagation of this beam ellipse depends on the line
- A transfer line optics is different for different input beams:
 - Synchrotrons are often multi-purpose, accelerating different beams but extracting through a common line transfer line: optics must switch to match the input and output conditions for each beam type

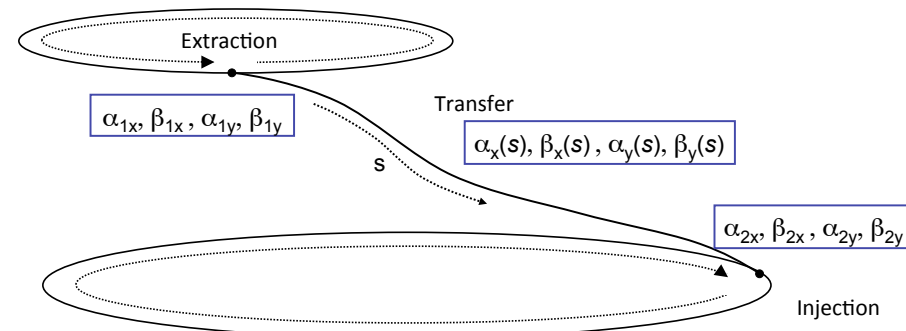
Transfer line

- On a single pass of a finite transfer line there is no regular motion from entrance to exit
 - Periodicity is not enforced: it's actually a design choice
 - Infinite number of possible starting ellipses are transported to an infinite number of final ellipses



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Linking Machines



The Twiss parameters can be propagated when the transfer matrix \mathbf{M} is known

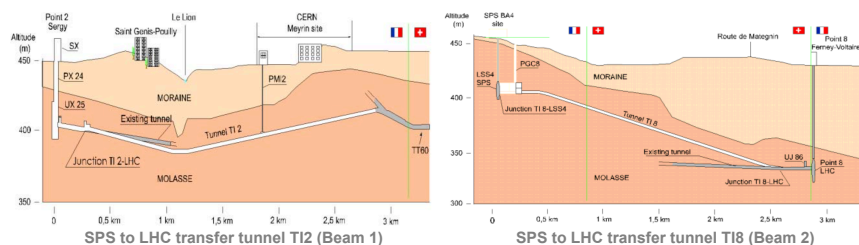
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1 \rightarrow 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

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Linking Machines

- Beams have to be transported from extraction of one machine to injection of the next machine:
 - Trajectory must be matched in all 6 geometric degrees of freedom ($x, y, z, \theta, \Phi, \psi$)
- Other important constraints can include:
 - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology or other obstacles, etc.



An example of how geology can influence transfer line design

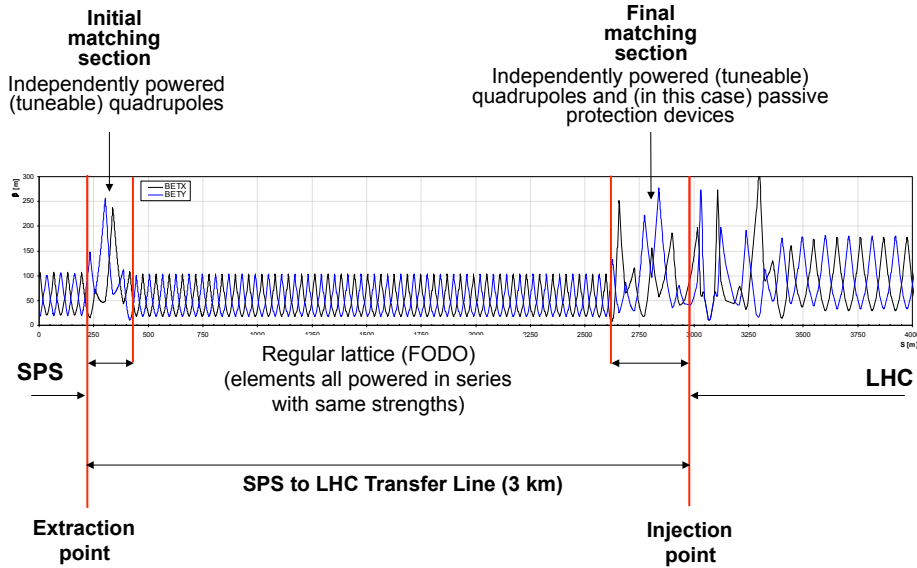
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Linking Machines

- Linking the optics is a complicated process:
 - Parameters at start of line have to be propagated to matched parameters at the end of the line (injection to another machine, fixed target etc.)
 - Need to “match” 8 variables ($\alpha_x, \beta_x, D_x, D'_x$ and $\alpha_y, \beta_y, D_y, D'_y$)
 - Matching done with number of independently power (“matching”) quadrupoles
 - Maximum β and D values are imposed by magnetic apertures
 - Other constraints exist:
 - Phase conditions for collimators
 - Insertions for special equipment like stripping foils
 - Low beam energy ($\beta \ll 1$) re-bunching cavities might be necessary, i.e. RF gymnastics in the transfer line
- Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error.

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Optics Matching



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Blow-up from betatron mismatch

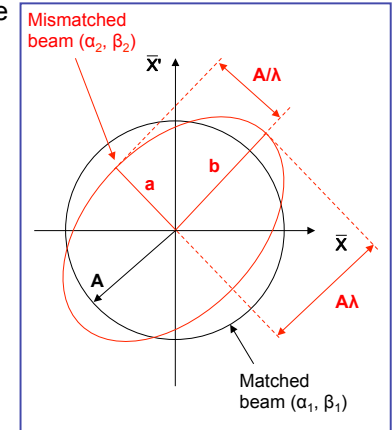
- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch
- Filamentation will produce an emittance increase
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse
- The emittance after filamentation:

$$\epsilon_{diluted} = \frac{\epsilon_{matched}}{2} \left(\lambda^2 + \frac{1}{\lambda^2} \right) \quad \text{where } \lambda = \sqrt{b/a}$$

- Writing λ as a function of the matched and mismatched Twiss parameters is an exercise in geometry:

$$\epsilon_{diluted} = \frac{1}{2} \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right) \epsilon_{matched}$$

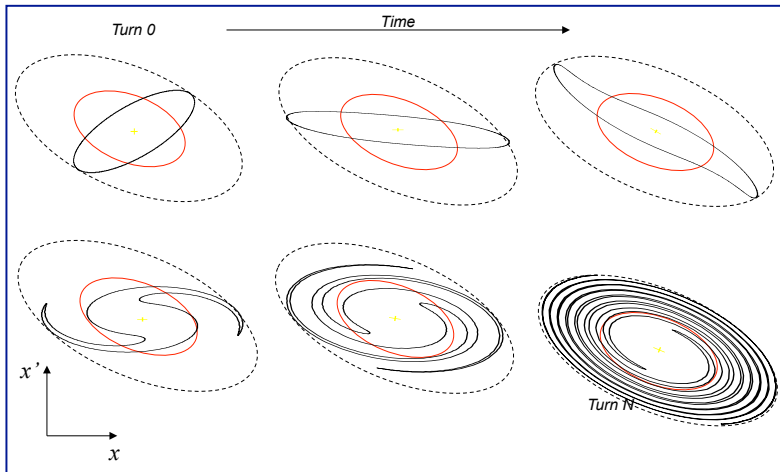
See appendix for derivation



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Optical Mismatch at Injection

- Filamentation fills larger ellipse with same shape as matched ellipse



- Dispersion mismatch at injection will also cause emittance blow-up

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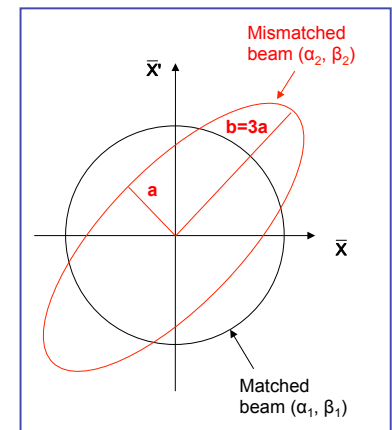
Blow-up from betatron mismatch

- A numerical example...
- Consider $b = 3a$ for the mismatched ellipse:

$$\lambda = \sqrt{b/a} = \sqrt{3}$$

$$\epsilon_{diluted} = \frac{\epsilon_{matched}}{2} \left(\lambda^2 + \frac{1}{\lambda^2} \right)$$

$$= 1.67 \epsilon_{matched}$$

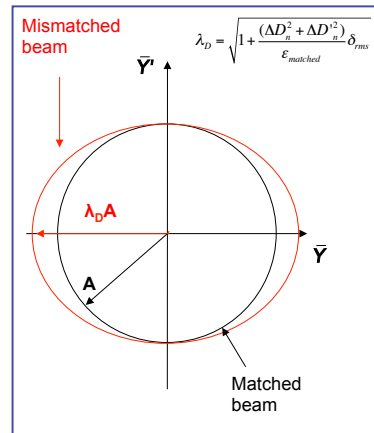


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Blow-up from dispersion mismatch

- Dispersion mismatch will also introduce emittance blow-up through filamentation much like optical mismatch
- Introducing normalised dispersion: $D_n = \frac{D}{\sqrt{\beta}}$ $D'_n = \frac{\alpha}{\sqrt{\beta}} D + \sqrt{\beta} D'$
- With a momentum error of $\delta = \frac{\Delta p}{p}$ the mismatch is:
 $\bar{X} = \bar{X} + \Delta D_n \delta$ $\bar{X}' = \bar{X}' + \Delta D'_n \delta$
- Rotating the reference frame to a convenient reference (see plot):
 $\bar{Y} = \bar{Y} + \sqrt{\Delta D_n^2 + \Delta D_n'^2} \delta$ $\bar{Y}' = \bar{Y}'$
- And averaging over a distribution of particles, one can write the emittance blow-up as:

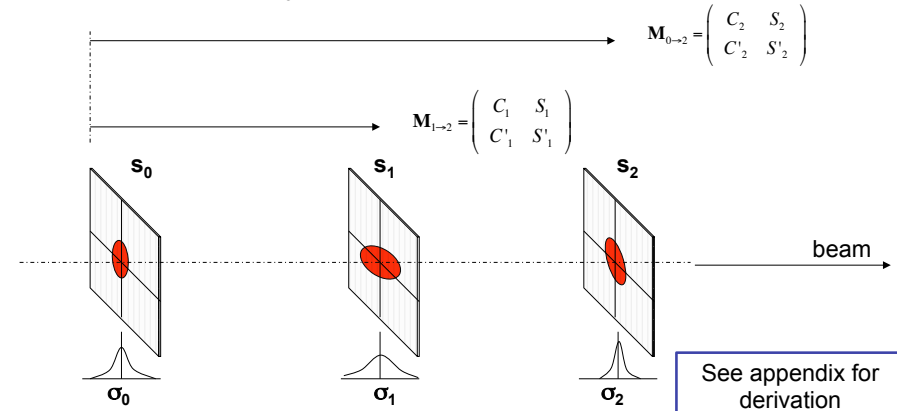
$$\epsilon_{\text{diluted}} = \epsilon_{\text{matched}} + \frac{\Delta D_n^2 + \Delta D_n'^2}{2} \delta_{\text{rms}}^2$$



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Optics measurement with 3 screens

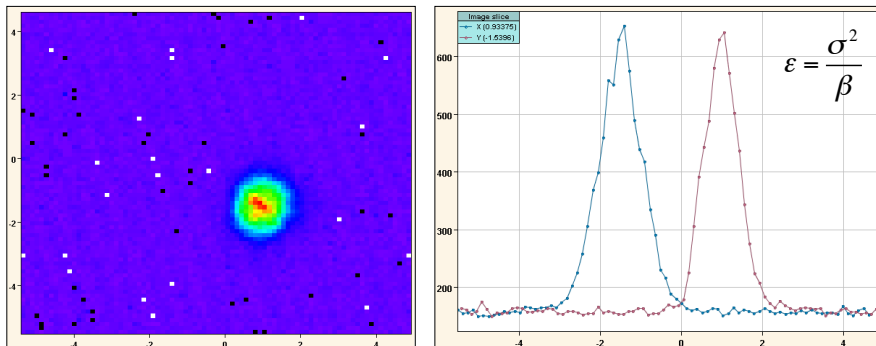
- Assume 3 screens in a dispersion free region and that the emittance is constant along the line: $\epsilon = \frac{\sigma_0^2}{\beta_0} = \frac{\sigma_1^2}{\beta_1} = \frac{\sigma_2^2}{\beta_2}$
- Measurements of σ at s_1, s_2, s_3 plus knowledge of the two transfer matrices M_{12} and M_{13} allows determination of ϵ, α and β



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Optics measurement with screens

- A profile monitor is needed to measure the beam size
 - e.g. beam screen (luminescent) provides 2D density profile of the beam
- Profile fit gives transverse beam size: σ
- If optics (Twiss parameters) are known, ϵ can be calculated from a single screen:



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Summary

- Depending on the injection/extraction concept we chose a dedicated combination of septa (spatial separation of fields) and kickers (temporal separation of fields)
- Transfer lines present interesting challenges and differences from circular machines:
 - No periodic condition mean optics is defined by transfer line element strengths and by initial beam ellipse
 - Matching is subject to many constraints
 - Emittance blow-up is an important consideration, and arises from several sources: mis-steering, mismatch (betatron and dispersion)
 - Measurement of beam parameters is important for ensuring beams are well matched between machines and/or experiments

Thank you for your attention

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Example parameters for kickers at CERN

Kicker Location	Beam momentum (GeV/c)	# Magnets	Gap Height [V_{ap}] (mm)	Current (kA)	Impedance (Ω)	Rise Time (ns)	Total Deflection (mrad)
CTF3	0.2	4	40	0.056	50	~4	1.2
PS Inj.	2.14	4	53	1.52	26.3	42	4.2
SPS Inj.	13/26	16	54 to 61	1.47/1.96	16.67/12.5	115/200	3.92
SPS Ext. (MKE4)	450	5	32 to 35	2.56	10	1100	0.48
LHC Inj.	450	4	54	5.12	5	900	0.82
LHC Abort	450 to 7000	15	73	1.3 to 18.5	1.5 (not T-line)	2700	0.275

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Example parameters for septa at CERN

Septum Location	Beam momentum (GeV/c)	Gap Height (mm)	Max. Current (kA)	B (T)	Deflection (mrad)	Septum thickness (mm)
LEIR/AD/CTF (13 systems)	Various	25 to 55	1 DC to 40 pulsed	0.5 to 1.6	up to 130	1.7 - 19.2
PS Booster (6 systems)	1.4	25 to 60	28 pulsed	0.1 to 0.6	up to 80	1 - 15
PS complex (8 systems)	26	20 to 60	2.5 DC to 33 pulsed	0.2 to 1.2	up to 55	3 - 11.2
SPS Ext.	450	20	24	1.5	2.25	4.2 - 17.2

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Blow-up from betatron mismatch

- General betatron motion:

$$x_2 = \sqrt{a_2 \beta_2} \sin(\varphi + \varphi_o), \quad x'_2 = \sqrt{a_2 / \beta_2} [\cos(\varphi + \varphi_o) - \alpha_2 \sin(\varphi + \varphi_o)]$$

- Applying the normalisation transformation for the matched beam...

$$\begin{bmatrix} \bar{X}_2 \\ \bar{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

...an ellipse is obtained in normalised phase space:

$$A^2 = \underbrace{\bar{X}_2^2 \left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 \right]}_{\gamma_{new}} + \underbrace{\bar{X}_2'^2 \frac{\beta_2}{\beta_1}}_{\beta_{new}} - 2 \bar{X}_2 \bar{X}_2' \underbrace{\left[\frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right) \right]}_{\alpha_{new}}$$

Blow-up from betatron mismatch

- We can evaluate the square of the distance of a particle from the origin as:

$$\mathbf{A}_{new}^2 = \bar{\mathbf{X}}_{new}^2 + \bar{\mathbf{X}}_{new}'^2 = \lambda^2 \cdot \mathbf{A}_0^2 \sin^2(\varphi + \varphi_1) + \frac{1}{\lambda^2} \mathbf{A}_0^2 \cos^2(\varphi + \varphi_1)$$

- The new emittance is the average for all particles with positions A_i over all phases:

$$\begin{aligned} \varepsilon_{diluted} &= \frac{1}{2} \langle \mathbf{A}_{new}^2 \rangle = \frac{1}{2} \left(\lambda^2 \langle \mathbf{A}_0^2 \sin^2(\varphi + \varphi_1) \rangle + \frac{1}{\lambda^2} \langle \mathbf{A}_0^2 \cos^2(\varphi + \varphi_1) \rangle \right) \\ &= \frac{1}{2} \langle \mathbf{A}_0^2 \rangle \left(\lambda^2 \langle \sin^2(\varphi + \varphi_1) \rangle + \frac{1}{\lambda^2} \langle \cos^2(\varphi + \varphi_1) \rangle \right) = \frac{1}{2} \varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right) \end{aligned}$$

- If we're feeling diligent, we can substitute back for λ :

$$\varepsilon_{diluted} = \frac{1}{2} \varepsilon_{matched} \left(\lambda^2 + \frac{1}{\lambda^2} \right) = H \varepsilon_{matched} = \frac{1}{2} \varepsilon_{matched} \left(\frac{\beta_1 + \beta_2}{\beta_2} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right)$$

where subscript 1 refers to the matched and 2 refers to mismatched cases

Blow-up from betatron mismatch

- From general ellipse properties one can write:

$$a = \frac{A}{\sqrt{2}} (\sqrt{H+1} + \sqrt{H-1}), \quad b = \frac{A}{\sqrt{2}} (\sqrt{H+1} - \sqrt{H-1}) \quad \text{where} \quad H = \frac{1}{2} (\gamma_{new} + \beta_{new})$$

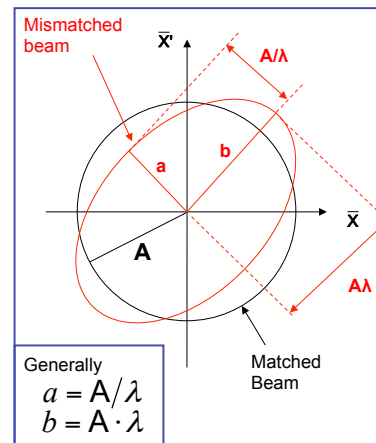
Giving:

$$\lambda = \frac{1}{\sqrt{2}} (\sqrt{H+1} + \sqrt{H-1}),$$

$$\frac{1}{\lambda} = \frac{1}{\sqrt{2}} (\sqrt{H+1} - \sqrt{H-1})$$

- The co-ordinates of the mismatched beam can be expressed:

$$\bar{\mathbf{X}}_{new} = \lambda \cdot \mathbf{A} \sin(\varphi + \varphi_1), \quad \bar{\mathbf{X}}'_{new} = \frac{1}{\lambda} \mathbf{A} \cos(\varphi + \varphi_1)$$



Optics measurement with 3 screens

- Remember how we propagate Twiss parameters from s_0 to s_1 :

$$\begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} C_1^2 & -2C_1S_1 & S_1^2 \\ -C_1C_1' & C_0S_0' + S_0C_0' & -S_1S_1' \\ C_1'^2 & -2C_1'S_1' & S_1'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

- Giving us three simultaneous equations and three unknowns ε_0 , α_0 and β_0 :

$$\begin{aligned} \beta_0 &= C_0^2 \cdot \beta_0 - 2C_0S_0 \cdot \alpha_0 + S_0^2 \cdot \gamma_0 \\ \beta_1 &= C_1^2 \cdot \beta_0 - 2C_1S_1 \cdot \alpha_0 + S_1^2 \cdot \gamma_0 \\ \beta_2 &= C_2^2 \cdot \beta_0 - 2C_2S_2 \cdot \alpha_0 + S_2^2 \cdot \gamma_0 \end{aligned} \quad \times \varepsilon \quad \begin{cases} \sigma_0^2 = \beta_0 \varepsilon \\ \sigma_1^2 = C_1^2 \cdot \beta_0 \varepsilon - 2C_1S_1 \cdot \alpha_0 \varepsilon + S_1^2 \cdot \frac{(1 + \alpha_0^2)}{\beta_0} \varepsilon \\ \sigma_2^2 = C_2^2 \cdot \beta_0 \varepsilon - 2C_2S_2 \cdot \alpha_0 \varepsilon + S_2^2 \cdot \frac{(1 + \alpha_0^2)}{\beta_0} \varepsilon \end{cases}$$

- After a bit of algebra... we find:

$$\alpha_0 = -\frac{\beta_0}{2} W \quad W = \frac{(\sigma_2/\sigma_0)^2/S_2^2 - (\sigma_1/\sigma_0)^2/S_1^2 - (C_2/S_2)^2 + (C_1/S_1)^2}{(C_1/S_1) - (C_2/S_2)}$$

Optics measurement with 3 screens

- Some (more) algebra with the above equations and we can finally express the beta function at the first screen:

$$\beta_0 = 1 / \left| \sqrt{(\sigma_2/\sigma_0)^2 / S_2^2 - (C_2/S_2)^2 + W (C_2/S_2)^2 - W^2/4} \right|$$

- And therefore also the emittance and the divergence of the beta function:

$$\varepsilon = \frac{\sigma_0^2}{\beta_0} \quad \alpha_0 = \frac{\beta_0}{2} W$$

- Other methods of emittance measurement:
 - Extension of the above method to multiple screens: tomography
 - Quad scan: same as above but use one screen and change $M_{\text{quad} \rightarrow \text{screen}}$
 - Direct measurements (lower intensity/energy beams):
 - slit-grid or pepper-pot, laser “wire” for H- beams